Chapter 2
Vagueness, Communication, and the Sorites Paradox

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Abstract In this paper, I model first-order and higher-order vagueness, look at certain aspects of vague communication, and offer an intuitively appealing resolution of the sorites paradox.

Keywords Vagueness · Cognitive models · Theory of communication · Sorites' paradox

2.1 Introduction

Practically every word in a natural language is vague. Despite a large philosophical literature, the attention this fact has received within semantics is surprisingly scant, perhaps because quite new ideas and tools are required to accommodate it.

Parikh (1994) may have been the first person to study how communication with vague expressions can be fruitful even when the speaker and addressee interpret them differently. His paper contributed three main things. First, it showed that, within limits, vagueness is not by itself an obstacle to effective communication, a pervasive but relatively unnoticed problem. Second, it challenged the orthodoxy that communication involves the transmission of a single *objective* proposition from one agent to another. Last, it made *agency* central in such considerations in a simple and appealing way. These Wittgensteinian—and, arguably, ancient Jaina[1]—insights helped move the conversation about vagueness away from purely metaphysical and logical issues towards questions of use.

In this paper, I attend to three problems he left open in his analysis. I start by modeling vagueness, that is, characterizing clear cases and borderline cases of a

[1] See Mohanty (2000, p. 4).

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vague concept. Then I ask: *how* do speakers and addressees decide to use or interpret a vague expression in the first place? Borderline cases and, indeed, borders that shift with context pose additional difficulties in understanding natural language generation and interpretation. Finally, I tackle the *sorites paradox*. It is perhaps this puzzle that has occupied most writers on vagueness, especially philosophers and logicians. Parikh (1983) has also discussed variants of this paradox.

I approach this task by adapting models from cognitive psychology as it appears that psychologists understand this domain better than philosophers or linguists owing to their emphasis on experimental data. I also apply these adapted models in new philosophical and linguistic ways. Once I describe vagueness, it becomes easy to incorporate it in a larger theory of communication and meaning. With the right models, the sorites paradox also yields to a natural and intuitive resolution.

2.2 Vagueness

I first clarify some basic terminology. Concepts are taken to be mental representations of collections of things and categories the collections themselves. Concepts correspond to properties or attributes or features, all terms referring to abstract entities. So an agent's vague concept *bald* corresponds to a vague property of baldness. Since each agent will have a slightly different concept *bald*, the corresponding vague property can be thought of in two ways: as a kind of social and abstract average of these individual representations[2] and as a kind of abstract individual counterpart to the concept, one for each agent. Both kinds of property are important, the average kind and the individual kind. Vague words such as BALD have vague concepts conventionally attached to them that serve as their conventional meanings.

It has been taken for granted from classical times until Wittgenstein (1953/1968, sections 66 and 67, pp. 31–32) questioned it that most concepts have clear definitions, that is, non-circular necessary and sufficient conditions. This implies every object is or is not a member of the corresponding category. As discussed by Smith and Medin (1981), this classical view and its variants are untenable primarily because most concepts are vague and have borderline cases.[3] Here is Murphy's (2004, p. 21) account of *why* vagueness is ubiquitous:

> The Necessity of Category Fuzziness
>
> The existence of unclear examples can be understood in part as arising from the great variation of things in the world combined with the limitations on our concepts. We do not wish to have a concept for every single object—such concepts would be of little use and would require enormous memory space. Instead, we want to have a relatively small number of concepts that

[2] In general, properties are individuated from *reality* by agents and so are social but abstract constructs that nevertheless have a certain objectivity. For example, the number 5 can be thought of as being abstracted from collections of five objects just as the property of being blue can be thought of as being discriminated from blue objects.

[3] The other important theoretical reason is that they are unable to account for typicality effects. And much of the experimental evidence disconfirms them.

> are still informative enough to be useful (Rosch 1978). The ideal situation would probably be one in which these concepts did pick out objects in a classical-like way. Unfortunately, the world is not arranged so as to conform to our needs.
>
> The gradation of properties in the world means that our smallish number of categories will never map perfectly onto all objects: the distinction between members and nonmembers will always be difficult to draw or will even be arbitrary in some cases. If the world existed as much more distinct clumps of objects, then perhaps our concepts could be formed as the classical view says they are. But if the world consists of shadings and gradations and of a rich mixture of different kinds of properties, then a limited number of concepts would almost have to be fuzzy.

As described by Murphy (2004, Chap. 3), there are three new views that have emerged: the exemplar approach, the prototype approach, and the knowledge approach. The first uses the information provided by each encounter with an exemplar for a category separately; the second works with a *summary representation* of a category derived from the experience of exemplars; and the third integrates concepts with the broader knowledge schemata in which they must reside via plausible reasoning.

Because concepts are used in very diverse tasks, none of these approaches is able to account for all the empirical data. Indeed, as Murphy (2004, Chap. 13) concludes, some amalgam of the three will probably be required for a "Big Theory of Concepts" as they also appear to be somewhat complementary in their explanatory adequacy. In other words, each approach focuses on a different source of information, and any one or more of these sources may be summoned for a particular task based on their suitability.[4]

I will adapt the first two approaches for my purposes. The third knowledge approach is, in a sense, not really an independent stand-alone approach but one that operates by combining knowledge effects with one or both of the other approaches. I construct the simplest models required for the tasks at hand and do not aim at more comprehensive versions.

Basic Setup

Both the exemplar and prototype approaches rely on the idea that each exemplar of a concept has multiple properties that take on particular values. For example, the concept *bald* may involve features such as the number of hairs on the scalp, the number of completely hairless patches on the scalp, the fraction of the scalp that is hairless, and so on;[5] each exemplar will instantiate these attributes with particular numbers. In other words, each concept is associated with an n-dimensional attribute space and each exemplar can be represented by a point in this space. Some dimensions may be continuous and some may be discrete but this should not affect the basic idea.

Let b_i with $i = 1, 2, \ldots, N$ be clear exemplars of *bald* that an agent $\mathcal{A}$ has encountered in his experience. Likewise, let $b'_{i'}$ with $i' = 1, 2, \ldots, N'$ be clear exemplars of

[4] I apologize to the reader for this extremely abstract summary of a very large and fascinating field.

[5] Such a listing of features is an idealization as they are not entirely independent of one another. It is not clear, however, whether agents actually operate with completely independent attributes. Presumably, this depends on what they know and this is one way in which knowledge effects enter.

not bald. A negative category such as the latter is a little unusual in that it contains not just persons with a full head of hair but also random items such as clocks and cars. There is no problem with this as all potential exemplars are assessed relative to the relevant attributes which come from the corresponding positive category. Thus, only persons with relatively full heads of hair will qualify as exemplars and items such as clocks and cars will be discarded as junk. A different way to think about negative categories is that in any particular situation where its exemplars are accessed there will always be a default reference category that will automatically limit the possibilities to the relevant types of individuals. In the case of *not bald*, the possibilities will be limited to persons; in the case of *not tall* to men or women or basketball players depending on the situation; and in the case of *not chair*, the default category might be items of furniture.

Let x_{ij} be the value of the jth attribute of b_i and, similarly, let $x'_{i'j}$ be the value of the jth attribute of $b'_{i'}$. That is, $b_i = (x_{ij})$ and $b'_{i'} = (x'_{i'j})$, the right-hand side of both equalities being vectors with $j = 1, 2, \ldots, n$.

Now suppose $\mathcal{A}$ has to judge whether the candidate a is bald or not bald or borderline bald in some situation u. Then a will also be a point in the same space with value x_{aj} in the jth dimension. That is, $a = (x_{aj})$.

The basic idea underlying both approaches is to see how "far" a is from all the exemplars taken separately or from an "average"exemplar (i.e. the prototype) and, based on this, to see how similar a is to the other members in the category. This computation allows $\mathcal{A}$ to decide where a stands with respect to *bald*.

The Exemplar Model

This model has its roots in Medin and Schaffer (1978), Nosofsky (1992), and Nosofsky and Palmeri (1997). I build upon the description in Murphy (2004, pp. 65–71). Schiffer (2010) informally mentions the possibility of using weighted distance in the context of vagueness.

In order to get at the psychological distance between a and b_i, we need to first note the following. For certain attributes such as the number of hairs on an individual's scalp, if a's value x_{a1} is less than b_i's value x_{i1} then the psychological difference between these values along this dimension is not $|x_{a1} - x_{i1}|$ but 0 because b_i is an *exemplar* and a has, so to speak, met the *bar* set by b_i. Likewise, if the attribute is the number of completely hairless patches on the individual's scalp, if $x_{a2} > x_{i2}$ then again the difference is 0 by the same reasoning. There may, of course, be attributes where only an exact equality $x_{aj} = x_{ij}$ results in a zero difference.[6] Which of these cases obtains depends on the particular concept and the attribute in question. So we can define a psychological difference function $\delta(x_{aj}, x_{ij})$ which is either 0 or $|x_{aj} - x_{ij}|$ based on the nature of the concept and attribute being considered.[7]

[6]For example, the category *blue* is such because overshooting the relevant color frequency in either direction counts as a non-zero difference. With such attributes, only an exact equality results in a zero difference.

[7]There is some empirical warrant for such a result as reported in, for example, Hampton et al. (2005).

Now define the weighted psychological distance between a and b_i as follows:

$$d_u(a, b_i) = \sqrt{\sum_{j=1}^{n} w_j(u)\,\delta(x_{aj}, x_{ij})^2}$$

Here, $w_j(u)$ are weights issuing from the situation u. The psychological distance that $\mathcal{A}$ perceives between a candidate and an exemplar thus varies with the situation he is in. This variation implies that certain attributes and therefore certain exemplars will play a more or less important role in $\mathcal{A}$'s judgment.

This distance function is *not* a metric in the technical sense as it is not symmetric: $d_u(a, b_i)$ may not equal $d_u(b_i, a)$ because the underlying psychological difference function δ is not symmetric. Also, many different forms for it can be used; I have restricted myself to the commonest Euclidean variety.

Correspondingly, the weighted psychological distance between a and $b'_{i'}$ will be:

$$d_u(a, b'_{i'}) = \sqrt{\sum_{j=1}^{n} w'_j(u)\,\delta(x_{aj}, x'_{i'j})^2}$$

Shepard (1987) has shown that behavioral similarity between items is an exponentially decreasing function of their psychological distance.

$$s_u(a, b_i) = e^{-c(u)d_u(a,b_i)}$$

where $c(u) > 0$ is a situation-based parameter. Again, a larger or smaller $c(u)$ will determine the relative importance of items that are near and items that are far.

Analogously:

$$s_u(a, b'_{i'}) = e^{-c'(u)d_u(a,b'_{i'})}$$

Finally, define the psychological probability that a is bald rather than not bald for the agent $\mathcal{A}$ as follows:

$$P(bald \mid a; u) = \frac{\sum_{i=1}^{N} s_u(a, b_i)}{\sum_{i=1}^{N} s_u(a, b_i) + \sum_{i'=1}^{N'} s_u(a, b'_{i'})}$$

Then:

$$P(not\ \ bald \mid a; u) = \frac{\sum_{i'=1}^{N'} s_u(a, b'_{i'})}{\sum_{i=1}^{N} s_u(a, b_i) + \sum_{i'=1}^{N'} s_u(a, b'_{i'})}$$

Note that $P(bald \mid a; u) + P(not\ \ bald \mid a; u) = 1$. These psychological probabilities are measured with respect to the agent $\mathcal{A}$ as they are based on his exemplars. So they are agent-relative probabilities. But they are not "subjective" probabilities in the usual sense of being $\mathcal{A}$'s beliefs. That is, an agent's beliefs are not related in the way psychological probabilities are to his exemplars. This means the latter are not open to the standard charges against epistemic probabilistic accounts that are based on an agent's beliefs.

The probabilities depend on the situation u which is a parameter, not a conditioning random variable. If we wish, we can introduce weights $v_i(u)$, $v'_{i'}(u)$ for each pair of similarities $s_u(a, b_i)$, $s_u(a, b'_{i'})$ but I will not. They would allow us to weight different exemplars directly.

I will return to these identifications after describing the prototype model.

The Prototype Model

This model has its roots in Rosch and Mervis (1975) but the account below is based on a certain natural construal of a summary representation of a category.

The only difference between the exemplar model and the prototype model is that the latter does not compute the psychological distance between the candidate and each exemplar separately as above but first averages the values of all the exemplars and then computes the distance.

So we first define the average values as follows:

$$\overline{x_j} = \frac{\sum_{i=1}^{N} w_i(u) x_{ij}}{N}$$

$$\overline{x'_j} = \frac{\sum_{i'=1}^{N'} w'_i(u) x'_{i'j}}{N'}$$

This tells us that the prototypes for *bald* and *not bald* are just $\overline{b} = (\overline{x_j})$ and $\overline{b'} = (\overline{x'_j})$. The weights $w_i(u)$, $w'_i(u)$ are different from the earlier weights above described in the exemplar model, and are also indexed with respect to i and not j as before, but I have used the same letters to avoid using an excessive number of symbols. These weights play an important role because with many categories such as *bald* it is not ordinary averages that count as prototypes but extreme examples such as a completely hairless person. Such extreme exemplars can be selected as the relevant prototypical average by adjusting the weights suitably. Alternatively, they can be selected as the minimum or maximum of the relevant attribute values. Generalized means are a family of functions for aggregating sets of numbers and we can draw upon any of these based on the nature of the concept and its attributes.[8]

We can use the same idea for the psychological difference as before except that it is measured with respect to the average values. So $\delta(x_{aj}, \overline{x_j})$ is either 0 or $|x_{aj} - \overline{x_j}|$ based on the nature of the concept and attribute being considered and likewise with $\delta(x_{aj}, \overline{x'_j})$.

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[8] See, for example, http://en.wikipedia.org/wiki/Generalized_mean.

Now the weighted psychological distance from the prototype is defined as follows:

$$d_u(a, \overline{b}) = \sqrt{\sum_{j=1}^{n} w_j(u)\ \delta(x_{aj}, \overline{x_j})^2}$$

$$d_u(a, \overline{b'}) = \sqrt{\sum_{j=1}^{n} w'_j(u)\ \delta(x_{aj}, \overline{x'_j})^2}$$

This in turn leads to similarity.

$$s_u(a, \overline{b}) = e^{-c(u)d_u(a,\overline{b})}$$

$$s_u(a, \overline{b'}) = e^{-c'(u)d_u(a,\overline{b'})}$$

And finally to the psychological probabilities for $\mathcal{A}$ as above.

$$P(bald \mid a; u) = \frac{s_u(a, \overline{b})}{s_u(a, \overline{b}) + s_u(a, \overline{b'})}$$

$$P(not\ \ bald \mid a; u) = \frac{s_u(a, \overline{b'})}{s_u(a, \overline{b}) + s_u(a, \overline{b'})}$$

As stated above, the key difference is that distances and similarities are measured with respect to a "summary representation," an *average* (or generalized mean) of all the exemplars.

Both models give us somewhat different ways to compute the same psychological probabilities $P(bald \mid a; u)$ and $P(not\ bald \mid a; u)$. I now put them to use.

Characterizing Vagueness

Intuitively, if a candidate is sufficiently similar to clear exemplars of both *bald* and *not bald*, it is reasonable to think it is a borderline case. This suggests the following definitions.

Definition 1 A candidate a is a borderline case of a concept C for an agent $\mathcal{A}$ in situation u if and only if $|P(C \mid a; u) - P(not\ C \mid a; u)| < \epsilon_u$ where $0 < \epsilon_u < 1$ is $\mathcal{A}$'s threshold in u.[9] If a is not a borderline case, then it is classified as clearly belonging to C if and only if $P(C \mid a; u) > P(not\ C \mid a; u)$ and as clearly belonging to *not C* if and only if $P(C \mid a; u) < P(not\ C \mid a; u)$.

[9] I have deliberately suppressed the agent in specifying the probabilities and threshold to avoid notational clutter.

Definition 2 A concept is vague for an agent $\mathcal{A}$ in situation u if and only if it has borderline cases. Otherwise, it is precise or "classical."

The decision to count an item as borderline or clear is *derived from* agent-relative psychological probabilities but is not itself probabilistic or belief-based (e.g. a is borderline with "subjective" probability q). So it is doubly immune to the charge against subjective probability views made by Schiffer (2003, Chap. 5): it is not based on beliefs and it is deterministic.

It also does not follow the Luce Choice Rule as described by Murphy (2004, p. 69) which enjoins the agent to make judgments that are themselves probabilistic. This rule has some empirical support for a task such as classifying an animal as a dog, cat, or burro, but has probably not been tested against my way of using these probabilities to get at complementary categories such as *bald* and *not bald*.

The threshold ϵ_u can be thought of in different ways. It can be conceived as a precise number or a fuzzy number.[10] It should also be seen as something the agent does not know for himself in different situations. That is, it operates sub-personally or non-consciously and the agent may lack a firm conviction about a particular decision to count an item as borderline or clear. Lastly, the situated threshold arises through communicative interactions and so agents in the same community tend to share it to a greater degree than intuition might suggest. Here, knowledge effects of the kind alluded to earlier when I described the knowledge approach to concepts may play an important role as such thresholds tend to partly arise also from the goals and interests of agents.

$\mathcal{A}$'s cognitive system determines when a case is borderline or clear. It is not entirely a conscious decision. This is confirmed by the familiar feeling of being stymied when we are asked to make a conscious judgment about a borderline case. There is simply no way to reason *decisively* about it based on the external facts. In a sense, the exemplars and the thresholds are the inaccessible "hidden variables" of the decision process. Indeed, when we scrutinize a judgment we have arrived at we may become uncertain about it as there is *no non-vague argument* available for the *feeling* that a is borderline. It is largely sub-personal. And all this is intensified when the threshold is a fuzzy number rather than a precise number.

An item a that is a borderline case for one agent need not be so for another agent. Likewise, the borderline cases of a concept shift with u for the same agent because all aspects of the definition depend on u, the probabilities as well as the threshold. The same agent $\mathcal{A}$ may choose to call Alex bald in one situation but not in another as the sentence ALEX IS BALD may be true for $\mathcal{A}$ in one situation, false

[10] A fuzzy number A is generally expressed as a function $A : \mathbb{R} \to [0, 1]$ such that:

$$A(x) = \begin{cases} f(x) & \text{for } x \in [a, b] \\ 1 & \text{for } x \in [b, c] \\ g(x) & \text{for } x \in [c, d] \\ 0 & \text{for } x < a \text{ and } x > d \end{cases}$$

where $a \le b \le c \le d$, f is a continuous function that increases to 1 at point b, and g is a continuous function that decreases from 1 at point c. See Klir et al. (1997, p. 170).

for $\mathcal{A}$ in another, and indeterminate for $\mathcal{A}$ in a third. Consider the sentences SHE WON'T DATE ALEX—HE'S BALD and ALEX ISN'T BALD—HE NEEDS A HAIRCUT. In the first case, the situation u is such that a less stringent membership condition for *bald* is operative, either because relatively less bald exemplars are weighted more or because the magnitude of the threshold ϵ_u is relatively smaller making the penumbra, the region of borderline cases, correspondingly smaller as well, or possibly because both factors apply simultaneously. In the second case, the situation u is the opposite—a more stringent membership condition is used. This explains what Schiffer (2010) calls *penumbral shift* in a natural way.

In order to obtain "natural" concepts of the kind that would be useful in thinking and communication, we must assume that the positive exemplars (i.e. the set $\{b_i\}$ for *bald*) are so distributed that all members of their convex closure are instances of C. Otherwise, we would get strange and seemingly arbitrary outcomes for what belongs to C, what is borderline C, and what is not C. This requirement translates into a restriction on ϵ_u: it must be sufficiently small. In other words, if $b = (b_i)$, $b' = (b'_{i'})$, the latter in each case being the vectors of positive and negative exemplars of *bald*, then $0 < \epsilon_u < \kappa_u(b, b') < 1$ where $\kappa_u(b, b')$ is a function of the N positive exemplars and N' negative exemplars that derives from the convexity assumption. The same kind of condition is obviously not required for the negative exemplars (i.e. the set $\{b'_{i'}\}$ for *bald*) as they can, in general, lie anywhere outside the convex closure of the positive exemplars in the n-dimensional attribute space. A candidate a that is judged to belong to C *need not* lie within the convex closure. All that is required is that it be sufficiently close to it. Indeed, subsequently, a would become a positive exemplar itself and the convex closure could be correspondingly enlarged. This suggests a dynamic model of concept learning that results in possibly expanded convex closures as more exemplars are encountered. After a while, the category would converge to a convex polytope in the attribute space with somewhat different boundaries for different situations u.

This convexity assumption is very similar to the convexity assumption made by Gärdenfors (2000) and Warglien and Gärdenfors (2013). The approach I have adopted of using exemplars to derive concepts seems to allow a clearer development of these ideas from a more foundational starting point. Also, their decision to banish the *external* significance of language from their model seems unnecessary and raises too many problems (e.g. Barwise and Perry 1983, pp. 28–31; Putnam 1975). As I discuss briefly in the next section, conventional meanings are mental representations but referential meanings are external entities such as the individuals and properties that make up propositions. It is possible to have one's cake and eat it too.

What may be true or false or indeterminate for one agent may not be so for another. There is *no* agent-independent or objective truth value in other words. However, because the agents must belong to the same linguistic community and the exemplars each agent draws upon are often shared through communication, they may agree more often than expected. In fact, for a concept to be socially useful as most are, its exemplars must be *sufficiently* shared among the community. This points to a community model of interacting agents where concepts are constantly being revised to have sufficient overlap of the kind explored by Parikh (1994).

The definitions above suggest that if the threshold is a precise, non-fuzzy number there is no *higher-order vagueness*, the phenomenon that the borderline between clear and borderline cases is itself unclear, resulting in borderline borderline cases and so on ad infinitum. The foregoing implies an epistemic view of this as agents are typically unaware of their thresholds in particular situations.

If we wish to allow for higher-order vagueness, we can identify the threshold with a fuzzy number. In this case, there is no precise cut-off between clear and borderline cases and higher-order vagueness can be admitted. I believe my model is richer than Hampton (2007) because it allows for both graded membership in a category as well as fuzzy judgments of when a case is clear or borderline whereas Hampton (2007, p. 377) only allows for the former while pointing to the latter as important experimental evidence. Something akin to Definition 1 would have to be introduced into his model.

Much of the evidence seems to indicate that higher-order vagueness is real. This implies that the threshold ϵ_u must be a fuzzy number and not a precise number. However, for the purposes of this paper, I keep the matter open and continue to discuss both possibilities.

The mistake "epistemicists" such as Williamson (1994) seem to make is to assume the existence of sharp thresholds between clear cases of a concept and its complement. That is, they not only reject higher-order vagueness but also first-order vagueness, which is completely unrealistic. It is easy enough to introduce precise thresholds between clear and borderline cases and thus allow for first-order vagueness without higher-order vagueness; further, for greater realism, by taking the threshold to be a fuzzy number, we can accommodate higher-order vagueness as well.

Indeed, it is possible to *characterize* higher-order vagueness by treating Definition 1 as a base case for an inductive definition. The key idea is to identify exemplars at each level, and therefore, psychological distance, similarity, and psychological probabilities at each level. For example, Definition 1 provides a precise or fuzzy account of first-order borderline cases. Then we can identify positive and negative exemplars for what is clearly borderline and what is clearly not borderline which, in turn, gives rise to psychological distance and similarity and psychological probabilities at the next level. The latter can then be used in a manner analogous to Definition 1 to define borderline borderline cases or, in other words, second-order vagueness. And so on to higher-order vagueness for all n.

In the definition below, I assume that we have the exemplars for nth order vagueness and therefore the nth order psychological probabilities $P_n(C_n \mid a; u)$ and $P_n(not\ C_n \mid a; u)$ where C_n is the nth order concept of being nth order borderline C and $not\ C_n$ is the corresponding complementary concept. When $n = 0$, this is understood as just standing for the concepts C and $not\ C$.

Definition 3 A candidate a is an $(n + 1)$st order borderline case of a concept C for an agent $\mathcal{A}$ in situation u if and only if $|P_n(C_n \mid a; u) - P_n(not\ C_n \mid a; u)| < \epsilon_{n,u}$ where $0 < \epsilon_{n,u} < 1$ is $\mathcal{A}$'s threshold in u. If a is not an $(n + 1)$st order borderline case, then it is classified as being clearly nth order borderline or clearly not nth order borderline according as $P_n(C_n \mid a; u)$ or $P_n(not\ C_n \mid a; u)$ is greater.

Combining this definition with Definition 1 yields a characterization of higher-order vagueness for all n. Now, the threshold $\epsilon_{n,u}$ has to be understood as fuzzy and this gives rise to a fuzzy *fractal*[11] set with no crisp boundaries even in the limit. There is some indirect evidence for the fractal nature of higher-order vagueness in Hampton et al. (2012). In practice, of course, an agent will not actually possess the threshold $\epsilon_{n,u}$ for all n, only for the first and possibly second order borderline case as there is no practical utility in having such thresholds.

A slightly different way to approach vagueness is described below.[12]

Definition 4 $P(\mathit{borderline}\ C \mid a; u) = 1 - |P(C \mid a; u) - P(\mathit{not}\ C \mid a; u)|$.

Now we have $P(C \mid a; u)$, $P(\mathit{not}\ C \mid a; u)$, and $P(\mathit{borderline}\ C \mid a; u)$.[13] These psychological probabilities can be used to define borderline cases in different ways. One possible decision rule is that $\mathcal{A}$ judges a to be borderline $P(\mathit{borderline}\ C \mid a; u) \times 100$ percent of the time. For example, if $P(\mathit{borderline}\ C \mid a; u) = 0.6$ then $\mathcal{A}$ will judge a to be borderline C 60 % of the time.

Definition 5 A candidate a is a borderline case of a concept C for an agent $\mathcal{A}$ in situation u $P(\mathit{borderline}\ C \mid a; u) \times 100$ percent of the time. If a is not judged to be a borderline case, then it is classified as clearly belonging to C if and only if $P(C \mid a; u) > P(\mathit{not}\ C \mid a; u)$ and as clearly belonging to *not* C if and only if $P(C \mid a; u) < P(\mathit{not}\ C \mid a; u)$.

This decision rule implies that there is no sharp cut-off between clear and borderline cases and so also leaves open the possibility of higher-order vagueness which can be defined in analogy with Definition 3. However, this definition faces certain problems as it may lead to odd results when a case that would ordinarily (e.g. by Definition 1) be judged to be borderline is not so judged owing to the decision rule being probabilistic. Incidentally, to the extent Definition 5 involves a probabilistic rule, it does follow something like the Luce Choice Rule referred to above.

So far, only vague concepts have been defined. Vague properties depend on the community's vague concepts.

Definition 6 A property is vague if and only if it is based on the community's corresponding vague concepts.

Definition 6 covers both types of property, the average kind and the individual kind. It is deliberately vague as there are somewhat messy issues relating to what happens if some members of the community have incorrect concepts and also if some members are vague and others are precise about the same concept. As should be obvious, the same sorts of observations, mutatis mutandis, follow for vague properties. However, aggregate vague properties are objectively true or false or indeterminate as they are derived from the concepts of all the individuals in a community. But there is no way

[11] A fractal is an object or quantity that displays "self-similarity" on all scales.

[12] The general idea was suggested to me by Gregory Murphy. I have fleshed out the details.

[13] There are overlaps among these probabilities so they do not sum to 1.

to know with certainty which of these truth values actually obtains as we have only approximate epistemic access to such a property.

The alert reader will have noticed that the apparatus of similarity and similarity-based probabilities is in fact unnecessary for these definitions. One could directly define borderline and clear cases once the notion of psychological distance is available. However, I do not do this in order to more closely mimic the psychological literature for greater realism and also to connect with other non-distance-based ways of capturing similarity as the latter seems like the fundamental idea. That is, instead of dispensing with similarity, we could dispense with distance, and define similarity on some different underlying basis.

We are now ready to apply these ideas to communication and to the sorites paradox.

2.3 Communication

As argued in Parikh (2010), communication and meaning involve four constraints: phonetic, syntactic, semantic, and flow. Here, I will confine myself to the semantic constraint that says that every word in an utterance is transformed by a conventional map into its conventional meaning(s) and then further transformed by an informational map into its referential meaning(s). This scheme is a generalization and refinement of Frege (1892/1980) system of sense and reference by which a word is converted first into its sense which is then converted into its reference. Each conventional meaning is the word's conventionally associated concept for the speaker and the addressee, and each referential meaning is the corresponding individuals and properties into which the former is mapped relative to the utterance situation u.

Pictorially:

$$\text{word} \longrightarrow \text{conventional meaning(s)} \xrightarrow{u} \text{referential meaning(s)}$$

Symbolically:

$$\omega \longrightarrow C^{\omega} \xrightarrow{u} P^{\omega}$$

assuming the concept C^{ω} is converted into the corresponding property P^{ω}.[14] The u on top of the second arrow implies that it is an argument of the informational map together with the conventional meaning. This schema combines the internal and external significance of language.

Consider now an utterance by $\mathcal{A}$ to $\mathcal{B}$ in u of the sentence $\varphi =$ ALEX IS BALD. Then the two maps above will apply to each of the three words in the utterance. As our interest is in BALD, we get the picture:

$$\text{BALD} \longrightarrow C^{\text{BALD}} \xrightarrow{u} P^{\text{BALD}}$$

[14]The symbol P is being used for both the property and for probability but there should be no confusion.

Each concept and property can be understood as marked by the relevant agent $\mathcal{A}$ or $\mathcal{B}$ implying that there are actually *two* such arrow diagrams for the word. The vague concept C^{BALD} can be more or less any of the many situated concepts that $\mathcal{A}$ (or $\mathcal{B}$) has used in the past or it can be some average of these. Further, in the current situation u, it gets transformed via a new u-relative concept into its corresponding vague property. That is, the concept that is the conventional meaning shifts to a related concept relative to u and thence to the corresponding property.

The property in question can also be taken to be the *intersubjectively derived* property rather than the subjective property and it can be assumed that each agent has just a partial understanding of the content of the utterance. Also, in practice, there would be more than two diagrams as the word BALD is *ambiguous* besides being vague. For example, it can also mean *plain* or *blunt* as in A BALD STATEMENT. So there will be two or more conventional meanings, each of which will be mapped into their respective referential meanings. But we can ignore this complexity here.

For our purposes, we need to ask what made $\mathcal{A}$ utter φ in the first place. In order to do so, he would have had to determine that Alex *is* bald and to make that determination he would have resorted to the calculations of the previous section. That is, he would have ascertained that $|P(C^{\text{BALD}} \mid \text{Alex}; u) - P(not\ C^{\text{BALD}} \mid \text{Alex}; u)| \geq \epsilon_u$ and that $P(C^{\text{BALD}} \mid \text{Alex}; u) > P(not\ C^{\text{BALD}} \mid \text{Alex}; u)$ as Definition 1 requires. Because the threshold is more likely to be fuzzy, and fuzzy numbers involve membership functions, the calculation will involve some situated rule based on interval arithmetic for deciding what degree of membership is sufficient for counting someone bald. A similar approach would be followed if we used Definition 5. In other words, such calculations are an integral part of natural language generation. Since most words in language are vague, this shows that speakers have quite a bit to do and it is something of a psycholinguistic mystery how so much is accomplished so quickly. Perhaps the human brain's parallel processing just is very fast with such probabilistic comparisons.

On $\mathcal{B}$'s side, she simply has to access her concept C^{BALD} and then her property P^{BALD} but if she wants to decide whether to agree or disagree with $\mathcal{A}$, then she too has to go through the same arithmetic with her own threshold.

Penumbral shift, the change of truth value of the same utterance in different situations, does not seem to raise any special problems once the general context-sensitivity of language is accounted for as in my book cited above.

It turns out, therefore, that the conceptual difficulties vagueness poses do not overly complicate our semantic frameworks. There is, of course, a great deal more to communication and meaning than I have indicated here but many of those complexities occur with non-vague language as well. One somewhat new phenomenon that arises with vague concepts is how precise an agent needs to make his utterance to balance the conflicting demands of costs and benefits. This is similar but not identical to the issue of how much to explicitly disambiguate the lexical and structural ambiguities in a sentence. I address these game-theoretic issues in Parikh (2018) as they would take up too much space here. So once one has the right way to view vagueness, its apparent hurdles seem to melt away.

2.4 The Sorites Paradox

The sorites paradox can be formulated for any vague property. It consists of the following type of argument:

1. A hairless person is bald.
2. For all k, if a person with k hairs is bald then a person with $k + 1$ hairs is bald.
3. Therefore, all persons are bald.

Most proposed solutions to the paradox deny the second premise on either semantic or epistemic grounds. Schiffer (2003, Chap. 5) does an able job of dispelling such proposals. My resolution also denies the same premise but on *psychological* grounds. First, consider an agent-relative concept-based (or individual property-based) restatement of this premiss:

- For all k, if $\mathcal{A}$ judges a person with k hairs to be bald then he would judge a person with $k + 1$ hairs to be bald.

The key to the resolution is that such judgments are made on the basis of *multiple* exemplars or a category prototype which is also based on multiple exemplars. So it is quite possible for $\mathcal{A}$ to judge a person with k hairs to be bald and then judge a person with $k + 1$ hairs to be only borderline bald for some definite or fuzzy $k^\star$. This would be true even if we restricted our attention to all the exemplars in the sequence that have already been judged by $\mathcal{A}$ to be bald. That is, as the value of k increases, the distance from the early members of the series (or from the dynamically changing prototype) keeps growing, their similarity keeps dropping, and, at $k^\star$, $\mathcal{A}$ finds himself with a borderline case that is not clearly bald. This follows easily from Definition 1.

I am *not* saying that there is a definite cut-off between bald and not bald as many solutions do; I am saying there is either a precise or fuzzy cut-off between clearly bald and borderline bald (and also between borderline bald and not bald). Whether $k^\star$ is a precise or fuzzy number depends on whether $\mathcal{A}$'s threshold ϵ_u is a precise or fuzzy number. Moreover, $\mathcal{A}$'s own cut-off will change with the situation u in which he is asked to make the judgments because ϵ_u depends on u. Finally, different agents will have different cut-offs because their thresholds will generally be a little different.

The reason why the sorites argument seems plausible is that its formulation tricks us into consciously focusing on just a single exemplar: the previous case k in the second premise. Because our judgments about vague concepts are typically sub-personal and non-conscious, we are not aware of the multiple exemplars and thresholds that go into our judgments. Indeed, our judgments also lack the kind of firm conviction we have in judging that $2 + 2 = 4$ and we may waffle over the exact value of $k^\star$. So the sorites works by forcing us to make intermediate judgments in a conscious and unnatural way and then freeing us to judge the conclusion that all persons are bald in a non-conscious natural way.

Since properties are abstract social constructs built out of a community's individual concepts, it follows that the agent-independent property version of the sorites paradox will also have the same kind of resolution. That is, there will be some function of the individual $k^\star$ values for each agent that yields some social cut-off $K^\star$ although it will not be possible for anyone to know what its precise or fuzzy value is.

If Definition 5 is used instead, the argument is less smooth because $\mathcal{A}$ may judge a person with k hairs to be borderline and then the next person with $k+1$ hairs to be bald. And $\mathcal{A}$ may never judge a candidate to be borderline because that decision is probabilistic and may go directly from baldness to non-baldness. But, barring these odd cases, the basic idea works because $P(\mathit{borderline}\ C \mid a; u)$ keeps growing as the number of hairs grows.

Since Definition 1 gives us a knockdown argument against the sorites and allows us to consider higher-order vagueness as well, it appears that it is to be preferred to Definition 5. In any case, the key point underlying both accounts is the same: new candidates are judged against multiple exemplars, not just the previous exemplar in the sequence.

2.5 Conclusion

I have construed the exemplar and prototype approaches of cognitive psychology in certain ways to characterize vagueness, describe certain aspects of communication, and approach the sorites paradox in a new way. The models appear to have some empirical support though I want to also emphasize the *kind* of reasoning that is involved in addressing these problems. More realistic models will doubtless become available but the underlying structure of explanation is not likely to change materially.

All three themes of Rohit Parikh's paper—vague communication, the lack of objectivity, and the centrality of agency—are echoed in the present paper.

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