Preface to the Second Edition

This second edition contains over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators (Chapter 24) including two sections on proximity operators of matrix functions, as well as new sections on cocoercive operators (Section 4.2), quasi-Fejér monotone sequences (Section 5.4), nonlinear ergodic theorems (Section 5.5), recession functions (Section 9.4), the subdifferential of a composition (Section 16.5), directional derivatives and convexity (Section 17.5), the partial inverse (Sections 20.3, 26.2, and 28.2), local maximal monotonicity (Section 25.2), parallel compositions of monotone operators (Section 25.6), the Peaceman-Rachford algorithm (Sections 26.4 and 28.4), algorithms for solving composite monotone inclusion problems (Section 26.8) and composite minimization problems (Section 28.8), subgradient projections (Section 29.6), and Halpern's algorithm (Section 30.1). Furthermore, many existing results have been improved. Finally, the list of references has been updated.

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We grieve the passing of Jonathan Borwein (1951–2016) and Jean Jacques Moreau (1923–2014), who have deeply influenced and inspired us in our work.

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Paris, France and Raleigh, NC, USA — Patrick L. Combettes
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Preface to the First Edition

Three important areas of nonlinear analysis emerged in the early 1960s: convex analysis, monotone operator theory, and the theory of nonexpansive mappings. Over the past four decades, these areas have reached a high level of maturity, and an increasing number of connections have been identified between them. At the same time, they have found applications in a wide array of disciplines, including mechanics, economics, partial differential equations, information theory, approximation theory, signal and image processing, game theory, optimal transport theory, probability and statistics, and machine learning.

The purpose of this book is to present a largely self-contained account of the main results of convex analysis, monotone operator theory, and the theory of nonexpansive operators in the context of Hilbert spaces. Authoritative monographs are already available on each of these topics individually. A novelty of this book, and indeed, its central theme, is the tight interplay among the key notions of convexity, monotonicity, and nonexpansiveness. We aim at making the presentation accessible to a broad audience and to reach out in particular to the applied sciences and engineering communities, where these tools have become indispensable. We chose to cast our exposition in the Hilbert space setting. This allows us to cover many applications of interest to practitioners in infinite-dimensional spaces and yet to avoid the technical difficulties pertaining to general Banach space theory that would exclude a large portion of our intended audience. We have also made an attempt to draw on recent developments and modern tools to simplify the proofs of key results, exploiting heavily, for instance, the concept of a Fitzpatrick function in our exposition of monotone operators, the notion of Fejér monotonicity to unify the convergence proofs of several algorithms, and the notion of a proximity operator throughout the second half of the book.

The book is organized in 29 chapters. Chapters 1 and 2 provide background material. Chapters 3–7 cover set convexity and nonexpansive operators. Various aspects of the theory of convex functions are discussed in

Chapters 8–19. Chapters 20–25 are dedicated to monotone operator theory. In addition to these basic building blocks, we also address certain themes from different angles in several places. Thus, optimization theory is discussed in Chapters 11, 19, 26, and 27. Best approximation problems are discussed in Chapters 3, 19, 27, 28, and 29. Algorithms are also present in various parts of the book: fixed point and convex feasibility algorithms in Chapter 5, proximal-point algorithms in Chapter 23, monotone operator splitting algorithms in Chapter 25, optimization algorithms in Chapter 27, and best approximation algorithms in Chapters 27 and 29. More than 400 exercises are distributed throughout the book, at the end of each chapter.

Preliminary drafts of the first edition of this book have been used in courses in our institutions, and we have benefited from the input of postdoctoral fellows and many students. To all of them, many thanks. In particular, HHB thanks Liangjin Yao for his helpful comments. We are grateful to Hédy Attouch, Jon Borwein, Stephen Simons, Jon Vanderwerff, Shawn Wang, and Isao Yamada for helpful discussions and pertinent comments. PLC also thanks Oscar Wesler. Finally, we thank the Natural Sciences and Engineering Research Council of Canada, the Canada Research Chair Program, and France's Agence Nationale de la Recherche for their support.

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