Chapter 2
RHC of Networked Nonlinear Systems with Two-Channel Packet Dropouts

2.1 Introduction

This chapter is focused on the RHC problem of nonlinear NCSs with two-channel packet dropouts and system constraints. In the literature, most of the available results on NCSs are focused on systems with linear models (i.e., linear NCSs). There is little effort in the study of nonlinear dynamical systems in network environments, though many practical systems are of nonlinear dynamics. Generally speaking, the literature on nonlinear NCSs can be classified into two classes. In the first class such as [4, 15, 16, 28], the method is the so-called emulation approach [26]. This approach normally requires that the nonlinear system without networks is stable and there exists a stabilizable controller to stabilize it. Using this stabilizable controller, the maximally allowable transmission interval and the maximally allowable delay can be figured out to ensure closed-loop stability and system performance [4].

In the second class, the RHC-based scheme [13] is utilized to deal with nonlinear NCSs, where the prediction property is used to compensate for communication constraints. The feasibility for using RHC-based networked controller is due to the development of Ethernet-like networks, allowing to pack data into large packets and then transmit them [2]. In the RHC-based approach, a future control sequence at each step can be generated by solving an optimization problem. With this control sequence, the communication delays and data packet dropouts can be effectively compensated if a mechanism is designed to pick up the appropriate control inputs. It is this unique feature that makes the RHC-based approach very effective for NCSs. The RHC-based approach for linear NCSs can be found in many results, such as to [2, 5, 10, 24, 27, 29, 30], to name a few.

The design and analysis of RHC for nonlinear NCSs is more challenging, and only a few results are available. In [17], the RHC strategy is designed for a class of nonlinear NCSs with state and input constraints, where the detailed compensation strategy is proposed to deal with time delays, and the theoretical results including

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feasibility and the regional ISS have been proved under certain conditions. In [14], a Lyapunov MPC strategy is proposed for the nonlinear NCSs with the sensor-to-controller (S-C) and the controller-to-actuator (C-A) packet dropouts, and the closed-loop stability is ensured. Note that [14] does not take the state constraint and input constraint into consideration. In [19], a novel RHC strategy is utilized to handle the C-A packet dropouts by make use of the properties of disturbances, and the ISS of the closed-loop system is established. Due to the fact that the communication constraints occur randomly in practice, the packet dropouts are modeled as Bernoulli processes in [18, 21, 23], where several RHC-based strategies are developed. Besides, the result on RHC-based control for nonlinear NCSs subject to Markovian packet dropouts is reported in [20].

This chapter will consider the RHC-based control problem for a class of nonlinear NCSs, where the control input constraints are considered, and both the C-A and S-C packet dropouts are allowed. The network is assumed to work in an Ethernet-like network environment. In the designed control strategy, the control packets including a control sequence are first designed by the RHC strategy. Using this control packets, a novel compensation strategy including control selection and transmission strategy is then designed. The designed strategy needs the TCP-like protocol and can alleviate the data-loss effects over the two channels. Different from the existing results in [18–21, 23], the designed compensation strategy considers the joint effects of the C-A and S-C packet dropouts explicitly, making use of the joint information into the controller design.

The main results of this chapter have been published in [8]. The main features of this chapter include

- A novel RHC-based control strategy is developed, and the novelty of this strategy lies in the new control packet design and transmission mechanism. Unlike the conventional RHC-based on control strategy that requires solving optimization problem at each time instant, the optimization problem is required to be solved only when new information is available. (The new information is figured out by jointly using data of the S-C packet dropouts and the acknowledgment packets.) Therefore, the optimization problem does not need to be solved for all the time instants, reducing computational load significantly. In addition, a new transmission mechanism over the C-A channel is elegantly designed such that only necessary control packets are transmitted from the controller node to the actuator node (and the control packet is not needed to be transmitted for all the time instants). The designed new mechanism can largely save communication resources.
- The theoretical analysis and results are provided. In this study, since the packet dropouts on the C-A and S-C channels and the new transmission strategy, the conventional optimal objective function is hard to prove to be an ISpS-type Lyapunov function. As a result, we develop a new approach to prove the closed-loop stability by designing an auxiliary optimization problem and a new ISpS-type Lyapunov function. We show that under certain conditions, the designed RHC algorithm ensures the regional input-to-state practical stability (ISpS) of the closed-loop systems.

The remainder of this chapter is organized as follows. In Sect. 2.2, the preliminaries including the network model and the stability for constrained nonlinear systems are introduced. In Sect. 2.3, the RHC-based control algorithm including the packet generation, the transmission and compensation strategy is designed. In Sect. 2.4, the theoretical results on the closed-loop stability is carried out and the regional ISpS is established. In Sect. 2.5, the application and simulation study is provided. Finally, the conclusions are given in Sect. 2.6.

The following notations are used in this chapter. The superscripts "T" and "−1" stand for the matrix transposition and the matrix inverse, respectively. $\mathbb{Z}$ ($\mathbb{Z}_{\geqslant 0}$) denotes the set of integers (non-negative integers) and $\mathbb{R}$ ($\mathbb{R}_{\geqslant 0}$) represents the real space (non-negative real space). Let $\|x\|$ denote the Euclidean norm of a given vector x and $\mathrm{col}\{x_1, x_2, \cdots, x_n\}$ denote the column operation as $[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_n^{\mathrm{T}}]^{\mathrm{T}}$ for column vectors $x_1, x_2, \cdots, x_n$. For any given N bounded discrete-time signals $\mathbf{v} = \{v_0, v_1, v_2, \cdots, v_N\}$, define the subsequence as $\mathbf{v}_{k_1,k_2} \triangleq \{v_{k_1}, v_{k_1+1}, \cdots, v_{k_2}\}$ with $k_1, k_2 \in \mathbb{Z}_{\geqslant 0}$; define the truncation as $\mathbf{v}_{[k-1]} \triangleq \{v_0, v_1, v_2, \cdots, v_{k-1}\}$ and the norm as $\|\mathbf{v}\|_\infty \triangleq \sup_{k\geqslant 0}\|v_k\|$. Given a vector $x \in \mathbb{R}^n$ and a compact set $\Omega \subset \mathbb{R}^n$, denote the point-to-set distance as $\mathrm{d}|x|_\Omega \triangleq \inf\{\|\xi - x\|, \xi \in \Omega\}$. Given two sets $A \subseteq B \subseteq \mathbb{R}^n$, the difference between the two set is defined as $A \setminus B \triangleq \{x | x \in A, x \notin B\}$. Given two sets $A \subseteq \mathbb{R}^n$, $B \subseteq \mathbb{R}^n$, the Pontryagin difference set C is denoted as $C = A \sim B \triangleq \{x \in \mathbb{R}^n | x + \xi \in A, \forall \xi \in B\}$. A closed ball centralized at a given point $x_0 \in \mathbb{R}^n$ with a radius of $r \geqslant 0$ is denoted as $\mathscr{B}^n(x_0, r) \triangleq \{\xi \in \mathbb{R}^n | \|\xi - x_0\| \leqslant r\}$ and the shorthand is written as $\mathscr{B}^n(r)$ when $x_0 = 0$. A continuous function $\alpha : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be a $\mathscr{K}$-function, if it is strictly increasing and $\alpha(s) > 0$ for $s > 0$ with $\alpha(0) = 0$. A continuous function $\alpha(.)$ is said to be a $\mathscr{K}_\infty$-function, if it is a $\mathscr{K}$-function, and $\alpha(s) \to \infty$ as $s \to \infty$. A continuous function $\beta : \mathbb{R}_{\geqslant 0} \times \mathbb{Z}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is said to be a $\mathscr{K}\mathscr{L}$-function, if $\beta(s, k)$ is a $\mathscr{K}$-function in s for every given $k \in \mathbb{Z}_{\geqslant 0}$, and it is strictly decreasing in k with $\beta(s, k) \to 0$ as $k \to \infty$. Let Id denote the identity function, i.e., $\mathrm{Id}(x) = x$.

2.2 Preliminary Results and Modeling

The nonlinear dynamics is given as follows:

$$x_{k+1} = f(x_k, u_k, \omega_k), \quad k \in \mathbb{Z}_{\geqslant 0}, \quad x_0 = \bar{x}, \tag{2.1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $u_k \in \mathbb{R}^m$ is the control input and $\omega_k \in \mathbb{R}^r$ is the external disturbance. The system state and the control input are constrained as

$$x_k \in \mathscr{X}, \quad u_k \in \mathscr{U}, \quad \forall k \in \mathbb{Z}_{\geqslant 0}, \tag{2.2}$$

where $\mathscr{X}$ and $\mathscr{U}$ are compact sets such that $\{0\} \subset \mathscr{X} \subseteq \mathbb{R}^n$ and $\{0\} \subset \mathscr{U} \subseteq \mathbb{R}^m$, respectively. The external disturbance belongs to a compact set Υ with $\{0\} \subset \Upsilon \subseteq$

$\mathbb{R}^r$, and $\rho_\omega \triangleq \max_{\omega \in \Upsilon}\{\|\omega\|\}$. For the system in (2.1), denote the nominal system model as

$$\hat{x}_{k+1} = \hat{f}(x_k, u_k) \triangleq f(x_k, u_k, 0), \quad k \in \mathbb{Z}_{\geqslant 0},$$

where $\hat{f}(0, 0) = 0$. It is assumed that $f(x, u, \omega)$ is locally Lipschitz in x and u, such that

$$\|f(x_1, u_1, \omega) - f(x_2, u_2, 0)\| \leqslant L_{f_x}\|x_1 - x_2\| + L_{f_u}\Delta_u + \mu(\|\omega\|),$$

for all $x_1, x_2 \in \mathscr{X}$, $u_1, u_2 \in \mathscr{U}$ and $\omega \in \Upsilon$, where L_{f_x} and L_{f_u} are local Lipschitz constants, $\Delta_u \triangleq \max\{\|u_1 - u_2\|\}$ and μ is a $\mathscr{K}$ function. Note that the assumption of local Lipschitz continuity is to guarantee the existence of unique solution to the system in (2.1).

To facilitate the controller design, the results on the invariant set [1, 9] are recalled.

Definition 2.1 For the nonlinear system $x_{k+1} = f(x_k, \omega_k)$ with the uncertainty $\omega_k \in \Upsilon$, if there exists a set $\Omega \subset \mathbb{R}^n$ such that $f(x_k, \omega_k) \in \Omega$ for all $x_k \in \Omega$ and $\omega_k \in \Upsilon$, then the set Ω is called a robust positively invariant (RPI) set.

Definition 2.2 For the system in (2.1) with the constraints in (2.2) and a set Ω, if there exists an admissible control input $u_k \in \mathscr{U}$ such that $f(x_k, u_k, \omega_k) \in \Omega$ for all $x_k \in \Omega$ and all $\omega_k \in \Upsilon$, then the set Ω is called a robust control invariant (RCI) set.

Definition 2.3 Consider the system in (2.1) with the constraints in (2.2) and associated with an RPI set Ω. The i-th step robustly stabilizable set $X_i(\Omega)$ is denoted by all the admissible states which can be steered into the target set Ω in i steps by using an admissible control sequence $\mathbf{u}_{[i]}$ for all $\boldsymbol{\omega}_{[i]} \in \Upsilon^i$.

2.2.1 Regional ISpS

To facilitate the stability analysis, the results on the regional ISpS for the discrete-time nonlinear system is recalled. The system in (2.1) can be rewritten as

$$x_{k+1} = g(k, x_k, \omega_k) \triangleq f(x_k, u_k, \omega_k),\ x_0 = \bar{x}, \tag{2.3}$$

where $x_k \in \mathbb{R}^n$ is the same system state as in (2.1), $\omega_k \in \mathbb{R}^m$ is the same external disturbance and the argument k in function g represents the time-varying property of the argument u_k in function f. Denote $x(k, \bar{x}, \boldsymbol{\omega}_{0,k-1})$ as the solution to the system in (2.3) at time instant k. For the system in (2.3), the definition of the regional ISpS is recalled [17, 22].

Definition 2.4 Given a compact set $\Omega \in \mathbb{R}^n$, if it is an RPI set for the system in (2.3) with $\omega_k \in \Upsilon$, and if there exist a $\mathscr{KL}$ function β, a $\mathscr{K}$ function γ and a constant $c \geqslant 0$ such that

$$\|x(k, \bar{x}, \boldsymbol{\omega}_{0,k-1})\| \leqslant \max\{\beta(\|\bar{x}\|, k), \gamma(\|\boldsymbol{\omega}_{[k-1]}\|_\infty)\} + c, \tag{2.4}$$

$\forall k \in \mathbb{Z}_{\geqslant 0}$, $\bar{x} \in \Omega$, then the system in (2.3) is said to be regional ISpS in Ω.

An effective tool of establishing input-to-state stability (ISS) and ISpS is the comparison function [6, 25]. For the constrained nonlinear systems with two-channel packet dropouts, we recall the following regional ISpS-type Lyapunov function (a type of comparison function) [17, 22].

Definition 2.5 For the system in (2.3), given two compact sets $\mathscr{X}$ and Ω with $\{0\} \subset \mathscr{X} \subseteq \mathbb{R}^n$, $\{0\} \subset \Omega \subseteq \mathscr{X}$ and $\mathscr{X}$ being an RPI set,

- (C1) if there exists a positive definite function $V(.,.) : \mathbb{R}^n \times \mathbb{Z}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ such that the following conditions hold:

$$V(x_k, k) \geqslant \alpha_1(\|x_k\|) \quad \forall x_k \in \mathscr{X}, \tag{2.5}$$

$$V(x_k, k) \leqslant \alpha_3(\|x_k\|) + c_3 \quad \forall x_k \in \Omega, \tag{2.6}$$

$$V(x_{k+1}, k+1) - V(x_k, k) - \alpha_2(\|x_k\|) + \gamma(\|\omega_k\|) + c_2 \; \forall x_k \in \mathscr{X}, \tag{2.7}$$

  for all $k \in \mathbb{Z}_{\geqslant 0}$ and with α_1, α_2 and α_3 being $\mathscr{K}_\infty$ function, γ being $\mathscr{K}$ functions, and $c_2, c_3 \geqslant 0$;
- (C2) if there exist a $\mathscr{K}$ function α_c, a constant $c_0 \geqslant 0$ and a function $\tilde{\gamma}$ with $(\mathrm{Id} - \tilde{\gamma})$ being $\mathscr{K}$ function, and define a compact set

$$\Omega_\omega \triangleq \{x_k | V(x_k, k) \leqslant \theta(\gamma(\rho_\omega) + c_4), \forall k \in \mathbb{Z}_{\geqslant 0}\}, \tag{2.8}$$

  such that $\Omega_\omega \subseteq \Omega \sim \mathscr{B}^n(c_0)$ with $\theta \triangleq \alpha_4^{-1} \circ \tilde{r}$, $\alpha_4 \triangleq \underline{\alpha}_2 \circ \bar{\alpha}_3^{-1}$, $\underline{\alpha}_2(s) \triangleq \min\{\alpha_2(\frac{s}{2}), \alpha_c(\frac{s}{2})\}$, $\bar{\alpha}_3 \triangleq \alpha_3 + \mathrm{Id}$ and $c_4 = c_2 + \alpha_c(c_3)$,

then the function $V(x_k, k)$ is a regional ISpS-type Lyapunov function for the system in (2.3) with $\omega_k \in \Upsilon$.

Similar as in [17, 22], the continuity of the trajectory for the system in (2.3) is needed for establishing the regional ISpS.

Assumption 1 For the system in (2.3), the trajectory $x(k, \bar{x}, \boldsymbol{\omega}_{0,k-1})$ is continuous at $\bar{x} = 0$ and $\boldsymbol{\omega}_{0,k-1} = 0$ with respect to the initial state and the disturbances for all $k \in \mathbb{Z}_{\geqslant 0}$.

Remark 2.1 Assumption 1 is a prerequisite for analyzing the solution to the nonlinear system under investigation [7]. This assumption can be guaranteed as long as the nonlinear systems satisfy the local Lipschtiz conditions according to Theorem 3.5 in [7].

Theorem 2.1 ([22]) *Suppose that Assumption 1 holds. Given a compact set Ω and an RPI set $\mathscr{X}$ with $\{0\} \subset \Omega \subseteq \mathscr{X} \subset \mathbb{R}^n$ for the system in (2.3), if it admits an ISpS-type Lyapunov function associated with sets $\mathscr{X}$ and Ω, then the trajectory of the system in (2.3) satisfies* $\lim_{k\to\infty} \mathrm{d}|x(k, \bar{x}, \boldsymbol{\omega}_{0,k-1})|_{\Omega_\omega} = 0$ *and the system in (2.3) is regional ISpS in $\mathscr{X}$.*

2.2.2 Network Model

Consider an NCS with Ethernet-like communication networks connecting the sensor, the controller and the actuator. In these NCSs, assume that the data can be transmitted via large time-stamped (TS) packets [27], and the Transmission Control Protocol (TCP)-like protocol is adopted in the C-A communication network. (The TCP allows to send an acknowledgement packet to the controller when the smart actuator receives the data packet, which is different from the User Datagram Protocol (UDP)-like protocol. Physically, the acknowledgement is sent by the feedback link from the actuator to the controller, which is shown in Fig. 2.1. We assume that feedback link from the actuator to the controller is perfect and free of packet dropouts and time delays. This assumption is practically feasible due to the fact that the transmission load of this link is very small. We further assume that the time synchronization among the sensor, the controller and the actuator can be ensured.

In the NCS shown in Fig. 2.1, the system plant is governed by the discrete-time dynamics in (2.1) with the constraints in (2.2), and the communication links over the S-C channel and the C-A channel are subject to randomly packet dropouts. In particular, the S-C packet dropout process is denoted as $\{\tau_{sc}(k)\}_{k\in\mathbb{Z}_{\geqslant 0}}$ and the C-A packet dropout process as $\{\tau_{ca}(k)\}_{k\in\mathbb{Z}_{\geqslant 0}}$, respectively. More specifically, the random process $\{\tau_{sc}(k)\}_{k\in\mathbb{Z}_{\geqslant 0}}$ is defined as

$$\tau_{sc}(k) \triangleq \begin{cases} 0 & \text{if the S-C packet dropout occurs at time instant } k, \\ 1 & \text{if no S-C packet dropout occurs at time instant } k. \end{cases}$$

The C-A dropout process is defined as

$$\tau_{ca}(k) \triangleq \begin{cases} 0 & \text{if the C-A packet dropout occurs at time instant } k, \\ 1 & \text{if no C-A packet dropout occurs at time instant } k. \end{cases}$$

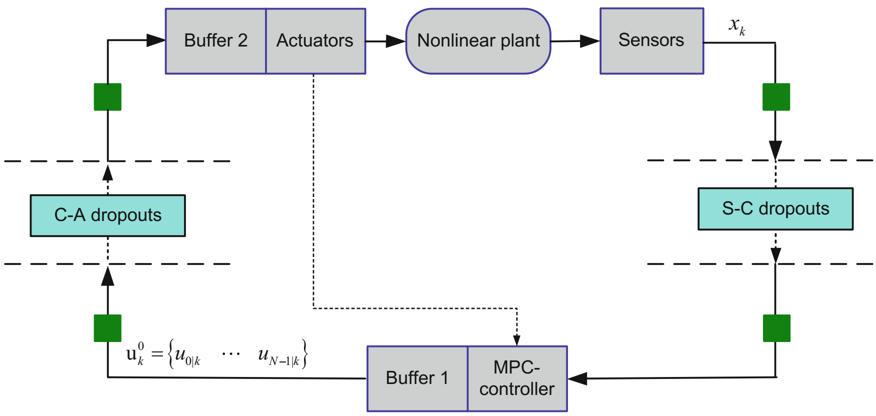

Fig. 2.1 Nonlinear NCS configuration

It is assumed that the maximum time durations of the consecutive S-C packet dropout and the consecutive C-A packet dropout are N_{sc} and N_{ca}, respectively. The maximum length of the consecutive packet dropouts is a measurement of the reliability of the communication networks, which will be used in the control packet design. The values of N_{sc} and N_{ca} are the network property parameters that can be determined by experiments.

2.2.3 *Buffer Model*

Two buffers are deployed in the configuration depicted in Fig. 2.1; one is situated in the control node denoted by Buffer 1 and the other is in the actuator node named Buffer 2. The buffer lengths for the Buffer 1 and Buffer 2 are both N_c with $N_c \geqslant N_{ca} + N_{sc}$. Denote the states of Buffer 1 and Buffer 2 as $\mathbf{B}_k^c$ and $\mathbf{B}_k^a$, respectively. The control packet passes from the controller to Buffer 1 sequentially, then the control information is sent through the network. By denoting the controller input to Buffer 1 is $\mathbf{u}_k^c$ at time k, the operation of Buffer 1 can be modeled as

$$\mathbf{B}_k^c = \begin{cases} \mathbf{u}_k^c & \tau_{sc}(k) = 1, \\ \mathrm{col}\{\mathbf{B}_{k-1}^c(2), \mathbf{B}_{k-1}^c(3), \cdots, \mathbf{B}_{k-1}^c(N_c), 0\} & \tau_{sc}(k) = 0. \end{cases}$$

The deployment of Buffer 1 is to reduce the controller load according to the S-C packet dropouts, which will be seen in the controller design procedure. Denoting the input to Buffer 2 is $\mathbf{u}_k$ at time k, the operation of Buffer 2 can be described as

$$\mathbf{B}_k^a = \begin{cases} \mathbf{u}_k & \tau_{ca}(k) = 1, \\ \mathrm{col}\{\mathbf{B}_{k-1}^a(2), \mathbf{B}_{k-1}^a(3), \cdots, \mathbf{B}_{k-1}^a(N_c), 0\} & \tau_{ca}(k) = 0. \end{cases}$$

2.3 Predictive Networked Controller Design

In this section, the realization of the networked control strategy is presented. First, the control packet is designed by the constrained RHC-based algorithm, where both the state constraint and the input constraint are satisfied. Then an effective control transmission and compensation mechanism is presented. Finally, an explicit control law is derived.

2.3.1 *Constrained Optimization Problem*

In order to compensate for data losses due to the packet dropouts and simultaneously take into account the input and state constraints, the constrained RHC strategy is adopted here. For the nonlinear system in (2.1), the cost function at time k is defined as

$$J(\mathbf{u}_k, x_k) \triangleq \sum_{i=0}^{N_c-1} L(\hat{x}_{k+i|k}, u_{i|k}) + F(\hat{x}_{k+N_c|k}), \tag{2.9}$$

where $\mathbf{u}_k \triangleq \mathrm{col}\{u_{0|k}, u_{1|k}, \cdots, u_{N_c-1|k}\}$, $\hat{x}_{k+i+1|k} = \hat{f}(\hat{x}_{k+i|k}, u_{i|k})$ and $\hat{x}_{k|k} = \hat{x}_k$. In the cost function (2.9), $L(\hat{x}_{i|k}, u_{i|k})$ is the stage cost and $F(\hat{x}_{N_c|k})$ is the terminal cost. The control packet is designed by solving the receding horizon optimization problem as follows.

Problem 2.1 $\mathbf{u}_k^o \triangleq \arg\min_{\mathbf{u}_k} J(\mathbf{u}_k, x_k)$ subject to: (1) the state constraint and the input constraint $\hat{x}_{k+i|k} \in \mathscr{X}$, $u_{i|k} \in \mathscr{U}$, for all $i = 0, 1, \cdots, N_c - 1$; (2) the terminal state constraint $\hat{x}_{N_c|k} \in \Omega_f$ with Ω_f being a compact set satisfying $\{0\} \subset \Omega_f \subset \mathbb{R}^n$; (3) the nominal model $\hat{x}_{k+i+1|k} = \hat{f}(\hat{x}_{k+i|k}, u_{i|k})$ for all $i = 0, 1, \cdots, N_c - 1$, and $\hat{x}_{k|k} = \hat{x}_k$.

Remark 2.2 Although the constrained optimization Problem 2.1 shares the same form as these in nonlinear constrained RHC strategies without communication networks in [12, 22], two essential differences exist. (1) The initial state $\hat{x}_{k|k}$ of the optimization problem is different. The initial state for standard nonlinear RHC is always the actual system state x_k. But for the networked nonlinear RHC, the initial state $\hat{x}_{k|k}$ can be the actual system state x_k or the state estimate $\hat{x}_k$ due to the S-C packet dropouts. (2) The actual control input for the closed-loop system is different. At time k, the control input of RHC with network-free nonlinear systems takes only the first element $u_{0|k}^o$ of the optimal control sequence $\mathbf{u}_k^o$ as the actual control input for the closed-loop system. But for this study, at time k, any control sequences generated from time instant $k - N_{ca} + N_{sc}$ to k can be taken, then any element from the first to the $(N_{ca} + N_{sc})$-th of these control sequences may be chosen as the actual control input for the closed-loop nonlinear NCS, which can be seen in (2.11) in the next section. It is well known that $u_{0|k}^o$ is the best choice for the common RHC algorithms and it provides very nice results on feasibility and stability [13]. In this study, since the actual system state x_k and the best control input $u_{0|k}^o$ cannot be utilized, the resulting nonlinear system dynamics is more complicated. Therefore, we need establish new results on feasibility and stability.

2.3.2 *Control Packet Generation*

In the control node, the control packet is generated by making the available information from the S-C packet dropouts and the information of the C-A packet dropouts provided by the acknowledgement packets. Meanwhile, a very efficient manner will be designed to produce the control packets, and the optimization Problem 2.1 will not be carried out for all the time instants. To describe in this manner, we denote a variable $r(k)$ to indicate whether the optimization problem is needed or not. The algorithm to determine the indicator $r(k)$ can be described by the following formula:

$$r(k) = \begin{cases} 1 & \text{if } \tau_{sc}(k) = 1, \\ 1 & \text{if } 1 - \tau_{sc}(k) = 1, \tau_{sc}(k-1) = 1, 1 - \tau_{ca}(k-1) = 1, \\ 1 & \text{if } \prod_{i=0}^{1}(1 - \tau_{sc}(k-i)) = 1, \tau_{sc}(k-2) = 1, \prod_{i=1}^{2}(1 - \tau_{ca}(k-i)) = 1, \\ \vdots & \\ 1 & \text{if } \prod_{i=0}^{N_{sc}-2}(1 - \tau_{sc}(k-i)) = 1, \tau_{sc}(k - N_{sc} + 1) = 1, \prod_{i=1}^{N_{sc}-1}(1 - \tau_{ca}(k-i)) = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{2.10}$$

In particular, whenever $r(k) = 1$, the optimization problem is required to be solved; the control sequence will be generated and packaged, and the information $r(k) = 1$ lets $\mathbf{B}_k^u$ in Buffer 1 be updated by $\mathbf{u}_k^o$; otherwise, the optimization is not conducted and the control packet is not generated, and the information $r(k) = 0$ commands $\mathbf{B}_k^c$ in Buffer 1 to be updated from the previous stored control sequence.

It should be pointed out that, at time k, the optimization Problem 2.1 can only be directly solved if the system state x_k is received, i.e., $\tau_{sc}(k) = 1$. But it also needs to conduct optimization in some situations (i.e., $r(k) = 1$) while the current system state is not received. To handle this issue, it needs to estimate the system state based on the previous system state and control inputs. The latest (previous) system state has been stored in $\mathbf{B}_k^x$ in Buffer 1 according to buffer model, and the corresponding (previous) control input sequence can be found in $\mathbf{B}_k^c$ in Buffer 1. Thus, the current system state can be estimated using the prediction of the nominal system for several steps.

Remark 2.3 It is worth noting that the above control packet generation approach is different from the existing results in [17, 19, 27], where the packets are generated for all the time instants. The principle under this approach is that it is not necessary to take any action if there is no further information provided (i.e., the measurement information is not accepted when the plant input has already been exploited by the predictions in solving the optimization problem, or simply, $r(k) = 0$). The proposed control packet generation method offers two benefits (1) since the constrained optimization does not require being conducted for all the time instants, the computation load of the controller can be reduced; (2) the prediction action is not needed when $r(k) = 0$, which can further increase the computational efficiency of the controller node.

2.3.3 Packet Transmission and Compensation Strategy Design

In this subsection, we are going to design the mechanism for transmitting the designed packets and implementing the actual control input according to the data losses.

For the control node, an efficient packet transmission method is designed by making use of the acknowledgement information of C-A channel and the information of the S-C packet dropouts. Specifically, the packet transmission operation is carried out according to the packet generation indicator $r(k)$. Case (1): $r(k) = 1$. At the beginning, a newly generated control packet will be pushed into $\mathbf{B}_k^u$ in Buffer 1 and simultaneously sent out through C-A channel to the actuator node; then once the successful transmission of the control packet over C-A channel is sent back by the acknowledgement link, i.e., $\tau_{ca}(k) = 1$, the content of $\mathbf{B}_k^u$ will be moved to replace $\mathbf{B}_k^c$. Case (2): $r(k) = 0$. No control packet is pushed into $\mathbf{B}_k^u$ or sent out through C-A channel. The buffer content $\mathbf{B}_k^c$ is shifted.

For the actuator node, if it receives the control packet, then control sequence will be updated by Buffer 2, and the acknowledgement packet will be sent back to the control node as $\tau_{ca}(k) = 1$. If the actuator node does not receive the packet, then no information will be sent, and the control sequence in Buffer 2 is shifted. After the updated operations, the first element of the control sequence in Buffer 2 will be used as the actual control input feeding the actuator.

Remark 2.4 It is noted that the proposed control packet transmission mechanism is physically built on the TCP-like protocols which can provide the information of the C-A packet dropouts back to the controller node. The detailed control packet transmission algorithm indicated by $r(t)$ is designed by the principle of providing all the useful control information using only the necessary transmission load over the C-A channel. In the existing literature in [14, 19], the control packets are transmitted for all the time instants without taking into account the information of the C-A packet dropouts and the S-C packet dropouts simultaneously. These methods actually transmit a lot of redundant data through the C-A channel, which increases the burden and degrades the performance of the network. In contrast, the proposed control packet transmission mechanism only sends all the necessary (new) information to the actuator node. Thus, it can significantly reduce the transmission load of the C-A channel, and may further alleviate the packet dropouts or time delays.

2.3.4 Explicit Control Law and Closed-Loop Model

To explicitly describe the actual control input, some variables need to be defined first. Define $\{p(j), \forall j \in \mathbb{Z}_{\geqslant 0}\}$ as an ordered time instant sequence to describe the events that there are successful transmissions for the C-A channel; define the ordered time instant sequence $\{q(l), \forall l \in \mathbb{Z}_{\geqslant 0}\}$ to describe the events $\{r(k) = 1\}$; define $m_{q(l)} \triangleq \inf_j\{p(j)|q(l) \leqslant p(j) < q(l+1)\}$ with $\inf(\emptyset) = \infty$; denote the ordered time instant sequence $\{m_{q(l_i)}, \forall i \in \mathbb{Z}_{\geqslant 0}\}$ as a subsequence of $\{m_{q(l)}, \forall l \in \mathbb{Z}_{\geqslant 0}\}$ by deleting all the elements equal to ∞. By combining the buffer model, the packet transmission and compensation mechanism, the actual control input can be derived as

$$u_k = \mathbf{u}^o_{m_{q(l_i)}}(k - m_{q(l_i)}), \quad m_{q(l_i)} \leqslant k < m_{q(l_{i+1})}, \tag{2.11}$$

where $\mathbf{u}^o_{m_{q(l_i)}} \triangleq \mathrm{col}\{u^o_{0|q(l_i)}, u^o_{1|q(l_i)}, \cdots, u^o_{N_c-1|q(l_i)}\}$. The resulting closed-loop system is given by

$$x_{k+1} = f(x_k, \mathbf{u}^o_{m_{q(l_i)}}(k - m_{q(l_i)}), \omega_k), \quad m_{q(l_i)} \leqslant k < m_{q(l_{i+1})}. \tag{2.12}$$

2.4 Stability Analysis

In this section, the regional ISpS of the nonlinear NCSs rendered by the proposed RHC strategy will be investigated. Before proving the regional ISpS of the resulting nonlinear NCSs, some notations and hypotheses are introduced.

Assumption 2 The stage cost function $L(x, u)$ is locally Lipschitz in x and u, i.e., there exist constants $0 < L_x < \infty$ and $0 < L_u < \infty$ such that for all $x_1, x_2 \in \mathscr{X}$ and $u_1, u_2 \in \mathscr{U}$,

$$\|L(x_1, u_1) - L(x_2, u_2)\| \leqslant L_x \|x_1 - x_2\| + L_u \Delta_u,$$

and $L(x, u)$ is lower bounded by

$$L(x, u) \geqslant \alpha_L(\|x\|) + \rho_L,$$

where $\rho_L \leqslant 0$ and α_L is a $\mathscr{K}_\infty$ function.

Assumption 3 The terminal cost function $F(x)$ is locally Lipschitz as $\|F(x_1) - F(x_2)\| \leqslant L_F ||x_1 - x_2\|, \forall x \in \mathscr{X}$ and $F(0) = 0$. Further, for all $x \in \Omega_f$, there exists an auxiliary control law $K_f(x) \in \mathscr{U}$ such that

$$F(\hat{f}(x, K_f(x))) - F(x) \leqslant -L(x, K_f(x)),$$

and $\hat{f}(x, K_f(x)) \in \Omega_f$.

Remark 2.5 Assumptions 2 and 3 are quite standard for the stage cost function and the terminal cost function, which have been adopted for the non-networked nonlinear systems [9, 12, 22] and the nonlinear NCSs [17, 19].

To present Assumption 4, let us define a mapping as

$$\begin{aligned} &f^j(x, \mathbf{u}^o_{m_{q(l_i)}}([0 : j-1]), \boldsymbol{\omega}_{[0:j-1]}) \\ \triangleq &f(f^{j-1}(x, \mathbf{u}^o_{m_{q(l_i)}}([0 : j-2]), \boldsymbol{\omega}_{[0:j-2]}), \mathbf{u}^o_{m_{q(l_i)}}(j-1), \omega), \end{aligned}$$

$\forall j = 1, 2, \cdots, m_{q(l_{i+1})} - m_{q(l_i)}$ and $i \in \mathbb{Z}_{\geqslant 0}$, where $\boldsymbol{\omega}_{[0:j-1]} \in \Upsilon^j$ and $f^0(x, \mathbf{u}^o_{m_{q(l_i)}}$ $([0 : -1]), \omega) = x$.

Assumption 4 There exists a compact set $\mathscr{R}_X$ with $\{0\} \subset \mathscr{R}_X \subseteq \mathscr{X}$, such that $\mathscr{R}_X \subseteq X_{N_c}(\Omega_f) \sim \mathscr{B}^n(L_{f_x}^{N_c-1} L_\omega \rho_\omega)$ and $\mathscr{R}_X$ is an RPI set for the mappings $f^j(x, \mathbf{u}^o_{m_{q(l_i)}}([0 : j-1]), \boldsymbol{\omega}_{[0:j-1]})$, for all $j = 1, 2, \cdots, m_{q(l_{i+1})} - m_{q(l_i)}$ with $i \in \mathbb{Z}_{\geqslant 0}$, where $X_{N_c}(\Omega_f)$ is the N_c-step RCI set of the system in (2.1).

Remark 2.6 Assumption 4 is to guarantee the feasibility of the proposed constrained RHC algorithm in $\mathscr{R}_X$; see Proposition 2.1. The similar assumption is made for the nonlinear NCSs only with the C-A packet dropout in [19] and the nonlinear systems without networks in [12, 22]. The main difference is that Assumption 4 can address both the S-C packet dropout and the C-A packet dropout simultaneously using state estimation. In fact, if there is no S-C packet dropout which is the case in [19] and no state estimation, then $q(l) = l$. In particular, if at some time instant l, there is a C-A packet dropout $p(j) = l$, then the control packet $\mathbf{u}^o_{m_{q(l_i)}}$ becomes $\mathbf{u}^o_l = \mathrm{col}\{u^o_{0|l}, u^o_{1|l}, \cdots, u^o_{N_c-1|l}\}$, and the control input (2.11) is given by $u_k = \mathbf{u}^o_l(k-l)$, for $l \leqslant k < p(j+1)$. As a result, the mapping $f^j(x, \mathbf{u}^o_{m_{q(l_i)}}([0 : j-1]), \boldsymbol{\omega}_{[0:j-1]})$ becomes exactly the corresponding mapping in [19], and the same assumption recovers from Assumption 4.

Before proceeding, a constrained minimization problem needs to be stated, based on which the ISpS-type Lyapunov function candidate can be constructed.

Problem 2.2 Minimize the following function $\bar{J}(\hat{x}_{k|k}, \mathbf{u}_k)$ as

$$\bar{J}(\hat{x}_{k|k}, \mathbf{u}_k) \triangleq \sum_{i=0}^{N_c} L(\hat{x}_{k+i|k}, u_{i|k}) + F(\hat{x}_{N_c+1|k}), \quad \hat{x}_{k|k} = x_k,$$

subject to

$$\begin{cases} \hat{x}_{k+i|k} \in \mathscr{X}, & i = 0, 1, 2, \cdots, N_c - 1, \\ \hat{x}_{k+N_c|k} \in \Omega_f, \ \hat{x}_{k+N_c+1|k} \in \Omega_f, & \\ u_{i|k} \in \mathscr{U}, \ \hat{x}_{k+i+1|k} = \hat{f}(\hat{x}_{k+i|k}, u_{i|k}), & i = 0, 1, 2, \cdots, N_c, \end{cases} \tag{2.13}$$

where $\mathbf{u}_k = \mathrm{col}\{u_{0|k}, u_{1|k}, \cdots, u_{N_c|k}\}$.

Proposition 2.1 *Suppose Assumptions 3 and 4 hold. Then Problem 2.1 is feasible for all $x_0 \in \mathscr{R}_X$ and Problem 2.2 is feasible in $\mathscr{R}_X$.*

Proof According to the state estimation algorithm, it can be verified that the state estimate $\hat{x}_k$ belongs to $X_{N_c}(\Omega_f)$ whenever $x_k \in \mathscr{R}_X$ for all k. Thus, according to Assumption 4, Problem 2.1 is feasible for all $x_0 \in \mathscr{R}_X$. For Problem 2.2, for all state x_k in $X_{N_c}(\Omega_f)$, there exists a sequence of control action $\mathbf{u}^o_k$ steering the predicted state $\hat{x}_{k+N_c|k}$ into the terminal set Ω_f according to the definition of $X_{N_c}(\Omega_f)$. Define the control sequence $\bar{\mathbf{u}}^o_k \triangleq \mathrm{col}\{\mathbf{u}^o_k, K_f(\hat{x}_{k+N_c|k})\}$ at time k. Then $\bar{\mathbf{u}}^o_k$ is a feasible control sequence for Problem 2.2 in terms of Assumption 3. Since $x_0 \in \mathscr{R}_X$ implies $x_k \in \mathscr{R}_X \subseteq X_{N_c}(\Omega_f)$, Problem 2.2 is feasible in $\mathscr{R}_X$.

For the simplicity of presenting the stability conditions, the definitions of some parameters which will be used in the sequel, are given as follows. Define $b_1 \triangleq \sum_{i=1}^{N_c} L_x \frac{L_{f_x}^i - 1}{L_{f_x} - 1} L_{f_u} \Delta_u + (N_c + 1) L_u \Delta_u + L_F \frac{L_{f_x}^{N_c+1} - 1}{L_{f_x} - 1} L_{f_u} \Delta_u - \rho_L$, $b_2 \triangleq (\frac{N_c L_x}{2} + L_F)(N_c+1) L_{f_u} \Delta_u + (N_c+1) L_u \Delta_u - \rho_L$, $b_3 \triangleq (L_F \frac{1-L_{f_x}^{N_c+2}}{1-L_{f_x}} + L_x \sum_{i=1}^{N_c+1} \frac{1-L_{f_x}^i}{1-L_{f_x}}) L_{f_u} \Delta_u + (N_c+2) L_u \Delta_u$ and $b_4 = (L_F \frac{1-L_{f_x}^{N_c+2}}{1-L_{f_x}} + L_x \sum_{i=1}^{N_c+1} \frac{1-L_{f_x}^i}{1-L_{f_x}}) L_{f_u} \Delta_u + (N_c+2) L_u \Delta_u$. Furthermore, we define parameters

$$\rho_1 \triangleq \begin{cases} b_1 & \text{if } L_{f_x} \neq 1, \\ b_2 & \text{if } L_{f_x} = 1, \end{cases} \quad \rho_2 \triangleq \begin{cases} b_3 & \text{if } L_{f_x} \neq 1, \\ b_4 & \text{if } L_{f_x} = 1. \end{cases}$$

Further, define the function

$$\gamma_{f_x}(s) \triangleq \begin{cases} (L_F L_{f_x}^{N_c+1} + \sum_{i=0}^{N_c} L_x L_{f_x}^i) \cdot \mu(s) & \text{if } L_{f_x} \neq 1, \\ (L_F + (N_c+1) L_x) s & \text{if } L_{f_x} = 1. \end{cases}$$

Assumption 5 Suppose that the disturbance set Υ is such that the condition C2 in Definition 2.5 is satisfied with $V(x_k, k) = L(x_k, u_k) + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}_k^*)$, $\Omega = \Omega_f$, $\alpha_2 = \alpha_L$, $\alpha_3 = \alpha_F$, $\gamma = \gamma_{f_x}$, $c_2 = \rho_1$ and $c_3 = \rho_2$, where $\hat{x}_{k+1|k} = \hat{f}(x_k, u_k)$ and $\mathbf{u}_k^*$ is the optimal solution to Problem 2.2.

Remark 2.7 This assumption is essentially to impose the condition on the bound of the disturbance set. That is, if one wants to achieve the regional ISpS, the external disturbance cannot be too “large”. The similar assumptions have been made for the robust nonlinear RHC in [12, 17, 22]. In fact, one can always find a small enough disturbance set to meet this assumption [17, 22].

The regional ISpS of the resulting nonlinear NCSs based on the proposed RHC control strategy is reported in the following Theorem 2.2.

Theorem 2.2 *Suppose that all the Assumptions hold with the compact sets Ω_f and $\mathscr{X}$, then the resulting NCSs in (2.12) is regional ISpS in $\mathscr{R}_X$ with respect to the disturbance $\omega_k \in \Upsilon$, $\forall k \in \mathbb{Z}_{\geqslant 0}$, and the system trajectory satisfies*

$$\lim_{k \to \infty} \mathrm{d}|x(k, \bar{x}, \boldsymbol{\omega}_{0,k-1})|_{\Omega_\omega} = 0,$$

$\forall \bar{x} \in \mathscr{R}_X$.

Proof According to Proposition 2.1, for all $x_0 = \bar{x} \in \mathscr{R}_x$, we can define the optimal solution to Problem 2.2 as $\mathbf{u}_k^*$ with the initial state $\hat{x}_{k+1|k}$ at time k, namely,

$$\mathbf{u}_k^* \triangleq \arg\min_{\mathbf{u}_k} \{\bar{J}(\hat{x}_{k+1|k}, \mathbf{u}_k)\}, \quad \text{subject to (2.13)},$$

where $\hat{x}_{k+1|k} = \hat{f}(x_k, u_k)$ with u_k given by (2.11) and $\mathbf{u}_k^* = \mathrm{col}\{u_{0|k}^*, u_{1|k}^*, \cdots, u_{N_c|k}^*\}$. It follows that the optimal solution to Problem 2.2 with the initial state $\hat{x}_{k+2|k+1}$ is

$\mathbf{u}_{k+1}^*$ at time $k+1$. Based on $\mathbf{u}_{k+1}^*$, we can construct a feasible control sequence $\bar{\mathbf{u}}_{k+1}^* = \mathrm{col}\{u_{0|k+1}^*, u_{1|k+1}^*, \cdots, u_{N_c-1|k+1}^*, K_f(\hat{x}_{k+N_c+2|k+1})\}$ for Problem 2.2 with the initial state $\hat{x}_{k+2|k+1}$ at time $k+1$. Choose $V(x_k, k) \triangleq L(x_k, u_k) + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}_k^*)$ as the ISpS-type Lyapunov function candidate at time k and define an auxiliary function $\bar{V}(x_{k+1}, k+1) \triangleq L(x_{k+1}, u_{k+1}) + \bar{J}(\hat{x}_{k+2|k+1}, \bar{\mathbf{u}}_{k+1}^*)$. The difference can be evaluated as

$$
\begin{aligned}
&\bar{V}(x_{k+1}, k+1) - V(x_k, k) \\
=& L(x_{k+1}, u_{k+1}) + \bar{J}(\hat{x}_{k+2|k+1}, \bar{\mathbf{u}}_{k+1}^*) - L(x_k, u_k) - \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}_k^*) \\
=& - L(x_k, u_k) + L(x_{k+1}, u_{k+1}) - L(\hat{x}_{k+1|k}, u_{0|k}^*) \\
&+ \sum_{i=2}^{N_c+1} L(\hat{x}_{k+i|k+1}, u_{i-2|k+1}^*) - L(\hat{x}_{k+i|k}, u_{i-1|k}^*) \\
&+ L(\hat{x}_{N_c+k+2|k+1}, K_f(\hat{x}_{k+N_c+2|k+1})) + F(\hat{x}_{N_c+k+3|k+1}) \\
&- F(\hat{x}_{N_c+k+2|k+1}) + F(\hat{x}_{N_c+k+2|k+1}) - F(\hat{x}_{N_c+k+2|k}).
\end{aligned} \tag{2.14}
$$

Since $\hat{x}_{k+N_c+2|k+1} \in \Omega_f$, it can be obtained that

$$
L(\hat{x}_{N_c+k+2|k+1}, K_f(\hat{x}_{k+N_c+2|k+1})) + F(\hat{x}_{N_c+k+3|k+1}) - F(\hat{x}_{N_c+k+2|k+1}) \leqslant 0. \tag{2.15}
$$

By using the Lipschitz condition, we get

$$
\begin{aligned}
&\sum_{i=2}^{N_c+1} L(\hat{x}_{k+i|k+1}, u_{i-2|k+1}^*) - L(\hat{x}_{k+i|k}, u_{i-1|k}^*) \\
\leqslant& \sum_{i=2}^{N_c+1} \left(L_x L_{f_x}^{i-1} \mu(\|\omega_k\|) + L_x \sum_{j=0}^{i-2} L_{f_x}^j L_u \Delta_u + L_u \Delta_u \right), \\
=& \sum_{i=1}^{N_c} \left(L_x L_{f_x}^i \mu(\|\omega_k\|) + L_x L_u \Delta_u \frac{L_{f_x}^i - 1}{L_{f_x} - 1} + L_u \Delta_u \right) \quad (L_{f_x} \neq 1).
\end{aligned} \tag{2.16}
$$

Similarly, we have

$$
L(x_{k+1}, u_{k+1}) - L(\hat{x}_{k+1|k}, u_{0|k}^*) \leqslant L_x \mu(\|\omega_k\|) + L_u \Delta_u. \tag{2.17}
$$

The terminal cost can be bounded as

$$
\begin{aligned}
&F(\hat{x}_{N_c+k+2|k+1}) - F(\hat{x}_{N_c+k+2|k}) \\
\leqslant& L_F \|\hat{x}_{N_c+k+2|k+1} - \hat{x}_{N_c+k+2|k}\|, \\
\leqslant& L_F (L_{f_x}^{N_c+1} \mu(\|\omega_k\|) + L_{f_u} \Delta_u \frac{L_{f_x}^{N_c+1} - 1}{L_{f_x} - 1}) \quad (L_{f_x} \neq 1).
\end{aligned} \tag{2.18}
$$

Substituting (2.15)–(2.18) into (2.14) and applying $L(x_k, u^o_{0|k}) \geqslant \alpha_L(\|x_k\|) + \rho_L$ result in

$$\begin{aligned}
&\bar{V}(x_{k+1}, k+1) - V(x_k, k)\\
\leqslant& -\alpha_L(\|x_k\|) + (L_F L_{f_x}^{N_c+1} + \sum_{i=0}^{N_c} L_x L_{f_x}^i) \cdot \mu(\|\omega_k\|) + b_1 \quad (L_{f_x} \neq 1).
\end{aligned}$$

Similarly, we can obtain

$$\begin{aligned}
&\bar{V}(x_{k+1}, k+1) - V(x_k, k)\\
\leqslant& -\alpha_L(\|x_k\|) + (L_F + (N_c+1)L_x) \cdot (\mu(\|\omega_k\|)) + b_2 \quad (L_{f_x} = 1).
\end{aligned}$$

By the optimality of Problem 2.2, $V(x_{k+1}, k+1) \leqslant \bar{V}(x_{k+1}, k+1)$. Consequently, we have

$$V(x_{k+1}, k+1) - V(x_k, k) \leqslant -\alpha_L(\|x_k\|) + \gamma_{f_x}(\|\omega_k\|) + \rho_1, \tag{2.19}$$

for all x_k in the feasible set. Next, it is easy to derive that

$$V(x_k, k) = L(x_k, u_k) + \sum_{i=1}^{N_c+1} L(\hat{x}_{k+i|k}, u^*_{i-1|k}) + F(\hat{x}_{k+N_c+2|k}) \geqslant \alpha_L(\|x_k\|) + \rho_L. \tag{2.20}$$

Finally, we need to establish the upper bound of $V(x_k, k)$ for all $x_k \in \Omega$. To this end, given the initial state x_k at time k, we define an auxiliary control sequence for Problem 2.2 as

$$\mathbf{u}'_k \triangleq \mathrm{col}\{K_f(\hat{x}'_{k+1|k}), K_f(\hat{x}'_{k+2|k}), \cdots, K_f(\hat{x}'_{k+N_c+1|k})\},$$

where $\hat{x}'_{k+j|k} = \hat{f}(\hat{x}'_{k+j-1|k}, K_f(\hat{x}'_{k+j-1|k}), \forall j \geqslant 2$, and $\hat{x}'_{k+1|k} = \hat{f}(x_k, K_f(x_k))$. Then we have

$$\begin{aligned}
V(x_k, k) =& L(x_k, u_k) - L(x_k, K_f(x_k)) + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}^*_k)\\
&- \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}'_k) + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}'_k) + L(x_k, K_f(x_k))\\
\leqslant& L_u \Delta_u + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}^*_k) - \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}'_k) + V_f(x_k),
\end{aligned} \tag{2.21}$$

where $V_f(x_k) \triangleq \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}'_k) + L(x_k, K_f(x_k))$. By applying the Lipschitz condition again, we have

$$\begin{aligned}
&\sum_{i=1}^{N_c+1} L(\hat{x}_{k+i|k}, u^*_{i|k}) - L(\hat{x}'_{k+i|k}, K_f(\hat{x}'_{k+i|k})) \\
&\leqslant \sum_{i=1}^{N_c} (L_x \|\hat{x}_{k+i|k} - \hat{x}'_{k+i|k}\| + L_u \Delta_u) \\
&\leqslant \sum_{i=1}^{N_c} L_x \frac{1 - L^i_{f_x}}{1 - L_{f_x}} L_{f_u} \Delta_u + (N_c + 1) L_u \Delta_u \quad (L_{f_x} \neq 1),
\end{aligned}$$

and

$$\begin{aligned}
F(\hat{x}_{k+N_c+2|k}) - F(\hat{x}'_{k+N_c+2|k}) \leqslant & L_F \|\hat{x}_{k+N_c+2|k} - \hat{x}'_{k+N_c+2|k}\|, \\
\leqslant & L_F \frac{1 - L^{N_c+2}_{f_x}}{1 - L_{f_x}} L_{f_u} \Delta_u \quad (L_{f_x} \neq 1).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\bar{J}(\hat{x}_{k+1|k}, \mathbf{u}^*_k) - \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}'_k) \\
&= \sum_{i=1}^{N_c+1} L(\hat{x}_{k+i|k}, u^*_{i|k}) - L(\hat{x}'_{k+i|k}, K_f(\hat{x}'_{k+i|k})) + F(\hat{x}_{k+N_c+2|k}) - F(\hat{x}'_{k+N_c+2|k}) \\
&\leqslant \sum_{i=1}^{N_c+1} L_x \frac{1 - L^i_{f_x}}{1 - L_{f_x}} L_{f_u} \Delta_u + (N_c + 1) L_u \Delta_u + L_F \frac{1 - L^{N_c+2}_{f_x}}{1 - L_{f_x}} L_{f_u} \Delta_u \quad (L_{f_x} \neq 1).
\end{aligned} \tag{2.22}$$

On the other hand, since $x_k \in \Omega_f$, then $\hat{x}'_{k+i+1|k} \in \Omega_f$, for all $i = 0, 1, 2, \cdots, N_c+1$. As a result,

$$F(\hat{x}'_{k+i+1|k}) - F(\hat{x}'_{k+i|k}) \leqslant -L(\hat{x}'_{k+i|k}, K_f(\hat{x}'_{k+i|k})), \tag{2.23}$$

for all $i = 0, 1, 2, \cdots, N_c$, where $\hat{x}'_{k|k} = x_k$. Summing up both sides of (2.23) from 0 to N_c gives rise to

$$V_f(x_k) \leqslant F(x'_{k|k}) \leqslant \alpha_F(\|x_k\|), \tag{2.24}$$

where $\alpha_F(s) = L_F s$. By plugging (2.22) and (2.24) into (2.21), we get

$$\begin{aligned}
V(x_k, k) \leqslant & \alpha_F(\|x_k\|) + (L_F \frac{1 - L^{N_c+2}_{f_x}}{1 - L_{f_x}} + L_x \sum_{i=1}^{N_c+1} \frac{1 - L^i_{f_x}}{1 - L_{f_x}}) L_{f_u} \Delta_u \\
& + (N_c + 2) L_u \Delta_u \quad (L_{f_x} \neq 1).
\end{aligned}$$

Analogously, we can obtain

$$V(x_k,k) \leqslant \alpha_F(\|x_k\|) + (L_F(N_c+2) + \frac{(N_c+2)(N_c+1)}{2}L_x)L_{f_u}\Delta_u \\ + (N_c+2)L_u\Delta_u \quad (L_{f_x}=1).$$

By combining the cases of $(L_{f_x}=1)$ and $(L_{f_x}\neq 1)$, we have

$$V(x_k,k) \leqslant \alpha_F(\|x_k\|) + \rho_2.$$

Therefore, $V(x_k,k)$ is an ISpS-type Lyapunov function. According to Theorem 2.1, the resulting nonlinear NCS is ISpS in $\mathscr{R}_X$ and all the trajectories starting in $\mathscr{R}_X$ converge to set Ω_ω. The proof is completed.

Remark 2.8 Unlike the regional ISS established in [17, 19], the regional ISpS is proven for the resulting nonlinear NCSs. The insight that we resort to the regional ISpS can be seen from the proof procedure: the joint effects of the C-A packet dropouts and the S-C packet dropouts make the actual control input u_k be not a function of the current system state x_k. Therefore, the deviation or bias exists for the lower bound of the stage cost function $L(x_k,u_k)$, which finally results in the regional ISpS.

Remark 2.9 It is worth noting that a novel approach has been proposed to prove the regional ISpS for the resulting nonlinear NCSs by accommodating the joint effects of the C-A packet dropouts and the S-C packet dropouts as well as the compensation strategy. In the existing results in [9, 12, 17, 19, 22], the regional ISpS or ISS is generally established by proving that the optimal index performance $J(\mathbf{u}_k^*, x_k)$ is the ISpS-type or ISS-type Lyapunov function; see the detailed techniques in the stability of the MPC approach [13]. However, the communication constraints and the compensation strategy prevent us from using the optimal index performance $J(\mathbf{u}_k^*, x_k)$ as the ISpS-type Lyapunov function directly. To circumvent this issue, we first construct the auxiliary Problem 2.2, based on which a novel $L(x_k,u_k) + \bar{J}(\hat{x}_{k+1|k}, \mathbf{u}_k^*)$ is proposed and verified. It is worthwhile to point out that Problem 2.2 is only an auxiliary vehicle for the proof of the regional ISpS, and it does not require being solved when applying the designed network-based RHC strategy.

2.5 Simulation

This section provides an example to verify the effectiveness of the proposed RHC strategy. The considered model is the cart-and-spring system working in an Ethernet-like environment. The diagram of the cart-and-spring system is drawn in Fig. 2.2. This model has been adopted in [11, 22] in the non-network environment.

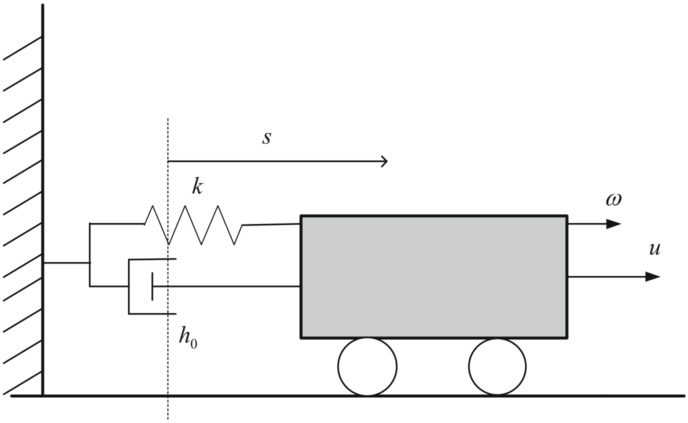

Fig. 2.2 Cart and spring system

In the spring-and-cart system, s represents the displacement of the carriage with respect to the equilibrium point. The spring factor is nonlinear and modeled as $k = k_0 e^{-s}$, and the damp factor is h_d. Using the state $x_1 = s$ and $x_2 = \dot{s}$, the dynamic model can be obtained as [11, 22]

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\frac{k_0}{M} e^{-x_1(t)} x_1(t) - \frac{h_d}{M} x_2(t) + \frac{u(t)}{M} + \frac{\omega(t)}{M}. \end{cases}$$

With a period T_c, the model is discretized as

$$\begin{cases} x_1(k+1) = x_1(k) + T_c x_2(k), \\ x_2(k+1) = x_2(k) - T_c \frac{k_0}{M} e^{-x_1(k)} x_1(k) - T_c \frac{h_d}{M} x_2(k) + T_c \frac{u(k)}{M} + T_c \frac{\omega(k)}{M}. \end{cases}$$

The numerical values of each parameter are given as follows: $k_0 = 0.10$ N/m, $M = 4.5$ kg, $h_d = 1.1$ N·s/m and $T_c = 0.4$ s. The control input constraints are $|u| \leqslant 4.5$ N and the states are constrained as $|x_1(k)| \leqslant 2.65$ m and $|x_2(k)| \leqslant 10.0$ m/s. Like the objective function in [22], stage cost function is designed as $L(x, u) = 0.01 x^{\mathrm{T}} x + 0.01 u^{\mathrm{T}} u$. The terminal cost $F(x) = x^{\mathrm{T}} P x$. Following the method in [3], $P = 10^{-2} \times [4.83, 2.19; 2.19, 2.34]$ and $\Omega_f = \{x | x^{\mathrm{T}} P_\omega x < 1\}$, where $P_\omega = [0.6032, 0.2739; 0.2739, 0.2927]$. An auxiliary control law is designed as $K_f(x) = -Kx$, where $K = [0.9050, 2.1179]$. The disturbance bound $\rho_\omega = 0.008$.

It can be seen that the stationary point is $x_1^o = 0.2$ m, $x_2^o = 0$ s/m and $u^o = 0.0540$ N. In the simulation, the packet dropouts in the S-C channel and the C-A channel are simulated by a random process, and the sample subsequences of the S-C and C-A packet dropouts are given as in Figs. 2.3 and 2.4, respectively.

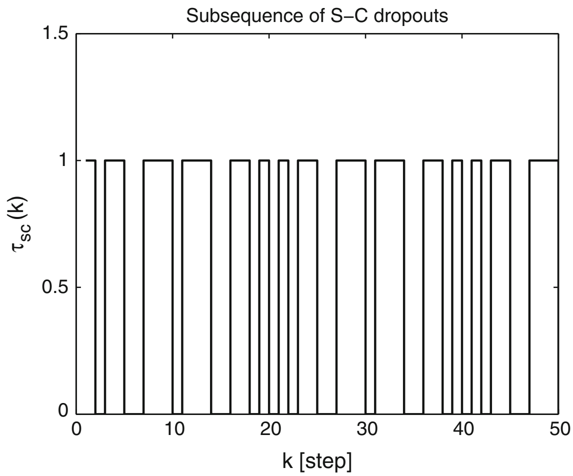

Fig. 2.3 Subsequence of the S-C dropouts

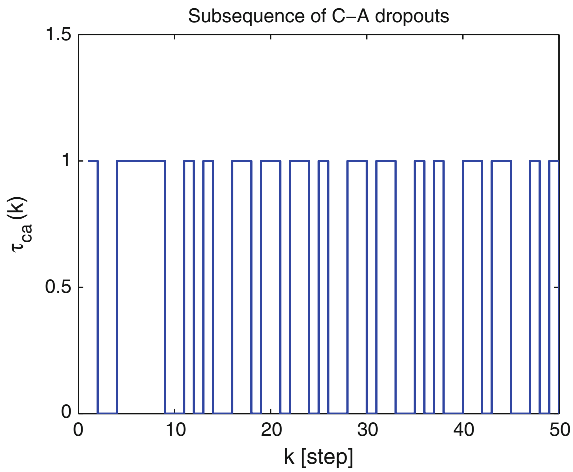

Fig. 2.4 Subsequence of the C-A dropouts

Four types of comparison studies are carried out. (1) The proposed RHC algorithm is proposed for the case of two-channel packet dropouts. (2) The RHC algorithm in [19] is simulated under the same condition except that only the C-A packet dropouts are compensated, and the S-C packet dropouts are not compensated. (3) The RHC algorithm is implemented and both the C-A and the S-C packet dropouts are not compensated. (4) The perfect system without any packet dropouts is simulated using the standard RHC strategy.

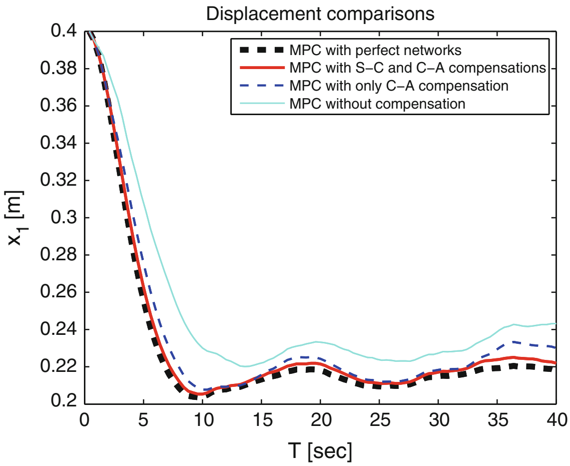

Fig. 2.5 Comparisons of displacements

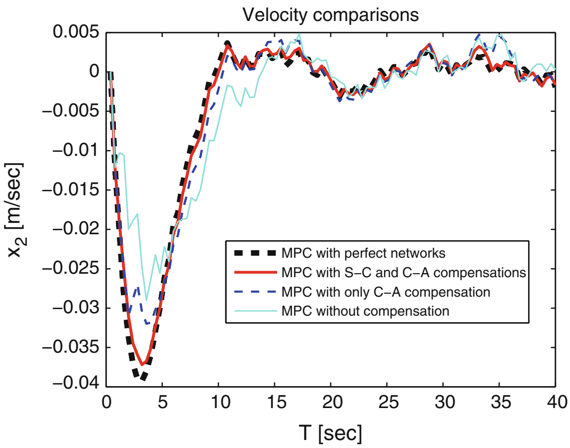

Fig. 2.6 Comparisons of velocities

The simulation results are reported in Figs. 2.5, 2.6 and 2.7. From these figures, it can be seen that: (1) the proposed RHC strategy is able to stabilize the closed-loop systems and the system constraints are satisfied. (2) The system performance of the proposed algorithm is comparable to that of the perfect system, outperforms that of C-A channel compensation strategy, and is much better than that of the case for no-compensation strategy.

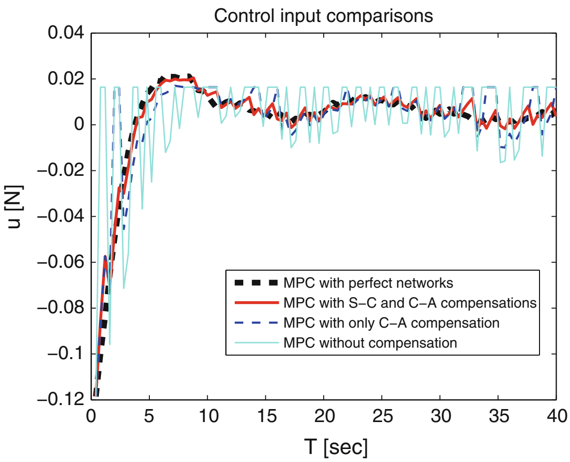

Fig. 2.7 Comparisons of control input

2.6 Note and Summary

This chapter studied the networked control problem for the nonlinear NCSs with two-channel random packet dropouts and bounded disturbances. The new RHC-based control algorithm including the control packet design, new transmission and compensation strategy, has been developed to stabilize the closed-loop system. Furthermore, a new method that builds on the new Lyapunov function is developed. Based on this idea, the regional ISpS of the closed-loop system is established. Finally, a simulation and comparison studies are conducted, verifying the effectiveness of the proposed algorithm.

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