

Chapter 2

Flight Mechanics

An object moving through the atmosphere of the Earth is subjected primarily to aerodynamic and gravitational forces. Although gravity acts on this object independent of shape and mass, the object's size and shape *do* determine its aerodynamic characteristics. Therefore, to modify the relative influence of the aerodynamic loads, the influence of the external shape and its internal mass can be modified. For a free-falling vehicle, even a slight variation of size and shape may result in a significantly different trajectory due to the non-linear nature of the problem at hand.

To safeguard a re-entry vehicle, it is typically equipped with a guidance and control (G&C) system, so that changes in the trajectory due to uncertainties in operational aspects and vehicle characteristics can be compensated for. The basis for a G&C system is the availability of a nominal trajectory that – taking the vehicle characteristics and constraints on the thermo-mechanical loads into account – shows whether a particular vehicle design is feasible or not.

Before we dive into the actual vehicle-shape optimization, we will cover the basics of re-entry flight mechanics. We begin in Sect. 2.1 by discussing the flight environment, namely the shape of the Earth, its gravity field and its atmosphere. Following this, the equations of motion for the vehicle will be defined in Sect. 2.2. To do so we will define the relevant reference frames, discuss the external forces and the resulting translational equations of motion. We conclude this chapter by showing the guidance and control approaches that we employ in our simulations in Sect. 2.3. As capsule-shaped and winged vehicles require a different approach they will be discussed separately.

2.1 Flight Environment

The vehicle-shape optimization that is the topic of this book is limited to entering the Earth's atmosphere. Therefore, the flight environment that we need to introduce is that of the Earth, consisting of its shape, gravity field, and atmosphere. This discussion is by no means meant to be exhaustive, as we only seek to discuss the underlying models

for the optimization process. For more information concerning environment models, the reader is referred to the vast and excellent collection of literature that is available on this topic, e.g., the books by Vinh et al. (1980), Regan and Anandakrishnan (1993) and Vallado (2007).

2.1.1 Central Body Shape

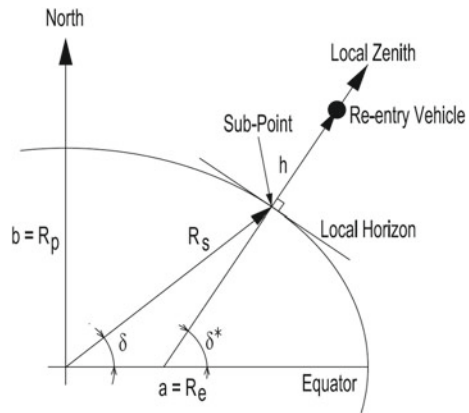
The shape of the Earth has a distinct effect on the atmospheric quantities at a given Earth-centered state, and is therefore a crucial part of the entry environment. Specifically, it is used to determine the vehicle's altitude from its Earth-centered Cartesian state. For aircraft and re-entry studies alike, one typically chooses one of several simplified shape models for the Earth: a flat-Earth approximation, a sphere, a spheroid or an ellipsoid. The particular choice depends on the application, where the vehicle's velocity, mission duration, and flight distance are the key decision drivers. For re-entry studies, one usually chooses between a spheroid. The flattening of the Earth has a noticeable effect on the altitude h of a vehicle for a given Cartesian position. Here, we use an ellipsoidal shape, shown schematically in Fig. 2.1.

The flattening of the central body is typically represented by a parameter f , also called the ellipticity. The flattening follows from:

$$f = \frac{R_e - R_p}{R_e} \quad (2.1)$$

where R_e represents the radius at the equator and R_p at the poles. For defining the location on the Earth's surface, either the geocentric latitude δ or the geodetic latitude δ^* may be used, shown in Fig. 2.1. The two are related as follows:

Fig. 2.1 Schematic representation of difference between the vertical and radial vector



$$\tan \delta^* = \frac{\tan \delta}{(1-f)^2} \quad (2.2)$$

So that δ and δ^* are very similar in magnitude, since f is small. From this value the local surface radius of the body, R_s , can be approximated, starting with the general expression of an ellipse and using polar coordinates:

$$R_s = R_e \left[1 - \frac{f}{2}(1 - \cos 2\delta^*) + \frac{5}{16}f^2(1 - \cos 4\delta^*) - \dots \right] \quad (2.3)$$

Since, to first order, δ can be approximated by δ^* , we may approximate Eq. (2.3) by

$$R_s \approx R_e \left[1 - \frac{f}{2}(1 - \cos 2\delta) \right] = R_e(1 - f \sin^2 \delta) \quad (2.4)$$

The altitude now simply follows from the following approximation:

$$h \approx r - R_s \quad (2.5)$$

Clearly, this relation is the same as what would be used for a perfect sphere, but then by substituting R_e for R_s .

2.1.2 Gravity

The gravitational force is one of the main external forces acting on a re-entry vehicle. Depending on the application one can model this force in different ways, ranging from a constant to a position-dependent quantity that includes the mass distribution inside the Earth.

Two point masses, M and m , separated by a vector $\mathbf{r}$, attract each other with a force given by Newton's law of gravitation as

$$\mathbf{F}_g = \frac{GMm}{r^2} \hat{\mathbf{r}} \quad (2.6)$$

where G is the universal gravity constant ($= 6.668 \cdot 10^{-11} \text{ m}^3/\text{kg s}^2$) and $\hat{\mathbf{r}}$ is the normalised position vector. Let us now assume that M is the mass of the Earth. It is then convenient to define the Earth's gravitational constant μ as GM , with $\mu = 3.9860047 \cdot 10^{14} \text{ m}^3/\text{s}^2$. Equation (2.6) is thus rewritten as

$$\mathbf{F}_g = m \frac{\mu}{r^2} \hat{\mathbf{r}} = m\mathbf{g} \quad (2.7)$$

with $\mathbf{g}$ being the acceleration due to gravity at a distance, r .

To first approximation, a spherical, radially mass-symmetric body can be assumed, taking the entire mass to be concentrated at its center allowing Eq. (2.6) to be used. When considering the mass distribution inside the body, for (near-) spheroidal bodies the so-called *spherical harmonics* gravity model can be used, which is an expansion of the gravitational potential in terms of Legendre polynomials. In this model, a set of coefficients is used to represent the irregularities in the body's mass distribution (e.g. Vallado 2007).

For precise-orbit determination of satellites orbiting the Earth, expansion to high order and degree are typically required. For entry missions into the Earth atmosphere, though, either central-field models or a model with a first-order corrections for the flattening of the Earth is usually sufficient. This stems from the fact that the aerodynamic forces are orders of magnitude larger than the higher-order gravitational terms. Therefore, for the current study we assume a rotationally symmetric mass distribution and only account for the flattening by including the so-called J_2 zonal harmonic ($J_2 = 1.082626 \cdot 10^{-3}$ for Earth). The gravitational acceleration in the vertical frame (see Sect. 2.2.1) is then given by:

$$\mathbf{g}_V = (g_n \ 0 \ g_d)^T \quad (2.8)$$

with g_n being the gravitational acceleration in north direction, and g_d the (radial) component in down direction. The two components g_n and g_d are given by:

$$\begin{aligned} g_n &= -3J_2 \frac{\mu}{r^2} \left(\frac{R_e}{r} \right)^2 \sin \delta \cos \delta \\ g_d &= \frac{\mu}{r^2} \left[1 - \frac{3}{2} J_2 \left(\frac{R_e}{r} \right)^2 (3 \sin^2 \delta - 1) \right] \end{aligned} \quad (2.9)$$

These components are only dependent on the radial distance to the center of the Earth, r , and the latitude, δ . Because of the rotational symmetry the longitude τ is not present in the potential, hence the component of $\mathbf{g}_V$ in east direction is zero.

2.1.3 Atmosphere

For calculations involving the aero-thermodynamic loads on a vehicle, knowledge regarding the state and composition of the atmosphere is required. Two different categories of atmosphere models can be distinguished. The first category deals with so-called *standard* atmospheres, which represent a distribution of the (average) state of the atmosphere with altitude that is representative for the Earth (or any other body with an atmosphere). The second category includes so-called *reference* atmospheres, which define this state of the atmosphere as a function of the actual position (altitude, longitude, and latitude). Additionally, temporal variations in the state, due to seasonal

and diurnal effects, for instance, can be taken into account. A model of the wind velocity vector can also be included as a function of time and position.

Despite the fact that a reference atmosphere is in general a more precise model of the actual state at any given point in space and time, for (conceptual) entry studies the use of a standard atmosphere is often preferred. First, the conceptual nature of the end-to-end model that is to be developed will most likely include deviations from the actual physics that are greater than those introduced by using a standard atmosphere in favor of a reference atmosphere. Second, the use of a standard atmosphere allows for a better comparison of the results produced by different simulations, as the entry point and time have no influence on the atmosphere that is encountered.

An often sufficiently accurate model of the atmosphere is the so-called US Standard Atmosphere of 1976 (typically abbreviated US76), described by NOAA/NASA (1976). This model is averaged over the diurnal and annual cycles as well as latitudes and longitudes, and represents the mean annual and global atmosphere at 45 degrees latitude, assuming a dry atmosphere (0 % humidity). For the definition of this atmosphere up to 86 km, the atmosphere is divided into seven regions, or strata, with each having its local temperature profile.

For the definition of the thermodynamic variables, two preliminary concepts are required. First of all, the geopotential altitude h_g , as opposed to the ‘actual’ or geometric altitude h is defined as follows:

$$h_g = \left(\frac{R_E}{R_E + h} \right)^2 h \quad (2.10)$$

Secondly, the molecular temperature T_M is defined, in addition to the kinetic temperature T (which corresponds to the conventional notion of temperature) as follows:

$$T_M = \left(\frac{M_0}{M} \right) T \quad (2.11)$$

where M is the molecular weight of the atmosphere at some altitude, with the 0 subscript denoting sea level conditions. Below 86 km, the value of M is almost invariant, having a value $0.999579M_0$ at 86 km, so that the molecular temperature and kinetic temperature can be assumed to be equal there. The definitions of Eqs. (2.10) and (2.11) have the advantage that they both lump the dependency on two variables (h and g ; T and M) into one new, transformed variable, simplifying the calculations.

The basic assumption of the 1976 standard atmosphere is that the variation of T_M in each of the seven layers up to 86 km is constant, with a thermal lapse rate L_{h_i} defined in each layer i as dT_M/dh . The values of these thermal lapse rates are given in Table 2.1. Above 86 km, the thermal lapse rate is not constant in all of the remaining layers. Below 86 km, the hydrostatic equation (which relates the weight of a column of air with the pressure it exerts on a reference surface) is assumed to be valid. Above 86 km however, its validity breaks down, as diffusive mixing and photochemical reactions become more pronounced. If the hydrostatic equation is

Table 2.1 Polytropic exponent and thermal lapse rates in the first seven strata, from NOAA/NASA (1976)

Geopotential altitude h , km	Thermal lapse rate L_h , K/km	Polytropic exponent n
0.0	−6.5	1.2350
11.0	0.0	1.0000
20.0	+1.0	0.9716
32.0	+2.8	0.9242
47.0	0.0	1.0000
51.0	−2.8	1.0893
71.0	−2.0	1.0622
84.8520		

valid, the pressure and density can be determined as follows, by rewriting it by using the definition of h_g and T_m , and assuming a perfect gas (see Sect. 3.1):

$$\frac{dp}{p} = -\frac{g_0 dh}{R_0 T_M} \quad (2.12)$$

Above 86 km, the pressure is obtained by summing the partial pressures of the various species. Due to the very limited interest of the precise atmospheric composition at such altitudes, though, these details are not repeated here.

2.2 Equations of Motion

This section will describe the equations of motion that are used to determine the trajectory of the entry vehicles. First, the reference frames that are used in the computations will be presented in Sect. 2.2.1 as well as the transformation matrices between them. Subsequently, the forces used in the trajectory determination are given in Sect. 2.2.2. Finally, the resultant equations of motion in spherical coordinates for a re-entry problem are presented in Sect. 2.2.3.

2.2.1 Reference Frames

To describe the equations of motion, it is necessary to define a number of reference frames, as the various external forces are obtained in different frames. Besides the definition of the reference frames, conversions between these frames are required to express all relevant components in one and the same frame. The following, right-handed reference frames will be considered here.

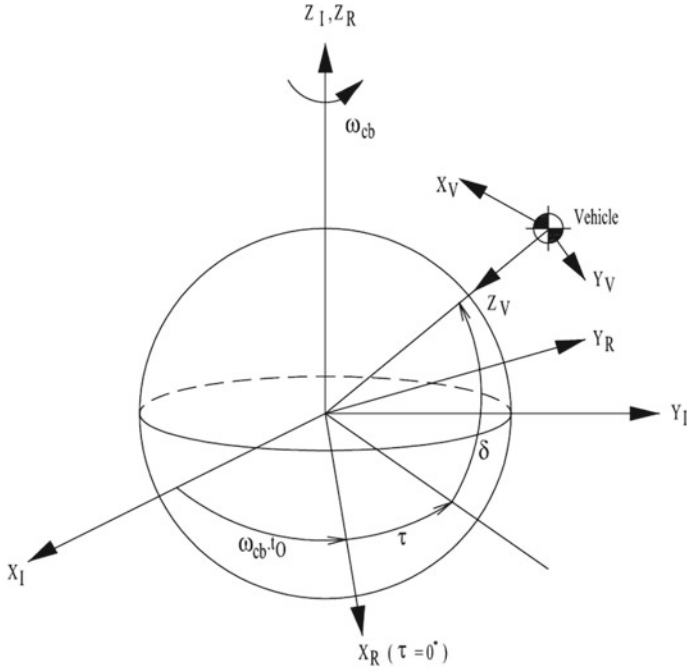


Fig. 2.2 Schematic representation of the inertial (index I) and rotating (index R) planetocentric frames, and the vertical frame (index V)

Inertial planetocentric frame I (Fig. 2.2):

The inertial frame used here has its origin in the Earth's center of mass, with the z -axis pointing in the direction of rotation (north as positive), the x -axis pointing in a reference direction and the y -axis completes the frame. For Earth the reference direction for the x -axis is typically the direction of vernal equinox (First Point of Aries, Υ). The reference frame that is commonly used is the J2000 reference frame.

Rotating planetocentric frame R (Fig. 2.2)

The rotating frame has its origin at the same position as the inertial frame, but is fixed to the Earth. The z -axis is again the rotation axis. The x -axis typically intersects the Greenwich meridian ($\tau = 0^\circ$, with τ being the longitude). The angle between the x - and y -axes of the I and R frames is termed the Greenwich Mean Sidereal Time (GMST) and denoted θ_{GMST} . From its value at $t = 0$ and the angular velocity ω_{cb} , the transformation between the inertial and planet-fixed frame can be determined. However, since this angle is constant for all simulations, it may also be put to zero and have no effect on the results. So, for simplicity, we assume that the I - and R -frame are collinear at $t = 0$. Also the Earth's angular velocity, as well as the rotation axis orientation, are considered to be constant, so any nutation or precession is ignored here.

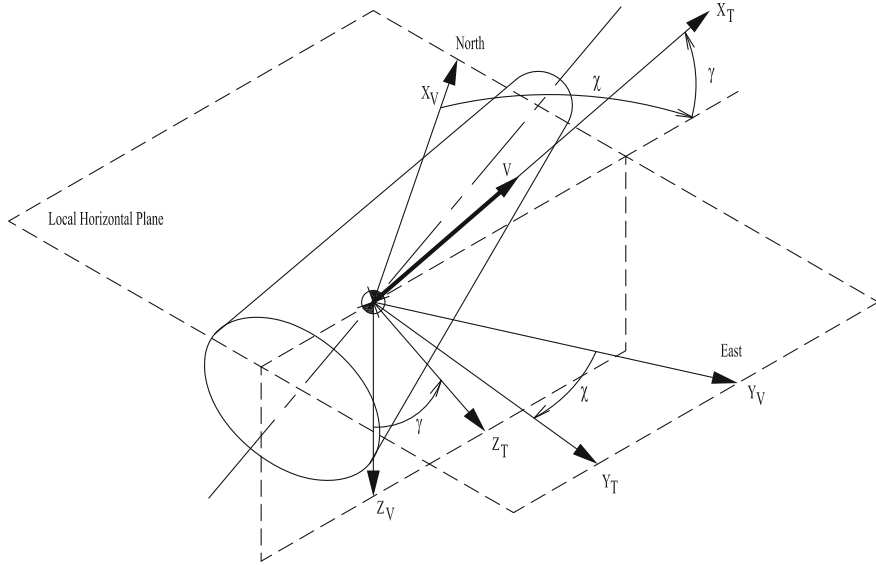


Fig. 2.3 Schematic representation of the vertical (index V) and trajectory (index T) frames

Vertical frame V (Figs. 2.2 and 2.3)

The vertical frame is vehicle-body centered, with its origin chosen at the center of mass of the vehicle. The directions of the axes based on the position relative to the central body. The z -axis points to the center of mass of the central body. The xy -plane is a locally horizontal plane (for a spherical central body), with the x -axis pointing to the north and the y -axis completing the system.

Trajectory reference frame T (Figs. 2.3 and 2.4)

The trajectory frame is body-centered, so again has the origin at the vehicle's center of mass. The trajectory reference frame has the x -axis pointing in the direction of flight, the z -axis lying in the vertical plane, pointing downwards and the y -axis completing the system. It should be noted that since the z -axis must be perpendicular to the x -axis and lie in the vertical plane, its direction is already defined, except for the sign (which is defined positive downwards).

Aerodynamic reference frame A (Fig. 2.4)

The aerodynamic frame has the origin at the center of mass of the vehicle. The x -axis is in the direction of the velocity vector, the z -axis is colinear with the lift force (see Fig. 2.5), but opposite in direction, and the y -axis completes the system. Two aerodynamic frames can be defined, one referenced to the airspeed and one to the groundspeed. However, since wind is not considered in this study, these two frames coincide and no distinction between the two will be made. It is noted that when the bank angle σ is zero, the A and T frames coincide.

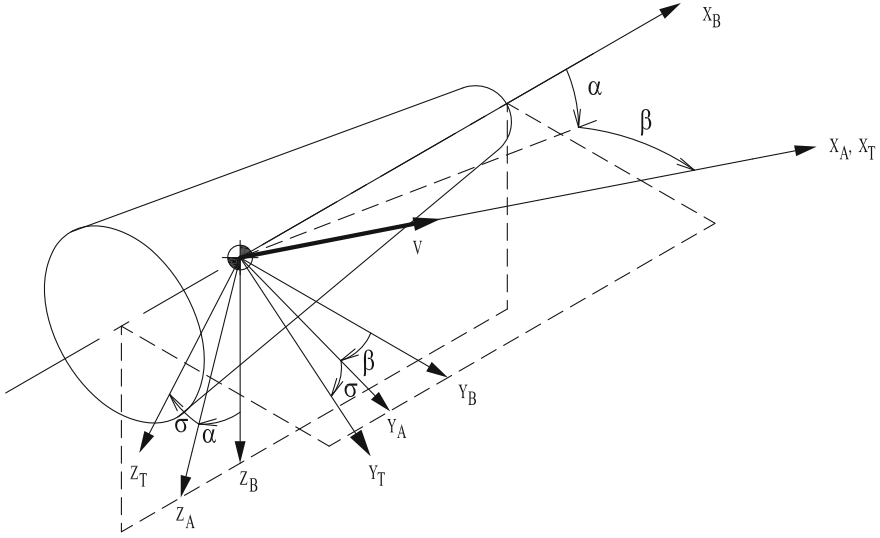


Fig. 2.4 Schematic representation of the *B*, *A* and *T* frames

Body frame B (Fig. 2.4)

The origin of the body frame is located in the center of mass of the vehicle, but, unlike the aerodynamic frame, are also fixed to the vehicle. The *x*-axis typically points in the direction of the front of the vehicle, the *z*-axis points down and the *y*-axis completes the system (corresponding to the direction of the right wing of an aircraft).

The transformations between the frames are handled using Euler angles. By using these, successive rotations about the axes are used to represent the full rotation. It should be stressed that the order of the rotations is not trivial, as changing them will result in a different transformation. The directional cosine matrices of rotation for a positively defined angle θ_i about the i^{th} axes are:

$$\mathbf{C}_x(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_x & \sin \theta_x \\ 0 & -\sin \theta_x & \cos \theta_x \end{pmatrix} \quad (2.13)$$

$$\mathbf{C}_y(\theta_y) = \begin{pmatrix} \cos \theta_y & 0 & -\sin \theta_y \\ 0 & 1 & 0 \\ \sin \theta_y & 0 & \cos \theta_y \end{pmatrix} \quad (2.14)$$

$$\mathbf{C}_z(\theta_z) = \begin{pmatrix} \cos \theta_z & \sin \theta_z & 0 \\ -\sin \theta_z & \cos \theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.15)$$

These transformation are called unit-axis transformations, since each of these gives the rotation about the *x*, *y* or *z* axis.

The total transformation matrix from system A to system C is constructed from these individual matrices, e.g., for rotation order j, k, l (first around the axis j , then the axis k and finally the axis l) as follows:

$$\mathbf{C}_{C,A} = \mathbf{C}_l(\theta_l)\mathbf{C}_k(\theta_k)\mathbf{C}_j(\theta_j) \quad (2.16)$$

The order of the matrices follows from the fact that one should successively left-multiply with the relevant rotation matrix. Similarly, when transforming from system A to C , an intermediate step of transformation to system B can be used (in case $\mathbf{C}_{B,A}$ and $\mathbf{C}_{C,B}$ are directly available, for instance) as follows:

$$\mathbf{C}_{C,A} = \mathbf{C}_{C,B}\mathbf{C}_{B,A} \quad (2.17)$$

Although a number of sets of three Euler angles will exist through which the system A can be rotated to find system C , it is possible that these angles will not have a clear physical interpretation. This could make an intermediate step to a system B the preferred method. It should be understood that in a series of three rotations as in Eq. (2.16), two non-successive rotations about the same axis may not be simply taken together as a single rotation over the sum of the angles. This is due to the fact that the orientation of this axis will have changed due to the intermediate rotation.

The rotation order for a number of relevant transformations are the following, from Mooij (1998):

$$\mathbf{C}_{R,I} = \mathbf{C}_z(\omega_{cb}t) \quad (2.18)$$

$$\mathbf{C}_{V,R} = \mathbf{C}_y(-\pi/2 - \delta)\mathbf{C}_z(\tau) \quad (2.19)$$

$$\mathbf{C}_{V,T} = \mathbf{C}_z(-\chi)\mathbf{C}_y(-\gamma) \quad (2.20)$$

$$\mathbf{C}_{T,A} = \mathbf{C}_x(\sigma) \quad (2.21)$$

$$\mathbf{C}_{A,B} = \mathbf{C}_z(\beta)\mathbf{C}_y(-\alpha) \quad (2.22)$$

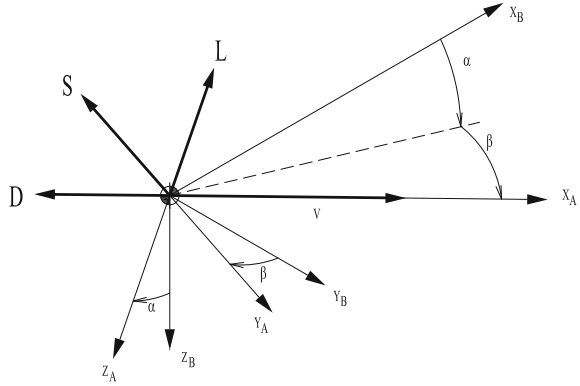
with γ and χ being the flight-path and heading angle, and α and β being the angle of attack and sideslip. Each of the angles has been defined in this section – see Figs. 2.2–2.4. It is noted that inverse transformation matrices are easily obtained. Because the transformation matrices are orthonormal, the inverse of such a matrix is simply its transpose.

2.2.2 Forces

This section will describe forces that describe the translational motion of the vehicle. The assumptions that we use to arrive at them are stated, and the physical implications of the resulting equations are discussed.

It will be assumed here that the aerodynamic and gravitational forces are the only external forces acting on the body, so:

Fig. 2.5 The aerodynamic drag (D), side (S) and lift (L) force



$$\mathbf{F}_{\text{ext},I} = \mathbf{F}_{a,I} + \mathbf{F}_{g,I} \quad (2.23)$$

where the I denotes that the inertial frame is used to evaluate the forces. No inclusion of propulsion is warranted due to the fact that only unpowered entry will be considered.

The gravitational force has been defined previously, Sect. 2.1.2, as follows:

$$\mathbf{F}_{g,V} = m\mathbf{g}_V \quad (2.24)$$

where $\mathbf{g}_V = (g_n \ 0 \ g_d)^T$; these gravity components are expressed in the V -frame, so to obtain the equivalent in the I frame we need to use the static transformation:

$$\mathbf{F}_{g,I} = m\mathbf{C}_{I,V}\mathbf{g}_V \quad (2.25)$$

with $\mathbf{C}_{I,V} = \mathbf{C}_{I,R}\mathbf{C}_{R,V}$, and the individual matrices given by the inverse matrices of $\mathbf{C}_{R,I}$ and $\mathbf{C}_{V,R}$ as shown in Eqs. (2.18)–(2.22).

The aerodynamic forces are defined in the aerodynamic reference frame A and given by:

$$\mathbf{F}_{a,A} = - \begin{pmatrix} C_D \\ C_S \\ C_L \end{pmatrix} \frac{1}{2} \rho V^2 S_{ref} \quad (2.26)$$

where C_D , C_S and C_L represent the drag, side force and lift coefficient, respectively, and V being the vehicle's velocity relative to the atmosphere. For some aerodynamic databases, the side force is defined positive in the positive y_A direction. However, the convention of having the three force coefficients in the aerodynamic frame defined in a right-handed manner, as in the above equation, will be observed here. A schematic representation is given in Fig. 2.5. Alternatively, the aerodynamic force may be represented in the body reference frame B by:

$$\mathbf{F}_{a,B} = \begin{pmatrix} C_X \\ C_Y \\ C_Z \end{pmatrix} \frac{1}{2} \rho V^2 S_{ref} \quad (2.27)$$

where the relation between the two follows from the transformations described in Sect. 2.2.1.

The aerodynamic force is transformed to the I -frame with

$$\mathbf{F}_{a,I} = \mathbf{C}_{I,A} \mathbf{F}_{a,A} \quad \text{or} \quad \mathbf{F}_{a,I} = \mathbf{C}_{I,B} \mathbf{F}_{a,B} \quad (2.28)$$

with, for instance, $\mathbf{C}_{I,A} = \mathbf{C}_{I,R} \mathbf{C}_{R,V} \mathbf{C}_{V,T} \mathbf{C}_{T,A}$, a combination of the (inverse) matrices defined earlier.

2.2.3 Entry Equations

The equations of translational motion follow directly from Newton's Laws of Motion. These laws are only directly valid when considering the motion in an inertial frame. However, by introducing relative rotations and including the resulting apparent forces (e.g., Coriolis; centripetal) between a target frame and the inertial frame, the equations can also be expressed in a rotating frame.

The resulting formulation of the equations is determined by the choice of state variables. For the translational motion through the atmosphere in three dimensions, six state variables are required to fully determine the motion, notably three position coordinates and three velocity components. In our numerical integration, the state is given by the Cartesian position and the velocity in the I -frame.

In its simplest form the state variables describing the inertial motion are $\mathbf{r}_I = (x_I \ y_I \ z_I)$ for the position, and $\mathbf{v}_I = (\dot{x}_I \ \dot{y}_I \ \dot{z}_I)$ for the velocity. With the gravitational and aerodynamic forces transformed to the I reference frame, the general equations of motion are:

$$\frac{d}{dt} \begin{pmatrix} \mathbf{r}_I \\ \mathbf{v}_I \end{pmatrix} = \begin{pmatrix} \mathbf{v}_I \\ \frac{1}{m} (\mathbf{F}_{a,I} + \mathbf{F}_{g,I}) \end{pmatrix} \quad (2.29)$$

To determine the trajectory of the vehicle from this set of six first-order, non-linear, coupled ordinary differential equations, no general analytical solution exists, so that numerical integration methods must be resorted to. A wide variety of such integration methods exists, with varying accuracy, computational complexity and numerical stability. Due to the conceptual nature of this study, the required accuracy of the entry trajectories is limited. This is due to the fact that modeling inaccuracies in, for instance, the aerodynamic coefficients and vehicle mass will also result in errors in the trajectories. Since a large number of trajectories has to be propagated, computational time should be kept to a minimum. It is possible to use a variable step-size integrator to allow for larger step sizes in regions of little variation in the solution. However, the fact that a guidance system with a certain update frequency is

used, makes these long time steps less desirable for a comparative analysis of vehicle behaviour.

A common numerical integrator for studies such as the one we discuss here is the Runge Kutta 4th-order method. Alternatively, a more accurate variable-step size methods can be used. Typical examples are the Dormand-Prince 4(5) method – a fourth-order method that is combined with a fifth-order method to determine the stepsize, given a predefined relative and/or absolute tolerance – or the family of Runge–Kutta–Fehlberg methods with different order pairs. Details on these and additional methods, including matters related to method stability, consistency and convergence can be found in a variety of texts, such as Shampine (1994), Lambert (1991), and Press et al. (2007).

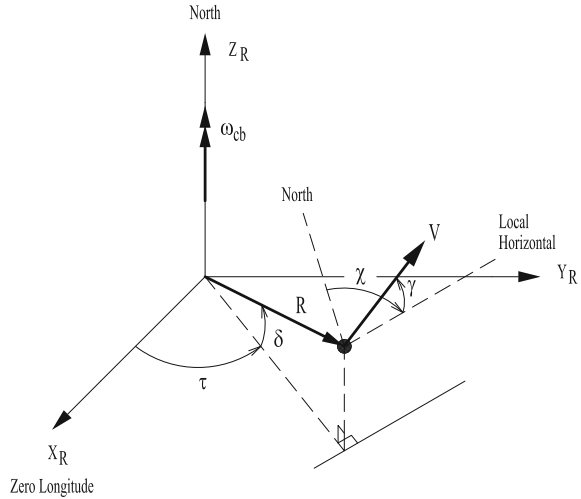
Although Eq. (2.29) may be used to efficiently simulate the trajectory of a vehicle, different representations can be more useful, either from a physical or computational viewpoint. A good example is the use of spherical position and velocity, which leads to a very intuitive formulation and representation of the equations of motion. These equations are commonly used for the development of guidance systems, which is the approach we take here (Sect. 2.3.2).

The spherical position coordinates and velocity components are shown in Fig. 2.6. These six state variables are defined as follows:

- Radial position r , the scalar distance from the center of the Earth to the vehicle's center of mass.
- Longitude τ , measured from the Greenwich meridian, positive in east direction ($-180^\circ \leq \tau < 180^\circ$).
- Latitude δ , measured along the local meridian starting from the equator, positive in north direction ($-90^\circ \leq \delta \leq 90^\circ$).
- (Relative) velocity V . The scalar velocity is measured w.r.t. the ground, which in this work equals the airspeed, since there is no wind. Note that this velocity differs from the velocity in inertial coordinates due to the rotation of the Earth, so that $V < V_I$ for posigrade entries.
- Flight-path angle γ , which is the angle between the ground velocity vector and the local horizontal plane ($-90^\circ \leq \gamma \leq 90^\circ$).
- Heading angle χ , which is the angle between the north direction in the local horizontal plane and the ground velocity vector projected onto this plane ($-180^\circ \leq \chi < 180^\circ$). It is measured positive clockwise. A heading of 0° is due north, and 90° is due east.

The velocity vector shown here is relative to the rotating central body, so Eq. (2.29) needs to be adapted to take the rotation of the central body with respect to which the velocity is being evaluated into account. The position vector of the vehicle combined with the tangent to the local curvature of the Earth defines the vertical plane; the plane perpendicular to this is called the local horizontal plane and contains the center of mass of the vehicle as well as its horizontal velocity component. Also, this plane is defined by the x - and y - axes of the V -frame. When the vehicle is moving, the local horizontal plane is co-moving with the vehicle, and the rotation of this plane is the second correction to Eq. (2.29) that must be applied.

Fig. 2.6 Definition of spherical position and velocity with respect to a rotating frame



As the derivation of the equations of motion using r , τ , δ , V , γ , and χ is rather long and tedious, we suffice by stating the final result. Note that we assume that $S = 0$, as we set $\beta = 0$ throughout our analysis. For more information, the reader is referred to textbooks, such as those by Vinh et al. (1980), Regan and Anandakrishnan (1993) and Mooij (1998).

$$\begin{aligned} \dot{V} = & -\frac{D}{m} + g_d \sin \gamma - g_n \cos \gamma \cos \chi + \dots \\ & \dots + \omega_{cb}^2 R \cos \delta (\sin \gamma \cos \delta - \cos \gamma \sin \delta \cos \chi) \end{aligned} \quad (2.30)$$

$$\begin{aligned} V \dot{\gamma} = & \frac{L \cos \sigma}{m} - g_d \cos \gamma + g_n \sin \gamma \cos \chi + 2\omega_{cb} V \cos \delta \sin \chi + \dots \\ & \dots + \frac{V^2}{r} \cos \gamma + \omega_{cb}^2 r \cos \delta (\cos \delta \cos \gamma + \sin \gamma \sin \delta \cos \chi) \end{aligned} \quad (2.31)$$

$$\begin{aligned} V \cos \gamma \dot{\chi} = & \frac{S \sin \sigma}{m} + 2\omega_{cb} V (\sin \delta \cos \gamma - \cos \delta \sin \gamma \cos \chi) + \dots \\ & \dots + \frac{V^2}{r} \cos^2 \gamma \tan \delta \sin \chi + \omega_{cb}^2 r \cos \delta \sin \delta \sin \chi \end{aligned} \quad (2.32)$$

$$\dot{r} = \dot{h} = V \sin \gamma \quad (2.33)$$

$$\dot{\tau} = \frac{V \sin \chi \cos \gamma}{r \cos \delta} \quad (2.34)$$

$$\dot{\delta} = \frac{V \cos \chi \cos \gamma}{r} \quad (2.35)$$

It should be noted that the above equations have two singularities: in Eq. (2.31) this singularity is encountered for $\gamma = \pm 90^\circ$, which means a purely vertical flight (up or down). The second singularity is met when $\delta = \pm 90^\circ$, which is at either north or south pole. In this study, neither situation will occur, so these equations will serve the purpose as baseline for guidance-logic design.

Often, the equations are rewritten in terms of the $L/D(=C_L/C_D)$ ratio, along with a ballistic parameter or coefficient, defined as follows:

$$B = \frac{m}{S_{ref} C_D} \quad (2.36)$$

where sometimes the weight (at sea level) instead of the mass is used in this definition. The ballistic parameter denotes the relative influence of gravitational and aerodynamic drag forces. It can be used to give an indication of the altitudes at which maximum heating, deceleration, etc., take place. It will, in general, not be constant, however, due to changes in drag coefficient due to varying flight angles, velocities and altitudes.

2.3 Guidance Approach

To limit the computational time required for the trajectory calculation, targeting of a landing site or TAEM interface is not performed here. Instead, entry conditions are specified, while the end conditions will be kept free. A trajectory optimization is not deemed cost-effective, since this would require an optimization for each of the vehicle shapes that is generated. Since the optimization of a single trajectory can be rather time-consuming, such an approach is unlikely to be cost-effective here. Coupled trajectory and shape optimization has been performed by (Armellin 2007; Grant et al. 2011), but at the expense of a more simplified dynamical/aerodynamics model, which we do not deem to be a good balance of model fidelity for this particular study.

The guidance and control approach differs between the ballistic and winged entry vehicles, which are treated in Sects. 2.3.1 and 2.3.2. Finally, we briefly discuss stability issues in Sect. 2.3.3. Both the longitudinal behaviour, characterized by the angle of attack, α , and the lateral behaviour, characterized by the bank angle, σ , will be discussed. The sideslip angle, β , is always assumed to be zero for both types of vehicles. For definitions of the attitude angles, see Fig. 2.4.

2.3.1 Capsule

Capsule-shaped vehicles are typically controlled by a Reaction Control System (RCS) only, although some concepts using a flap for aerodynamic control have also

been proposed (Andersen and Whitmore 2007). However, attitude propagation will not be included in the simulations and the details of the control system will therefore not be considered. Instead, for the capsule vehicles trimmed conditions will be assumed. The trimmed angle of attack follows from:

$$\alpha_{tr} = \alpha|_{C_m=0} \quad (2.37)$$

That is, the angle of attack is chosen such that the pitch moment coefficient (see Sect. 3.2) equals zero. If such an angle of attack cannot be found within certain bounds, the vehicle will be labeled as untrimmable (see Sect. 6.3.2). In the context here, we define a trim angle to be the angle of attack that fulfills Eq. (2.37).

For symmetric vehicles with the center of mass on the center line, the trim attitude will be $\alpha_{tr} = 0$. For an offset center of mass in z -direction, e.g., as was the case for the Apollo capsule, the trimmed angle of attack will be non-zero. As described in Sect. 5.1, such an offset will also be used here as a design variable. As the moment-coefficient curve is a function of Mach number, the trimmed angle will also be dependent on Mach number. The change in coefficient with Mach number remains limited, though (see Sect. 7.2.1). Since the trajectory propagation has three degrees of freedom (only translational), the time-dependent process by which the capsule changes attitude is not analyzed. Considering the minor changes in angle of attack that are expected to occur, this will not influence the results significantly (assuming the vehicle to be stable). Attitude stability of the vehicle is important, however, as instability will make it unlikely for the vehicle to retain its trimmed conditions throughout the flight. For this reason, the following condition for static stability will be imposed on the capsule:

$$C_{m_\alpha}|_{\alpha=\alpha_{tr}} < 0 \quad (2.38)$$

Whether this relation is fulfilled or not depends on the location of the center of mass with respect to the center of pressure. This constraint is discussed further in Sect. 6.3.2. For the rare cases where a capsule is analyzed with two values of α_{tr} that fulfill Eq. (2.37), only the point that fulfills condition (2.38) is used. Since such cases will always have one controllable and one uncontrollable trim angle of attack, they will thus always be controllable.

In addition to pitch guidance the vehicle also requires lateral guidance, which can be achieved by bank-angle modulation. By modulating this angle, the direction of the lift vector can be controlled, so that influence on the flight-path and heading angles can be exerted, see Eqs. (2.31) and (2.32). To prevent the vehicle from skipping out of the atmosphere, the flight-path angle is controlled in such a manner that the following condition will hold:

$$\dot{\gamma} \leq 0 \quad (2.39)$$

From Eq. (2.31), neglecting the centrifugal term due to the Earth's rotation and the latitudinal gravity term, the following relation is obtained:

$$V\dot{\gamma} = \frac{L}{m} \cos \sigma - \left(g_d - \frac{V^2}{r} \right) \cos \gamma + 2\omega_{cb} V \cos \delta \sin \gamma \quad (2.40)$$

By setting $\dot{\gamma} = 0$, condition (2.39) is satisfied, while keeping the flight-path angle as large as possible, which will increase the vehicle's range. It follows that:

$$\cos \sigma = \frac{m}{L} \left[\left(g_d - \frac{V^2}{r} \right) \cos \gamma - 2\omega_{cb} V \cos \delta \sin \gamma \right] \quad (2.41)$$

From the Earth's rotation rate ($\approx 7.27 \cdot 10^{-5}$ rad/s), it can be seen that the Coriolis acceleration can reach values of up to 1.1 m/s² for orbital velocity. When considering the fact that the term $g_d - V^2/r$ equals zero for orbital velocity, it can be concluded that the Coriolis term is an important contributor to the flight-path angle derivative. Therefore, it must not be neglected in the bank-angle determination.

The bank angle is modulated according to Eq. (2.41), until $\cos \sigma > 1$, at which point the vehicle no longer has sufficient lift to be able to fly at constant flight-path angle. When this occurs, the bank angle is set to 0°. It has been chosen to use the solution of positive σ of the above equation. This corresponds with banking to the left, as can be seen in Fig. 2.4.

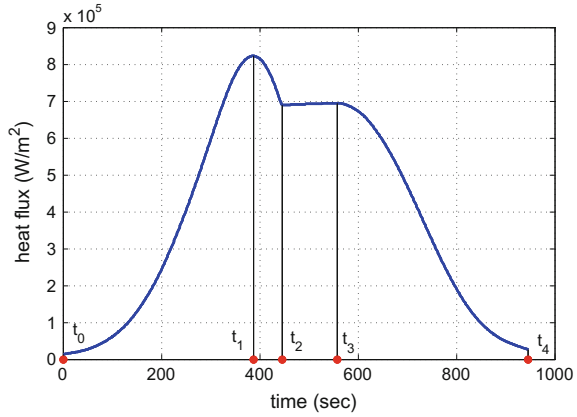
2.3.2 Winged Vehicle

The winged vehicles will be guided to fly a maximum time at a given reference stagnation-point heat rate $q_{c,s,ref}$, which can be seen as a typical mission profile for a class of experimental vehicles. This can yield valuable information of both flow and material behaviour at such a heat flux, aiding in the design effort of future re-entry vehicles (Mooij and Hänninen 2009). It has the virtue of allowing much of the guidance law to be expressed analytically.

For the first portion of the trajectory, the vehicle is commanded to fly at maximum angle of attack, so as to minimize the maximum heat flux. After the maximum value of $q_{c,s}$ is reached, the heat-flux tracking is activated after $q_{c,s,ref}$ is reached, which will guide the vehicle to maintain a constant stagnation-point heat rate. A typical flight profile is shown in Fig. 2.7. In this figure, a number of characteristic times is indicated. These times are: i) t_0 , initiation of entry, ii) t_1 , point of maximum stagnation point heating, iii) t_2 , point of initiation of heat-flux tracking, iv) t_3 , end of heat-flux tracking, and v) t_4 end of hypersonic entry phase ($M < 3$).

If the heat rate at t_1 is smaller than $q_{c,s,ref}$, the heat rate at the heating peak will be tracked instead. In such a case t_1 and t_2 will coincide. A similar approach could be used at the end of the trajectory to maintain constant dynamic pressure, as both rely (approximately) on keeping

Fig. 2.7 Typical heat flux profile for described guidance algorithm



$$K = \rho V^n \quad (2.42)$$

constant, where $n \approx 6$ (exact value depends on the choice of model, see Eq. (3.77)) for constant heat rate and $n = 2$ for constant dynamic pressure. Taking the first derivative with respect to time yields the following:

$$0 = \frac{d\rho}{dt} V^n + \rho n V^{n-1} \frac{dV}{dt} \quad (2.43)$$

$$\frac{dV}{dt} = -\frac{1}{n} \frac{V}{\rho} \frac{d\rho}{dt} \quad (2.44)$$

To enforce this condition, we modulate α to cause the drag to be such that Eq. (2.44) is enforced (see Eq. (2.30)). The bank angle is modulated to enforce Eq. (2.41).

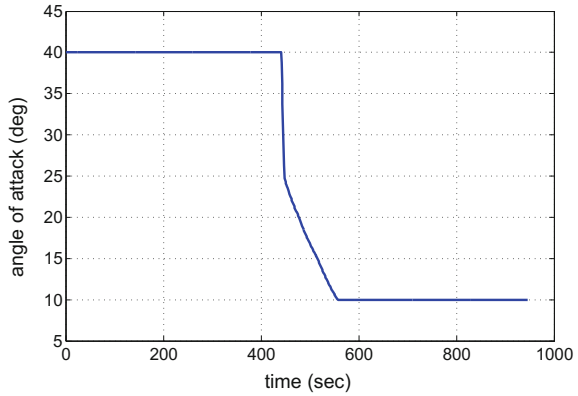
The angle-of-attack profile corresponding to the heat-flux profile shown in Fig. 2.7 is shown in Fig. 2.8, with minimum and maximum values of α set to 10° and 40° , respectively. It can be seen that the initiation of the heat-flux tracking corresponds to the initiation of the reduction of the angle of attack, and the end of the tracking corresponds to the minimum angle of attack.

Neglecting the centrifugal term due to the rotation of the Earth and latitudinal dependency of the gravitational acceleration, substitution of Eq. (2.44) into Eq. (2.35) leads to the following relation for the drag:

$$D_{K=const} = \frac{mV}{n\rho} \frac{d\rho}{dt} - mg \sin \gamma \quad (2.45)$$

From the aerodynamic coefficient database of the vehicle and the flight conditions, this value of the drag can be matched to a required angle of attack. A maximum angle-of-attack rate will be imposed to avoid discontinuities in the angle of attack. The winged-vehicle shapes that are to be analyzed will have active pitch control capability by the use of a body flap and elevons. Since only symmetric ($\beta = 0$)

Fig. 2.8 Typical angle of attack profile for described guidance algorithm



entries are considered, the yaw and roll moments will be zero by virtue of the vehicle symmetry w.r.t. the vertical center plane. The guidance scheme will attempt to trim the pitch moment by body flap and elevon deflections. First, trim by only the body flap is attempted. If this fails, the elevons are also used. The resulting values of δ_{bf} and δ_e lead to the following expressions for the aerodynamic coefficients:

$$C_D = C_{D,0}(\alpha) + \Delta C_{D,bf}(\alpha, \delta_{bf}) + \Delta C_{D,e}(\alpha, \delta_e) \quad (2.46)$$

$$C_L = C_{L,0}(\alpha) + \Delta C_{L,bf}(\alpha, \delta_{bf}) + \Delta C_{L,e}(\alpha, \delta_e) \quad (2.47)$$

$$C_m = C_{m,0}(\alpha) + \Delta C_{m,bf}(\alpha, \delta_{bf}) + \Delta C_{m,e}(\alpha, \delta_e) (= 0) \quad (2.48)$$

If the final non-zero condition on the moment coefficient is not satisfied by the control system, the solution is marked as infeasible in the optimization process. Again, the time-dependent process by which the attitude changes is not included in the simulation. Also, the control surface deflections are assumed to occur instantaneously.

The lateral guidance is performed in a manner similar to the capsule-shaped vehicle, with bank angle modulation based on Eq. (2.39), resulting in Eq. (2.41). Again, this is to prevent the vehicle from skipping out of the atmosphere. This is, due to the high lift of the winged vehicle, a very likely scenario if bank-angle modulation is not included. The bank-angle modulation is only started once the lift becomes the dominant term in Eq. (2.31). For the initial portion of the entry, the density is very low, so the lift will not have a significant influence on the vehicle's behaviour compared to the other forces.

2.3.3 Vehicle Stability

Related to the control capabilities and characteristics of entry vehicles are their attitude-stability characteristics. Although the analysis of attitude stability will be

rather simplified here, as no rotational motion is included in the trajectory propagation, a number of general vehicle characteristics are used that are indicative for stability.

When discussing stability, two different types are distinguished, static and dynamic. For static vehicle stability, only the tendency of the vehicle to move to an equilibrium position is considered, whereas dynamic stability also considers the time-dependent process that is involved. When the vehicle is trimmed, the equilibrium attitude corresponds to its current attitude, as the center of mass coincides with the center of pressure and the resulting moment about this point is zero.

Now, we consider what will happen to the vehicle's attitude when there is a slight difference between the center of pressure and center of mass is introduced, *i.e.*, when the current attitude is slightly perturbed from equilibrium. If the vehicle has the tendency to move back to its equilibrium position, it is considered to be statically stable. This behaviour is characterized by a number of stability derivatives, which denote the resulting moments as a result of a change of attitude. Three stability derivatives will be used here, C_{m_α} , C_{l_β} and C_{n_β} , which denote the changes in pitch, roll and yaw moment due to a change in angle of attack, angle of sideslip and angle of sideslip, respectively, so:

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \quad (2.49)$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta} \quad (2.50)$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta} \quad (2.51)$$

The value of C_{m_α} is a measure for the pitch stability of the vehicle. To clarify this, consider the situation when the angle of attack of the vehicle is slightly increased from its equilibrium position. If this causes the value of C_m to increase (corresponding to positive C_{m_α}), this would cause a positive value of $\dot{\alpha}$, which increases the angle of attack further, as well as C_m , etc. As such, the equilibrium point is unstable for pitch motion. Similarly, deviations in sideslip angle must not cause the yaw or roll moment to increase.

The influence of control-surface deflections on the vehicle stability can be quite substantial. Specifically, upward and downward deflections of the control surfaces yield different contributions to the pitch moment derivative. A clear explanation of this is given by Hirschel and Weiland (2009), where a flat plate with a control surface at the end is analyzed. In general, an upward control-surface deflection will aid in pitch stabilizing the vehicle, whereas a downward deflection will decrease the pitch stability. However, since a downward deflection of the control surface will increase the L/D of the vehicle, a trade-off must be made between these conflicting requirements. The results of this trade-off, as performed by the optimizer, will be discussed in Sect. 9.2.

For dynamic stability, the dynamic behaviour of the attitude dynamics must be considered. However, this behaviour is typically characterized by the derivatives of the moment coefficients with respect to attitude rates and quantities involving the moments and products of inertia. To include such matters, the trajectory simulation could be extended to six degrees of freedom, to include attitude propagation. Such an extension of the analysis would introduce the need for the determination of dynamic aerodynamic derivatives. For hypersonic vehicles, the embedded Newtonian method (Ericsson 1975; East and Hutt 1988) is a good candidate in the context of the methods we use. Alternatively, the theory described by McNamara et al. (2010) for non-rigid structures at hypersonic velocities could be adapted. In addition, attitude dynamics would require knowledge of the inertia tensor of the vehicles. This would require that the mass model be extended, as the specific mass distribution is then needed. The additional required computational cost would be extensive, at a limited addition to the model fidelity. Consequently, we limit ourselves to static stability considerations here.

Conceptual Shape Optimization of Entry Vehicles
Applied to Capsules and Winged Fuselage Vehicles

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