

Chapter 2

Electromagnetic Fields in Electro-technologies

Abstract This chapter is the second introductory chapter of the book. It deals with the basic laws of electromagnetic fields and describes some specific phenomena occurring in electro-heat technologies. In particular, the electromagnetic wave diffusion in a conductive half-space is presented and the basic quantities that characterize the thermal processes based on internal heating sources are given. The penetration depth of the electromagnetic wave is a parameter that characterizes all the relevant phenomena in these applications. A qualitative description is provided of several effects influencing the distribution of the current density and, as a consequence, the heating sources in the heated workpieces: the proximity effect (that occurs between two conductors that carry electrical current), the ring effect (that occurs in bended conductors), the slot effect (that occurs in conductors placed in the slot of a magnetic yoke), and the end and edge effects (that occur due to the finite length of inductor and load and their relative position).

In electro-technological processes, electromagnetic phenomena are the basis for generation of heat and forces inside the body to be heated (in the following *workpiece*).

This chapter recalls the basic laws of electromagnetic fields and describes some relevant phenomena occurring in these technologies.

2.1 Basic Equations

Maxwell's equations is a set of equations which fully describe electromagnetic phenomena and, in particular, allow to calculate spatial distribution and direction of field vectors, including their time dependence, as a function of current density and characteristics of medium.

The field vectors are:

$\vec{E}$ electric field strength (V/m)

$\vec{D}$ electric flux density (As/m²)

$\vec{H}$ magnetic field intensity (A/m)

$\vec{B}$ magnetic induction (magnetic flux density) (T)

$\vec{J}$ current density (A/m²)

Maxwell's equations can be written in the form:

$$\text{rot } \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (\text{Ampere's law}) \quad (2.1)$$

$$\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (\text{Induction law}) \quad (2.2)$$

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \epsilon_r \vec{E} \quad (2.3)$$

$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} \quad (2.4)$$

where the quantities ϵ and μ , characteristics of the material, are respectively *electrical permittivity* (or dielectric constant) and *magnetic permeability*.

The parameters $\epsilon_r = \epsilon/\epsilon_0$ and $\mu_r = \mu/\mu_0$ denote relative permittivity and relative magnetic permeability.

Magnetic permeability and dielectric constant have known constant values in vacuum [$\mu_0 = 4 \cdot \pi \cdot 10^{-7}$ (H m⁻¹); $\epsilon_0 = c^2/\mu_0$ (F m⁻¹); $c = 299792.458$ (km/s)—velocity of light], but in many practical cases they depend on process parameters, such as temperature or, for permeability of ferromagnetic materials, also on magnetic field intensity.

In Eq. (2.1), the first term of r.h.s. represents the conduction current density due to an electric field $\vec{E}$ in a medium:

$$\vec{J} = \sigma \vec{E} = \frac{\vec{E}}{\rho} \quad (\text{Ohm's law}), \quad (2.5)$$

while the second term $\partial \vec{D}/\partial t$ is the displacement current density which represents the dominant mechanism of electrical conduction in dielectric materials. The quantity $\sigma = 1/\rho$ (Ω^{-1} m⁻¹), denotes material's *electrical conductivity* and ρ its *electrical resistivity* (Ω m).

If an electrically conductive body is placed in an electric field, Joule heat will develop in the body. The electric power converted into heat within each volume element is the power density w :

$$w(t) = \sigma \cdot E^2(t) = \rho \cdot J^2(t) \text{ (W/m}^3\text{)} \quad (2.6)$$

The total active power converted into heat is calculated by integration of w over the volume v of the body:

$$P(t) = \int_v \sigma E^2(t) dv = \rho \int_v J^2(t) dv \quad (2.7)$$

The *Poynting's vector* $\vec{S}$ is defined as:

$$\vec{S} = \vec{E} \times \vec{H}^*. \quad (2.8)$$

It describes the change of energy density in space. It can also be interpreted as describing the flow of energy since it specifies at any point in space the surface power density (in VA m⁻² or W m⁻²) and the direction of flow.

Thus considering the rate of the energy flow out from a volume v enclosed by a closed surface A , and denoting with $\dot{P}_S$ the complex power, the rate of complex power flow is:

$$-\dot{P}_S = \oint_A \vec{S} \cdot d\vec{A} = \int_v \text{div } \vec{S} dv \quad (2.9)$$

Expressing the term $\text{div } \vec{S}$ by Eq. (2.8), on the basis of vector identities and Maxwell's equations it is possible to show that Eq. (2.9) can be re-written as follows:

$$\begin{aligned} \oint_A \vec{S} \cdot d\vec{A} &= \int_v \text{div } \vec{S} dv = \\ &= -\frac{\partial}{\partial t} \int_v \left(\frac{1}{2} \mu H^2\right) dv - \frac{\partial}{\partial t} \int_v \left(\frac{1}{2} \varepsilon E^2\right) dv - \int_v \vec{J} \cdot \vec{E}^* dv = \\ &= -\frac{\partial}{\partial t} W_m - \frac{\partial}{\partial t} W_e - \int_v \vec{J} \cdot \vec{E}^* dv \end{aligned} \quad (2.10)$$

where is:

$W_m = \int_v \left(\frac{1}{2} \mu H^2\right) dv$ the energy in the magnetic field;

$W_e = \int_v \left(\frac{1}{2} \varepsilon E^2\right) dv$ the energy in the electric field;

$\int_v \vec{J} \cdot \vec{E}^* dv$ a term which represents *either* the power dissipated as ohmic losses *or* the power generated by a source inside v

Therefore, the power delivered by a generator or absorbed by a consumer, can be evaluated by the integral of Poynting vector over a surface enclosing the generator or the consumer.

2.2 Equations for AC Fields

In most applications steady-state AC fields are used and all field quantities $\vec{H}$, $\vec{E}$, $\vec{B}$ vary sinusoidally in time:

$$\vec{H} = \vec{H}_m \cdot \sin \omega t, \quad \vec{B} = \vec{B}_m \cdot \sin \omega t, \quad \vec{E} = \vec{E}_m \cdot \sin(\omega t + \phi)$$

In this case, using the complex exponential form ($e^{j\omega t}$), the field quantities can be rewritten in the form:

$$\dot{\vec{H}} = \vec{H}_m \cdot e^{j\omega t}, \quad \dot{\vec{B}} = \vec{B}_m \cdot e^{j\omega t}, \quad \dot{\vec{E}} = \vec{E}_m \cdot e^{j(\omega t + \phi)}$$

and the general form of Maxwell's equations for harmonic processes becomes:

$$\text{rot } \dot{\vec{H}} = (\sigma + j\omega\epsilon) \dot{\vec{E}} \quad (2.11a)$$

$$\text{rot } \dot{\vec{E}} = -j\omega\mu \dot{\vec{H}} \quad (2.11b)$$

with: $j = \sqrt{-1}$; ω —angular frequency (rad/s).

Equations (2.11a) and (2.11b) are written in a simpler form in the following cases of practical interest:

(a) *Electromagnetic field in electrical conductors*

In applications concerning conductive materials with high electrical conductivity or electromagnetic fields at relatively low frequency (e.g. less than 10 MHz), it is $\sigma \gg \omega\epsilon$ and the conduction current density is much greater than displacement current density, so that Eqs. (2.11a) and (2.11b) have the form:

$$\text{rot } \dot{\vec{H}} = \sigma \dot{\vec{E}} \quad (2.12a)$$

$$\text{rot } \dot{\vec{E}} = -j\omega\mu \dot{\vec{H}} \quad (2.12b)$$

Applying the rotational operator to each member of Eqs. (2.12a) and (2.12b) and taking into account Eq. (2.5), we can write:

$$\text{rot rot } \dot{\vec{E}} + j \omega \mu \sigma \dot{\vec{E}} = 0 \quad (2.13a)$$

$$\text{rot rot } \dot{\vec{H}} + j \omega \mu \sigma \dot{\vec{H}} = 0 \quad (2.13b)$$

Using the rules of vector algebra and taking into account that the electrical field intensity $\dot{\vec{E}}$ and the magnetic field intensity $\dot{\vec{H}}$ satisfy zero divergence conditions, can be obtained the differential equations of field distributions in electrical conductors (passive conducting regions):

$$\nabla^2 \dot{\vec{E}} + \dot{k}^2 \dot{\vec{E}} = 0 \quad (2.14a)$$

$$\nabla^2 \dot{\vec{H}} + \dot{k}^2 \dot{\vec{H}} = 0 \quad (2.14b)$$

where: $\dot{k}^2 = -j \omega \mu \sigma = \frac{2}{\delta^2} (1 - j)$ —eddy current constant (m^{-2}); δ —penetration depth of electromagnetic field (m); ∇^2 —the Laplacian, which has different forms in different coordinates system.

(b) *Electromagnetic field in non-conducting materials*

In non-conducting materials, where $\sigma \approx 0$, it is $\sigma \ll \omega \epsilon$ and the Maxwell's Eqs. (2.11a) and (2.11b) take the form:

$$\text{rot } \dot{\vec{H}} = j \omega \epsilon \dot{\vec{E}} \quad (2.15a)$$

$$\text{rot } \dot{\vec{E}} = -j \omega \mu \dot{\vec{H}}. \quad (2.15b)$$

As in (a), we can obtain the wave equations:

$$\nabla^2 \dot{\vec{E}} + \dot{k}_0^2 \dot{\vec{E}} = 0 \quad (2.16a)$$

$$\nabla^2 \dot{\vec{H}} + \dot{k}_0^2 \dot{\vec{H}} = 0 \quad (2.16b)$$

with: $k_0 = \omega \sqrt{\epsilon \mu}$ —real constant (m^{-1}).

Equations (2.16a) and (2.16b) are used for calculating the field distribution in non-conducting materials, e.g. in dielectric or microwave heating applications.

2.3 Electromagnetic Wave Diffusion in Conductive Half-Space

2.3.1 Electromagnetic Field Distribution

To describe how the electromagnetic wave propagates inside an electrically conductive body, consider the homogeneous semi-infinite metal body of Fig. 2.1, infinitely extended along x , y , z , with interface in the plane x - z and depth along y .

We look for the solution of Eq. (2.14b) with the following assumptions:

- homogeneous material with constant electrical resistivity ρ and magnetic permeability μ ;
- sinusoidal field quantities;
- exciting plane wave of magnetic field incident on the plane xoz .

Since in Cartesian coordinates the Laplace's operator has the form

$$\nabla^2 \dot{\vec{H}} = \frac{\partial^2 \dot{\vec{H}}}{\partial x^2} + \frac{\partial^2 \dot{\vec{H}}}{\partial y^2} + \frac{\partial^2 \dot{\vec{H}}}{\partial z^2},$$

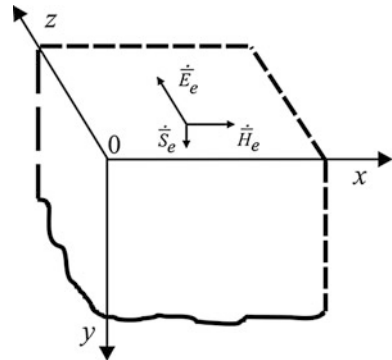
and the magnetic field intensity $\dot{\vec{H}}$ is constant in any plane of coordinate y , then it is $\partial^2 \dot{\vec{H}} / \partial x^2 = \partial^2 \dot{\vec{H}} / \partial z^2 = 0$ and Eq. (2.14b) becomes:

$$\frac{\partial^2 \dot{\vec{H}}}{\partial y^2} = k^2 \dot{\vec{H}} \quad (2.17)$$

with

$$k^2 = j\omega\sigma\mu.$$

Fig. 2.1 Schematic of a semi-infinite homogeneous conductive body



The general solution of this linear differential equation of second order is:

$$\dot{\vec{H}} = \dot{\vec{C}}_1 \cdot e^{-ky} + \dot{\vec{C}}_2 \cdot e^{ky}, \quad (2.18)$$

and the spatial modulus of the magnetic field intensity vector is

$$\dot{H} = \dot{C}_1 \cdot e^{-ky} + \dot{C}_2 \cdot e^{ky}, \quad (2.18a)$$

with $\dot{\vec{C}}_1, \dot{\vec{C}}_2, \dot{C}_1, \dot{C}_2$ —integration constants, $\dot{\vec{C}}_1, \dot{\vec{C}}_2$ —complex vectors and $\dot{C}_1, \dot{C}_2$ —moduli of the complex vectors.

With the assumptions made, the vector of magnetic field intensity is directed along ox , while its components along the axes oz and oy are equal to zero, i.e. $\dot{H}_z = \dot{H}_y = 0$.

Moreover it is:

$$\frac{\partial \dot{H}_z}{\partial y} = \frac{\partial \dot{H}_y}{\partial z} = \frac{\partial \dot{H}_x}{\partial z} = \frac{\partial \dot{H}_z}{\partial x} = \frac{\partial \dot{H}_y}{\partial x} = 0$$

and

$$\text{rot } \dot{\vec{H}} = \vec{u}_z \cdot \text{rot}_z \dot{\vec{H}} = -\vec{u}_z \cdot \frac{\partial \dot{H}_x}{\partial y}$$

with $\vec{u}_z$ —unit vector along the coordinate x .

We are able now to write the general solution for the electric field intensity since—omitting the subscript x of $\dot{H}_x$ —from Eq. (2.12a) we have:

$$\dot{\vec{E}} = \rho \cdot \text{rot } \dot{\vec{H}} = -\vec{u}_z \cdot \rho \cdot \frac{\partial \dot{H}}{\partial y} \quad (2.19a)$$

Using Eq. (2.18a), finally we obtain:

$$\dot{\vec{E}} = \vec{u}_z \cdot k \cdot \rho \cdot (\dot{C}_1 \cdot e^{-ky} - \dot{C}_2 \cdot e^{ky}). \quad (2.19b)$$

The integration constant $\dot{C}_1$ and $\dot{C}_2$ of Eqs. (2.18a) and (2.19b) can be determined from the boundary conditions. In particular,

- the constant $\dot{C}_2$ must be equal to zero, since the field intensity cannot increase when y increases. In fact, the first term $\dot{C}_1 \cdot e^{-ky}$ of Eq. (2.19b) describes the incident wave, the second term $\dot{C}_2 \cdot e^{ky}$ —the reflected one. But in the propagation of an electromagnetic wave inside a semi-infinite half space there is not any reflective surface and the reflected wave is absent.

- the constant $\dot{\bar{C}}_1$ is determined from the condition that on the surface $y = 0$ is known the amplitude of the exciting magnetic field vector $\dot{\bar{H}} = \dot{\bar{H}}_e$ tangential to the metal surface. In this hypothesis the magnetic field intensity does not vary at the crossing of the surface between the conductive medium and air.

Therefore, it results $\dot{\bar{C}}_1 = \dot{\bar{H}}_e$ and $\dot{\bar{C}}_2 = 0$.

Finally we obtain:

$$\dot{\bar{H}} = \bar{u}_x \cdot \dot{\bar{H}}_e \cdot e^{-ky} \quad (2.20a)$$

$$\dot{\bar{E}} = \bar{u}_z \cdot k \cdot \rho \cdot \dot{\bar{H}}_e \cdot e^{-ky} \quad (2.20b)$$

where k is the damping coefficient of the electromagnetic wave and δ the penetration depth:

$$k = \sqrt{\frac{j\omega\mu}{\rho}} = \sqrt{2j} \sqrt{\frac{\omega\mu}{2\rho}} = \frac{1+j}{\delta} = \frac{\sqrt{2}}{\delta} e^{j\frac{\pi}{4}}. \quad (2.21a)$$

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2\rho}{\pi f \mu_0 \mu_r}} \quad (2.21b)$$

The penetration depth δ has dimension of a length; its value varies with the square root of the electrical resistivity and inversely with the square root of the relative magnetic permeability and the frequency. Therefore, it depends upon the electromagnetic material properties and the applied frequency and, for the same frequency, it has different values in different materials.

For example, at 50 Hz: $\delta_{Cu50} \approx 1$ cm; at 5 kHz: $\delta_{Cu5000} \approx 1$ mm.

In trigonometric form Eqs. (2.20a) and (2.20b) can be written as follows:

$$\dot{\bar{H}} = \bar{u}_x \cdot H_{em} \cdot e^{-\frac{y}{\delta}} \cdot \sin\left(\omega t - \frac{y}{\delta}\right) \quad (2.22a)$$

$$\dot{\bar{E}} = \bar{u}_z \cdot \frac{\sqrt{2}}{\delta} \cdot \rho \cdot H_{em} \cdot e^{-\frac{y}{\delta}} \cdot \sin\left(\omega t - \frac{y}{\delta} + \frac{\pi}{4}\right) \quad (2.22b)$$

and the amplitudes of the electric and magnetic field intensities are

$$\left. \begin{aligned} H_m &= H_{em} \cdot e^{-\frac{y}{\delta}} \\ E_m &= H_{em} \cdot \frac{\sqrt{2}}{\delta} \cdot \rho \cdot e^{-\frac{y}{\delta}} \end{aligned} \right\}. \quad (2.23)$$

As shown by Eqs. (2.20a), (2.20b), (2.22a), (2.22b) and (2.23), the amplitudes of the magnetic and electric field intensities decay exponentially as the electromagnetic wave penetrates in the conducting body, with a decay rate proportional to the

damping factor $k = (1 + j)/\delta$, which is an index of the phenomenon of skin effect in conductors.

The factor $e^{-y/\delta}$ describes the damping, while the second term $e^{-jy/\delta}$ says that the maximum field strength is achieved at different times in different places.

2.3.2 Current and Power Distributions

The instantaneous values of the current density vector $\vec{J} = \vec{E}/\rho$ in the thickness of the semi-infinite metal body, evaluated from Eq. (2.22b), are:

$$\vec{J} = \vec{u}_z \cdot \frac{\sqrt{2}}{\delta} \cdot H_{em} \cdot e^{-\frac{y}{\delta}} \cdot e^{j(\omega t - \frac{y}{\delta} + \frac{\pi}{4})}. \quad (2.24)$$

By introducing the notation

$$J_{em} = \frac{\sqrt{2}}{\delta} \cdot H_{em}, \quad (2.25a)$$

for the amplitude of the current density vector inside the semi-infinite metallic body we can write:

$$J = J_{em} \cdot e^{-\frac{y}{\delta}}. \quad (2.25b)$$

The r.m.s. value J of modulus of the current density vector therefore is:

$$J_m = \frac{J_{em}}{\sqrt{2}} \cdot e^{-\frac{y}{\delta}}. \quad (2.25c)$$

The maximum amplitude of the current density vector, given by Eq. (2.25a), occurs at the surface $y = 0$.

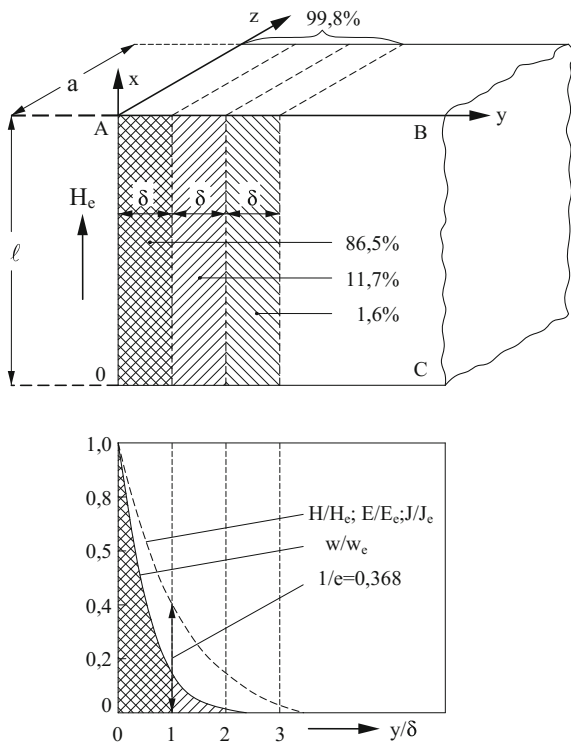
The distribution of the volume power density inside the semi-infinite body can be evaluated according Eq. (2.25c) by the relationship:

$$w(y) = \rho \cdot J^2 \cdot e^{-\frac{2y}{\delta}} = \frac{1}{2} \cdot J_{em}^2 \cdot e^{-\frac{2y}{\delta}} = w_e \cdot e^{-\frac{2y}{\delta}} \quad (2.26)$$

with w_e —value of w at $y = 0$.

At $y = \delta$ it is $w_\delta = w_e \cdot e^{-2} = 0.135 \cdot w_e$, i.e. the heat source density falls within one penetration depth to about 14 % of its surface value.

Fig. 2.2 Decay of magnetic field (H), electric field (E), current density (J) and power density (w) inside the conducting medium, as a function of distance from the surface



By integration of the heat source density along the y coordinate the following results are obtained:

- active power dissipated in a layer below the surface of length ℓ along x and depth a along z (see Fig. 2.2):

$$P_{wy} = \ell a \int_0^y w dy = \ell a w_e \int_0^y e^{-2y/\delta} dy = \ell a H_e^2 \frac{\rho}{\delta} (1 - e^{-2y/\delta}) \quad (2.27a)$$

- total active power within an element of the semi-plane of length ℓ (along x) and depth a (along y):

$$P_{w\infty} = \ell a \int_0^\infty w dy = \ell a w_e \int_0^\infty e^{-2y/\delta} dy = \ell a H_e^2 \frac{\rho}{\delta} \quad (2.27b)$$

- ratio $P_{wy}/P_{w\infty}$:

$$\frac{P_{wy}}{P_{w\infty}} = (1 - e^{-2y/\delta}) \quad (2.27c)$$

- power dissipated in the first layer of thickness δ below the surface

$$P_{\delta} = P_{w\infty}(1 - e^{-2}) = 0,865 \cdot P_{w\infty}. \quad (2.27d)$$

This means that more than 86 % of the total power is developed in the first layer of thickness δ . From the previous equations it's easy to demonstrate that in the subsequent two layers of thickness δ the power is 11.7 and 1.6 % of the total power, as shown in Fig. 2.2.

The diagram of the figure also shows that at a depth greater than 3 times δ , the values of all the quantities of interest (current density, volume specific power and magnetic field intensity) are practically negligible.

2.3.3 Wave Impedance

The electrical impedance of one square meter of the metallic semi-infinite body, which is called *wave impedance*, is equal to:

$$\dot{Z}_{e1} = R_{e1} + jX_{e1} = \frac{\dot{E}}{\dot{H}} = \frac{\dot{H}_e \cdot \rho \cdot \frac{1+j}{\delta} \cdot e^{-\frac{1+j}{\delta}y}}{\dot{H}_e \cdot e^{-\frac{1+j}{\delta}y}} = \frac{\rho}{\delta}(1+j) \quad (2.28)$$

The corresponding resistance and reactance therefore are:

$$R_{e1} = \frac{\rho}{\delta}; \quad X_{e1} = \frac{\rho}{\delta}.$$

As a consequence, the power factor of the semi-infinite body is:

$$\cos \varphi = \frac{R_{e1}}{\sqrt{R_{e1}^2 + X_{e1}^2}} = \frac{1}{\sqrt{2}} = 0.707 \quad (2.29)$$

It is noteworthy to note that the power factor is equal to 0.707 and does not depend on material characteristics and frequency.

2.4 Special Electromagnetic Phenomena in Electro-technologies

2.4.1 Skin-Effect

The skin-effect is the effect which produces a decrease of the current density from the surface towards the internal part of the body in conductors where AC current or magnetic flux in magnetic cores flow.

In a conductor supplied with DC current, the current density J is constant in the cross-section and is given by:

$$J = \frac{I}{A},$$

with: I —current in the conductor (A); A —conductor cross-section (m^2).

A different situation occurs in a conductor where AC current flows.

As example consider the infinitely long cylindrical conductor of Fig. 2.3a, where the AC current I flows.

The direction of the magnetic flux lines $\vec{B}_I$ produced by the current I , is determined by the advancement of the right-hand screw rule.

This AC magnetic flux $\vec{B}_I$ induces, in ideal rectangular paths inside the conductor, the eddy current i which in turn produces a reaction magnetic field, characterised by the vector $\vec{B}_i$.

According to Lenz's law, the direction of the magnetic field $\vec{B}_i$ produced by the current i , is opposite to that of the magnetic induction created by the current I and the direction of the induced current i in any path encircling the lines of magnetic induction also will obey the right-hand screw law.

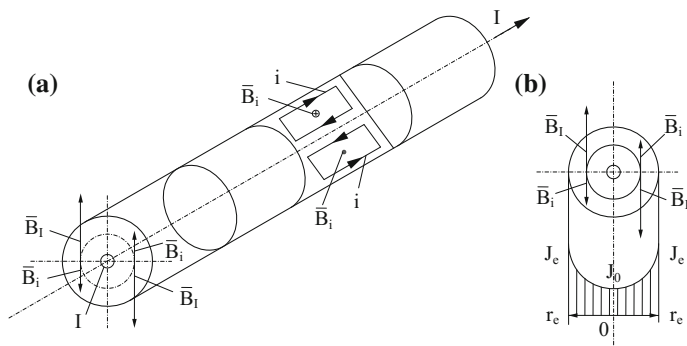


Fig. 2.3 Skin-effect in a cylindrical conductor [1, 2]

Therefore, the induced eddy currents i will increase the total value of the current near the surface and weaken it in the central part of the conductor, producing in this way an uneven distribution of the current density in the cross-section. This phenomenon is illustrated qualitatively in Fig. 2.3b.

2.4.2 Exponential Distributions and Equivalent Surface Active Layer

As shown by Eqs. (2.25a)–(2.25c), in the conducting semi-infinite body or, more generally, when the skin-effect is pronounced, the distribution of the current density from the surface towards the interior of the conductor follows the exponential law:

$$J = J_{em} e^{-\frac{y}{\delta}}, \quad (2.30a)$$

where J —current density at a distance y from the conductor's surface (A/m^2); J_{em} —corresponding value at the conductor's surface (A/m^2); δ —penetration depth of the electromagnetic wave (m).

Introducing in Eq. (2.21b) the values $\omega = 2\pi f$ and $\mu_0 = 4\pi \cdot 10^{-7}$ (H/m), the penetration depth can be expressed in the more convenient form:

$$\delta = 503 \sqrt{\frac{\rho}{\mu_r f}} \text{ (m)}. \quad (2.30b)$$

From Eq. (2.30a), the penetration depth can be defined as the distance from the surface of the conductor at which the current density is reduced to a value $e = 2.718$ times lower than the current density at the surface.

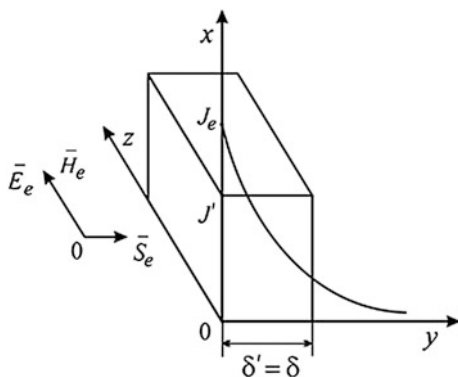
It has been also shown that, in this case, the distribution of volumetric power density from the surface to the conductor interior is expressed by the relationship:

$$w = w_e e^{-\frac{2y}{\delta}} \quad (2.30c)$$

with: $w_e = \rho J_{em}^2$.

Considering that, as shown by Eq. (2.27d), nearly all active power is concentrated in the surface layer of thickness δ , can be sometimes useful in practical calculations to make reference to an *equivalent surface active layer* of thickness δ' . This equivalent surface layer is obtained by replacing the actual exponential current density distribution in the heated body by a fictitious distribution, such that in the equivalent active layer the current density is uniformly distributed with value J' , while it is equal to zero in the remaining part of the cross-section (see Fig. 2.4).

Fig. 2.4 Equivalent surface active layer



The thickness δ' of the equivalent layer and the value of the current density J' are determined by the condition that the total active power dissipated in the conductor by the real exponential current density distribution must be equal to the one dissipated in the equivalent active layer.

Choosing arbitrarily the value of δ' equal to the penetration depth δ , it's easy to demonstrate that the current density at the surface of the body with exponential distribution and the current density in the equivalent active layer are linked by the relationship $J_e = \sqrt{2} J'$ [1].

Note: The introduction of the active layer thickness δ' and the equivalent current density J' is an artificial operation without physical meaning.

2.4.3 Non-exponential Distributions of Current Density and Heat Sources

As stated before, the exponential distributions of the current density and heat sources described by Eqs. (2.30a)–(2.30c) occur in homogeneous and linear bodies in case of pronounced skin effect (e.g. at relative high frequency).

At lower frequencies, also in case of homogeneous and linear materials, the distributions differ from exponentials, depending on frequency, material characteristics and geometry of conductor.

A typical example occurs in the longitudinal flux induction heating of non-magnetic cylindrical work-pieces, where—with constant resistivity—the radial distributions of induced current density are those illustrated in Fig. 2.5. In this case the distribution becomes nearly exponential only for values of $m = \sqrt{2}r_c/\delta \geq 12$.

In many applications of rapid induction heating or direct resistance heating, during the thermal transient the temperature may have considerable different values in the work-piece cross-section and—as a consequence—the local resistivity, which is temperature dependent, is different from point to point.

Fig. 2.5 Radial distributions of induced current density in cylindrical workpieces ($\xi = r/r_e$; $m = \sqrt{2}(r_e/\delta)$; r_e —external radius of cylinder) [3]

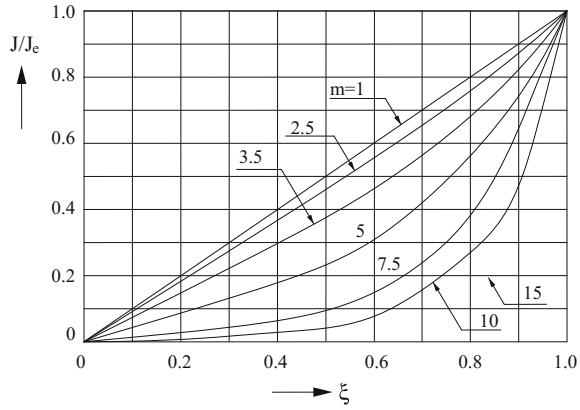
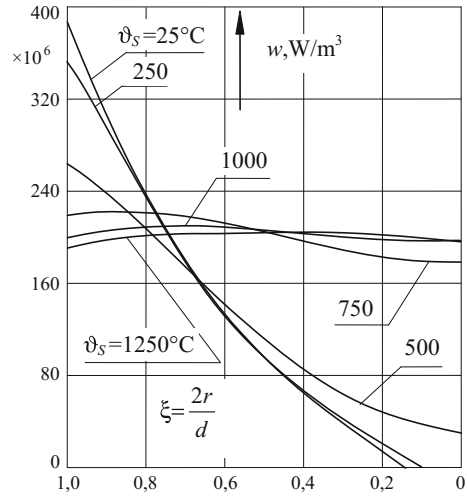


Fig. 2.6 Power density distributions along radius in DRH of a cylindrical rod, at different instants when surface temperature is ϑ_s ($r_e = d/2 = 30$ mm; $\xi = r/r_e$; $\ell = 6$ m; $U = 100$ V; steel C45) [3]



This reflects on the distribution in the cross-section of specific power per unit volume, i.e. the heat sources, which may differ significantly from the exponential one even in case of pronounced skin effect.

This phenomenon is more evident in ferromagnetic work-pieces, where the current density distribution is influenced not only by the temperature dependent resistivity, but also by the local values of magnetic permeability, which in turn depend not only on temperature but also on local magnetic field intensity.

Figure 2.6 shows an example of the specific power radial distribution in the direct resistance heating (DRH) of a magnetic steel rod. The curves correspond to different instants of the process, when surface temperatures were those specified in the figure. They show that at the beginning of the process the power distribution is nearly exponential. In subsequent instants of the heating transient—due to the variations of resistivity and permeability with temperature and local magnetic field

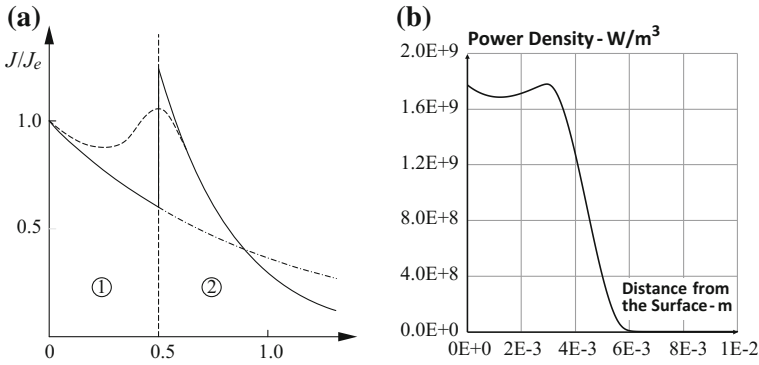


Fig. 2.7 **a** Schematic representation of the magnetic wave phenomenon; **b** radial distribution of power density in induction heating at 4 kHz of a carbon steel shaft, 100 mm diameter, with $H_0 = 240$ kA/m (r.m.s. value)

intensity—the power distributions remain highly non-uniform below Curie point, while in the final stages above Curie temperature they have a quite different shape, becoming more “flat” and showing the influence at the surface of radiation losses.

A particular phenomenon, known as “*magnetic wave*”, occurs in induction surface hardening where the power density distribution inside the workpiece may show a peculiar shape, significantly different from the commonly assumed exponential distribution. In this case, as usual, the power density has its maximum value at the surface, and decreases toward the core. But, at a certain distance from the surface, it starts to increase again, reaching a maximum value before finally declining (Fig. 2.7) [3, 4].

This behavior can be explained considering that, during rapid induction heating, like in surface hardening of a carbon steel workpiece, it may occur a heating stage where the core has still magnetic properties (region 2 in Fig. 2.7a), while a surface layer (region 1) is non-magnetic having exceeded the Curie temperature.

In this situation, the current density distribution can be schematically represented by two exponentials corresponding to two different values of δ due to the material characteristics of the magnetic and non-magnetic regions, as it is illustrated in Fig. 2.7a.

In particular, in some cases, the values of the current and power density at the surface of separation of the two layers, can be even higher than those at the surface.

However, since in practice the values of ρ and μ change gradually from layer to layer (*natura non facit saltus*), the real distributions are more complex, as indicated qualitatively by the dashed curve in the same figure or as shown by the example of Fig. 2.7b, which gives the radial distribution of power density during the transition of Curie point in the induction heating of a carbon steel shaft.

2.4.4 Proximity Effect

The proximity effect consists in a re-distribution of the current density in the conductor cross-section as a result of the interaction of the electromagnetic field produced by the conductor itself and the fields of all other nearby current carrying conductors.

In a single conductor carrying AC current, the current is distributed unevenly over the cross-section, symmetrically about the plane of symmetry for conductors of rectangular cross section (Fig. 2.8a) or the axis for cylindrical conductors.

In case of two conductors in which alternating currents flow, the current density distribution in the cross-sections depends on the distance between them and the direction of currents.

For conductors with currents flowing in opposite directions (i.e. with phase difference 180°), the maximum of the current density occurs on the two sides placed in front to each other (Fig. 2.8b). For currents in the same direction (phase difference equal to 0°), the highest current density occurs on the far side of conductors (Fig. 2.8c).

This phenomenon is similar to the skin-effect where the current density distribution is higher at the surface of the conductor where the magnetic field intensity is maximum (see Eqs. 2.23 and 2.25a–2.25c) and decreases where the magnetic field is lower.

In this case the magnetic field H produced by one conductor is modified by the influence of the magnetic field H' produced by the current in the other conductor.

Consider for example the case of Fig. 2.8b where there are two conductors with opposite directed currents of the same intensity I . For the super-position principle, the total magnetic field intensity produced by the two conductors is the sum of H and H' .

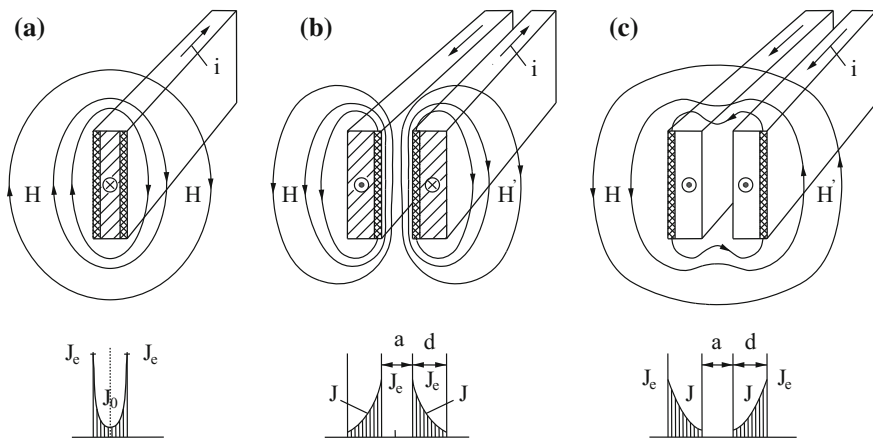


Fig. 2.8 Skin (a) and proximity effects (b, c) in rectangular straight conductors [5]

If the distance (a) between the conductors is much less than their height (h) and is comparable with their thickness (d), the magnetic field intensity H_i in the internal region between the two conductors is nearly the double of H_1 , i.e. is

$$H_i = H + H' \approx 2 \cdot H = 2 \cdot (I/2h) = I/h,$$

while on the opposite sides of the conductors it is practically nil, i.e.

$$H_e = H - H' \approx 0.$$

Obviously the influence of the field H' on the total field depends on the value of the current producing it: it is maximum when the conductors are very close to each other and decreases with the increase of the distance (a).

In any case the current density inside the conductors concentrates towards the surfaces where the magnetic field is maximum, as schematically shown in the figure.

Similarly, we can explain the redistribution of the current density when the currents in the conductors are in the same direction (Fig. 2.8c).

Arguing similarly, in case of equal currents the magnetic field intensities in the internal and external regions respectively are:

$$H_i = H - H' \approx 0 \quad \text{and} \quad H_e = H + H' \approx 2 \cdot H$$

In this case the current density inside the conductors is maximum on the external sides of the conductors and decreases inside the cross sections as illustrated in the figure.

The proximity effect depends on the ratio a/d and is more pronounced as the distance between the conductors decreases (i.e., for smaller ratios a/d) and the greater is the thickness of the conductor in comparison with the penetration depth.

A special case of proximity effect occurs in induction heating where the conductor of the inductor, carrying the current I_1 , is located near the surface of an electrically conductive body (the workpiece), in which the eddy current i_2 is induced by the exciting magnetic field produced by the inductor. (see the sketch of Fig. 2.9).

For the right-hand screw rule, the direction of the induced current i_2 is opposite to the direction of the current I_1 in the inductor. Thus, the phase difference of the currents I_1 and i_2 will be 180° .

In this situation, uneven distributions of current density will result both in the inductor and the heated body. This effect corresponds to the proximity effect in two parallel conductors, with the maximum of the current density located at the surfaces of the inductor and the heated body.

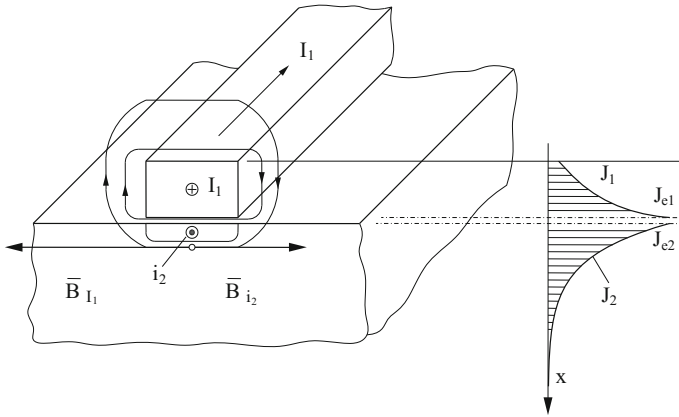


Fig. 2.9 Proximity effect in induction heating [1]

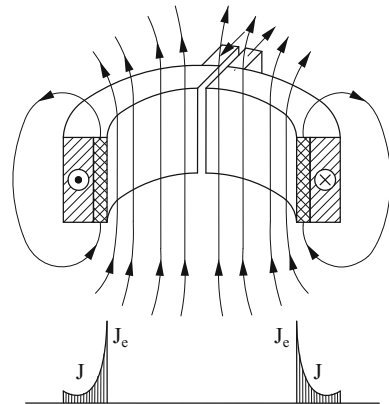
2.4.5 Ring Effect

In ring shaped or bent conductors in which DC or AC current flows, the current density tends to concentrate on the inner part of the ring or the bending radius, as shown in Fig. 2.10. This phenomenon is known as ring- or coil-effect.

Due to the asymmetrical pattern of the magnetic field, the current density will be higher in the section of the conductor having the smallest reactance (and resistance), i.e. near the inner surface of the ring, and lower on the opposite side of the cross-section.

The effect is stronger the greater is the ratio d/r_0 —for a conductor of rectangular cross section (with d —radial thickness of the conductor and r_0 —bending radius), or the ratio r_1/r_0 —for a conductor of circular cross-section of radius r_1 . Moreover, it is more pronounced the higher is the frequency of the current flowing in the

Fig. 2.10 Ring effect in a rectangular conductor and current density distribution in the cross-section [1, 5]



conductor. In particular, when the skin effect is very high, the current density is practically concentrated in the internal surface layer of thickness δ .

2.4.6 Slot Effect

The presence of a magnetic core or a magnetic yoke near an AC current carrying conductor, strongly modifies the current density distribution in the conductor cross-section.

As an example, we will illustrate this phenomenon, which is known as *slot effect*, in the case of a conductor placed in an open slot of a magnetic circuit made of magnetic laminations or other magnetic material suited for the frequency of the current (Fig. 2.11).

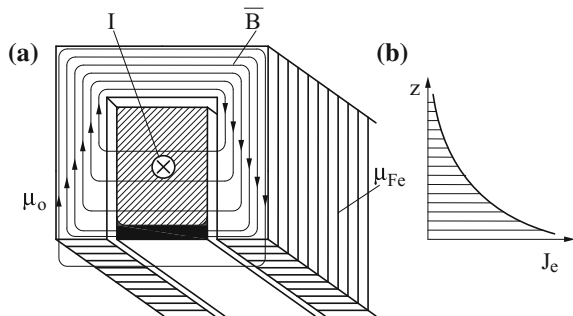
Since the magnetic material has permeability much higher than copper, the magnetic field lines tend to flow mainly in magnetic paths of lower magnetic reluctance. In particular, in the upper part of the slot the magnetic flux lines are confined in the magnetic material, while in the lower part they reclose partly through the conductor and partly in air in the lower air gap external to the conductor. This last part of flux lines is totally linked with the conductor cross-section.

For a qualitative description, the conductor cross-section can be ideally subdivided into several elementary layers. The layers closer to the magnetic material (at the bottom of the slot), are ‘linked’ with a magnetic flux density higher than those close to the open side of the slot.

Therefore, in the inner elements of the slot, stronger counter-electromotive forces will be induced in comparison with those near the open side. As a consequence the internal current paths are characterized by higher reactance values, and their current density will be of lower intensity in comparison with the elements in the regions of the conductor with lower reactance near the open side of the slot, where the maximum current density will occur.

The power density concentration near the opening of the slot will be greater the higher are the frequency and the slot depth.

Fig. 2.11 Slot effect [2, 5]



2.4.7 Load and Coil End- and Edge-Effects

As described in previous paragraphs, the current and power density distributions have a fundamental influence on the transient and final temperature pattern in the workpiece to be heated.

These distributions, in turn, are influenced not only by the above mentioned electromagnetic phenomena and the heat transfer conditions but also by the distortion of the magnetic field that occur at the end regions of the inductors and at the edges of workpieces in induction heating, or near the contact system in direct resistance heating, where the current lines have a complex two dimensional pattern also in axis-symmetric geometries.

These field distortions are known as *end-* and *edge-effects*.

End- and edge-effects act differently depending on the application, the type of load, magnetic or non-magnetic, cylindrical or with rectangular cross-section (slabs) and—in induction heating—relative dimensions and position of inductor and load.

A detailed discussion of this topic can be found in the bibliography [4, 6–10]. Here we will illustrate only some few examples in order to underline the importance of these phenomena.

2.4.7.1 Longitudinal End- and Edge-Effect

These effects occur typically in the induction heating of cylindrical workpieces with solenoidal coils or circular plates with pancake coils. They may be illustrated by the sketch of Fig. 2.12, which refers to the heating of a long homogeneous workpiece with a multi-turn inductor.

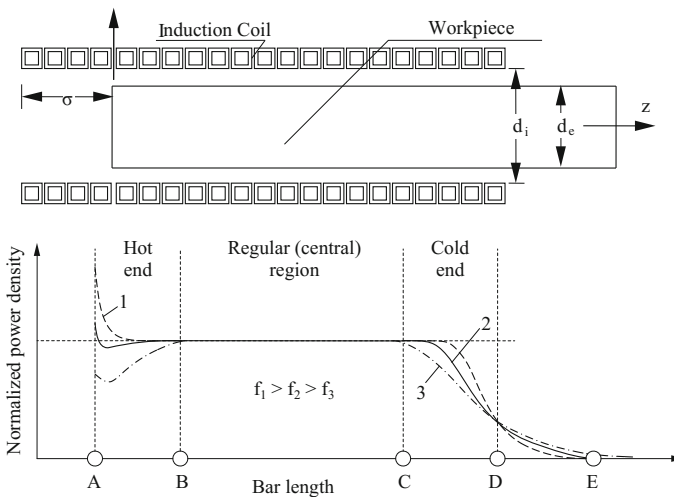


Fig. 2.12 Longitudinal end-effect (zone CE) and edge-effect (zone AB) in a workpiece

As regards the end-effect of the coil, it is known that the density of the induced current under the coil end (zone CE of Fig. 2.12) is approximately two times lower than in the central part. Accordingly, in this end zone of the workpiece the power density is equal to one quarter of that in the central part.

Depending on the work-piece and coil geometrical dimensions and the frequency, the inductor coil overhang σ and the material characteristics, the end effect (or load edge effect) can produce under heating or overheating of the work-piece end regions (zone A-B in Fig. 2.12).

Basically, this effect in non-magnetic workpieces is defined by three variables:

- d_e/δ ratio charactering the skin effect;
- σ/δ normalized coil overhang;
- d_i/d_e coupling ratio between inductor and load

If the coil and workpiece lengths are two times larger than the length of the end effect zones, then the end effects at the opposite ends do not mix and in the “*central part*” there is a uniform magnetic field zone, which corresponds to the field in an infinitely long system.

Figure 2.13 shows, as an example, the normalized power density distributions referred to the value of the central part, along the length of a typical aluminium induction heating system. It shows that an appropriate choice of the above parameters allows to obtain a reasonable uniform power density distribution, like the one in the curve with overhang $\sigma/d_e = 0.7$.

At the workpiece butt-end there is a local surplus of power, while in the adjacent internal zone there is a power deficit. This power profile combined with material thermal conductivity and heat losses from the workpiece front end provides a thermal equalization which can produce a nearly uniform temperature distribution along the workpiece length.

Fig. 2.13 Power distribution in the end zone along a typical aluminium heating system for different coil overhangs, normalized to the value of the central part ($d_e/\delta = 6$; 1 $\sigma/d_e = 1.5$; 2 $\sigma/d_e = 0.7$; 3 $\sigma/d_e = 0.3$; 4 $\sigma/d_e = 0$) [8]

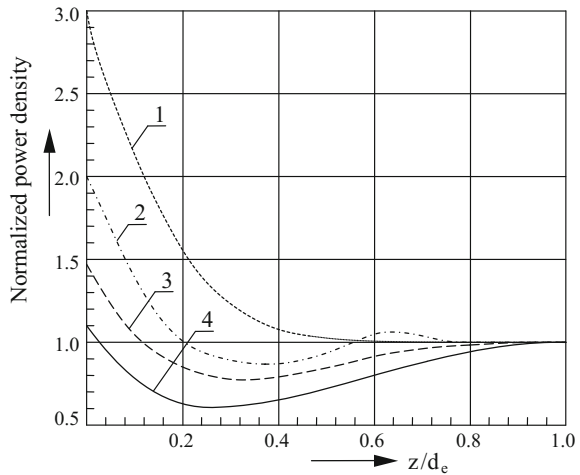
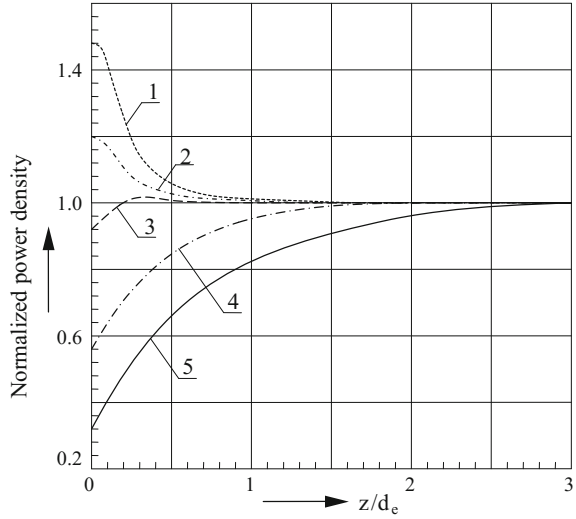


Fig. 2.14 Power distribution, normalized to the value of the central part, in the end zone of a ferromagnetic workpiece for different surface permeability in central part ($d_e/\delta = 6$; $l/\mu = 10$; 2 $\mu = 15$; 3 $\mu = 30$; 4 $\mu = 90$; 5 $\mu = 180$) [8]



The end-effects have quite different features in magnetic workpieces, since they are affected by two opposing factors: on one side the demagnetizing effect of eddy currents which tends to force the magnetic field out of the workpiece, on the other side, the magnetizing effect of the material, which tends to gather the magnetic field lines within the work-piece.

The first factor tends to increase the power at the workpiece ends, like in non-magnetic materials; the second one, on the contrary, causes a power reduction. Therefore, unlike from non-magnetic material, the ends of ferromagnetic workpieces, even in magnetic fields produced by coils with large overhangs, may be either overheated or underheated.

Figure 2.14 shows an example of the power distribution, normalized to the value of the central part, in the end zone of a ferromagnetic workpiece, for different surface permeability values in the central part. As reference is taken the surface permeability of the central part because μ varies along the workpiece length, and the skin-effect parameter d_e/δ accordingly varies in the axial direction also.

The diagram shows that the power deficit in the end zone is more pronounced with high values of μ , while a surplus of power can occur with the lower permeability values.

2.4.7.2 Longitudinal and Transverse End and Edge Effects in Rectangular Slabs

In the induction heating of rectangular slabs end- and edge-effects occur both in the transversal and longitudinal direction, due to the finite size of the workpiece.

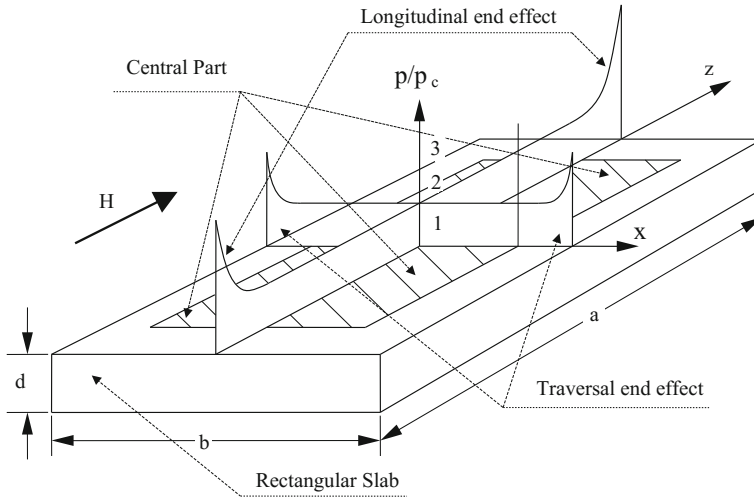


Fig. 2.15 Electromagnetic end and edge effects in induction heating of a slab heated in longitudinal uniform magnetic field

Suppose that the slab of Fig. 2.15, which has length (a) and width (b) much larger than its thickness (d), is placed in an initially uniform magnetic field, oriented along the z -axis.

As schematically shown in the figure, the electro-magnetic field distribution in the slab can be subdivided in three zones: the *central part*, the zone of *transversal edge effect* and the zone of *longitudinal end effect*. In the central part the field distribution is the same of an infinite plate excited with the same surface magnetic field intensity.

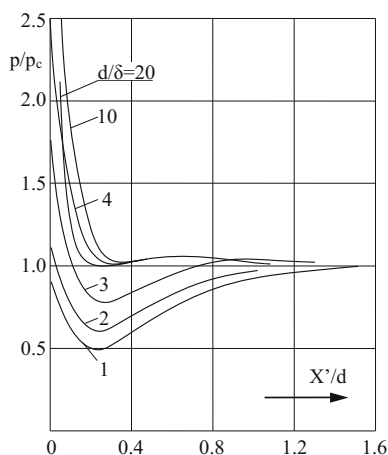
Basically, end and edge-effects have a two dimensional character and are determined only by the parameter (d/δ). The only exception occurs at the three-edge corners, where the field distribution is three-dimensional and is the result of combined end and edge effects [4, 7].

The same figure shows the distributions along the axes x and z of the power density p induced on the upper surface of the slab, normalized to value p_c in the central point of the centre line of the longer side.

Figure 2.16 gives an example of such distributions along the slab width as a function of the relative distance (x'/d) from the edges of the slab, for different values of d/δ . They are characterised near the edges by a region with induced power values lower than the ones in the “central” zone (under-heated area) and an external area with higher power density values (possible overheating).

In conclusion, the figures underline on one hand the strong influence that edge- and end-effects may have on the power density distributions and, as a consequence, on the heating pattern in the workpiece; on the other hand, they suggest that—in

Fig. 2.16 Relative power density distribution along slab width for different values of d/δ (x' -distance from slab edge) [7]



some cases—a convenient choice of the frequency allows to obtain an induced power pattern where the overheated areas can compensate to a certain extent the under-heated ones, by taking advantage of the material's thermal conductivity.

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