Chapter 2
Elements of Map Contents with (0D) Point Reference Units

Abstract Although in the visual perception of a map, point symbols give way to lines, which cut the map surface, or colourful surface fillings, the role of point symbols in the analyses of deployment of real linear and surface objects—in activating the modelling features of the map—cannot be underestimated. This chapter presents different types of point signatures and their functions. An overview of the qualitative signature features is extended by examples illustrating the application of the original method for the construction of the scale isolines to increase the modelling value of old maps. The discussion of quantitative signatures focuses on the transfer of information and new map forms created on the basis of point data. An important part of the chapter is the section devoted to the design of dot maps and their usefulness in the evaluation of the spatial reach of phenomena and changes in phenomena intensity.

Keywords Reference unit · Point symbols · Accuracy · Relative location · Layout · Density · Potential

2.1 Cartographic (Objects/Phenomena) Symbols Characterised by (0D) Point Reference Units

A map is a model correlating to a geometrised observation of a set comprising of a number of elements varying in substance and matter. The criteria for the allocation of given categories constituting an object of modelling is a **dimension** and the corresponding symbols: point, linear and surface make up its conceptually coherent representation, where the distribution of symbols represents the perceived relations of objects establishing a system.

For a map user, the most accurate medium of information amongst all of the symbols are point symbols, dots and point diagrams. The qualitative symbols represent single objects or phenomena of one or several categories, whereas dots represent an established number of objects on maps of small and medium scale within the same category. The quantitative symbols represent single, numerical

E. Krzywicka-Blum, *Map Functions*, Springer Geography,
DOI 10.1007/978-3-319-47358-1_2

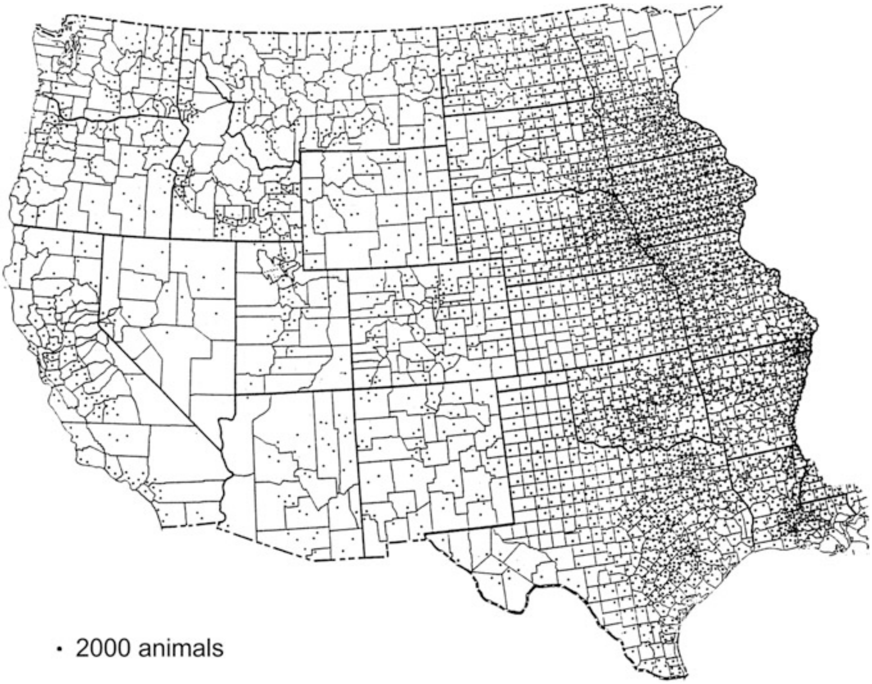

Fig. 2.1 Horses and mules (over 3 months' old). *Source* Atlas of Agriculture of the USA and Canada (1970)

characteristics of objects and phenomena (both isolated and linear) with point diagrams also representing structural sets instead of single characteristic.

The observation of the entire set of symbols or dots representing a given category of objects enables the recognition of their distribution as a whole, therefore, the range, characteristics of the layout and zones that are intricately different when it comes to the intensity of the distribution (Fig. 2.1).

2.2 Qualitative Point Symbols

2.2.1 Role of Qualitative Point Symbols in Revealing Function of Maps as Topological Models

Point symbols play an important role in the process of identifying the map contents and its correlation with the original which is fundamental in revealing the modelling function of a map. The role of point symbols comes from the particular importance of the objects represented by them among those objects that compose an 'area', however, the size in relation to those composing an area justifies the application of point representations on the map. Therefore, not only the accurately rendered points of geodetic control network make up an object represented on the map by a point symbol, but also a tower, sculpture, buffer strip, mine or a town. An object—situated in a relatively small area, yet unique in matter—plays an important role in **identifying** a model with an object of modelling. Its first stage is the recognition by a human of a known fragment of an area/object on the basis of a direct observation of a relevant 'scene' depicted on a map. The scale of a map constitutes its simplicity when it comes to the recognition of an environment on the basis of properly measured and distributed symbols in the same way as in the real world when a man using elements builds a geometric synthesis of a visible world or rather—tries to recreate how he perceives a structure as a whole focusing on mutual relations between point, linear and surface elements. In accordance with the convention of cartographic notation, the symbol of an imagined 'observer' of a given fragment of a map would be the point symbol distributed amongst the symbols of a real, known environment. The topological congruity of a model with an object of modelling can be conveyed in the, enabling an easier identification, congruity of relationships of particular point elements towards the linear (a city in the eastern river bank) and surface (a tree in the centre of a meadow) ones. The second stage of the recognition of the map contents rests in the examination of the graphic code learned by a map user in the first stage. The proper recognition of an unknown fragment of an area/object on the map means meeting the necessary condition of activating those practical functions of the map that are essential in everyday and professional endeavours of men such as: determining one's position in an unknown environment or finding the optimal way to one's destination. In the third stage, practical recognition of different relations between the deployment of groups of symbols indicating known elements and their corresponding objects of modelling allows the transfer of associative cartographic print onto the layout of elements singled out either structurally or functionally which make up a whole which is of interest to a human being. The above said can be actual (used to determine the northernmost position of a given species of beetle or slug) (Atlas Śląska Dolnego… 2008), or constitute a general category. It is the correlation of the relative location system of differentiated spatial elements of the map content with the object of modelling that makes up the feature which allowed the term "map" to spread throughout a number of scientific fields (i.e. biology, medicine, physics, astronomy and anthropology) as a determinant of a model type of the distribution of critical structure elements important for human beings (Fig. 2.2). Despite the basic purpose of models in revealing the structure of matter or inheritance conditioning, such a type is not always perceived as the most relevant one in terms of being the most suitable type of scientific documentation or rather an addition to formal descriptions in mathematical, chemical or biological terminology.

The ability to create integrated spatial projections on the basis of observation of standard layouts of elements of maps is broadly used in school education and social communication as well as in situations, where the application of topologically coherent model with the original is sufficient.

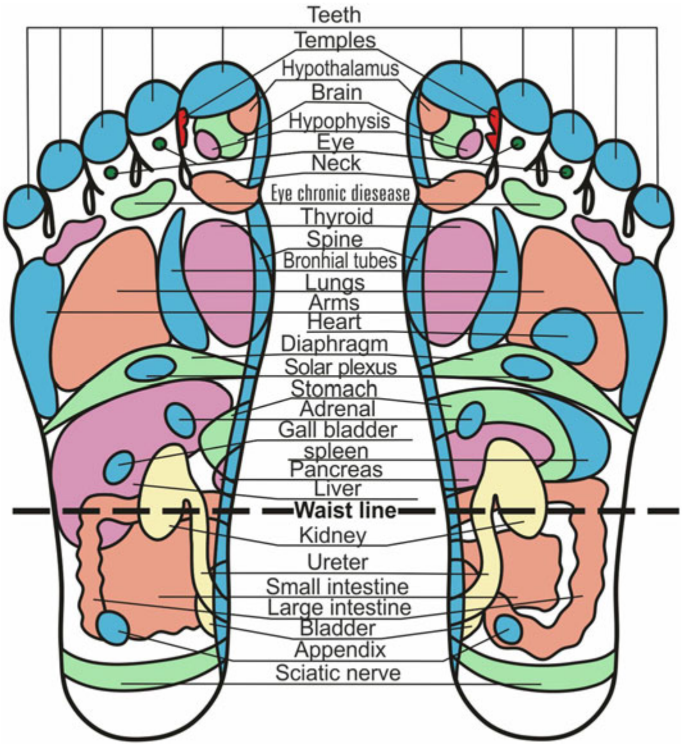

Fig. 2.2 Foot map. *Source* www.gazetawroclawska.pl (2011)

2.2.2 *Role of Qualitative Point Symbols in Revealing the Functions of Maps as Geometric Models*

The topological characteristics of a cartographic model do not justify treating a map as a perfect geometric representation of the 'original'. This is due to the discrepancy of the scale. It depends on the adopted representation, however, even on large-scale topographic maps, the distortion effects cannot be neglected if the map is to fulfil its function as a model metrically consistent with an object of modelling.

Point symbols play a crucial role in the evaluation of geometric and topological credibility of a map. This springs from structural nature of symbols which reflect a very intricate reality. In these types of models, open lines and contours confining the surface are treated as sequential layouts of points, therefore measurements of evaluation of the congruity between the model and the original are also defined as functions of base points location. It comes as no surprise that the scale of cartographic models is conceived as the relation between the distance of two points on the map and on the actual surface. And it was this relation that Tissot defined in differential geometry language, meaning the term of **the scale length in a given**

point and direction, by pointing out that **metric** is **the key to reveal the functional map purposes as a model of spatial relations in the real world**. Therefore, it is reasonable to identify the role of the base which allows the establishment of the scale in terms of two fundamental functions of point symbols sets.

The local variability of the scale of the map value depends on the property used in the preparation of maps. This often remains unknown.

A map is a synthetic medium of information characterised both qualitatively and quantitatively revolving around elements of the environment altered and occupied by men. It is often difficult, or even impossible to determine their accuracy in an analytical way. The map metric or complete descriptions of assumptions of the map are not always available. More often than not, the information regarding the circumstances in which the creation emerged can be found; however, things like references to the nature of the measurements, source materials, the identity of the author or the printmaker or even dating are often missing. Drawn up under various conditions and on a number of substrates using different techniques and not always stored properly, maps present a valuable material for a given area, object or period. However, such material cannot afford an unquestionable belief from its user. A map, as a model of spatial relations, must have a set scale in every point of the map where symbols are located. The local value of the scale is crucial to understanding the mutual relations between the distribution of objects,if it concerns a cycle of dated maps—also the key to assessing the location changes and therefore also that of the processes.

In general, the scale variation of large-scale maps does not limit their potential to deduce the spatial relations of observed elements, however, when the analysis has to do with changes in rivers or border locations or the range of plants over time, then it is crucial to determine the dating of the model and the local value of the scale in an area where these observed elements are located.

Predominantly, the base of points designating the scale is set by symbols depicting measurement points of known coordinates on the map. As far as topographic maps are concerned, the aforementioned base is set by geodetic control points; on geological maps, they are set by drillings; on hydrological, by water gauges; on climatic, by the observation posts and on sozological, by sites where the measurement of the contamination level takes place. Nowadays, in the era of localisers and various types of remotely sensed and scanned recordings being more and more common, the function of base points can also be fulfilled by single, easy to identify objects represented by point symbols.

It is the method of the **isoline scale** construction that allows the measurement of the scale's value in the most accurate way (Krzywicka-Blum 1994b). Its essence is the division of convex polygons with the vertices at 'base' points into the most measurable triangles whose sides, dividing the horizon into three constituent sections, are segments with lengths designating the scale values in relation to the real distance of base points (the vertices of triangles on the map). In each of the triangles, the length value of the scale is then determined in centre points of sides in the direction of a given side. The average value of these scales in three directions is assumed as a local scale in the point of intersection of medians of triangles and is

Fig. 2.3 Part of the map decreased 2.1 times ALEXANDRI MAGNI EXPEDITIO MACEDONIS taken from Ortelius' atlas from 1603 with a network of triangles. *Source* Krzywicka-Blum (1994b) (in Polish)

treated as a direction-independent value. Then, the network of triangles built on points with a set scale allows for linear interpolation to take place as well as the construction of an isoline model of scale variability (Figs. 2.3 and 2.4).

The isoline method is more reliable when it comes to locally differentiated quantitative propositions compared to a broad spectrum of methods used in the evaluation of environmental changes on the basis of analytical (Helmert transformation) or graphical (calibration) fittings of elements of two maps; differently dated and compiled on different mathematical basis indicated by the average error.

In practice, point symbols are most widely used as designating points on older maps when it comes to the evaluation of environmental changes over time [the analysis of changes on a F.F. Czaki's map of Wisla (Strzelecki 2012)], whereas on newer topographic maps, both the measuring points and structural components like castle walls or church towers are still in use (Konias 2000). Mentioned components in the form of operative tracks (as well as their intersections) or river influxes, do not ensure sufficient reliability in regard to evaluations of changes relatively determined on their basis.

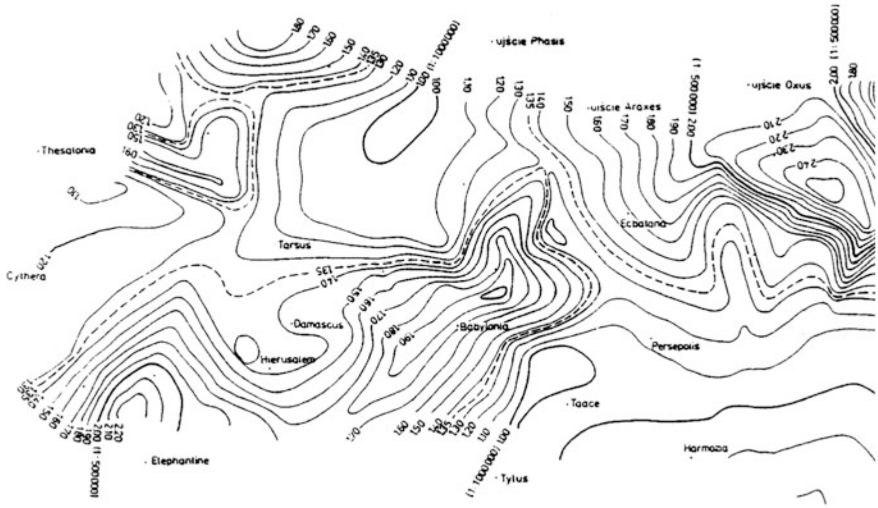

Fig. 2.4 The scale discrepancy model of the western part of the map: ALEXANDRI MAGNI EXPEDITIO MACEDONIS taken from Ortelius' atlas from 1603. *Source* Krzywicka-Blum (1994b)

2.2.3 Distribution of Objects/Phenomena Sets in Point Reference Units; Distribution Range, Proximity and Density

The term **distribution** requires an elaboration. If it concerns the set of points, the position of each element can be discussed either in terms of its relationship to the surface depicted on a map or in relationship to the distance between them and other points within the set.

The simplest quantitative characteristic of the distribution of objects in the area depicted on a map is the **spatial reach**, established by extreme coordinate values of geographic point sets. For a properly matched area, it is the spatial reach of phenomenon/process which is of interest to the map user, for example: an animal species (Fig. 2.5, Atlas Śląska Dolnego i Opolskiego 2008) or the architectural trend of the sacral buildings.

When direct observation enables a clear distinction of a few disjunctive sub-areas grouping all of the symbols of a given category within the area of the map, as an area of objects distribution of a given category, most often the sum of surfaces delimited by the smallest convex polygons containing respective groups is adopted. Such an approximate way of delimitation in the practice of cartographic modelling is replaced by building sub-areas of symbol distribution using the method of applying their circular rims where, in conformation to the envisaged function of a map, the length of the radius is determined. This procedure leads to the division of the map area into sub-areas grouping symbols and sub-areas without

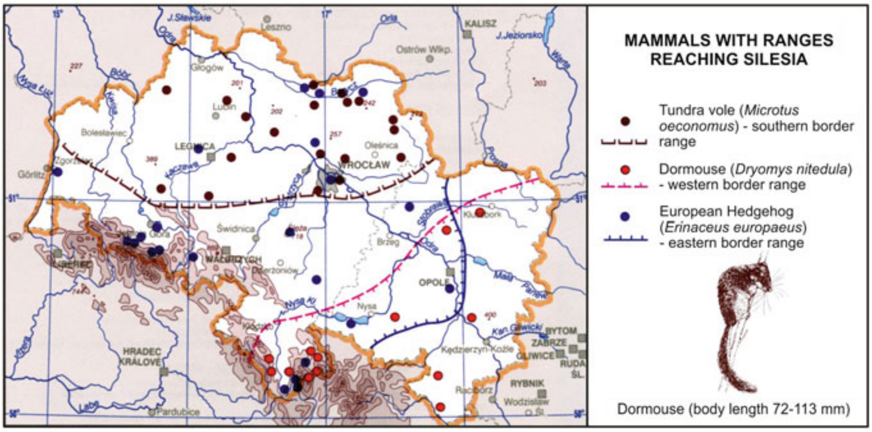

Fig. 2.5 Mammals reaching Silesia (R. Haitlinger). *Source* Atlas Śląska Dolnego i Opolskiego, ed. W. Pawlak (2008) (in Polish)

any symbols. The distance between any sign and its **closest neighbour** does not exceed the criterion value which usually means complying with the *a priori* established accessibility requirement (to the health centre, parking lot or water source). Signatureless sub-areas constitute terrain reserves which are of interest to planners and investors, however, they also enable the possibility of the existence of natural or artificial barriers.

The distribution of point objects in the area depicted on a map can be characterised by the deployment type of the set of the distance between the symbols towards their closest neighbours and to their furthest neighbours or by the average distance between a given point and all the other points. Histogram analysis enables the evaluation of the deployment discrepancy from the regularity after the aggregation of data into size classes takes place. After the aggregation into the size classes, the same sets of data can become the basis for the establishment of the entropy, meaning the measure of the distance differentiation (of a given type).

In addition to the distribution characteristics based on the distance between the points, it is the relative information, meaning the number of objects located in a given area (**density**, saturation, intensity), which is often important from a practical standpoint.

Once the grid is applied to the field of the map, the density value in respective fields of the grid is calculated and after the aggregation into classes, the choropleth model of density zones is prepared. The borders of the class segments are adjusted to the envisaged function of the map. If this function is the state evaluation, generally, either the principle of grouping similar values and separating more distinct ones is used or the design of segments equal in size. The latter is the most legible to the user. If the conversion of the signature map into the choropleth map is to contribute to the design of innovative changes regarding the natural environment or infrastructure, the bilateral series of density classes are created in which the classes

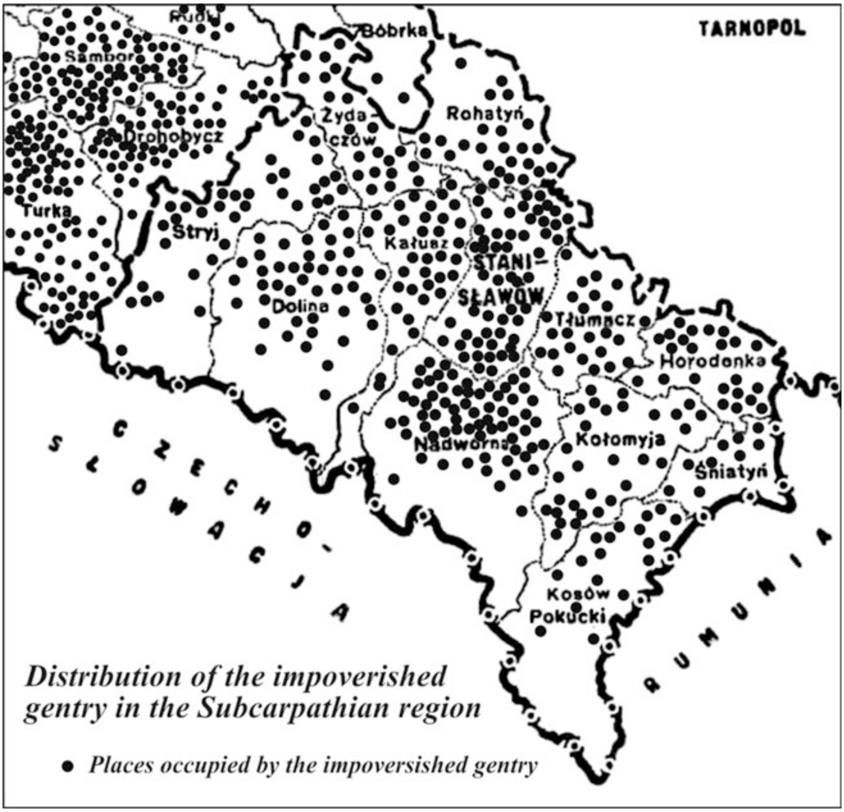

Fig. 2.6 The distribution of the impoverished gentry during the interwar period in the Subcarpathian region according to Plunarowicz. *Source* http://herbarz.net/Forumnobilium/mapy%20zoGalicji.htm

above the threshold indicate the sub-areas which fulfil the required conditions on the map while classes below the said threshold indicate sub-areas in need of transformations.

The informative effectiveness of the transformation of symbol-like maps into choropleth maps can be shown by a historical inventory of some type of settlement. On the symbol map (Fig. 2.6) in the 1:2,000,000 scale, the distribution of the gentry farmhouses clearly remains subject to the Subcarpathian topography, meaning the mountain range from the south and beds of river valleys.

When the grid with 5 mm (10 km × 10 km) sides is applied to the area shown on the map, the distribution of the number of areas corresponding to the variation of the incidence number (Table 2.1), (i.e. the density) is obtained in 205 fields of sub-areas with symbols.

Table 2.1 The density of gentry farmhouses in the Subcarpathian region during the interwar period

The number of objects/100 km^2	0	1	2	3	4	5	6	7	8	9	10	$\sum = 385$
Number of fields	84	29	19	30	18	11	6	3	2	2	1	$\sum = 205$

After the aggregation of fields into four classes: without objects; with 1 or 2 nobility residences and—respectively—with the number 3–5 and 6–10 of residences in 100 km^2 radius, the variability of density is easier to process by using the choropleth model (Fig. 2.7).

The use of surface symbols reveals the distribution range of intensity zones, the layout of smudgy sub-areas, the dominant location and the isolation of the borders indicating sudden changes in intensity. The relation of zones in terms of size, their adjacency and direction are of particular importance. The choropleth map (Fig. 2.7) represents a readable transfer of information on the linear layout of the intensity zones of gentry farmhouses density along with the decrease of altitude a.s.l., parallel to the upheavals of the Carpathian range towards the north, with three sub-areas of extreme density in central range, meaning the northernmost border area. The deployment is random in nature.

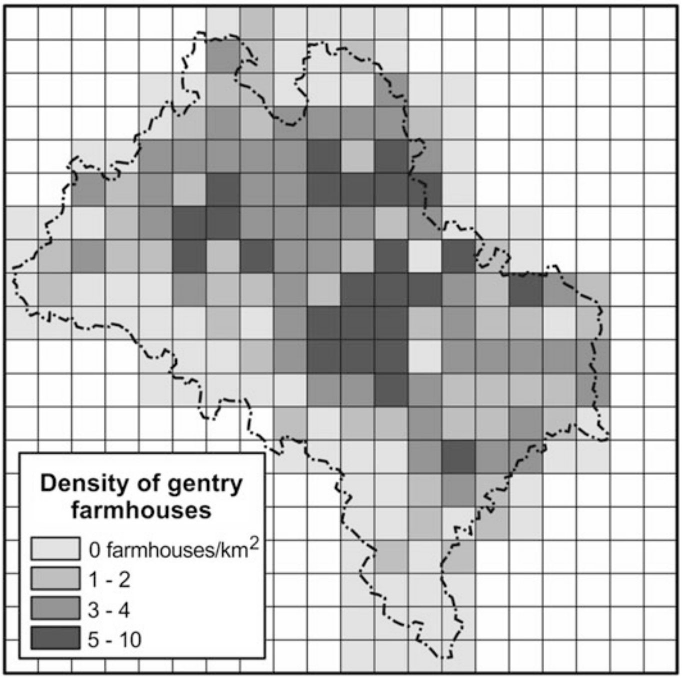

Fig. 2.7 The density of gentry farmhouses in the Subcarpathian region during the interwar period (Ł. Szymanek, ed.)

The relative entropy 'h' is a known informative measure of diversity indicating the ratio of entropy H of a given deployment and the maximum entropy which characterises the balanced layout with equal probability of distribution of elements in distinguished groups. In the following equation:

$$H = -3.319 * \sum_{i=1}^{n} w_i * \log w_i \tag{2.1}$$

w stands for the distribution incidence of elements in the group, whereas n for the number of classes.

The largest entropy value

$$H_{\max} = -3.319 * 1/\log n \tag{2.2}$$

In the case of the evaluation of the sub-areas, surface differentiation representing four distinguished levels of objects density, the distribution incidence of elements of the subsequent classes is as follows: $w(I) = 0.409756$, $w(II) = 0.234146$, $w(III) = 0.287805$ and $w(IV) = 0.068293$ and finally: $H = 1.57117$, and unto $H_{\max} = 2$, the relative entropy value equals $h = 0.786$. This pertains to the 80 % level of distribution diversification of gentry farmhouses in the area where they occur.

Cartographic analysis of the structure of distribution of windmills in Poland has another pragmatically oriented application. The materials provided by the Higher Banking School in Poznań (Wyższa Szkoła Bankowa w Poznaniu) include 493 windmills of the following types: post, smock, paltrok, sokolski (name derived from the town of Sokółka where they occur). The source information constituted the comparison of the number of objects in the fields of the grid of 5 mm × 5 mm on the map in 1:7,000,000 scale and depicted in Fig. 2.8. This can be treated as digital equivalent of a symbol map where location of symbols is determined with accuracy not lesser than the field of the grid.

There are 14 different levels when it comes to the numbers of objects in 284 fields of grid, which, considering the readability of presentation of distribution intensity differentiation, justifies the aggregation of quantitative characteristics set into several groups, which as choropleth map classes, designates the division of Poland into a mosaic with a clearly distinguishable layout of (intensity) levels of an observed phenomenon. The unveiling of the structure of spatial density instances or distribution of objects enables the map user to evaluate the conditions and such evaluation is an initial stage in devising changes.

The determination of the borders of choropleth map class segments in adjustment to the deployment structure of a phenomenon facilitates the observation of scheme depicted in Fig. 2.9. The density levels (known as the number of windmills in a 'mesh' of a grid) are designated on the horizontal axis; each of them corresponds to the vertical segment expressing (in the scale of vertical axis) the number of fields with a given density.

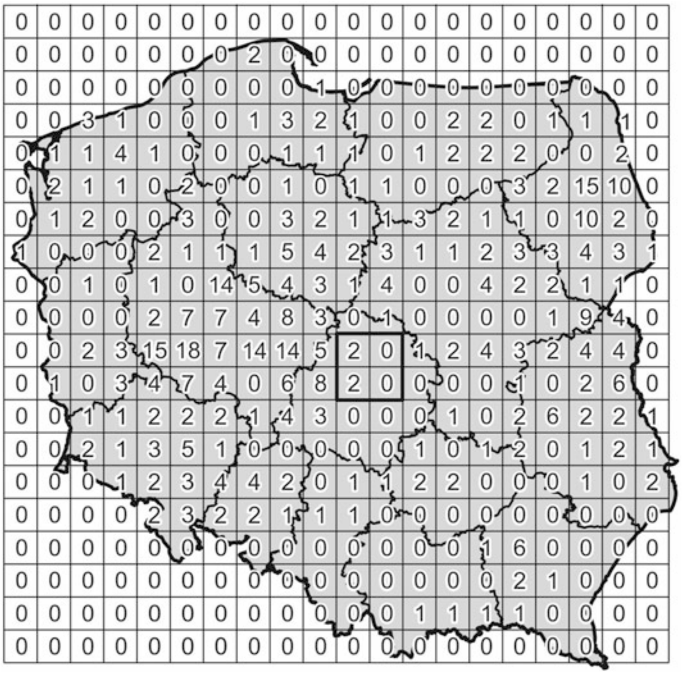

Fig. 2.8 The number of windmills in the sub-areas (35 km) of Poland

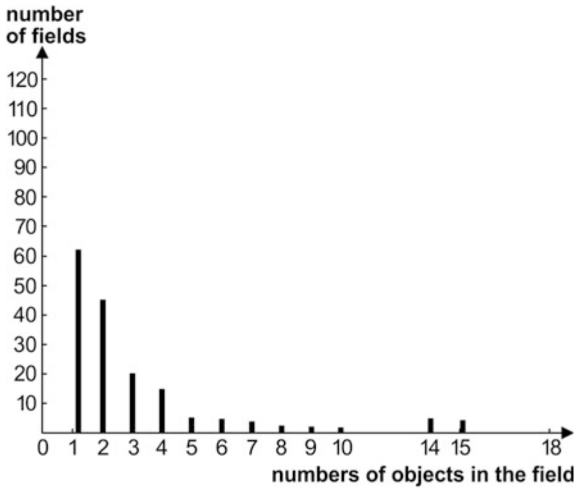

Fig. 2.9 The numbers of objects' density classes (cf. Fig. 2.8)

The borders of choropleth map classes should follow the dilacerations of sequences of density levels (10–14, 14–18 objects) as well as trigger points of changes in field numbers of neighbouring density levels (0–1, 2–3, 4–5). This

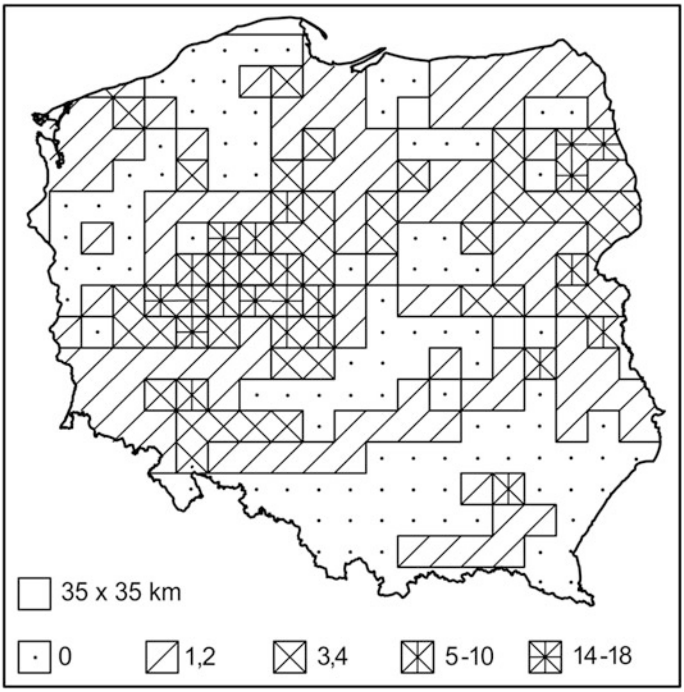

Fig. 2.10 The intensity of windmills distribution in Poland

corresponds to the presentation of intensity differentiation of windmills distribution in Poland on a map (Fig. 2.10).

The mosaic structure of distribution: sub-areas without windmills and sub-areas corresponding to one of classes with four levels of density of objects are characterised by location of two density centres of windmills and by the direction of three ranges from the South–West towards the North–East of Poland where there are no windmills.

Determination of the conditioning of the layout differentiation in terms of water relations and the agricultural use of land lies ultimately with the specialists.

In the case of the choropleth map shown in Fig. 2.10, the range division of windmills density variation is a structural one. The aggregation of 13 intensity classes, distinguished in the first stage of the chorochromatic model construction, into four classes, leaves the distributionless sub-areas intact and results in the plummeting of the relative entropy value from its initial level of 0.83 to 0.69 (due to the discrepancy in the number of classes, the comparison may involve only the relative entropy).

More often than not, the choropleth map user does not pay much attention to the most faithful rendition of a structure, as due to pragmatic reasons, the model with equal proportion of each object density class or the one with equally spread class segments is much more desired. The choropleth map in Fig. 2.11 presents the most

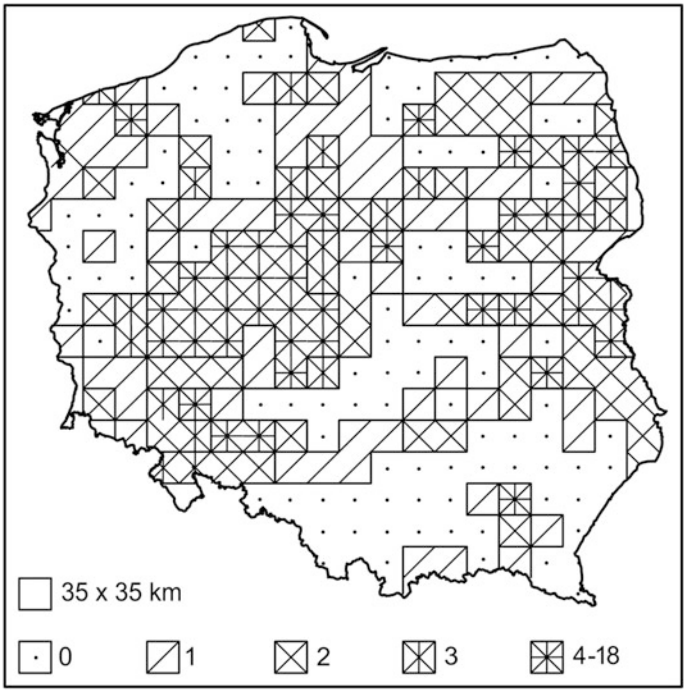

Fig. 2.11 The surface aligned areas of windmill density in Poland

equalised proportion of each of the four intensity classes. The adjustment can only be approximate. The borders of class segments are determined by balancing the total number of fields in the few consecutive density classes. Figure 2.11 depicts the division of the windmills distribution area in Poland into sub-areas of four density classes. In this division, the surface share of classes is optimally aligned.

In relation to the choropleth map (Fig. 2.10), the established assumption led to the plummeting of the lower border of class four which, perceptually, resulted in the 'effect' of increasing the evaluation numbers of windmills. This does not necessarily mean that the choropleth map is not a valid model but rather that it conforms to the a priori assumptions adjusted to the envisaged utility functions for example the grant distribution or the establishment of taxation. The equal proportions of classes, in relation to the structural distribution, results in a significant increase in the entropy value, and also, in the same maximum entropy value (which for four classes is −2), as well as the relative entropy.

For the intensity deployment depicted in Fig. 2.11, it amounts to 0.94. This value cannot be compared to the deployment corresponding to the initial model which did not represent a balanced involvement of classes (reaching a level $h = 0.83$, instead of $h = 1$).

In Fig. 2.12, a choropleth map of equally spread classes can be seen. The perceptual 'effect' of the evaluation of the number of windmills is in this case

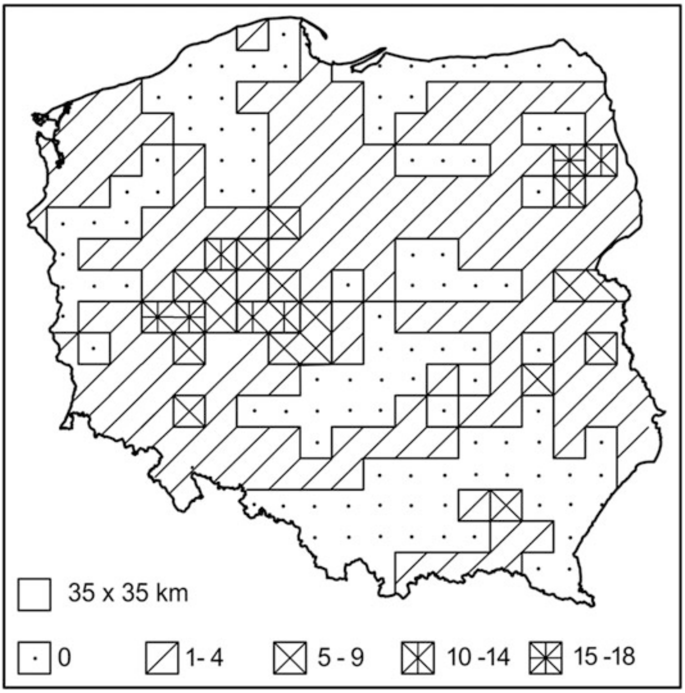

Fig. 2.12 Density of windmills in Poland—the cartogram of equally spread classes

opposite to the previous one, therefore the evaluation is downgraded in relation to evaluations on the basis of the 'structural' model. The value of the relative entropy is the lowest from all the other ones and amounts to 0.37.

The conducted analysis indicates the necessity to exercise caution when quantitatively deducing on the basis of contradictory derivative models in respect to the model presenting unprocessed source data.

2.3 Dot Symbols

2.3.1 Function of Dot Distribution Map, Dot Value

Out of the quantitative methods, the dot method was accepted by the cartographers only at the beginning of the Twentieth century. This type of presenting information is connected with the Swedish school (Sten de Geera).

The symbols indicating point location are used both on the general geographic and thematic maps. In addition, when it comes to the process of identifying the model with an object of modelling, the role played by the location accuracy of symbols representing the observed group of objects with unequal deployments sets up conditions which enable the reliable determination of the borders of objects distribution (Bugaj 2004).

What can be seen is that as the reduction of the scale of the maps takes place, the type of model undergoes a change by adjusting the symbols' code to presentations of more and more abstract observations corresponding to scale. This is reflected in the image symbols being replaced by geometric ones, as well as placemarks of single objects by **dots** which in a given scale constitute the most accurate rendition of a non-single object and the groups of objects of the same category with a strictly set quantity known as the **weight** of the dot. In the atlas of the world (Atlas świata PPWK 1974), this may correspond to 100,000 sheep on the map of Australia in 1:80 M scale or on the map of the world in 1:200 M scale, to 300,000 tonnes of oilseeds. In particularly-differentiated density deployments, point symbols are used which correspond to several levels of weight, this however, is not, according to some cartographers, a solution well adjusted to the characteristics of visual perception (Pasławski 2006).

In Atlases, a weight uniformity principle of a given category of objects is also used on all maps. While this may facilitate the comparison of deployments in different areas, it is not a solution which ensures an optimal adjustment of graphic code to the features of elements distribution belonging to an observed category. When on the map in 1:30 M scale located in a junior high atlas (Atlas gimnazjalny PPWK 1999), the population of Europe is represented by a set of dots weighting 1–3, 3–5 and more than 5 (millions o people), then in the world atlas (Atlas świata PPWK 1974) on the map of Europe in 1:20 M scale, the number of groups, thus also the weight of dot symbols, was determined on the ¼, ½, 1 and 3 (millions of people) level. The relationship of fields (4:9) points towards the possibility of applying more details on a map in 1:30 M scale, however, the adopted **levels** also apply to the population of Asia (1:60 M scale), where the range of density is much broader than in Europe. The determined consequences are brought by the determination of weight value for objects expressed in a given group of units regardless of categories of objects. In the Atlas of agriculture of USA and Canada (1970) the following weights were adopted: 5000 acres (1 acre = 0.4056 ha) of a surface, 10,000 tonnes, 5000 items, 100,000 bushels. This provided the map user with readable (due to separability) quantitative information about the cultivation area of meadows or potatoes, number of horses and mules over 3 months of age (Fig. 2.1) or haymaking, whereas in some sub-areas of intensive farming of cattle and calves, their number in the group above 3 months of age exceeded the possibility of representative presentation with isolated dots and areas of 'agglutination' appeared on the map. Visual evaluation is therefore restricted to the delimitation of agglomerates.

Whereas the inference about the distribution of objects in regard to symbols depends solely on the features of projection and the scale of the map, the evaluation of spatial deployment features represented by dot symbols is the function of the scale of the map, weights of the dots and the way in which they were distributed. When the preparation of a map depends on even distribution of dots in units of administrative division (choropleth map), be it political or proprietorial with assigned numerical source data, the borders of visually distinguished zones/sub-areas of intensity distribution of dots (therefore also objects which are of interest to the map user) are less reliable than when it comes to topographic distribution. The smaller the units referring to the data available to the editor of the map, the greater the precision of deduction in terms of characteristics of objects distribution. The selection of the weight size is also significant to the precision of a map's graphic code in a given scale. For example, in the preparation of the map of Poland at a scale of 1:7,500,000, the level of the size of object set in various sources of data (the Internet, CSO catalogues, industry materials) justifies, in accordance to the graphic separability of dots, the assumption of 10,000 weight (data from 2007) for crimes admitted in this year (91,152,993) and 1000 weight for the number of registered mails of a public operator (351,531) and subsequently: 12,500 for the population of geese (3,814,069), but only 150 for rural towns (45,000) and 10 for libraries (92,664). For the province area, the size of the map 10 cm × 10 cm implies, depending on the size of an area, the scale of 1:250,000 (Opole Province) to the scale of 1:2,000,000 (Mazovian and Lower Silesia Provinces). As the area gets smaller, the type of objects changes and the value of the weight decreases. The function as well as the detail of inference differ to a great extent. For example, in the design of the dot map representing accidents involving pedestrians (the total of 3122 in the year 2009 in Mazovian Province), it is reasonable to assume the weight of 75 and on the map of bridge objects on public roads in Lower Silesia Province (in the year 2004) the weight of 15; even at the county level, it may be justified to present some phenomena on the dot map. For example, having access to the data of respective municipalities, it is possible to prepare a map with an easy-to-analyze distribution of poultry farming (in Szczecinek county in the year 2004, the weight of 1000 items would be effective).

Even though the comparison of two differently-dated cartographic dot maps representing a spatial density deployment of observed set of objects (of phenomena or events) in a given area may be the basis for comprehensible, visual, evaluation of the change in state in the analysed period, the dot method is not a widespread one. To some extent, this may be due to the fact that automated compilation of dot maps on the basis of unit-transferred data of a given surface division does not ensure meeting the condition of choropleth map dot distribution in each of the separated fields of sub-areas because software currently available does not provide a uniform distribution of symbols.

The modern position of maps with the topographic dot distribution is completely different. The modern means of compiling maps with the use of properly organised spatial information systems offer the chance to analyze a given group of environmental elements by using a correct cartographic model. An example can be the presentation of AIDS cases among the chosen race of USA population. The application of logically compiled code of dot symbols (a circle with a dot inside, a circle without a dot, a dot) allows both the observation of the distribution of all AIDS cases in the USA and the evaluation of the participation level of the observed race in the group of patients. The model provides the *a priori* established accuracy, therefore also the detail (based on the selection of weight symbols) and location congruity of source data.

In addition to the informative functions, dot maps with topographically distributed symbols that are compiled using automated systems constitute the basic source in the development of strategic operations in case of an epidemic. Such cases can be noted in Egypt and USA. The rapid intervention (closing schools, cancellation of mass events) in the areas likely to be at risk chosen on the basis of analysis of the map indicating the incidence distribution has significantly reduced the spread of the epidemic.

Apart from maps that proved to be useful for emergency services, the dated information may be of use when it comes to the observation of a number of processes regarding the implementation of innovative solutions.

The density distribution and the spatial reach of the dots are, due to their reliability and simplicity, an efficacious source of information.

This is also very important from both the theoretical and practical standpoints in the **function of dot maps**. These models with topographical dot distribution and maps drawn up on the basis of data corresponding to the units of territorial division, constitute a form of data presentation which is often treated as the starting point in terms of preparing chorochromatic density maps in the form of choropleth or isoline maps. The processing of information related to the point reference units into the organised surface information enables activation of new utilitarian functions of models. In the case of dot maps characterised by choropleth dot distribution—a new presentation reveals information about initially assigned units of territorial division, naturally after the elimination of these parts from areas where the distribution of observed objects is excluded. The use of available numerical data regarding a number of object categories may be the basis for compiling density maps useful in the conditioning analysis of the location of zones with high or relatively low object distribution density. To illustrate this, a number of live births and the number of contracted marriages (in the years 2008 and 2009) in Poland can be compared. The zone with the highest density of live births and marriages concluded is in the areas around Warsaw, Wroclaw, Bydgoszcz and Opole. This can be associated with urbanization. The case is different when it comes to explaining the positioning of areas: the highest 'density' level of female students of technical faculties (in Silesia province and in the eastern part of Mazowieckie province) as well as the lowest (in Lubuskie province). It turns out that in Silesia, which is relatively small in terms of territory, there were as many as 59 higher education institutions (in a dispersed system) and in Lubuskie and Świetokrzyskie provinces the number was smaller and mostly concerned subsidiaries. The relationship between the positioning of zones with the largest number of point objects and environmental elements, bound to them either causally or functionally, can be seen on the choropleth model of bridges on public roads of the Lower Silesia.

The separation of zones was conducted on the basis of a dot map (the dot weight of 15) in 1:2,000,000 scale in counties layout. The high-density zone stretches along the rivers and streams (Odra, Bóbr, Barycz, Kwisa, Kaczawa). The concentration of significant magnitude occurs in the area of Wrocław, specifically in the river basins of Oława, Ślęża and Widawa. The weight is generally low when it comes to the study of objects in small areas. However, this does not mean that the method of processing point information into the surface one is not useful. An example would be the analysis of the spatial distribution of the number of dwellings commissioned in the Trzebnica county from 2004 to 2009. The choropleth map elaborated on the basis of a dot map (the dot weight of 5) shows the relation of the location of the zone with the highest level of density in two of its municipalities-cities with good communication to Wrocław (work places). The fact that the aspect of employment did not remain without influence on the density deployment levels can be indicated by the area location with the lowest level in agricultural provinces of Prusice and Żmigród.

The division of a given area into zones with a few levels of intensity of the instances where the objects or phenomena are interesting to the map user can prove to be helpful in space management. Most of the time, it has to do with the optimisation of the production localisation, investments, etc., then, the adjacency of high- and low- intensity zones may be the first step of locating the areas of 'reserve'. The case is no different when it comes to the protective 'buffer' in the areas either with intense population or with objects of significant value like monuments, specimens of plant or rare fauna.

It may be noticed that the scope of the utilitarian function of the dot maps, the choropleth maps compiled on their basis or the isoline maps has expanded significantly due to the availability of a number of sources of information and new techniques of data processing and imaging.

2.3.2 Distribution of Dot Symbols

A set of absolute information point distributed within the area of the map and regarding the number of objects of a given category makes up a spatial system on the map.

This system is characterised by a density uniformity where, usually, dots create a local concentration or zones varying in density.

The compilation of choropleth or isarithmic density models on the basis of dot maps expressed in the number of objects attributed to a conventionally-selected surface in terms of size which depends on the method: choropleth or topographic, of dot distribution on a source map. In the case of the choropleth distribution, the alignment of surface fields in reference units by joining the neighbouring municipalities, counties or countries is justified. This concerns, for example, the studies of population in global or continental scale (it is worth noting the 'reversed' principle of alignment in the Polish cartographic tradition, meaning the joining of areas in units equal in the number of objects, was used by Franciszek Uhorczak. It is he who can be considered a forerunner of anamorphic population models where the size of the sub-areas' surface area transforms according to the population number).

The accuracy of compiled derivative maps is raised by the choropleth adjustment of dot distribution on source maps.

It involves the exclusion of those area sections where existing natural conditions or housing make it impossible for the location of a given type of objects.

The conversion of absolute information into the relative one follows as a result of the division of a given area on the map on the reference units; mostly regular in shape (squares, hexagons) with a fixed-size surface area. The divisions into curvilinear trapezoids created by meridians and parallels are also used. Representation of maps depicting the variation of objects density should adhere to the equal area rule. In the corresponding reference units (grid "cells") the number of dots is calculated. The density deployment corresponds to the respective population levels (in dots) in the reference areas (grid "cells")—the numbers of reference units with identical density. It is therefore a division of fields into groups with a set density.

The usefulness of both statistical and informative symbols is diverse when it comes to the recognition of the spatial density deployment of a given group of objects. It can be evaluated using a set of nine maps in 1:200,000 scale which constitute educational examples of different types of deployment of 100 dots in 25 squares weighing 10 and 1 cm in size 1 cm spreading across the map (Fig. 2.13). Table 2.2 contains numerical data characterised by the dot distribution on the

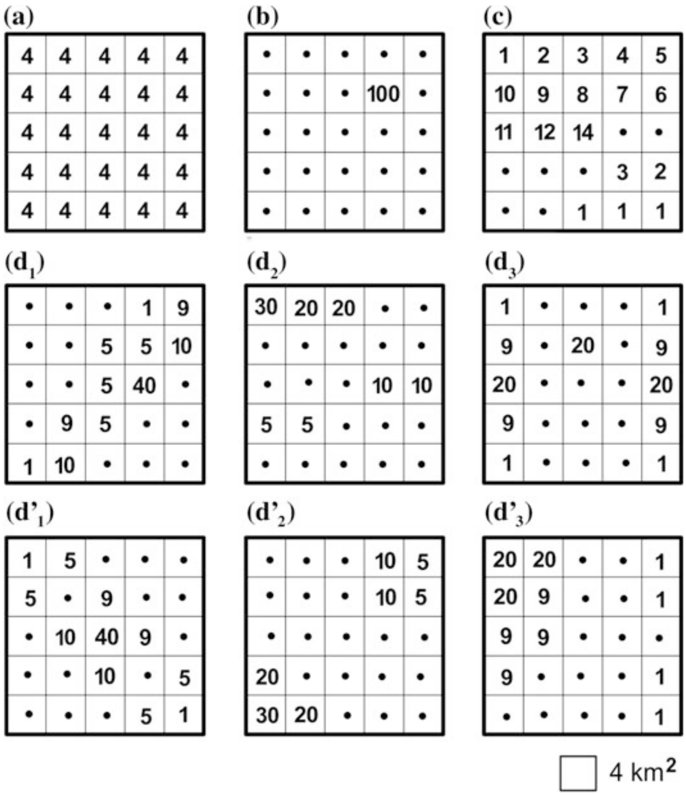

Fig. 2.13 Systems of dots on the a–d maps

Table 2.2 Density deployments of dots on the maps: "a"–"d" (Fig. 2.13)

	"a" map		"b" map		
Class number	i	1	i	1	2
Number of dots	k_i	4	k_i	0	100
Number of fields	n_i	25	n_i	24	1
Frequency	v_i	1	v_i	0.96	0.04
The sum of	$\sum_1^i v_i$	1	$\sum_1^i v_i$	0.96	1

	"c" map														
Class number	i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of dots	k_i	0	1	2	3	4	5	6	7	8	9	10	11	12	14
Number of fields	n_i	7	4	2	2	1	1	1	1	1	1	1	1	1	1
Frequency	v_i	0.28	0.16	0.08	0.08	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
The sum of	$\sum_1^i v_i$	0.28	0.44	0.52	60	0.64	0.68	0.72	0.76	0.80	84	0.88	0.92	0.96	1.00

	Maps																	
Class number	i	1	2	3	4	5	6	i	1	2	3	4	5	i	1	2	3	4
Number of dots	k_i	0	1	5	9	10	40	k_i	0	5	10	20	30	k_i	0	1	9	20
Number of fields	n_i	14	2	4	2	2	1	n_i	18	2	2	2	1	n_i	14	4	4	3
Frequency	v_i	0.56	0.08	0.16	0.08	0.08	0.04	v_i	0.72	0.08	0.08	0.08	0.04	v_i	0.56	0.16	0.16	12
The sum of	$\sum_1^i v_i$	0.56	0.64	0.80	0.88	0.96	1.00	$\sum_1^i v_i$	0.72	0.80	0.88	0.96	1.0	$\sum_1^i v_i$	0.56	0.72	0.88	1.0

corresponding maps. The following deployments correspond to 5 rows and '*i*' ($1 \le i \le 25$) columns. The number of columns depends on the number of different density levels of dot symbols in 25 areas 1 cm by 1 cm on a given map. The first row contains enumeration '*i*' of subsequent classes, structured in accordance with the increasing number of dots in the reference area. The second row contains information about the number of '*k*' dots in '*i*' unit section of a given class and the third row contains the number of '*i*' elements of said density class, therefore the number of fields with '*k*' dots.

Fourth and fifth rows contain numerical characteristics of frequency of the elements of density classes distribution; the fourth row containing each subsequent class and the fifth row the total frequency of all classes from the first to the '*i*' class.

The frequency of class '*i*' is the *n*/25 ratio, meaning the participation of *k* density fields in the entire map.

By comparing the data in Table 2.2 and sample distribution of dot symbols on the maps presented in Fig. 2.13 we can distinguish pairs of different types of spatial systems characterised by the same deployment (maps of the "d" group).

If the case is that the distribution uniquely determines the deployment, then the opposite inference applies to only one type of distribution due to the fact that even in reference to a condensed layout, it is impossible to determine the location of distribution without a map.

The distribution closest to random type of deployment is characterised by the largest number of density classes in a given set (of deployments) undergoing research. The comparison of graphs in Fig. 2.14 corresponding to the 'a'–'d' maps—the cumulative incidence of fields with density not exceeding the density of subsequent classes, enables the recognition of special features. These are: the reduced (to a point and a section) figures corresponding to an even distribution (the 'a' type) and a figure of condensed system (the 'b' type). The remaining four, corresponding to the distribution of dots on seven maps: $(c, d_1, d_1^/, d_2, d_2^/, d_3, d_3^/)$, are polylines with a shape constituting the approximation of the distribution shape of a random deployment probability, however, they differ in regard to the zero value reaching the lowest level for the deployment 'c'.

Among the deployment **statistics**, the **central moments** of different rows have a limited meaning in cartographic practice of characteristics of the type of sub-areas distribution with several distinguished levels of dot density.

Using the indications in Table 2.2, it is easy to notice that the value of **arithmetic average** of dot density on respective maps of exemplary map sets is constant. This is calculated on the basis of the following formula:

$$\bar{k} = \frac{\sum_1^m k_i \, * \, n_i}{\sum_1^m n_i} \tag{2.3}$$

where the number of "*m*" classes meets the condition: $1 \le m \le N$.

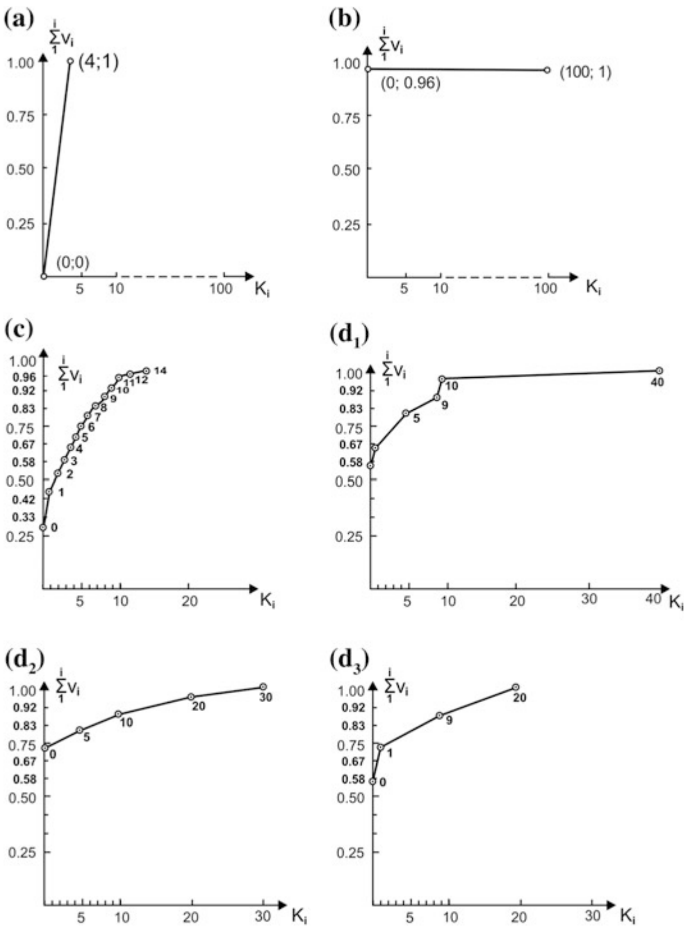

Fig. 2.14 The mass curves of distribution frequency of fields in accordance with density classes of maps: a–d, (Fig. 2.13)

As per the set value of the number of dots and fields, the arithmetic average of the density is not dependent on the type of dot distribution.

Among the **central moments of *k* numbers of rows which is** defined as arithmetic averages raised to the *k* power of variations of all ‘*k*’ values in the examined set from the general average ‘*k*’.

$$M_k = \frac{1}{N}\sum n_i\left(k_i - \overline{k}\right)^k \tag{2.4}$$

The second row moment plays a crucial role in the analyses of density deployment. For a small sample (number of "N" sub-areas), the measure of **empirical standard deviation** is the root from the following **variance**:

$$\sigma_{N-1}^2 = \frac{1}{N-1} \sum_{n}^{m} n_i \left(k_i - \bar{k}\right)^2 \quad (2.5)$$

where $1 \le m \le N$.

For the accidental deployment, the following condition applies: $\frac{\bar{k}}{\sigma} = 1$ and the zero value of variance indicates the uniformity of deployment (considering the following: 100 dots in 25 fields) The largest variation value is characterised by condensed deployment.

The central moments of higher rows, despite the fact that they allow the evaluation of 'skewness' (asymmetry coefficient), 'aggregation' or 'flatness', they do not enable the indication of spatial properties of distribution (fields with a set number of dots).

In Table 2.3, the following values were compared: k, i, $\frac{k}{\sigma}$ which characterise important to determine types of dot distributions on maps; 'a'–'d' (Fig. 2.13).

The data in Table 2.3 confirms the visual evaluation of the distribution type. The deployment 'c' and consequently 'd' is the closest to random. This confirms the shape of mass curves and the values of the variation factor. The largest value of the standard deviation is characterised by condensed deployment 'b'. The statistical evaluation only in a limited scope constitutes information about the type of the spatial variability of intensity distribution of a given category of objects or phenomena.

In the evaluation of elements **diversity** of a group, the characteristics of defining in theory the information as a 'measure of uncertainty' (unawareness) in relation to conveying a message (information) via a 'grainy source' from k number of different messages where the p probability of conveying each of the information is known. The entropy value is calculated according to the following formula:

$$H = -\sum_{1}^{\lambda} p_i \lg_2 p_i \quad (2.6)$$

where k designates the number of elements (information), p_i—the probability of disclosing i—this information.

Table 2.3 Mean value, standard deviation and coefficient variation of density deployment of 100 dots in 25 fields of maps (Fig. 2.13)

Maps	a	b	c	d1	d2	d3
(k)	4	4	4	4	4	4
a	0	20	4.39	8.33	8.04	6.85
$\frac{k}{\sigma}$	0	5	1.1	2.1	2.0	1.7

Table 2.4 Density distribution of dots on the maps: "a"–"d" (Fig. 2.13)

Indicator, map	a	b	c	d1	d2	d3
(H)	0	0.240	3.378	1.953	1.405	1.644
(h)	0	0.24	0.86	0.76	0.60	0.82

The adoption of logarithm of base 2 in the formula (2.6), explains the calculation of the measurement value of uncertainty in the case of 'binary source' (alternative feature) with the presumption of equal probability of an event where the information is either conveyed or not conveyed (the electricity will flow or not). The value then amounts to 1 and is called a **bit**.

In the evaluation of the diversity of groups of elements of a set, the numbers of elements in a group may designate the probability (as a part of the entire set of elements of all groups) whereas the criterion of distinguishing groups may either be qualitative (plant species on an observed meadow, races of people in a given country or sacral objects of a given style in Poland) or quantitative (the increase in population of Poles in the year 2000). For a given N number of groups (classes) the highest entropy value asserts the even number of elements in groups and regardless of their numbers the value is $H = 1/lg_2N$. Given two tile colours, the most diverse floor would be the one with alternately arranged tiles, regardless of the size of the bathroom.

The comparison of the sets diversity with a differing number of groups (classes) can be conducted on the basis of relative value rendition of entropy meaning the **relative entropy** defined as the relationship of the entropy to the maximum entropy: $h = H/H_{\max}$.

In Table 2.4, the values of entropy and relative entropy are compared as indicators of the level of dot density variation on maps: presented in Fig. 2.13.

The one-class deployment means no density variation (the 'a' map), condensed deployment means little variation, as much as 24/25 of the surface belongs to the zero density class (the 'b' map). The highest level of spatial density variation is characterised by dot distribution on map 'c', where a large number of classes means a rather even-spread quantity as indicated by value $h = 0.86$. Among the various deployment groups of the 'd' maps, the 'd_3' map stands out due to the fact that the nature of deployment is the closest to random which can be corroborated by the shape of the mass curve (Fig. 2.14).

2.3.3 Choropleth and Isoline Maps as Variation Models of Dot Symbols Density

The relational evaluations of the numerical variation of objects/phenomena are useful in a number of studies aimed at the natural, socio-demographic or cultural characterisation of an area. The transformation of the optimal—for given practical needs—dot model into the relational in nature form does not mean the change of the inference scope, however, the basic object of modelling undergoes a change.

In a given scale, the dot map covers the number of objects in any part of an area, whereas in relational models, the accuracy of indirect quantitative information is lower. On the other hand, when it comes to the division of areas into sub-areas/zones with a few levels of density (dots, objects), choropleth and isoline maps are characterised by their superiority, speaking both in respect to perception and accuracy as far as visual (indirect) process of distinguishing zones on a dot map is concerned.

Object density choropleth maps constituting the derivatives of dot models (Fig. 2.13) can be seen in Figs. 2.15 and 2.16. The original division into the density classes (indicated by the number of dots in an area) as a result of the analysis of the deployment and designated by the size of the primary area has been simplified in

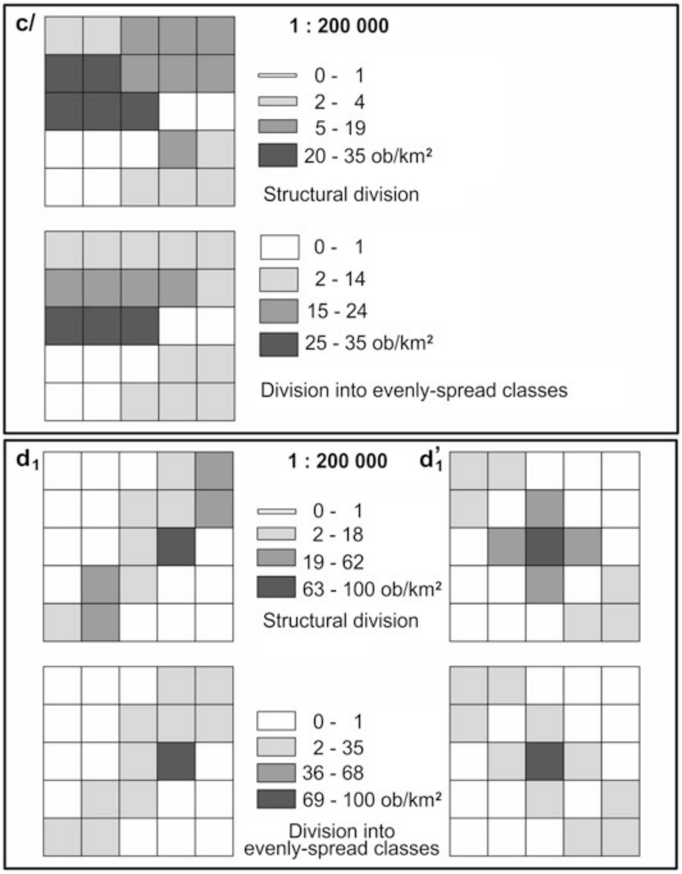

Fig. 2.15 Choropleth maps of objects density (see the map c, d_1, d'_1 in Fig. 2.13)

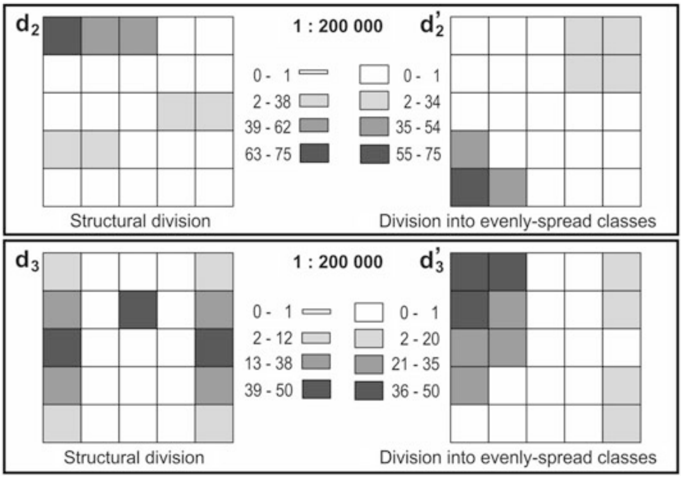

Fig. 2.16 Choropleth maps of objects density (see the map d_2, d'_2 in Fig. 2.13)

order for the smaller number of density levels to reveal the spatial characteristics of the layout of the classes more plainly.

The delineation of the zero class aims to demonstrate the boundaries of the objects occurrence. The borders of the class range were established using two methods.

The first one revolves around the principle of the separation of those values that are different and the grouping of those that are closer to each other with the adjustment of the number of classes to *a priori* assumed accuracy of approximation. In this way, the received structure of spatial (density) deployment becomes more explicit. It is worth noting that the selection of the size of the basic area has a significant impact on the primary density distribution of a given dot deployment (Krzywicka-Blum 2003). In order to recognise the structure of the density deployment accurately, the method of subsequent divisions of a given area would have to be implemented and the largest of a given distribution number of density classes received. Then such deployment would have to be treated as initial for the process of aggregating the density classes. However, considering the aim of the aggregation, the estimated degree of structure recognition allows the construction of a compatible model in this regard.

Continuous scale value division was implemented and in that respect the borders of segments correspond to the average: maximum value in class '*i*' and minimum in class '*i* + 1' This lowers the accuracy in respect to choropleth map whose borders are compatible with the extreme values of elements (Pasławski 1993, 2006), but allows the comparison when the state undergoes a change for example in the research of processes. The second method is the division of the variation range of the quantitative characteristics depicted on the choropleth map into evenly spread classes. This type of scale value is widely recognised as user-friendly in general

Table 2.5 The number of objects on dot maps and choropleth maps: (Figs. 2.13, 2.15 and 2.16)

Type of map	Number of objects	c	d1	d2	d3
Choropleth map	*L*1	3540	4900	4160	4320
Structural classes					
Choropleth map	*L*2	4000	4260	3770	4470
Evenly spread classes					
Dot map	*L*	3000	3000	3000	3000
The evaluation error	*L*1 (%)	18.0	63.3	38.7	44.0
The evaluation error	*L*2 (%)	33.3	42.0	25.7	49.0

evaluation. None of the above-mentioned methods meet the condition of even accuracy of determinations on the basis of average values from segment borders, determined by comparing the size of absolute or relative average error; such assumptions are met by proper computer software in automated procedures of preparing choropleth maps.

The differences between the evaluation of the number of objects on the basis of dot maps and on the basis of compiled (on their basis) choropleth maps depend both on the deployment of objects and the manner in which they were generalised. Table 2.5 presents the evaluations of the number of L_1 objects obtained on the basis of choropleth maps with a structural delimitation of classes and L_2 on the basis of choropleth maps with evenly spread classes c, d_1, d_2, d_3—compiled on the basis of aggregation of density classes characterising the dot maps (Fig. 2.13).

The accuracy of the evaluation on the basis of dot maps depends on the weight of a dot and the distribution of the aggregation of objects. In Table 2.5, the level of 'information loss' in the process of generalisation was determined in accordance with the **volumetric** criterion (Mościbroda 1999), where the dot map was treated as, corresponding to specific practical purposes, a form of coding the source information. The scale of differences which are the result of concluding on the basis of derivative maps does not enable the 'transfer' of the functions of dot maps into choropleth maps. The range of reliable concluding is assigned to each cartographic map. Unfortunately, both the compilation of maps as well as their application for practical purposes in solving various issues, especially in the view of disseminating the access to 'crude', automated methods of map compilation, is not always characterised by the sufficient level of necessary cartographic knowledge.

Whereas the presentation of density deployment in the form of choropleth map perfectly demonstrates the adjacency of areas with clearly differing intensity of object distribution belonging to the observed category, it is the isoline model which, in the process of continuing a feature, allows for the emergence of 'transient' zones on the map, which often results in incompatibility with reality of expanding the borders of areas of object distribution. Depending on the type of proximity of dots and the reference unit's surface size (generating the density level), the first or the second model turns out to be more suitable. This is visible once the choropleth map

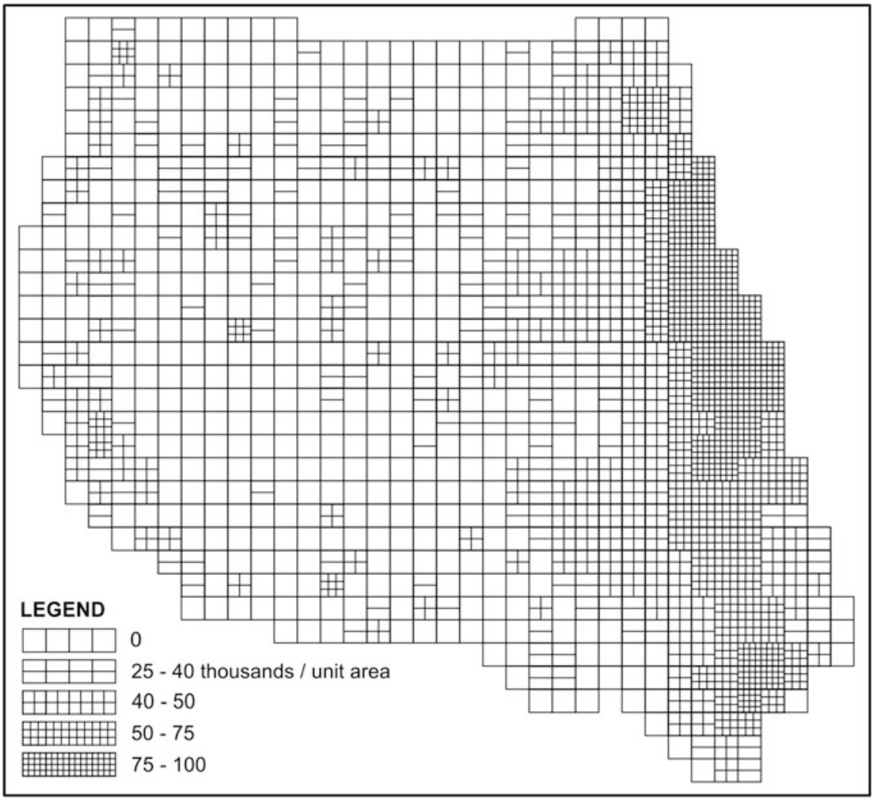

Fig. 2.17 Choropleth map: The cattle and calf population in the western states of the USA

(Fig. 2.17) and the isoline map (Fig. 2.18) are compared which constitute the derivatives of the map: The cattle and calf population in the western states of the USA (Atlas of Agriculture of the USA and Canada 1970).

2.4 Cartographic Symbols Containing Quantitative Information: Symbols, Diagrams, Graphs

2.4.1 Types of Point-Distributed Quantitative Symbols

Numeric symbols, diagrams and charts can be considered as point symbols constituting either simple or intricate quantitative characteristics of objects, phenomena or events. Numeric symbols may be expressed in size or contingent scales. The example of implementing the numerical size scale can be the land ordinates on topographic maps (Fig. 2.19) and symbols in contingent scale—symbols of cities in adjustment to the number of inhabitants in a given class (Pasławski 2006).

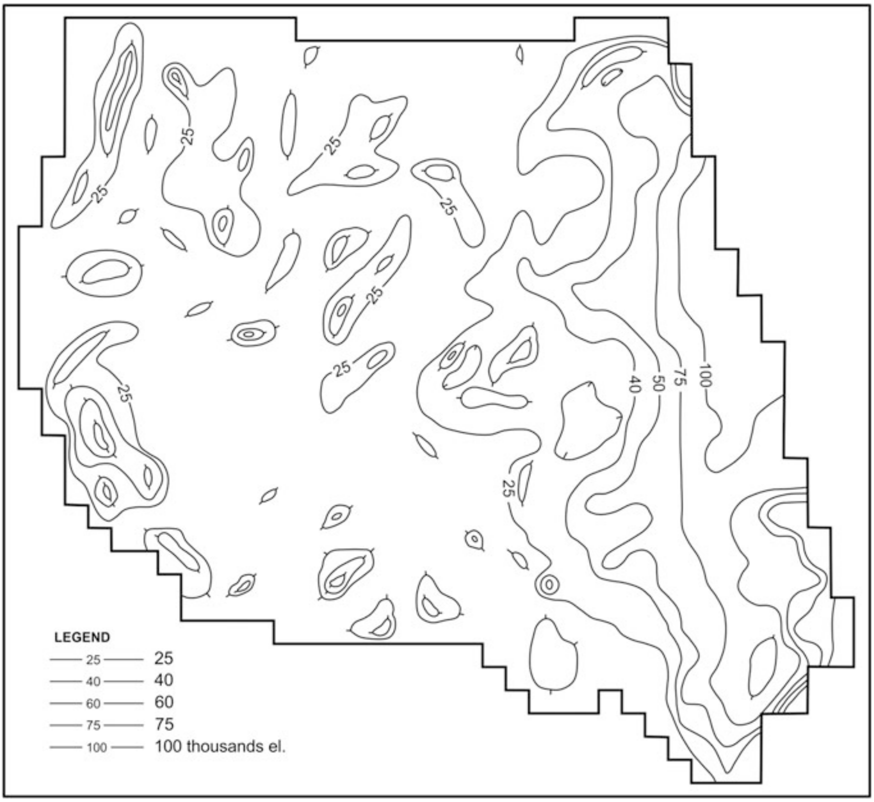

Fig. 2.18 The isoline map: the cattle and calf population in the western states of the USA

A separate group of quantitative symbols constitutes the dating of phenomena (phenological stages) located in a point manner or events (military campaigns, flood wave front) characterizing the processes.

Diagrams with point references show, with the use of metric characteristics of geometric figures (bar height or circle surface), numerical values of qualities that characterise point objects, both single and in groups but also ones in structural combinations.

The point-located graphs most often constitute the notation of observed values in time or measured environmental features.

2.4.2 Numerical Point Symbols

The numerical symbols can be divided into a group indicating object features or point-distributed phenomena and a group constituting the representation of variation level of object features or phenomena of linear or surface distribution.

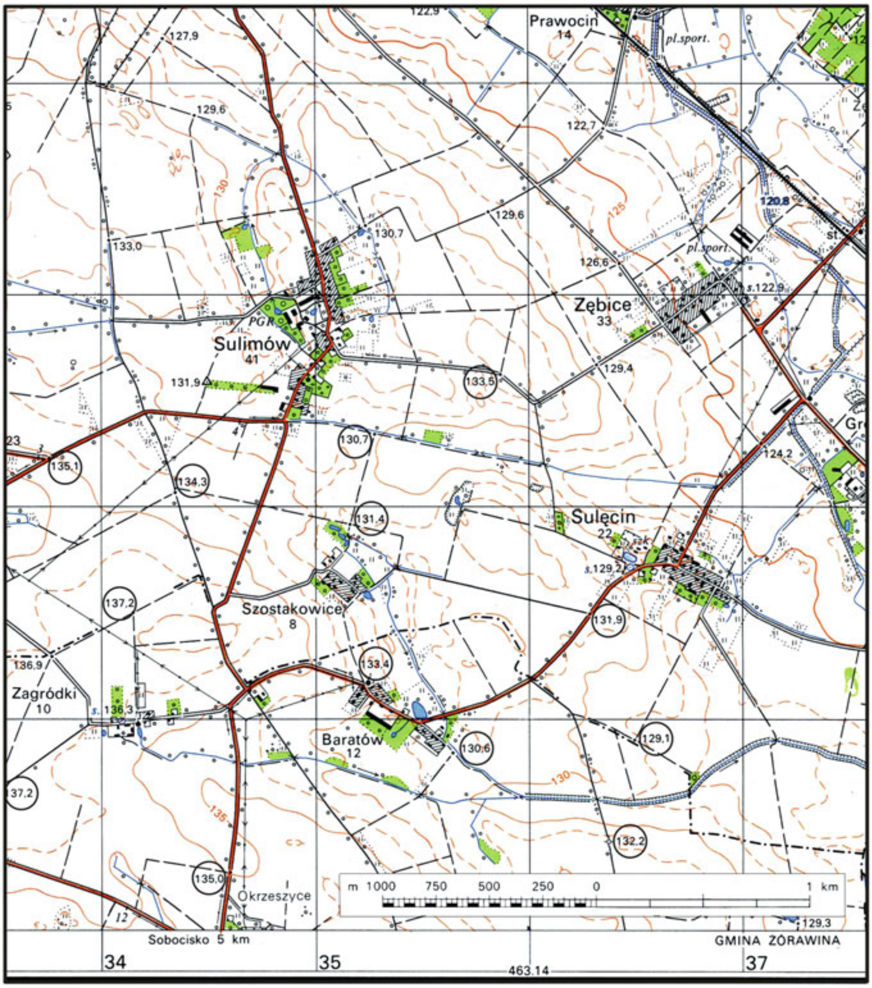

Fig. 2.19 Numeric symbols as height symbols (H a.s.l.) of surface area points. *Source* Mapa topograficzna (2005)

The representativeness of numerical symbols concerns the relation of features towards:

- characterised objects or phenomena;
- the surface variation of the feature;
- the credibility of indication.

The first of said relations does not constitute a cartographical domain but rather these disciplines in the scope of which, the categories of objects or phenomena characterised by quantitative features presented on maps are included. The population issues are covered by demographics, the importance of precipitation or temperature by climatology and also agriculture when it comes to their applications.

The representativeness of information contained in numerical symbols for surface variability of feature concerns the location of symbol sets. When the location of symbols and objects/phenomena, characterised by a given feature is the same then the representation is complete. The case is different with the point recognition of feature variation of phenomena or objects distributed linearly (a river or a road) or structurally (deposit footwall, the bottom of the sea) A discrete variation model is useful only when it constitutes an approximation to a given task with a preciseness enabling a practical consistency (Steinhaus 1954a, b). This applies especially to the principles of using models of symbol map derivatives, compiled with different, often simplified, assumptions regarding the deployment. The representativeness of the determined numerical values which indicate symbols, are to be perceived as an adjustment to the accuracy obtained using a given method of measurement or observation. It is the adjustment to the range and type of feature variability, map scale, symbol distribution and the accuracy of their values which determines the efficiency of a model intended for visual perception.

A given spatial deployment of a feature value corresponding to all objects of a given category within a field can be treated as a set of individual information or as a relational presentation constituting the basis for the analysis of potential (probable) changes of a given tendency towards compensating the differences in characterizing the natural and social environment of people.

In regard to the interaction between two objects, the concept of **potential** was defined in cartography as the function of their masses and distances (Kraak and Ormeling 1998). In a closed system of the N number of located '*i*' objects characterised by the w_1, w_2, …, w_N, values, the potential in each of the points is calculated in accordance with the following formula:

$$v_i = w_i + \sum_{k=1}^{N} w_k/d_{ik}, \tag{2.7}$$

where $k = 1, 2, \ldots, i-1, i+1, \ldots, N$; d_{ik} is the distance between objects '*i*' and '*k*'.

The spatial differentiation of potential in a given area can be presented by constructing a derivative isoline cartographic model using the method of interpolation. The potential maps are used in the studies of migration conditioned by the labour market or educational offer, in predicting the information transfer, in evaluating the potential epidemic risks, but also in the propagation of innovation. The potential models constitute an important 'research tool' of many current processes, whose direction and dynamics are determined to a large extent by concatenation or network of interactions between observable centres.

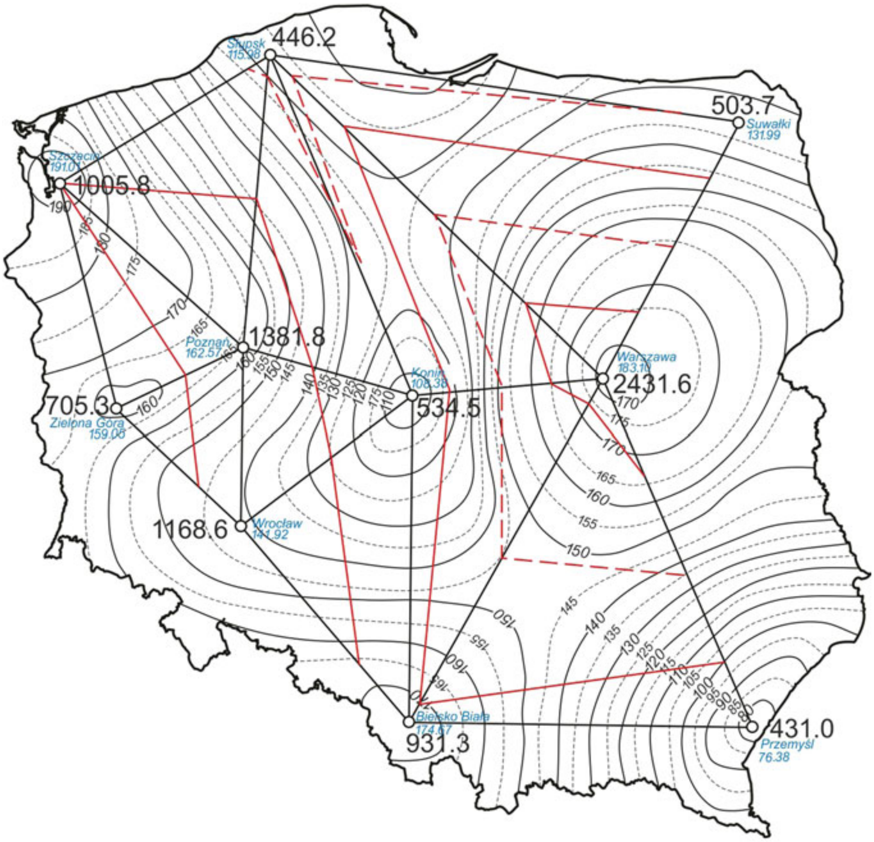

Fig. 2.20 The potential population—sample model

In Fig. 2.20, the isoline course of the population potential of 10 Polish cities and obtained as a result of linear interpolation is presented. The source data and relevant calculations are summarised in the following Tables 2.6 and 2.7.

The spatial continuity of the value of the potential determined in a finite number of points is an example of a widespread method of creation on the basis of a discrete recognition of continuous feature, a model of its course in an entire area in which a discrete set of determinations is deployed.

Whereas the potential map is an example of a realization of known information whose representativeness was ensured by a set of 10 point determinations, it is the model which in the case of applying the interpolative methods to determine the continuous point features, bearing unknown spatial variability, that has to be treated as a hypothesis.

The smaller values differences in a closer proximity and their increase as they move apart is intuitively assumed in the 'transfer' of information from designating points. The simplest model constitutes, in the case of linear object features,

Table 2.6 Source data (Rocznik Statystyczny 1998)

	Konin	Wroclaw	Poznań	Zielona G.	Kalisz	Przemyśl	Warszawa	Suwałki	Słupsk	Bielsko-B	Szczecin
Konin	–	175	100	225	55	505	200	145	320	340	340
Wrocław	175	–	170	155	120	520	345	620	450	230	370
Poznań	100	170	–	130	110	595	305	525	280	400	240
Zielona G.	225	155	130	–	210	675	425	655	410	385	215
Kalisz	55	120	110	210	–	470	255	380	380	300	350
Przemyśl	505	520	595	675	470	–	375	585	810	340	835
Warszawa	200	345	305	425	255	375	–	275	425	400	515
Suwałki	145	620	525	655	380	585	275	–	500	660	670
Słupsk	320	450	280	410	380	810	425	500	–	660	230
Bielsko-B	340	230	400	385	300	340	400	660	660	–	600
Szczecin	340	370	240	215	350	835	515	670	230	600	–

The population inhabiting the provinces in 1997 (in thousands)

Konin	Wrocław	Poznań	Zielona G.	Kalisz	Przemyśl	Warszawa	Suwałki	Słupsk	Bielsko-B	Szczecin
475.6	1131.6	1341.4	667.2	718.0	411.5	2409.1	480.1	421.1	907.3	981.4

Table 2.7 Calculation of the potential population of 10 Polish cities (1997)

	Konin	Wrocław	Poznań	Zielona G.	Kalisz	Przemyśl	Warszawa	Suwałki	Słupsk	Bielsko-B.	Szczecin
Konin	475.6	6.47	13.41	2.96	13.05	0.81	12.04	3.31	1.32	2.67	2.89
Wrocław	2.72	1131.60	7.89	4.30	5.98	0.79	6.98	0.77	0.93	3.94	2.65
Poznań	4.76	6.66	1341.40	5.13	6.53	0.69	7.90	0.91	1.50	2.27	4.09
Zielona G.	2.11	7.30	10.32	667.20	3.42	0.61	5.67	0.73	1.02	2.36	4.56
Kalisz	8.65	9.43	12.20	3.18	718.00	0.88	9.45	1.26	1.11	3.02	2.80
Przemyśl	0.94	2.18	2.25	0.99	1.53	411.50	6.42	0.82	0.52	2.67	1.18
Warszawa	2.38	3.28	4.40	1.57	2.82	1.10	2409.10	1.75	0.99	2.27	1.91
Suwałki	3.28	1.82	2.56	1.02	1.89	0.70	8.76	480.10	0.84	1.37	1.46
Słupsk	1.49	2.51	4.79	1.63	1.89	0.51	5.67	0.96	421.10	1.37	4.27
Bielsko-B.	1.40	4.92	3.35	1.73	2.39	1.21	6.02	0.73	0.64	907.30	1.64
Szczecin	1.40	3.06	5.59	3.05	2.05	0.49	4.68	0.72	1.83	1.51	981.40

segmented polyline determined by neighbouring values in points of observation or measurement. In this way, for example, the level of water table (point measured in 2 km intervals in a time synchronised manner of measurement) of the Odra river or the vertical alignment of the longitudinal route profile can be determined. The discrepancy of surface variation of the feature $w = w(x, y)$ on the basis of a finite number of its determinations (the values measured in located points) comes down to, in the case of linear variation, the joining of neighbouring triangular plane fragments designated by trios of points in which the values of the feature are known. A multidimensional surface has no tears but it is not 'smooth' either. In order to avoid the collapse of the course of the line which characterises the linear interpolation, algebraic polynomials of higher degrees were used. This ensured smoothness, however the possibility of oscillation between 'nodes' lowered the credibility of the determinations. The approximation theory constitutes a well-developed branch of theoretical mathematics, however, some of the claims have found practical application in geodetic and cartographic modelling.

The accuracy of interpolar approximations of an unknown function of a variable depends on the representative choice of point determinations. This, with a fixed number, pertains to location. The application of orthogonal polynomials in constructing the sectional **spline** functions on the basis of optimally distributed points meets the expected conditions of the accuracy of determining the spatial curve (Bałut 1988).

While modelling the surface on the basis of determinations in discrete set of localised points, the method of polynomial interpolation is used or, along with spline functions, the method of **distance-weight** or **autocorrelation** (**kriging**) methods are applied. The choice should be adapted to the nature of the feature variation in a given area. The precision in determining the values in 'base' points and their representative location towards the spatial deployment of a feature is designated by the reliability of a model and therefore, the range of its usefulness.

In the spatial studies of object differentiation or phenomena dependant on a number of factors, the map of separately treated components constitutes an important research tool. The number and location representativeness of observation points or the measurement of even a single element is not always ensured (Krzywicka-Blum 1994a) and it is even more difficult to ensure the representativeness in conditions when the nature of progress constitutes a tendency to reduce numbers and types of observation posts for the sake of automated posts of integrated measurement of a number of parameters which characterise a given object or phenomenon. The essence of variability in the case of respective features may differ to a great extent: the noise level in urban agglomeration depends on the layout of transport, the wind tunnels above the rivers change the wind direction and the housing has an impact on the local temperature variability (Bac-Bronowicz 1997, 2003). An approximate analysis of conditions in a given area which cause a clear differentiation of their spatial deployments is helpful when it comes to the optimisation of choosing the observation/measurement points of several components of a given phenomenon. The comparisons of variability can be made by calculating Pearson's **correlation coefficient**

$$V_x = \sigma_{N-1} * 100/|\overline{x}|, \tag{2.8}$$

where σ_{N-1} is an standard empirical deviation, $|\overline{x}|$—the arithmetic average.

Such percentage presentation is adjusted to the parallel of level variability of features expressed in different measures.

The construction of continuous model on the basis of discrete value recognition of a given feature is conducted with the use of various functions (Bac-Bronowicz and Grzempowski 2011). In the distance-weight methods the value of a feature in the point of the analysed area which is not a designating point, the 'weighted' average of value is calculated in 'base' points in accordance with the following formula:

$$w(x, y) = \frac{1}{\sum_{i=1}^{N} \frac{1}{d_i^e}} \sum_{i=1}^{N} \frac{w_i}{d_i^e}, \tag{2.9}$$

where

i = 1 …, N designates P_i *of* points with known feature values 'w_i';
P (x, y) is a point at which the value is determined;
d_i—designates the distance between points P and P_i;
e—the exponent which is determined depending on the established assumptions and does not exceed 4 (Mościbroda 1999).

The error of approximation depends on the feature deployment, therefore it is worth conducting its evaluation by excluding some of the closely located 'base' points from the calculations and treat them as control points. In general, the effect of approximation is the reduction of maximum and increase in minimum of values set.

The most widespread method of continuing the discrete model is the method derived from its author's surname, the South African geologist D.G. Krige, known as the method of kriging. The computational formulae are adjusted to the 'intuitive' assumptions that the designated values are less different from the values in close designating points than in the distant ones. The autocorrelation function has the form of semi-variation in regard to the assumption of isotropy;

$$f(d) = \frac{1}{2N} \sum_{i=1}^{N} [w(x_1) - w(x_i + d)], \tag{2.10}$$

where $w(x_i)$ designated the value of the feature in a point x_1; N designates the number of pairs of points; d—their distance.

The method is applicable to the evaluation of deposits when the $f(d)$ function allows the assignment of weighting factors to designating points which make up the nodes of the grid. The regular layouts of wells are characterised by numerous types of exploration works with the objective to determine the spatial reach of deposits underground. After the initial stage of diagnosis, the fragments of the grid become more compact in its adjustment towards the assumed accuracy (the evaluations of the coal stratum, the volume of groundwater) (Górnika 1972; Krzywicka-Blum 1980). Whereas the application of kriging may be justified when it comes to the continuous modelling of many point-recognised natural phenomena with natural isotropy, the autocorrelation assumption in the process of modelling the economic or socio-economic phenomena may lead to practically useless presentations.

Among the sets of spatial characteristics of objects and phenomena, one is of particular importance to the living and working conditions of a man. It is the height of land above sea level. Throughout cartographic history, the predominant position was that of the treatment of map contents as sets of symbols of objects distributed with the preservation of consistency with operationally sufficient surface location. The importance of information of horizontal and vertical objects positioning for a man can be indicated by the enormous usefulness of remote methods of imaging despite the unequal level of accuracy of obtained situational (x, y) and height (z, h) coordinates.

Ground-based methods of determining height differences useful in grand-scale engineering projects of bygone cultures did not create enough of an impact on the scope of the map content when it comes to the numerical characteristics of differentiation of objects location. The environmental elements were presented due to their significance in identifying the model with the reality or the availability barriers or as decorations filling the empty fragments of the map field. The breakthrough was the abstract presentation of feature levels in the form of isolines: in late XVI century isobaths and two centuries later isohips. The third coordinate, presented especially in the form of isohips model with coloured fillings, turned out to be a particular enrichment of situational presentations revealing the important relations between the environmental elements. Nowadays, it is hard to imagine a topographic map without height characteristics of the ground in the form of contour lines and numeric signatures distributed in points characteristic to the landmarks and next to selected objects. The combination of improved traditional ground-based and remote methods, optical and hydrostatic levelling, autogrammetric processing of aerial photos, methods of converting satellite images, the possibility of manipulating, joining and synchronizing data—all this added to the adjustment of accuracy in determining the height of points for the needs of modern engineering but also for the development of reliable cartographic models. Nowadays, the data indicating the height of the terrain is also acquired from a digital terrain model. Considering that in spatial information systems, the digital model compiled on the basis of numerical terrain model (of lesser accuracy) is often available in the form of regular surface grid, the analysis of available data regarding the numerical model and interpolar algorithm is of particular importance for the reliability of maps obtained on its basis (Wysocki 1997).

The discrete identification of the land surface in regard to the knowledge of values: z_1, z_2, …, z_n of the function $z = z(x, y)$ in a finite set of points: P_1, P_2, …, P_N, allows the construction of many different surfaces representing a spatial variability of land height above sea level with the implementation of different spline functions (Wasiljenko 1983; Mitasova et al. 1990). The following function proved to be easy-to-automate in the process of interpolation (Sjerbjenjuk and Musin 1986):

$$z(x, y) = \sum_{i=1}^{n} \lambda_i r_i(x, y) + ax + by + c, \tag{2.11}$$

where

$$r_i(x, y) = \left[(x - x_i)^2 + (y - y_i)^2\right] \ln\left[(x - x_i)^2 + (y - y_i)^2\right]$$
$$r_i(x_i, y_i) = 0$$

The system of $(n + 3)$ linear equations with $(n + 3)$ unknowns $\lambda_1, \ldots, \lambda_n, a, b, c$ is determined with the use of the following conditions:

$$z(x_i, y_i) = z_i \qquad \sum_{i=1}^{n} \lambda_i = 0$$
$$\sum_{i=1}^{n} \lambda_i x_i = 0 \qquad \sum_{i=1}^{n} \lambda_i y_i = 0$$

Determined in accordance with the formula (2.11) function ensures meeting the condition of 'smoothness' on the surface, expressed mathematically as a minimum condition of 'the energy integral', meaning the seminetrics

$$\iint_{\Omega} \left[\left(\frac{\partial^2 z}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 z}{\partial y}\right)^2\right] dxdy = minimum \tag{2.12}$$

The model obtained with the use of function type (II, 11) is characterised by hypothetical location of contour lines (isohips, isobaths). These lines are referred to as **real isarithms** on maps which is different than in the case of drawing isometric isohips in the processing of aerial photographs on autographs or mapping the results of levelling. It is the evaluation of reliability of available data that is of particular importance for quantitative information sets, which are compiled using different methods and have point references, which characterise objects and phenomena.

One of the major problems is the relationship between a discrete and continuous model of a quantitatively characterised phenomenon that is of interest to the user. Most commonly, the evaluation is made by comparing the hypothetical determinants, obtained by the continuous discrete model with a given method, and the empirical determinants with accuracy that is acceptable by the user. The correlation coefficients as well as error variance and Malow's statistics are used as measurements of adjustment (Kuchar 2001).

The comparison between the approximations accuracy with the application of neural networks of surface strings described with the shape functions, and the results of approximations of polynomial, spline functions and the kriging method (Mrówczyńska 2007) justifies the attempts to extend the application range of this group of methods from dam support and other engineering structures with shapes transformed by functions to modelling of terrain relief. The treatment of morphological terrain forms as moment representations of some stochastic processes, meaning the realisation of a complex variable function with setting the trend and the constantly changing microrelief (Borkowski 1994) not only allows the optimisation of discrete models but also enables states (relief) recognition and prediction whose reliability depends only on the stability of the nature of the process.

Just like the knowledge of the feature value characterising a full set of point objects of a given category enabled the construction of the isoline model of an important practical information (population potential) connected to the object distribution and feature value, the constant quantitative feature, recognised in a set of nodal points of a regular grid, can be observed in a relational presentation which, better than the set of absolute values, corresponds to specific practical notions.

The variability of quantitative characteristics of objects or phenomena with a continuous distribution can be presented in the form of a symbol or isoline model of, treated in an absolute manner, feature value (of temperature or height above sea level) but also relationally as a 'rate' model of local spatial variability (land slope) or a model of temporal variability (changes in temperature or land height between two measurement 'eras'). Among the models of spatial variation, the most widely used are the isoline maps of **gradient**, whereas the changes in the quantitative levels of feature between determinants in two (divided by a generating changes event) or several (important for the process) moments, are usually presented in the optimally perceptive form of diagrams.

In the case of quantitative feature characterizing a continuous phenomenon, the function series: $w = w(x, y)$ corresponds to the scalar field $D(x, y)$ where every point $P(x, y)$ corresponds to the numeric value of 'w'. The change $dw = 0$, when the motion of the point is on the equiscalar surface $w = \text{const.}$ (along the isoline), whereas the change of value dw (the increase in the function of the field) can be expressed by the exact differential

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy \tag{2.13}$$

The translation of *dn* in the perpendicular direction to the isoline of scalar field *w* (*x*, *y*) results in the following change:

$$\frac{dw}{dn} = \frac{\partial w}{\partial x}\frac{dx}{dn} + \frac{\partial w}{\partial y}\frac{dy}{dn} = tg\alpha, \qquad (2.13')$$

which in the case of $w = z(x, y)$ designates the largest decline 'rate' (in the direction of water drop falling due to gravity).

The Eq. (2.13) determines, in a given point of scalar field, the direction as well as the length of the gradient **vector**.

$$\left|\overrightarrow{grad \quad w}\right| = tg\alpha = \sqrt{\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2} \qquad (2.14)$$

with the vector being directed towards the decreasing value.

The gradient isoline maps of land height have the most widespread application. The isolines of identical land inclination make it easier to conduct preliminary area divisions on a map due to the erosion risk, crop conditions, mechanisation application, planned road, sport and recreational investments.

Isogradient maps are compiled on the basis of scalar determinants of given feature values in the nodes of a regular grid. The values of land height are derived from digital terrain models or directly from topographic maps, however, in the case of high accuracy expectations, they are derived from direct measurements.

As an example of gradient mapping of continuous feature, the most widespread instance is a vectorial or isoline gradient model of land height above sea level, built of point determinants of coordinate '*z*'. These points create a regular layout of grid nodes. The interpolation of 30 point values was conducted on the basis of topographic isohips (land ordinates) drawing. The side of the grid corresponds to the land value of 1.2 km. The area of 4.8 km × 6 km is located in the eastern part of Sudetes. The gradient vector in each central point of 30 squares of the grid is calculated by determining the approximate length value as

$$\left|\overrightarrow{grad \quad z}\right| = \frac{1}{1,2}\sqrt{(\Delta z)_x^2 + (\Delta z)_y^2}\ [\text{km}], \qquad (2.15)$$

where $(\Delta z)_x$ designates the difference of '*z*' value between the top-left and top-right node of the square and $(\Delta z)_y$—the difference between lower right and top-right node of the square. The azimuth is calculated as

$$\alpha = arctg\frac{(\Delta z)_y}{(\Delta z)_x} \qquad (2.16)$$

Table 2.8 The height differences, lengths and azimuths vectors of height gradient (fragment: 4.8 km × 6 km, Eastern Sudetes)

Node number row, column	Height differences			Azimuth
	$(z)\ x$ [m]	$(z)\ y$ [m]	$\sqrt{\left(\frac{\Delta x}{1{,}2}\right)^2 + \left(\frac{\Delta y}{1{,}2}\right)^2}$ [m/km]	α [degrees]
1.1	45	325	273.42	82.168
1.2	95	−165	158.66	299.932
1.3	−145	−225	223.06	237.200
1.4	−230	−185	245.97	218.811
2.1	185	200	227.04	47.231
2.2	−5	−215	179.22	268.668
2.3	310	15	258.64	2.770
2.4	240	−100	216.67	337.380
3.1	−55	85	84.37	122.905
3.2	5	−25	21.25	281.310
3.3	−60	−300	254.95	258.690
3.4	−10	−30	26.35	251.565
4.1	90	355	305.19	75.744
4.2	35	−80	72.76	293.630
4.3	−110	−240	220.01	245.376
4.4	−90	−80	100.35	221.634
5.1	80	240	210.82	71.565
5.2	255	−30	213.97	353.290
5.3	70	−90	95.01	307.875
5.4		−100	93.17	243.435

In Table 2.8 the value differences are compared between $(\Delta z)_x$ and—$(\Delta z)_y$ the 'z' values along the sides of subsequent squares and the calculated values of length and azimuths of gradient height vectors.

Figures 2.21 and 2.22 present two models of spatial variation of land height in the form of vectorial gradient field and isogradient map with layer tints. The directions of gradient vectors are perpendicular to isohiets. The isogradient lines have high values in places of steep slopes. The angle of inclination undergoes only a slight change in the valley of the Kamienica river (latitudinal stretch in the centre of the area) and in the proximity of eastern and western sides of the valley of Sucha Kopa and Młyńsko peaks, the change is much more visible.

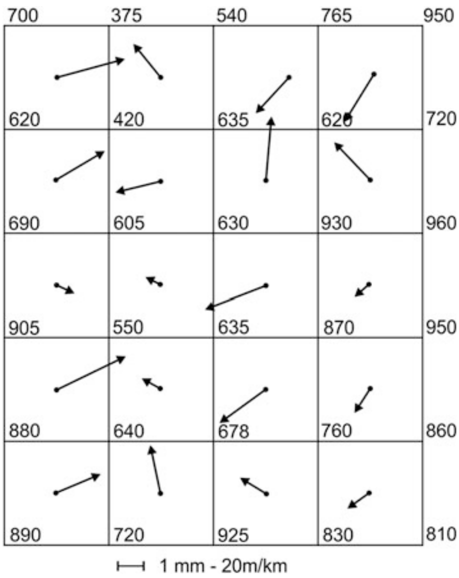

Fig. 2.21 Altitude and vectors; a fragment of 4.8, 6 km, Eastern Sudetes

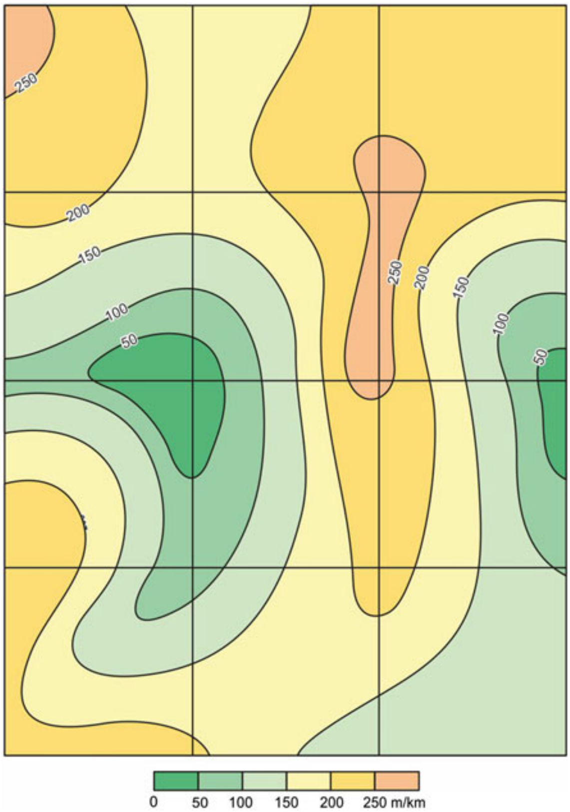

Fig. 2.22 Height isogradients, a fragment of 4.8 km × 6 km, Eastern Sudetes

References

Atlas of Agriculture of the USA and Canada (1970)
Atlas Śląska Dolnego i Opolskiego, 1997, vol II; (2008) Pawlak W (ed) Uniw. Wrocławski, Polska Akademia Nauk. [Bac-Bronowicz J., cf. Krzywicka-Blum 1991, 1992, 1994a]
Atlas gimnazjalny PPWK (1999) Warszawa-Wrocław
Atlas świata PPWK (1974) Warszawa-Wrocław
Bac-Bronowicz J (1997) Model symulacyjny warunków aerodynamicznych modyfikowanych przez zabudowę miejską. Zesz. Nauk. Ak. Rol. we Wrocławiu, Geod. i Urz. Rol., vol XIV, no. 324
Bac-Bronowicz J (2003) Określenie wiarygodności informacji uzyskanych z modeli numerycznych elementów przyrodniczych na przykładzie opadu atmosferycznego. Geoinformatica Polonica, vol 5, Kraków
Bac-Bronowicz J, Grzempowski P (2011) Klasyfikacja obszarów ze względu na wybrane warunki środowiskowe w relacji opadu na obszarze Dolnego Śląska. Rocz. Geom, vol. 9(43), no. 3, pp. 7–16
Bałut A (1988) Modelowanie odkształceń obiektów za pomocą funkcji giętych. Prace Naukowe Inst. Geotechn., no. 55
Borkowski A (1994) Stochastisch-geometrische Beschreibung, Filterung und Präsentation des Reliefs. Techn. Univ. Dresden, Abteilung Geod/Kart Geographie—Ph.D. thesis. München Verlag der Bayerischen Akademie der Wissenschaften, 1988
Bugaj A (2004) Percepcyjne i funkcjonalne uwarunkowania zastosowania metod kartodiagramu i punktowej na mapach regionalnych i krajowych. MA thesis, Uniwersytet Przyrodniczy we Wroclawiu
Górnika P (1972) Mapy oraz wykresy jakościowe zasobności. Geometryzacja złóż. 2nd edn., pp 1331–1337, Katowice
Konias A (2000) Kartografia topograficzna Śląska Cieszyńskiego i zaboru austriackiego od połowy XVIII wieku do początku XX wieku. Katowice
Kraak M-J, Ormeling F (1998) Kartografia i wizualizacja danych przestrzennych. PWN, Warszawa
Krzywicka-Blum E (1980) Konstrukcja izarytm na mapach miąższości złóż. Ses. Nauk. Wydz. Melior. Wodnych Akad. Roln. we Wrocławiu, vol 2, pp 3–11
Krzywicka-Blum E (1994a) Metody kartograficzne w ocenie zanieczyszczeń powietrza atmosferycznego (chapter: powietrze). In: Środowisko Wrocławia. Informator, Rada Miejska Wrocławia, Komisja Ochrony Środowiska
Krzywicka-Blum E (1994b) Nowa metoda analizy i prezentacji zmienności skali dawnych map dużych obszarów. Polski Przegl Kartogr 26(2):75–85
Krzywicka-Blum E (2003) Agregacja danych punktowych i pól odniesienia a informacyjne własności map gęstości. Polski Przegląd Kartograficzny, Vol 35 No. 3, Warszawa, pp. 175–184
Kuchar L (2001) Ocena modeli matematycznych na podstawie testu Cross Validation, Przegląd Naukowy, Warszawa, SGGW, Wydz. Inżyn. i Kszt. Środowiska, vol 21. [Kuchmister J., cf. Krzywicka-Blum 2005]. [Lam N., cf. Cao 1997]
Mapa topograficzna (2005)
Mitasova H, Hofierka M, Zlocha M (1990) Kartografice modelowanie ploch a telies splainami s tenziou. Geodeticky a kartograficky obzor, 36(78), Praga. [Mitdbe T., cf. Bjorke J.T.]
Mościbroda J (1999) Mapy statystyczne jako nośniki informacji ilościowej. Wyd. Uniw. M. Curie-Skłodowskiej w Lublinie
Mrówczyńska M (2007) Estymacja błędów modelu metodą sieci neuronowych. Acta Scientiarum Polonorum, Geodesia et Descriptio Terrarium, 6(1)

Pasławski J (1993) Jak opracować kartogram, vol II, Uniw. Warszawski, Wydz. Geogr i St. Regionalnych
Pasławski J (2006) Kartograficzne metody prezentacji. [in:] Wprowadzenie do kartografii i topografii, red., pp 196–234, Wyd. Nowa Era, Warszawa
Rocznik Statystyczny (1998) GUS, Warszawa
Sjerbjenjuk S, Musin O (1986) Awtomaticzeskoje postrojenie kart i proizwodnych ot nich izobrażjenij. Gjeodezija i Kartografija, no. 7
Steinhaus H (1954a) Length, shape and area. Colloqvium mathematicum, no. 3, pp 11–13
Steinhaus H (1954b) O ścisłości w matematyce. MATEMATYKA, no. 3, Wrocław
Strzelecki K (2012) Mapa Wisły Franciszka Floriana Czackiego z XVIII wieku w świetle analizy geograficznej i kartograficznej. Ph.D. thesis, Uniw. Wrocławski (2013)
Wasiljenko WA (1983) Spłajn-funkci: tjeorija, ałgoritmy, programy. Nowosybirsk, Nauka 2
www.gazetawroclawska.pl (2011)
Wysocki J (1997) Wykorzystanie numerycznego modelu terenu (NMT–DTM) dla potrzeb inżynierii środowiska. Konf. Kart. Akad. Roln., Wydz. Melioracji i Inż. Środ., Poznań. [Zbcha M., cf. Mitasova 1990]

Springer
http://www.springer.com/978-3-319-47357-4

Map Functions
Krzywicka-Blum, E.
2017, XIX, 207 p. 100 illus., 44 illus. in color., Hardcover
ISBN: 978-3-319-47357-4