

An Overview of Granular Computing Using Fuzzy Logic Systems

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Abstract As Granular Computing has gained interest, more research has lead into using different representations for Information Granules, i.e., rough sets, intervals, quotient space, fuzzy sets; where each representation offers different approaches to information granulation. These different representations have given more flexibility to what information granulation can achieve. In this overview paper, the focus is only on journal papers where Granular Computing is studied when fuzzy logic systems are used, covering research done with Type-1 Fuzzy Logic Systems, Interval Type-2 Fuzzy Logic Systems, as well as the usage of general concepts of Fuzzy Systems.

Keywords Granular · Fuzzy logic · Fuzzy systems · Type-2 fuzzy sets

1 Introduction

With an inspiration on the human cognitive process, where personal experience adds to choosing the best level of abstraction which can achieve a better decision; Granular Computing (GrC) [1] uses numerical evidence to set the best possible level of resolution for achieving a good decision model. GrC works on existing models, or modeling methods, by adding an abstraction layer based on its inspiration of human cognition and with such modifications obtains better models which can lead to improved model performance as well an improved interpretation of its parts, viz. Information Granules.

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For a GrC model to represent anything a medium is necessary. This medium is called an Information Granule, which is a grouping of similar information taken from some type of previous experimental evidence. These Information Granules can have any type of representation, e.g. intervals, rough sets [2], fuzzy sets [3], fuzzy rough sets [4], quotient space [5], intuitionistic fuzzy sets [6], neutrosophic sets [7], among others. Although each one has their own research surrounding their GrC-based implementations, as well as having their own strengths and weaknesses, in the end, choosing a representation is a matter of choice. Information Granules in the form of fuzzy sets (FS) have been chosen for the survey in this paper, since they have a high maturity level where they can be expertly designed or be created by an algorithm based on experimental evidence, they can be inferred via a fuzzy logic system (FLS) where a group of FS which form different rules in the IF...THEN format process inputs into outputs which ultimately represent a given model of information, knowledge, or behavior. Also, fuzzy sets have two basic trends in complexity which have been used with GrC: Type-1 Fuzzy Sets (T1 FS), and Interval Type-2 Fuzzy Sets (IT2 FS). Although a third trend exists, which are Generalized Type-2 Fuzzy Sets (GT2 FS), but no known research exists in journal form where they are used with GrC, hence its omittance.

In this paper, a survey was made which shortly examines all journal publications where GrC is used when Information Granules are represented by FS. For this case, the following combinations of keywords were searched for granular computing, information granule, fuzzy, type-1, and type-2.

This paper is divided into four main sections, first, a quick background look into T1 FLS and IT2 FLS; then, the general overview is presented, separated into research publication done with T1 FLS, IT2 FLS, and other fuzzy concepts; afterwards, some trends in current publications are shown; finally, some concluding remarks are added.

2 Fuzzy Logic Systems

With traditional logic, available values are $\{1,0\}$; with three-valued logic, available values are $\{1,0,1\}$, or other variations, e.g., $\{0,1,2\}$; whereas in the case of Fuzzy Logic (FL), available values are between $[0,1]$. This interval leads to the possibility expressing ambiguities, or imprecision, when dealing with decision making situations as no longer only did two or three choices exist, but an infinite of possible choices between the said intervals.

A FLS transforms inputs into outputs via its inference. This fuzzy inference system comprises of rules, which are defined by various membership functions. This inference gives the possibility of applying an FLS in any area where a decision might be required. an FLS can be manually or automatically designed; manual design [8–10] is usually done by experts where they choose the best rule description and distribution of individual membership functions which best describe the problem to be solved, on the other hand, automatic design [11–13] is based on

assessing the best rule description and membership functions based on previously obtained experimental data.

As previously stated, information granules can be represented by Fuzzy Sets. These fuzzy information granules can make use of the fuzzy inference's capabilities and transform inputs into outputs. This precedence has given Fuzzy Information Granules much attention, such that this paper is solely focused on implementations of Information Granules using FLSs.

2.1 Type-1 Fuzzy Logic Systems

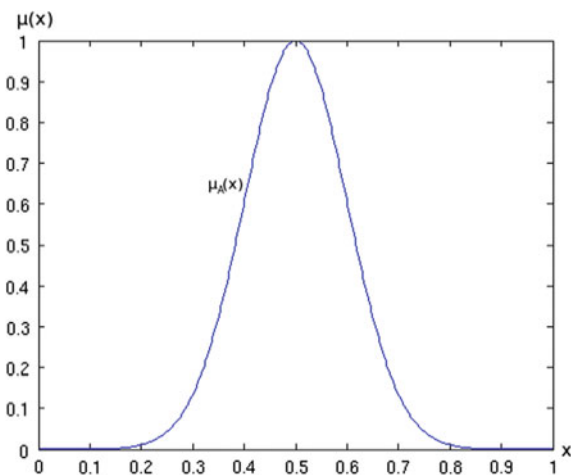
A Type-1 Fuzzy Logic System (T1 FLS) has membership functions which only represent a type of imprecision, or ambiguity. Shown in Fig. 1, is a sample generic Gaussian membership function. A Type-1 Fuzzy Set A , denoted by $\mu_A(x)$, where $x \in X$ and X is the Universe Of Discourse. Represented by $A = \{x, \mu_A(x) | x \in X\}$, this being an FS which takes on values in the interval $[0,1]$.

The rules for a T1 FLS take the form of Eq. (1), where a mapping exists from input space into output space. Where R^l is a rule, x_p is an input p , F_p^l is an input membership function on the l th rule and p th input; y is the output on membership function G^l in the l th rule. Here, F and G take on the form of $\mu_F(x)$ and $\mu_G(x)$.

$$R^l: \text{IF } x_1 \text{ is } F_1^l \text{ and } \dots \text{ and } x_p \text{ is } F_p^l, \text{ THEN } y \text{ is } G^l; \quad l = 1, \dots, M \quad (1)$$

The inference is first performed rule-wise by Eq. (2) when done with t-norm connectors ($\tilde{*}$). Where μ_{B^l} is the resulting membership function in the consequents per each rule's inference, and Y is the space belonging to the consequents.

Fig. 1 Sample type-1 fuzzy membership function



$$\mu_{B^l}(y) = \mu_{G^l}(y) \tilde{*} \left\{ \left[\sup_{x_1 \in X_1} \mu_{x_1}(x_1) \tilde{*} \mu_{F_1^l}(x_1) \right] \right. \\ \left. \tilde{*} \dots \tilde{*} \left[\sup_{x_p \in X_p} \mu_{x_p}(x_p) \tilde{*} \mu_{F_p^l}(x_p) \right] \right\}, \quad y \in Y \quad (2)$$

The defuzzification process can be done in various ways, all achieving a very similar result, for examples: centroid, center-of-sums, or height, described by Eqs. (3), (4), and (5), respectively. Where y_i is a discrete position from Y , $y_i \in Y$, $\mu_B(y)$ is a FS which has been mapped from the inputs, c_{B^l} denotes the centroid on the l th output, a_{B^l} is the area of the set, and $\bar{y}^l$ is the point which has the maximum membership value in the l th output set.

$$y_c(x) = \frac{\sum_{i=1}^N y_i \mu_B(y_i)}{\sum_{i=1}^N \mu_B(y_i)} \quad (3)$$

$$y_a(x) = \frac{\sum_{l=1}^M c_{B^l} a_{B^l}}{\sum_{l=1}^M a_{B^l}} \quad (4)$$

$$y_h(x) = \frac{\sum_{l=1}^M \bar{y}^l \mu_{B^l}(\bar{y}^l)}{\sum_{l=1}^M \mu_{B^l}(\bar{y}^l)} \quad (5)$$

2.2 Interval Type-2 Fuzzy Logic Systems

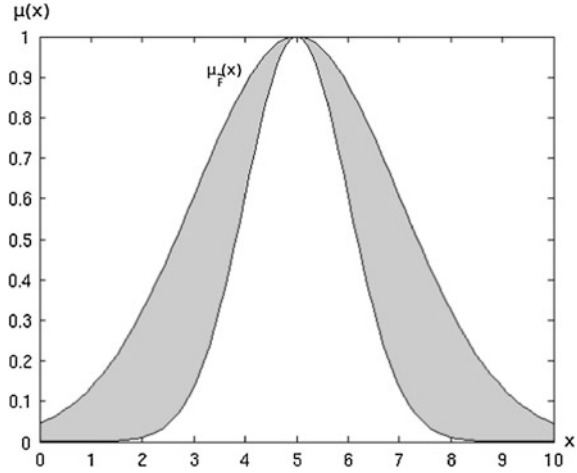
An Interval Type-2 Fuzzy Logic System (IT2 FLS) has membership functions which can represent varying degrees of uncertainty. Shown in Fig. 2, is a sample generic Gaussian membership function with uncertain standard deviation. An Interval Type-2 Fuzzy Set $\tilde{A}$ is denoted by $\mu_{\tilde{A}}(x) = [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)]$, where $\underline{\mu}_{\tilde{A}}(x)$ and $\bar{\mu}_{\tilde{A}}(x)$ are the lower and upper membership function which conform the IT2 FS. Represented by $\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u) = 1) | \forall x \in X, \forall u \in [0, 1]\}$, where x is a subset of the Universe Of Discourse X , and u is a mapping of X into $[0, 1]$.

The rule's format for an IT2 FLS is the same as that of a T1 FLS, as shown in Eq. (6). When inferred upon, it maps from an input space into an output space.

$$R^l: \text{IF } x_1 \text{ is } \tilde{F}_1^l \text{ and } \dots \text{ and } x_p \text{ is } \tilde{F}_p^l, \text{ THEN } y \text{ is } \tilde{G}^l; \quad l = 1, \dots, M \quad (6)$$

The Fuzzy Inference from an IT2 FLS is best represented by Eq. (7). Where μ_{B^l} is the resulting membership function in the consequents per each rule's inference, and Y is the space belonging to the consequents.

Fig. 2 Sample interval type-2 fuzzy membership function



$$\mu_{\tilde{B}}(y) = \left\{ \left[\sup_{x_1 \in X_1} \mu_{X_1}(x_1) \mu_{F_1^l}(x_1) \right] \right. \\ \left. \tilde{*} \cdots \tilde{*} \left[\sup_{x_p \in X_p} \mu_{X_p}(x_p) \mu_{F_p^l}(x_p) \right] \right\} \tilde{*} \mu_{G^l}(y), \quad y \in Y \quad (7)$$

The type reducer of an IT2 FLS can be done in various ways, one of these techniques is that of center-of-sets (cos), as shown in Eq. (8), where $Y_{\cos}$ is an interval set defined by two points $[y_l, y_r]$, which are obtained from Eqs. (9) to (10).

$$Y_{\cos}(x) = \int_{y^l \in [y_l^i, y_r^i]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \int_{f^1 \in [\underline{f}^1, \bar{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \bar{f}^M]} 1 / \frac{\sum_{i=1}^M f^i y^i}{\sum_{i=1}^M f^i} \quad (8)$$

$$y_l = \frac{\sum_{i=1}^M f_l^i y_l^i}{\sum_{i=1}^M f_l^i} \quad (9)$$

$$y_r = \frac{\sum_{i=1}^M f_r^i y_r^i}{\sum_{i=1}^M f_r^i} \quad (10)$$

Finally, the defuzzification process is done by Eq. (11), which obtains a crisp output value.

$$y(x) = \frac{y_l + y_r}{2} \quad (11)$$

3 Overview of Fuzzy Information Granule Research

Overviews have been separated into three categories: where T1 FLSs are used with granular concepts, where IT2 FLSs are used with granular concepts, and where general Fuzzy concepts are used with granular concepts.

3.1 *Fuzzy Information Granules Using Type-1 Fuzzy Logic Systems*

The following overviews show research done in the area of Granular Computing using Fuzzy Information Granules constructed from T1 FSs and inferred via an FLS.

In Pedrycz et al. [14] multiple fuzzy models are constructed by separate locally available data, in which later on a global model is formed at a higher level of hierarchy, therefore obtaining a granular fuzzy model. The principle of justifiable granularity is used throughout to show how granular parameters of the models are formed.

In Pedrycz and Homenda [15] the principle of justifiable granularity is introduced, which supports the creation of information granules by optimizing a design process where experimental data is balanced with granule specificity.

In Pedrycz [16] multiple items are revised. First, a goal of design concerned with revealing and representing structure in a data set is oriented toward the underlying relational aspects of experimental data; second, a goal that deals with their formation of quality mappings which is directly affected by the information granules over which is operates. A fuzzy clustering is used in order to obtain fuzzy models, in which a generalized version of the FCM is proposed that has an additive form of the objective function with a modifiable component of collaborative activities.

In Pedrycz et al. [17] an augmented FCM is proposed which models with the purpose of a balance between the structural content of the inputs and output spaces, with Mamdani and Takagi-Sugeno fuzzy rules. A direct link between the development of fuzzy models and the idea of granulation-degranulation is also studied.

In Pedrycz et al. [18] a fuzzy clustering scheme is proposed which uses viewpoints to represent domain knowledge. Where the user introduced a point of view of the data, treated as prototypes, the clustering takes place using these viewpoints and constructs its fuzzy model.

In Bargiela et al. [19], they define a granular mapping over information granules and are mapped into a collection of granules expressed in an output space. This is done by first defining an interaction between information granules and experimental data, and then using these measures of interaction in the expression of a granular mapping.

In Balamash et al. [20] they formulate information granules as an optimization task where selected information granules are refined into a family of more detailed

information granules, that way the partition requirement becomes satisfied. The Conditional FCM is used as a clustering method for information granulation.

In Wang et al. [21] a modified FCM is used in conjunction with information granulation in order to solve the problem of time series long-term prediction, obtaining more meaningful and semantically sound entities in the form of information granules that is interpretable and easily comprehended by humans.

In Lu et al. [22] a numeric time series is segmented into a collection of time windows for which each one is built into a fuzzy granule exploiting the principle of justifiable granularity. Subsequently, a granular model is reached by mining logical relationships of adjacent granules.

In Pedrycz et al. [23] a general study is made on several algorithmic schemes associated with Granular Computing. A study of the implications of using the principle of justifiable granularity and its benefits is made. The overall quality of granular models is also evaluated via the determination of the area under curve, where this curve is formed in the coverage-specificity coordinates. Various numerical results are discussed which display the essential features of each algorithmic scheme.

In Pedrycz et al. [24] a granular data description is proposed which characterizes numeric data via a collection of information granules, such that the key structure of the data, its topology and essential relationships are described in the form of a family of fuzzy information granules. An FCM clustering algorithm is used as a granulation method followed by the implementation of the principle of justifiable granularity as to obtain meaningful information granules.

In Rubio et al. [25] through the use of Neutrosophic Logic, which extends the concepts of Fuzzy Logic and uses a three-valued logic that uses an indeterminacy value, granular neural-fuzzy models are created. This leads to more meaningful and simpler granular models with better generalization performance when compared to other recent modeling techniques.

In Yu et al. [26] a fundamental issue of the design of meaningful information granules is covered. This issue is that of the tradeoff between the specificity of an information granule and its experimental evidence. For this, an optimization task is proposed for which it discusses its solution and properties.

In Pedrycz et al. [27] a concept of granular representation of numeric membership functions of fuzzy sets is introduced. It gives a synthetic and qualitative view of fuzzy sets and their processing. An optimization task is also introduced alongside, which refers to a notion of consistency of a granular representation. This consistency is also related to several operations, such as *and* and *or* consistency of granular representations, implemented by t-norms and t-conorms.

In Panoutsos et al. [28] a systematic modeling approach using granular computing and neuro-fuzzy modeling is presented. A granular algorithm extracts relational information and data characteristics from an initial set of information, which are then converted into a linguistic rule-base of a fuzzy system. Afterwards it is elicited and optimized.

In Zadeh [29] a theoretic proposal of fuzzy information granulation based on human reasoning is shown. The idea is centered on the logic of fuzzy boundaries

between granules when their sum is tightly packed, such that crisp information granules cannot depict a better description, but fuzzy granules can. Later on, various principal types of granules are proposed: possibilistic, veristic and probabilistic.

In Pedrycz et al. [30] a mechanism for fuzzy feature selection is introduced, which considers features to be granular rather than numeric. By varying the level of granularity, the contribution level of each feature is modified with respect to the overall feature space. This is done by transforming feature values into intervals, they consider broader intervals to be less essential to the full feature space. The quantification of the importance of each feature is acquired via an FCM model.

In Kang et al. [31] granular computing is introduced into formal concept analysis by providing a unified model for concept lattice building and rule extraction on a fuzzy granularity base. They present that maximal rules are complete and nonredundant, and users who want to obtain decision rules should generate maximal rules.

In Park et al. [32] a fuzzy inference system based on information granulation and genetic optimization is proposed. It focuses on capturing essential relationships between information granules rather than concentrating on plain numeric data. It uses an FCM to build an initial fuzzy model which then using a genetic algorithm optimizes the apex of the fuzzy sets in respect to its structure and parameters.

In Choi [33] a methodology for information granulation-based genetically optimized fuzzy inference system is proposed which has a tuning method with a variant identification ratio for structural and parametric optimization of the reasoning system. This tuning is done via a mechanism of information granulation. The structural optimization of the granular model is done via an FCM algorithm, afterward the genetic algorithm takes over in order to optimize the granular model.

In Novák [34] the concept that a granule can be modeled in formal fuzzy logic is introduced. It proposes the concepts of intension and extension of granules, internal and external operations, various kinds of relations, motion of granules, among other concepts. And proposes a more complex concept of association which is formed from granules.

In Bargiela et al. [35] a model of granular data emerging through a summarization and processing of numeric data is proposed, where the model supports data analysis and contributes to further interpretation activities. An FCM is used to reveal the structure in the data, where a Tchebyshev metric is used to promote the development of easily interpretable information granules. A deformation effect of the hyperbox geometry of granules is discussed; where they show that this deformation can be quantified. Finally, a two-level topology of information granules is also shown where the core of the topology is in the form of hyperbox information granules.

In Pedrycz et al. [36] a concept of granular mappings is introduced which offer a design framework, which is focused on forming conceptually meaningful information granules. The directional nature of the mapping is also considered during the granulation process. An FCM algorithm is used to facilitate the granulation of the input space, and the principle of justifiable granularity is used to construct the information granules in the output space.

In Pedrycz et al. [37] the concept of granular prototypes which generalize the numeric representation of clusters is shown, where more details about the data structure are captured through this concept. The FCM algorithm is used as a granulation method. An optimization of the fuzzy information granules is guided by the coverage criterion. Various optimal allocations of information granularity schemes are researched.

In Pedrycz [38] an idea of granular models is introduced, where generalizations of numeric models that are formed as a result of an optimal allocation of information granularity. A discussion is also made of a set of associated information granularity distribution protocols.

In Fazzolari et al. [39] a fuzzy discretization for the generation of automatic granularities and their associated fuzzy partitions is proposed, to by a multi-objective fuzzy association rule-based classification method which performs a tuning on the rule selection.

In Roh et al. [40] a granular fuzzy classifier based on the concept of information granularity is proposed. This classifier constructs information granules reflective of the geometry of patterns belonging to individual classes; this is done efficiently by improving the class homogeneity from the original information granules and by using a weighting scheme as an aggregation mechanism.

In Lu et al. [41] a method is proposed that partitions an interval information granule-based fuzzy time series for improving its model accuracy. Where predefined interval sizes partition the time series such that information granules are constructed from each interval's data, these constructions takes place from the amplitude-change space belonging to the corresponding trends in the time series. These information granules are continually adjusted to better associate corresponding and adjacent intervals.

In Dick et al. [42] information is translated between two granular worlds via a fuzzy rulebase, where each granular world employs a different granulations, but exist in the same universe of discourse. Translation is done by re-granulating and matching the information into the target world. A first-order interpolation implemented using linguistic arithmetic is used for this purpose.

In Lu et al. [43] a modeling and prediction approach of time series is proposed, based on high-order fuzzy cognitive maps (HFCM) and fuzzy c-means (FCM) algorithms, where the FCM constructs the information granules by transforming the original time series into a granular time series and then generates an HFCM prediction model. A PSO algorithm is used to tune all parameters.

In Sanchez et al. [44] a clustering algorithm based on Newton's law of gravitation is proposed, where the size of each fuzzy information granule is adapted to the context of its numerical evidence. Where this numerical evidence is influenced by gravitational forces during the grouping of information.

In Pedrycz et al. [45] a collaborative fuzzy clustering concept is introduced which has two fundamental optimization features. First, all communication is done via information granules; second, the fuzzy clustering done for individual data must take into account the communication done by the information granules.

In Dong et al. [46] a granular time series concept is introduced which is used for long-term and trend forecasting. A fuzzy clustering technique is used to construct the initial granules afterward a granular modeling scheme is used to construct a better forecasting model, this conforms a time-domain granulation scheme.

In Oh et al. [47] the concept of information granulation and genetic optimization is used a design procedure for a fuzzy system. A c-means clustering is used for initial granulation, where the apex of all granules are found. Afterward a genetic algorithm and a least square method tunes the rest of the fuzzy parameters.

In Pedrycz et al. [48] given a window of granulation, an algorithm is proposed which constructs fuzzy information granules. All fuzzy information granules are legitimized in terms of experimental data while their specificity is also maximized.

In Pedrycz [49] a dynamic data granulation concept done in the presence of incoming data organized in data snapshots is introduced, where each snapshot reveals a structure via fuzzy clustering. An FCM is used as granulation method; information granular splitting is later done by a Conditional FCM to merge two or more neighboring prototypes.

In Dai et al. [50] a fuzzy granular computing method is used to characterize signal dynamics within a time domain of a laser dynamic speckle phenomenon, where sequences of intensity images are obtained in order to evaluate the dynamics of the phenomena. The sequence of characterization is used to identify the underlying activity in each pixel of the intensity images.

In Pedrycz et al. [51] a scheme of risk assessment is introduced which is based on classification results from experimental data from the history of previous threat cases. The granulation process is done via a fuzzy clustering algorithm which reveals an initial structural relationship between information granules. Two criteria are also introduced for the evaluation of information granules, representation capabilities and interpretation aspects.

In Fig. 3, the total number of publications in the category of granular computing research done with T1 FLS is shown, although the amount of research was stable from 1997 to 2005 a temporary increasing trend in research is seen from 2005 to 2011, and again increases in recent years starting from 2012. The total amount of publications done in this area has been 38 research papers for the 1997–2015 time period.

3.2 Higher Type Fuzzy Information Granules Using Interval Type-2 Fuzzy Logic Systems

The following overviews show research done in the area of Granular Computing using Higher Type Fuzzy Information Granules constructed from IT2 FSs and inferred via an FLS.

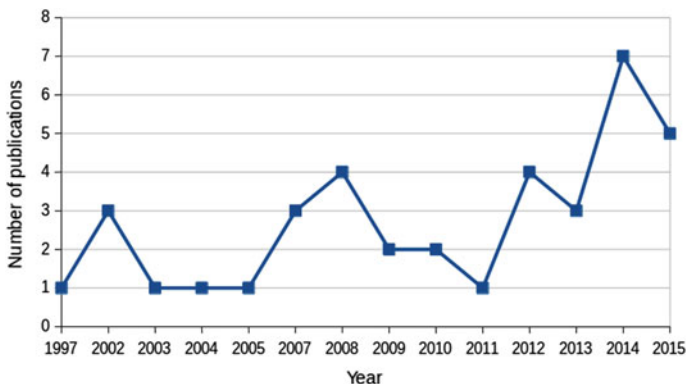


Fig. 3 Total publications per year on granular computing research done with T1 FLS, for the 1997–2015 time period

In Ulu et al. [52] an approach for the formation of Interval Type-2 membership functions is proposed, where the Footprint Of Uncertainty is formed by using rectangular Type-2 fuzzy information granules, providing more design flexibility in Interval Type-2 Fuzzy Logic System design.

In Sanchez et al. [53] a technique for the formation of Higher Type information granules represented by Interval Type-2 membership functions is proposed, which is based on the concept of uncertainty-based information, whereby measuring the difference between information granule samples the Footprint Of Uncertainty is extracted.

In Pedrycz et al. [54] the concept of granular fuzzy decision built on a basis formed by individual decision models is introduced. The principle of justifiable granularity is used to improve the holistic nature of a granular fuzzy decision, alongside an established feedback loop in which the holistic view is adjusted to the individual decision. Various optimization schemes are also discussed along with multiple examples of forming Type-2 and Type-3 Fuzzy Sets.

In Pedrycz [55] the capabilities of Interval Type-2 Fuzzy Sets as Higher Type information granules is discussed. Seen as aggregation of Fuzzy Sets, a framework is proposed which leads to the construction of Interval Type-2 Fuzzy Sets. The principle of justifiable granularity is used as a means to quantify the diversity of membership degrees.

In Fig. 4, the total number of publications in the category of granular computing research done with IT2 FLS is shown, in comparison to the previous research category of using T1 FLS, research done with IT2 FLS is still very lacking, especially since only four research papers can be found for the 2009–2015 time period.

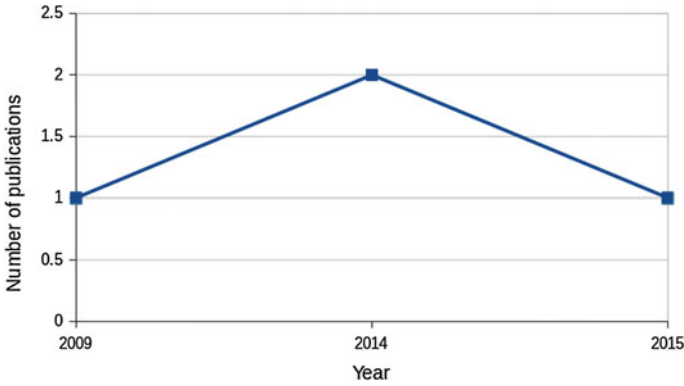


Fig. 4 Total publications per year on granular computing research done with IT2 FLS, for the 2009–2015 time period

3.3 Other Fuzzy Concepts Used in Granular Computing

The following overviews show research done in the area of Granular Computing using various concepts taken from Fuzzy Logic in general, fuzzy hybrid schemes, etc.

In Rubio-Solis et al. [56] a modeling framework for an Interval Type-2 radial basis function neural network is proposed. The functional equivalence of Interval Type-2 Fuzzy Logic Systems to radial basis neural networks is used as a modeling structure. The linguistic uncertainty is used as an advantage in the proposed Interval Type-2 radial basis function neural network. A Granular Computing approach is used as a means to find the Footprint of Uncertainty.

In Nandedkar et al. [57] a granular neural network which can learn to classify granular data is proposed, where granular data is represented by a Hyperbox Fuzzy Set. The proposed granular neural network is capable of learning online using granular or point data.

In Qian et al. [58] a viewpoint is studied which looks to answer the question: “what is the essence of measuring the fuzzy information granularity of a fuzzy granular structure.” A binary granular computing model is used to study and demonstrate such viewpoint, altogether five aspects are studied: fuzzy binary granular structure operators, partial order relations, measures for fuzzy information granularity, an axiomatic approach to fuzzy information granularity, and fuzzy information entropies.

In Kundu et al. [59] a fuzzy granular framework for representing social networks is developed. Based on granular computing theory and fuzzy neighborhood systems which provide the framework where fuzzy sets provide the representation of Information granules. New measures are also proposed alongside the provided framework: granular degree, granular betweenness, and granular clustering

coefficient of a node, granular embeddedness of a pair of nodes, granular clustering coefficient and granular modularity if a fuzzy granular social network.

In Dick et al. [60] a granular neural network which uses granular values and operations on each neuron is proposed. It is based on a multilayer perceptron architecture, uses fuzzy linguistic terms as neuron connection weights, and has a backpropagation learning algorithm.

In Mencar et al. [61] a survey is provided where they present multiple interpretability constraints for the generation of fuzzy information granules. Their intention is to provide a homogeneous description of all interpretability constants, as well as their respective overviews, and to identify potential different interpretation meanings.

In Pedrycz et al. [62] granular neural networks are introduced. By applying data granulation technique, various aspects of training neural networks with large data sets is addressed. Multiple granular neural network architectures are provided as well as their training scenarios. Several techniques to the design of information granules are also discussed.

In Leite et al. [63] a granular neural network framework is used for evolving fuzzy systems from fuzzy data streams is introduced. This evolving granular neural network obtains local models that are interpretable using fuzzy granular models, fuzzy aggregation neurons, and an online incremental learning algorithm.

In Pal et al. [64] a rough-fuzzy classifier based on granular computing is presented, where class-dependent information granules exist in a fuzzy environment. Feature representation belonging to different classes is done via fuzzy membership functions. The local, and contextual, information from neighbored granules is explored through a selection of rough sets of a subset of granulated features.

In Liu et al. [65] a fuzzy lattice granular computing classification algorithm is proposed, where a fuzzy inclusion relation is computed between information granules by means of a inclusion measure function based on a nonlinear positive valuation function and an isomorphic mapping between lattices.

In Pedrycz [66] a discussion on various research trends regarding Granular Computing is shown, focusing mainly in fuzzy regressions and its future developments, granular fuzzy models, higher order and highertype granular constructs.

In Ganivada et al. [67] a fuzzy rough granular neural network is proposed, based on the multilayer perceptron using backpropagation training for tuning of the network. With an input vector defined by fuzzy information granules and a target vector defined by fuzzy class membership values. The granularity factor is incorporated into the input level and is also used for determining the weights of the proposed network.

In Kaburlasos et al. [68] a fuzzy lattice reasoning classifier which works in granular data domain of fuzzy interval numbers, fuzzy numbers, intervals, and cumulative distribution functions is introduced. The classifier integrates techniques for dealing with imprecision in practice.

In Oh et al. [69] a granular oriented self-oriented hybrid fuzzy polynomial neural network is introduced, based on multilayer perceptrons with polynomial neurons or context-based polynomial neurons. Where a context-based FCM algorithm is used

for granulating numeric data for the context-based polynomials neurons, executed in input space.

In Bodjanova [70] axiomatic definitions of granular nonspecificity are proposed. Various approaches to measures of granular nonspecificity are suggested. Other measures are discussed, such as Pawlak's measure, Banerjee and Pal's measure, and Huynh and Nakamori's measure. Furthermore, relationships between roughness, granular nonspecificity, and nonspecificity of a fuzzy set are also discussed.

In Kundu et al. [71] the identification of fuzzy rough communities done via a community detection algorithm is introduced, it proposes that a node with different memberships of their association can be part of many groups and that the granule's node decreases as its distances itself from the granule's center.

In Pedrycz et al. [72] an architecture of functional radial basis function neural network is proposed. One of its core functions is its use of experimental data through a process of information granulation, these granules constructed by a context-driven FCM algorithm. The model values of the fuzzy sets of contexts and the prototypes of the fuzzy clusters from input space are a direct functional character of the proposed network.

In Pal [73] a general overview of data mining and knowledge discovery is shown from a point of view of soft computing techniques, such as computational theory of perceptions, fuzzy granulation in machine and human intelligence, and modeling via rough-fuzzy integration.

In Pedrycz et al. [74] a feature selection based on structure retention concept is proposed, which is quantified by a reconstruction criterion. A combinatorial selection reduced the feature space, where it is used for optimizing the retention of the original structure. An FCM algorithm is used as a granulation technique, and a genetic algorithm optimizes the medium.

In Li et al. [75] a measure for finding the similarity between two linguistic fuzzy rulebases is introduced, based on the concept of granular computing based on linguistic gradients. The linguistic gradient operator is used for the comparison of both linguistic structures.

In Ghaffari et al. [76] an overall flowchart based on information granulation used in the design of rock engineering flowcharts is proposed. Granulation is done by self-organizing neuro-fuzzy inference systems and by self-organizing rough set theory, knowledge rules are then extracted from both sets of granules, crisp granules, and fuzzy rough granules, respectively.

In Gacek [77] a general study is shown of the principles, algorithms and practices of Computational Intelligence and how they are used in signal processing and time series. Although focus is not solely on the scheme of Granular Computing, it is shown in the form of fuzzy granules, where in this regard, fuzzy information granules are found to be excellent in facilitating interpretation while also maintaining a solid modeling base.

In Pedrycz [78] the conceptual role of information granules in system modeling and knowledge management when dealing with collections of models is shown. Various algorithmic schemes are shown for this purpose, among which is the use of fuzzy sets expressed as fuzzy information granules.

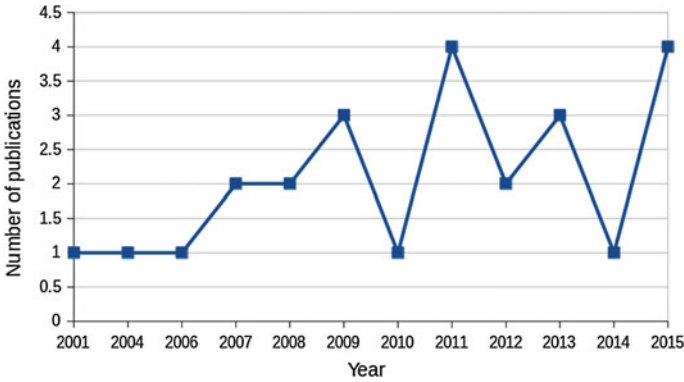


Fig. 5 Total publications per year on granular computing research done with other fuzzy concepts, for the 2001–2015 time period

In Pedrycz et al. [79] fuzzy proximity matrices induced by corresponding partition matrices are introduced, used as an optimization scheme for the variability of granular sources of knowledge. This knowledge networks offers a conceptual framework focused on knowledge aggregation, consensus building, and association building. An FCM algorithm is used as a means of constructing information granules.

In Degang et al. [80] a theory of granular computing based on fuzzy relations is proposed. The concept of granular fuzzy sets based on fuzzy similarity relations is used to describe granular structures of lower and upper approximations of a fuzzy set. Fuzzy rough sets can also be characterized via the proposed fuzzy approximations.

In Fig. 5, the total number of publications in the category of granular computing research done with other Fuzzy concepts is shown; from 2006 to 2015 research has varied in trend. The total amount of publications done in this area has been 25 research papers for the 2001–2015 time period.

4 Trends in Research

The general area of Granular Computing used with fuzzy systems has been steadily increasing in recent years. In Fig. 6, this trend is shown where from 1997 to 2006 the amount of research was fairly steady and minimal, yet after 2006 a steady increase in research exists up to 2015. The total amount of publications done in this area of Fuzzy Granular Computing has been 67 research papers for the 1997–2015 time period.

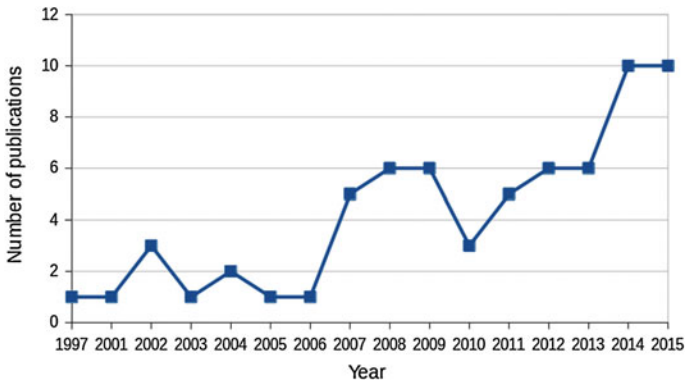


Fig. 6 Total publications per year on granular computing research done with fuzzy logic systems in general, for the 1997–2015 time period

Author-wise statistics show that a total of 97 different authors have participated in publishing papers in the area of Fuzzy Granular Computing, and as such, Table 1 shows a partial list of 21 authors whose names appear more often (2 or more papers), with the most prominent author being Dr. Witold Pedrycz as he surpasses appearance in papers by a considerable amount, 40 research papers from the 67 which were examined. Another trend worth mentioning is that only these 21 authors are recurring researchers in this area. The other 76 unlisted authors have appeared only once on all overviewed research papers.

Table 1 Author appearance in research papers related to granular computing using fuzzy systems in general

Author's name	Number of publications author has appeared in
Witold Pedrycz	40
Sankar K. Pal	5
Sung-Kwun Oh	5
Abdullah Saeed Balamash	4
Ali Morfeq	4
Rami Al-Hmouz	4
Xiaodong Liu	4
George Panoutsos	3
Jianhua Yang	3
Scott Dick	3
Wei Lu	3
Andrzej Bargiela	3
Adam Gacek	2
George Vukovich	2

(continued)

Table 1 (continued)

Author's name	Number of publications author has appeared in
Juan R. Castro	2
Kaoru Hirota	2
Keon-Jun Park	2
Mauricio A. Sanchez	2
Oscar Castillo	2
Soumitra Dutta	2
Suman Kundu	2

As an interesting note, the most widely used mechanism for granulating information is the Fuzzy C-Means and its variations.

5 Conclusion

This paper shows that an increasing trend in research of Granular Computing using Fuzzy Systems exists, be it using Type-1 Fuzzy Systems, Interval type-2 Fuzzy Systems, or any other Fuzzy concepts.

With the oldest research publication done in 1997 and considering that so far only 67 journal research papers exist, and that so far only 97 authors have works in this area, this area is still very alluring as many ideas and concepts can still be explored.

Acknowledgments We thank the MyDCI program of the Division of Graduate Studies and Research, UABC, and Tijuana Institute of Technology the financial support provided by our sponsor CONACYT contract grant number: 314258.

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Nature-Inspired Design of Hybrid Intelligent Systems

Melin, P.; Castillo, O.; Kacprzyk, J. (Eds.)

2017, XII, 838 p. 390 illus., 258 illus. in color.,

Hardcover

ISBN: 978-3-319-47053-5