Joint Treatment of Imprecision and Randomness in the Appraisal of the Effectiveness and Risk of Investment Projects

Bogdan Rębiasz, Bartłomiej Gaweł and Iwona Skalna

Abstract This paper proposes a new method for evaluating the effectiveness and risk of investment projects in the presence of both fuzzy and stochastic uncertainty. The main novelty of the proposed approach is the ability to take into account dependencies between uncertain model parameters. Thanks to this extra feature, the results are more accurate. The method combines non-linear programming with stochastic simulation, which are used to model dependencies between stochastic parameters, and interval regression, which is used to model dependencies between fuzzy parameters (possibility distributions). To illustrate the general idea and the effectiveness of the proposed method, an example from metallurgical industry is provided.

Keywords Risk of investment projects · Fuzzy random variable · Stochastic simulation · Interval regression

1 Introduction

It is well recognized that uncertainty is inevitable in economic evaluation models and consequently needs to be addressed appropriately in order that decision-makers have confidence in model's result. This would be possible only by using appropriate computational methods and tools for describing and processing uncertain data.

B. Rębiasz (✉) · B. Gaweł · I. Skalna
Faculty of Managementul, AGH University of Science and Technology,
Gramatyka 10, 30-067 Kraków, Poland
e-mail: Brebiasz@zarz.agh.edu.pl

B. Gaweł
e-mail: Bgawel@zarz.agh.edu.pl

I. Skalna
e-mail: iskalna@zarz.agh.edu.pl

Z. Wilimowska et al. (eds.), *Information Systems Architecture and Technology: Proceedings of 37th International Conference on Information Systems Architecture and Technology—ISAT 2016—Part IV*,
Advances in Intelligent Systems and Computing 524, DOI 10.1007/978-3-319-46592-0_2

For many years, probability theory was the only tool that allowed uncertainty to be expressed in mathematical language [5]. However, extensive research (see, e.g., [8, 9]) have shown that probabilistic approach faces several problems. For example, it is difficult to determine the probability distributions of economic parameters, when there is no sufficient data to perform statistical tests. Also, in many decision-making situations, the nature of uncertainty does not obey the assumptions of the probability theory. One can mention epistemic (reducible) uncertainty [14, 20], which stems from insufficient or imprecise information or the lack of knowledge or experts knowledge, which is usually expressed in linguistic terms. Therefore, alternative ways, such as fuzzy or intervals numbers, are increasingly often used to model uncertainty. The first works on using alternative ways to describe uncertain parameters in financial analyses have emerged in late '80s (see, e.g., [21]). Buckley [3] used fuzzy numbers to compute NPV of investment projects. Choobineh and Behrens [6] applied possibility distributions in economic analyses. Methods for computing effectiveness ratios of selected investment projects with fuzzy parameters were presented, e.g., in [7, 14, 13]. Kuchta [16] used fuzzy numbers in capital budgeting. Hui [11] discussed a capital budgeting problem in uncertain environment, where investment outlays and annual net cash flows of available projects are given subject to experts' estimations. The extend survey of using alternative methods of modelling uncertainty can also be found in [4].

Currently, probability theory and possibility theory are the most commonly used mathematical tools for describing uncertainty in effectiveness and risk appraisal problems. It is difficult to decide which approach is better [18], especially that the most common situation in practice is that some parameters are stochastic, whereas others are available in the form of imprecise knowledge, so that two types of uncertainty must be processed in a single framework. Most of the existing approaches handle this task by unifying different types of uncertainty [20], but this transformation is not one-to-one. The transition from possibility to probability introduces some (artificial) information, whereas in the opposite transition some information is lost [20], and thereby systematic errors are introduced in risk assessment. Therefore, it is better to process each type of uncertainty according to its nature.

In view of this, hybrid description of uncertainty, where some parameters are given by probability distributions, and others by possibility distributions was proposed, e.g., in [4, 6, 8, 9, 20]. A method for processing hybrid data, which combines stochastic simulation with fuzzy numbers was presented in [2, 10]. Hybrid approach was also successfully implemented in finance [13, 17], reliability theory [1] and LCA [19]. This direction of research becomes increasingly popular, but still has not been exhaustively investigated. One of the biggest deficiency of the existing methods for processing hybrid data, which limit their practical usage, is inability to take into account dependencies between hybrid parameters. In order to properly assess the risk, this problem must be solved. Baudrit et al. [2] have suggested how this problem can be tackled with rank correlations, copulas and some other techniques.

In this paper, a new method for processing hybrid data, which takes into account dependencies between uncertain parameters of the risk model is developed. The obtained results prove the truth of the following three hypotheses: (a) the use of both fuzzy numbers and probability distributions in assessment of the risk of investment projects provides a more adequate description of conditions under which decisions are made, (b) dependencies between hybrid data can be efficiently modeled by using correlation matrix and interval regression, (c) taking into account statistical dependence significantly improves the results, otherwise systematic errors in risk assessment may occur.

The rest of the paper is organized as follows. Section 2 describes a model for investment project risk appraisal. Section 3 presents an algorithm for processing hybrid data. Section 4 discusses a numerical example from the steel industry. The paper ends with concluding remarks.

2 Investment Project Risk Appraisal—Problem Statement

Effectiveness and risk of investment projects is estimated using a mathematical model, which consists of two groups of equations. The following notation is adopted:

$\boldsymbol{Pr}_{ij}^{t}$ - variable determining the quantity of the total output of product i in year t in department j,
$\boldsymbol{G}_{ia}^{t}$ - variable determining sales of the product i in year t in market a,
$\boldsymbol{ZO}^{t}$ - variable determining the operating profit in year t,
$\boldsymbol{NKO}^{t}$ - variable determining the change in working capital in year t,
IA - volume of investment outlays,
RV - the residual value of the project,
NPV - net present value
I, - index set of products
I_j - index set of products produced in department j,
J - index set of production departments,
A - index set of markets of the company,
B - index set of raw materials,
v_{j}^{t} - manufacturing capacity of department j in year t,
g_{ia}^{t} - forecasted sales for product i on market a in year t,
m_{ijz}^{t} - per unit consumption indicator of product i used for producing product z in department j in year t,
m_{bi}^{t} - per unit consumption indicator of raw material b used for producing product i in year t,

$\dddot{t}$ - economic life of project of project,
c_{ia}^{t} - selling price of product i in year t on market a,
c_{b}^{t} - price of raw material b in year t,
kz_{ij}^{t} - variable processing cost for product i manufactured in production department j in year t,
rk^t - interest rate of short-term credit in year t,
rd^t - interest rate of long-term credit in year t,
r_{dys} - discount rate,
kf^t - company's fixed costs without amortization in year t,
χ^t - value of amortization in year t.

The first group of equations includes:

- manufacturing capacities balance for production departments

$$\sum_{i \in I_j} \boldsymbol{Pr}_{ij}^{t} \leq v_j^t \text{ for } j \in J, t = 0,1,\cdots,\dddot{t} \tag{1}$$

- material balance:

$$\sum_{j \in J} \boldsymbol{Pr}_{ij}^{t} - \sum_{j \in J} \sum_{z \in I_j} m_{ijz}^{t} \boldsymbol{Pr}_{iz}^{t} = \sum_{a \in A} \boldsymbol{G}_{ia}^{t}, \text{ for } i \in I, \quad t = 0, 1, \cdots, \dddot{t}, \tag{2}$$

$$\boldsymbol{G}_{ia}^{t} \leq g_{ia}^{t} \text{ for } i \in I, \text{a} \in A, t = 0, 1, \cdots, \dddot{t}, \tag{3}$$

The Eq. (1) determines the quantity and structure of the production of each department in consecutive years of a project lifecycle. The Eq. (2) determines the distribution of production of specific products for sales and for internal production usage. The Eq. (3) is the limitations for the amount of sale of production of specific products sold.

The second set of equations consists of financial equations. These linear equations determine specific items of a company's balance sheet, P&L account and cash flows (NCF), which are used to calculate NPV. For example, the equation for a company's operating profit has the following form:

$$\boldsymbol{ZO}^{t} = \sum_{i \in I} \sum_{a \in A} c_{ia}^{t} \boldsymbol{G}_{ia}^{t} - \sum_{j \in J} \sum_{i \in I_j} kz_{ij}^{t} \boldsymbol{Pr}_{ij}^{t} - \sum_{b \in B} \sum_{j \in J} \sum_{i \in I_j} c_b^t m_{bi}^{t} \boldsymbol{Pr}_{ij}^{t} - \chi^t - \boldsymbol{kf}^t \tag{4}$$

For $t = 1, \ldots, \dddot{t}$.

$$NPV = \sum_{t=1}^{\overset{\dots}{t}} (ZO^t + \chi^t \pm ZKO^t) - IA - RV \tag{5}$$

In the Eq. (5), the variables ZO^t, χ^t, and ZKO^t are defined as increment in relation to the corresponding values without the investment project.

The remaining financial equations express commonly known dependencies. A detailed presentation of them would considerably increase the volume of the article, therefore, they are omitted.

In the proposed approach, it is assumed that some of the model parameters, $v_j^t, g_{ia}^t, m_{bi}^t, m_{ijz}^t, \overset{\dots}{t}, c_{ia}^t, c_{if}^t$, c_b^t, kz_{ij}^t, χ^t, kf^t, can be described by probability distributions (stochastic parameters), and some others by possibility distributions (fuzzy parameters). Additionally, it is assumed that there are dependencies (correlation) between certain parameters.

3 Procedure of Determining *NPV* and Risk

The proposed procedure of determining the effectiveness and risk combines stochastic simulation with interval regression and non-linear programming. The computational procedure is as follows. The values of stochastic parameters are drawn taking into account dependencies represented by a correlation matrix [2]. Next, each fuzzy parameter is represented by a finite family of α-cuts, and then, for each α-level, dependencies between the α-cuts are determined by using interval regression [12]. Hence, fuzzy parameters are replaced with real variables x_i and the following additional constraints are imposed on the model defined in Sect. 2:

$$\inf(\tilde{\boldsymbol{X}}_i)_\alpha \le x_i \le \sup(\tilde{\boldsymbol{X}}_i)_\alpha \tag{6}$$

$$\boldsymbol{x}_i \ge \inf(a_1^{iz}) \cdot \boldsymbol{x}_z + \inf(a_2^{iz}) \tag{7}$$

$$\boldsymbol{x}_i \le \sup(a_1^{iz}) \cdot \boldsymbol{x}_z + \sup(a_2^{iz}) \tag{8}$$

where:

$\inf(\tilde{\boldsymbol{X}}_i)_\alpha, \sup(\tilde{\boldsymbol{X}}_i)_\alpha$	- lower and upper bound of an α-cut of the fuzzy parameter $\tilde{\boldsymbol{X}}_i$,
$\sup(a_1^{iz}), \inf(a_1^{iz})$, $\sup(a_2^{iz}), \inf(a_2^{iz})$	- lower and upper bounds of interval regression coefficients describing the dependency between the fuzzy parameters $\tilde{\boldsymbol{X}}_z$ and $\tilde{\boldsymbol{X}}_i$

Algorithm 1. Method for evaluating the effectiveness and risk of investment projects

START

Step 1. $n = 1$ // set the number of the current iteration to 1

Step 2. Randomly generate a vector of probabilistic variables taking into account the dependencies given in the form of the correlation matrix

Step 3. $\alpha = 0$ // set the starting α-level

Step 4. Calculate α-cuts of the fuzzy parameters

Step 5. Calculate constraints (6)-(8) by using interval regression

Step 6. Solve the nonlinear programming problems (9), (10), s.t. constraints (1)-(8)

Step 7. $\alpha = \alpha + \wp$ // take next α-level

Step 8. If $\alpha \leq 1$ goto Step 4 else $n = n + 1$

Step 9. If $n \leq \ddot{n}$ goto Step 2 // $\ddot{n}$ - number of iterations

Step 10. Define the set possibility distributions ($\mu_1^{NPV}, \cdots, \mu_{\ddot{n}}^{NPV}$)

Step 11. Calculate a p-box for the random fuzzy number ($\mu_1^{NPV}, \cdots, \mu_{\ddot{n}}^{NPV}$)

STOP

Next, for each α-level the two following nonlinear programming problems are solved:

$$NPV_\alpha \rightarrow \min, \text{s.t.}(1)-(8), \tag{9}$$

$$NPV_\alpha \rightarrow \max, \text{s.t.}(1)-(8). \tag{10}$$

As a result a family of α-cuts $NPV_\alpha = [NPV_\alpha^{\min}, NPV_\alpha^{\max}]$ is obtained, which can be considered as a possibility distribution μ^{NPV}. The overall procedure is repeated $\dddot{n}$ times, and the set of possibility distributions $\left(\mu_1^{NPV}, \ldots, \mu_{\dddot{n}}^{NPV}\right)$, which can be considered as a random fuzzy set [15], is obtained. Based on this set, a p-box (probability box) is determined. It can be used to characterize uncertainty having both aleatory and epistemic nature. The details on computing a p-box can be found, e.g., in [2].

The hybrid procedure which implements the described approach is presented in Algorithm 1.

4 Numerical Example

The performance of the proposed method is verified on the example of a project of construction of sheet organic coating plant for metallurgical industry enterprise. The assessment of the effectiveness and the risk is performed for the production setup, which is presented in Fig. 1. This setup includes the four production departments (J) which need one raw material—continuous casting stand and produces four outputs. The value of v_j^t, which is put in Fig. 1 next to the department's name, is constant for each department.

In such investment projects, demand, sales prices, costs of materials and a value of investment outlays are potential sources of uncertainty. Table 1 shows fuzzy input parameters as trapezoidal possibility distributions and stochastic parameters as normal probability density functions specifying forecast of parameters for the year t = 0 of economic life of an investment project.

In order to simplify the computations, it is assumed that the apparent consumption of metallurgical products will increase by 1.5 % per year. Whereas, prices of products and material consumption indicator will be constant. It also assumed that production setup sells products on only one market (a = 1). The volume of investment outlays is assumed to be a triangular fuzzy number (42,000, 46,000, 50,000) given in thousands USD.

The value of the fixed cost kf^t is determined at the level of USD 315 090 thousand per year. The adjusted unit processing cost kz_{ij}^t for particular product

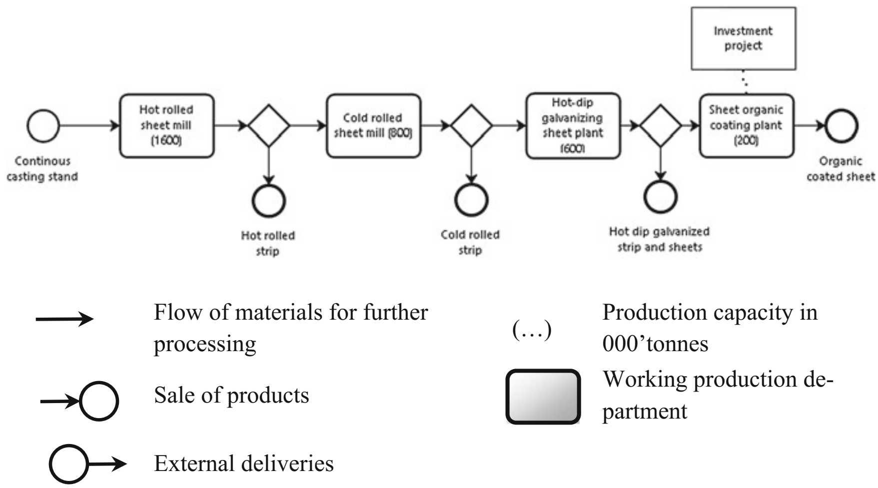

Fig. 1 Diagram of the analyzed production setup

Table 1 Trapezoidal fuzzy numbers representing forecasts of products and raw material prices, material consumption indicators and parameters of normal probability density function characterizing apparent consumption of metallurgical products

Purchasing c_i^t and selling prices c_i^t	Trapezoidal fuzzy numbers, USD/t
Continuous casting stands (purchase)	(440.3, 445.0, 450.7, 460.3)
Hot rolled strip (sell)	(666.7, 680.0, 711.7, 728.3)
Cold rolled sheets (sell)	(715.0, 730.0, 763.3, 781.7)
Hot dip galvanized strip and sheets (sell)	(805.0, 821.7, 860.0, 880.0)
Organic coated sheets (sell)	(1,080.0, 1,101.7, 1,153.3, 1,175.0)
Material consumption indicator m_{ijz}^t	Trapezoidal fuzzy numbers, tons/t
Continuous casting stands—hot rolled strip	(1,058, 1,064; 1,075, 1,078)
Hot rolled strip—cold rolled sheets	(1,105, 1,111, 1,124, 1,130)
Cold rolled sheets—hot dip galvanized strip and sheets	(1,010, 1,020, 1,026, 1,031)
Hot dip galvanized strip and sheets—organic coated sheets	(0,998, 0,999, 1.000, 1.001)
Apparent consumption g_i^t	(Average [thousand. tons]; standard deviation [thousand. tons])
Hot rolled strip	(2,704.0, 117.5)
Cold rolled sheets	(1,162.3, 51.4)
Hot dip galvanized strip and sheets	(1,147.9, 52.4)
Organic coated sheets	(708.4, 30.8)

Table 2 Unit processing cost for particular product ranges

Product	Hot rolled strip	Cold rolled sheets	Hot dip galvanized strip and sheets	Organic coated sheets
Adjusted unit variable processing cost, USD/tone	28.4	28.0	116.7	175.3

ranges is presented in Table 2. As soon as one department produces each product, unit processing cost may be simplified to four variables.

Table 3 presents coefficients of the interval regression equation characterizing relations between prices of product manufactured by analyzed producer and prices of continuous casting stands. Table 4 presents matrix of correlation of apparent consumption of particular product ranges manufactured by the producer.

Table 3 Interval regression equations coefficients depicting interrelations between prices of particular product ranges produced by the manufacturer in question prices of raw materials

		Dependent variable				
		Continuous casting strand v_1	Hot rolled strip v_2	Cold rolled sheets v_3	Hot dip galvanized strip and sheets v_4	Organic coated sheets v_5
v_1	a_1		[0.70, 0.88]	[0.39, 0.56]	[0.35, 0.45]	[0.31, 0.42]
	a_2		[3.66, 4.59]	[101.81,143.90]	[101.80,131.01]	[49.311, 66.675]
v_2	a_1	[1.05, 1.33]		[0.59, 0.79]	[0.48, 0.66]	[0.39, 0.66]
	a_2	[22.01,28.01]		[112.16,148.39]	[112.60,155.43]	[31.68, 53.31]
v_3	a_1	[0.49, 0.69]	[0.86, 1.40]		[0.753, 0.917]	[0.570, 0.955]
	a_2	[128.78,180.53]	[−10.59, −17.12]		[−2.17, −2.65]	[−95.30, −159.72]
v_4	a_1	[0.44, 0.56]	[0.96, 1.50]	[0.98, 1.22]		[0.59, 1.06]
	a_2	[127.51,162.70]	[45.66, 71.29]	[57.87, 72.08]		[−48.89, −87.59]
v_5	a_1	[0.34, 0.53]	[0.35, 0.86]	[0.80, 1.31]	[0.63, 1.12]	
	a_2	[69.20, 107.60]	[385.56, 941.97]	[248.17, 403.48]	[236.97, 420.88]	

Table 4 Correlation between the apparent consumption of metallurgical products manufactured by analyzed company

	Hot rolled strip	Cold rolled sheets	Hot dip galvanized strip and sheets	Organic coated sheets
Hot rolled strip	1.000	0.878	0.911	0.863
Cold rolled sheets	0.878	1.000	0.915	0.888
Hot dip galvanized strip and sheets	0.911	0.915	1.000	0.966
Organic coated sheets	0.863	0.888	0.966	1.000

Figure 2 compares lower and upper cumulative distribution functions for NPV_1 and NPV_2. (NPV_1 was calculated taking into account the correlation of the price and apparent consumption, and NPV_2 without taking this correlation into account).

The results indicate that the dependencies between the model parameters have a considerable impact on the *NPV* of a project.

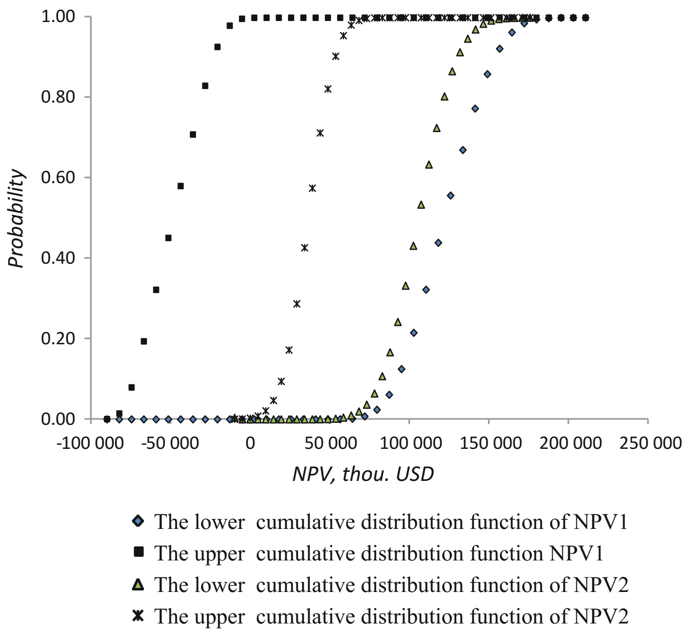

Fig. 2 Upper and lower cumulative distribution functions depicting *NPV1 and NPV2*

5 Conclusions

The new method for processing hybrid data is developed in this work. The proposed method equips decision-makers with a formal language for defining dependencies between uncertain parameters in decision making models. The method improves previous works concerning the processing of hybrid data. First of all this method allows to take into account dependencies between the fuzzy parameters. Moreover, it can be used in the case of arbitrary possibility distributions, since the calculations are performed on α-cuts. The method is flexible, since with minor changes it can be used to solve many other problems.

The obtained results indicate that the dependencies between the decision model parameters has a considerable impact on the estimated value of NPV. Omitting this dependencies can cause a considerable systematic error.

References

1. Baraldi, P., et al.: Monte Carlo and fuzzy interval propagation of hybrid uncertainties on a risk model for the design of a flood protection dike. Advances in Safety, Reliability and Risk Management: ESREL (2011)
2. Baudrit, C., Dubois, D., Guyonet, D.: Joint propagation and exploitation of probabilistic and possibilistic information in risk assessment. IEEE Trans. Fuzzy Syst. **14**, 593–607 (2006)

3. Buckley, J.: The fuzzy mathematics of finance. Fuzzy Sets Syst. **21**, 257–273 (1987)
4. Carmichael, D.: An alternative approach to capital investment appraisal. Eng. Econ. **56**(2), 123–139 (2011)
5. Carmichael, D., Balatbat, M.: Probabilistic DCF analysis and capital budgeting and investment—a survey. Eng. Econ. **53**(1), 84–102 (2008)
6. Choobineh, F., Behrens, A.: Use of intervals and possibility distribution in economic analysis. J. Oper. Res. Soc. **43**, 907–918 (1992)
7. Dimova, L., Sevastiyanov, D., Sevastiyanov, P.: Application of fuzzy sets theory, methods for the evaluation of investment efficiency parameters. Fuzzy Econ. Rev. **5**(1), 34–48 (2000)
8. Dubois, D., Guyonnet, D.: Risk-informed decision-making in the presence of epistemic uncertainty. Int. J. Gen Syst **40**(02), 145–167 (2011)
9. Ferson, S., Ginzburg, L.: Difference method are needed to propagate ignorance and variability. Reliab. Eng. Syst. Saf. **54**, 133–144 (1996)
10. Guyonnet, D., et al.: Hybrid approach for addressing uncertainty in risk assessment. J. Environ. Eng. **126**, 68–76 (2003)
11. Hiu, L., Peng, J., Shengguo, L.: Uncertain programming models for capital budgeting subject to experts' estimations. J. Intell. & Fuzzy Syst. **28**(2), 725–736 (2015)
12. Hladík, M., Černý, M.: Interval regression by tolerance analysis approach. Fuzzy Sets Syst. **193**, 85–107 (2012)
13. Kahraman, C., Ruan, D., Tolga, E.: Capital budgeting techniques using discounted fuzzy versus probabilistic cash flows. Inf. Sci. **142**, 57–76 (2002)
14. Kahraman, C., Tolga, E., Ulukan, Z.: Justification of manufacturing technologies using fuzzy benefit/cost ratio analysis. Int. J. Prod. Econ. **66**(1), 45–52 (2000)
15. Kaufmann, C., Gupta, M.: Introduction to Fuzzy Arithmetic: Theory and application. Van Nostrand Reinhold, New York (1985)
16. Kuchta, D.: Fuzzy capital budgeting. Fuzzy Sets Syst. **111**, 367–385 (2000)
17. Liu, B.: A survey of credibility theory. Fuzzy Optim. Decis. Mak. **5**, 387–408 (2006)
18. Rębiasz, B., Gaweł, B., Skalna, I.: Hybrid framework for investment project portfolio selection, In: 2014 Federated Conference on Computer Science and Information Systems (FedCSIS), pp. 1117–1122. IEEE (2014)
19. Skalna, I. et al.: Advances in Fuzzy Decision Making, pp. 75–77. Springer (2015)
20. Song, S., et al.: The uncertainty importance measures of the structural system in view of mixed uncertain variables. Fuzzy Sets Syst. **243**, 25–35 (2014)
21. Ward, T:, Discounted fuzzy cash flows analysis, In: Proceedings of the 1985 Fall Industrial Engineering Conference, pp. 476–481 (1985)

http://www.springer.com/978-3-319-46591-3

Information Systems Architecture and Technology: Proceedings of 37th International Conference on Information Systems Architecture and Technology – ISAT 2016 – Part IV

Wilimowska, Z.; Borzemski, L.; Grzech, A.; Świątek, J. (Eds.)

2017, XV, 304 p. 83 illus., 38 illus. in color., Softcover
ISBN: 978-3-319-46591-3