

Preface

The elastoplasticity has highly developed responding to the rapid advance of industries during the last half century. In the conventional plasticity, the interior of the yield surface was assumed to be an elastic domain so that the plastic strain rate due to the rate of stress inside the yield surface is not predicted, resulting in the following limitations and the difficulties: (1) The tangent modulus changes discontinuously at the yield point; (2) the yield judgment whether the stress reaches the yield surface is required with the determination of the offset (permanent, i.e., plastic strain) value at yielding which is accompanied with an arbitrariness (although 0.2 % plastic strain is often used); and (3) the operation to pullback the stress to the yield surface is required when it goes out from the yield surface by large loading increments in numerical calculation. Then, the fierce fight has started aiming at formulating the rigorous plasticity model capable of predicting cyclic loading behavior of solids and structures under the machine vibrations, the earthquakes, etc., at the middle of the last century. The fight would have come to the end by the creation of the *subloading surface model* by which the above-mentioned limitations and the difficulties are resolved, describing the cyclic loading behavior rigorously with the high efficiency in numerical calculation.

All the elastoplastic models other than the subloading surface model are the *cyclic kinematic hardening models*, e.g. the *multi surface model*, the *two (so-called bounding) surface model* and the *superposed-kinematic hardening (so-called Chaboche) model* in which a small yield surface enclosing the elastic domain translates as the plastic strain rate develops. The models inheriting the yield surface enclosing a purely-elastic domain are required to incorporate smaller and smaller yield surfaces one after another endlessly depending on the stress level and the stress amplitude, like an infinity mirror or a nest of boxes, so that they fall into the endless pit without any substantial resolution of the problem.

The basic features of the subloading surface model are itemized as follows:

- (1) It is based on the quite natural concept that the plastic deformation is developed as the stress approaches the yield surface and thus it possesses the high generality and the capability of describing accurately irreversible deformations.

- (2) It fulfills the smoothness condition, describing always continuous variation of tangent stiffness modulus and thus depicting the smooth elastic-plastic transition.
- (3) It possesses the automatic controlling functions to attract the stress to the normal-yield surface, so that the stress is pulled-back to the yield surface when it goes over the surface in numerical calculations. In addition, the normal-isotropic hardening surface varies so as to involve always the kinematic hardening variable, i.e. the back stress in the stagnation phenomenon of isotropic hardening observed after the re-yielding in the stress reversal event.
- (4) It is capable of describing the finite deformation and rotation under an infinitesimal elastic deformation.

By virtue of these rigorous physical backgrounds, the subloading surface model is endowed with the following rigorous descriptions distinguishable from the other elastoplastic constitutive models.

1. Plastic strain rate is predicted even for any low stress level. Then, cyclic loading behavior is predicted accurately even for infinitesimal loading amplitude. *The other models, e.g. the multi, the two and the superposed kinematic hardening models are incapable of describing the cyclic loading behavior appropriately.*
2. Plastic deformation of various solids unlimited to metals, e.g. soils, etc. can be described pertinently. *The other cyclic plasticity models are limited to the description of metal deformation.*
3. Inelastic strain rate induced by the stress rate tangential to the yield surface, i.e. tangential-inelastic strain rate is described appropriately, fulfilling the continuity condition, which is required to describe the non-proportional loading behavior. *All the other models assume the purely-elastic domain so that they predict the tangential-inelastic strain rate induced suddenly at the moment when the stress reaches the yield surface if the tangential-inelastic strain rate is incorporated, violating the continuity condition at the moment.*
4. Viscoplastic constitutive deformation is described pertinently in a general rate from the quasi-static to the impact loading. *The other models are incapable of describing the viscoplastic deformation behavior at high rate, predicting an elastic response for an impact loading.*
5. Damage phenomenon under cyclic loading is described appropriately, which leads to a softening behavior in general. *The other cyclic plasticity models are incapable of describing the cyclic damage phenomenon with a softening pertinently.*
6. Constitutive equation of fatigue would be described pertinently, which is required to describe plastic strain rate induced in low stress level and small stress amplitude. *The other models are incapable of describing the fatigue phenomenon, predicting only an elastic strain rate in low stress level and small stress amplitude.*
7. Constitutive equation of phase-transformation of metals can be described pertinently. *The other models are incapable of describing the phase-transformation phenomenon pertinently.*

8. Friction phenomenon is described pertinently, including the transition from static to kinetic friction by the sliding, the recovery of static friction with elapse of time and the both of positive and negative rate-sensitivities. The negative and the positive rate sensitivities are relevant to the dry and the lubricated (fluid) friction, respectively. *The other model is incapable of describing these friction phenomena.*
9. Crystal plasticity analysis can be executed pertinently, in which calculation of slips in numerous number of slip systems are required. The yield judgment is unnecessary since the smooth elastic-plastic transition is described and the resolved shear stress is automatically pulled-back to the critical shear stress. *The other models are inapplicable to the crystal plasticity analysis rigorously because the yield judgment and the operation to pull back the resolved shear stresses to the critical shear stress are required in numerous number of slip systems. Then, impertinent analysis using the creep model has been performed widely after Pierce et al. (1982, 1983). It is impertinent to adopt the rate-dependent model for the rate-independent deformation phenomenon. In addition, it should be noted that the creep model is impertinent such that it predicts a creep shear strain rate even in unloading process of the resolved shear stress from the critical shear stress.*
10. The multiplicative hyperelastic-based plasticity can be formulated exactly based on the subloading surface model, which is capable of describing the cyclic loading behavior exactly for finite elastoplastic deformation/rotation. *The multiplicative hyperelastic-based plasticity cannot be formulated by the other cyclic plasticity models.*

Thus, the subloading surface model is endowed inherently with the high generality and flexibility for the description of irreversible mechanical phenomena of solids.

Eventually, it can be stated that

“Subloading surface concept is to be the unified constitutive law which is inevitable to describe irreversible mechanical phenomena in wide classes of solids, ranging from monotonic to cyclic loadings, from quasi-static to impact loadings, from infinitesimal to finite deformations and from micro to macro phenomena”.

Then, the elastoplasticity theory will be developed steadily (breaking through the stagnation) by incorporating the exact formulation of the subloading surface model, although it has stagnated during a half century since the study on the unconventional (cyclic) elastoplasticity aiming at pertinent description of the plastic strain rate caused by the rate of stress inside the yield surface has started in the 1960s.

The subloading surface and the subloading-friction models have been implemented to the commercial FEM software Marc marketed by MSC Software Ltd. as the standard uploaded (ready-made) programs, so that Marc users can apply these models to their deformation analyses. The implementation was highly supported by Dr. Motohatsu Tateishi, MSC Software Ltd., Japan. Then, the author decided to

publish this book in order to provide the comprehensive explanation of the subloading surface model for readers ranging from beginners to specialists of the elastoplasticity so that they can apply it easily to their deformation analyses. In addition, the computer programs of the subloading surface and the subloading-friction models are released in Appendix J so as to use them capturing clearly the formulations and the calculation processes.

The main contents in each chapter will be delineated below in order.

As a foundation for the formulation of elastoplasticity theory, the mathematical and the physical ingredients of the continuum mechanics are provided in Chaps. 1–4. Chapter 1 addresses the vector-tensor analysis since physical quantities used in continuum mechanics are tensors; consequently, their relations are described mathematically by tensor equations. Explanations for mathematical properties and rules of tensors are presented to the extent that is sufficient to understand the subject of this book: elastoplasticity theory. Chapter 2 addresses the description of motion and strain (rate) and their related quantities. Chapter 3 presents conservation laws of mass, momentum, and angular momentum, and equilibrium equations and virtual work principles derived from them. In addition, their rate forms used for constitutive equations of inelastic deformation are explained concisely.

Chapter 4 specifically addresses the objectivity of constitutive equations, which is required for the description of deformation behavior under material rotation. The substantial physical meaning of the objective rate of tensor is explained incorporating the convected base. Then, the objectivities of various stress, strain, and their rates are described by examining their coordinate transformation rules. Then, the pullback and the push-forward operations are systematically explained, defining the Eulerian and the Lagrangian vectors and tensors. Further, all the objective and the corotational time derivatives of tensors are derived systematically from the convected (embedded) time derivative. The mathematical proof is given to the fact that the material time derivative of scalar-valued tensor function can be transformed to the corotational time derivative of that.

Chapter 5 specifically examines the description of elastic deformation. Elastic constitutive equations are classified into the hyperelasticity, the Cauchy elasticity, and the hypoelasticity depending on exactness in the description of reversibility. The mathematical and physical characteristics of these equations are explained prior to the description of elastoplastic constitutive equations in the subsequent chapters.

Elastoplastic constitutive equations are described comprehensively in Chaps. 6–9. In Chap. 6, the physical and mathematical backgrounds are first given to the additive decomposition of strain rate (symmetric part of the velocity gradient) into the elastic and the plastic parts and that of the continuum spin (antisymmetric part of the velocity gradient) into the elastic and the plastic parts from the standpoint of the multiplicative decomposition of deformation gradient which provides the exact decomposition of deformation gradient tensor into the elastic and the plastic parts by introducing the intermediate configuration as the hyper-elastically unloaded state to the stress-free state. In addition, the physical backgrounds are given to facts that the elastic spin designates the sum of the rigid-body rotational rate and the rotational rate due to the elastic distortion and that the plastic spin designates the rotational rate

of the intermediate configuration. Thereafter, the basic formulations of elastoplastic constitutive equation, e.g., the elastic and the plastic strain rates, the consistency condition, the plastic flow rule, and the loading criterion, are provided based on the physical interpretations. Descriptions of anisotropy and the tangential inelastic strain rate are also incorporated. However, they fall within the framework of conventional plasticity on the premise that the interior of the yield surface is an elastic domain. Therefore, they are incapable of predicting a smooth transition from the elastic to plastic state and a cyclic loading behavior of real materials pertinently.

In Chap. 7, the continuity and the smoothness conditions are described first. They are the fundamental requirements for the constitutive equations of irreversible deformation, especially to describe cyclic loading behavior accurately. The subloading surface model is described in detail, which falls within the framework of the unconventional plasticity excluding the assumption that the interior of yield surface is an elastic domain. It satisfies both the continuity and the smoothness conditions. In Chap. 8, cyclic plasticity models are classified into the models based on the translation of the small yield surface enclosing a purely elastic domain, i.e., the cyclic kinematic hardening models, and the model based on the expansion/contraction of loading surface, i.e., the subloading surface model. Further, their mathematical structures and mechanical features are explained in detail. It is revealed that the cyclic kinematic hardening models, e.g., the multi-, the two-, and the superposed nonlinear-kinematic hardening models, are the temporizing models, which do not possess a generality/pertinence and contain various serious deficiencies. First of all, the purpose for the creation of unconventional plasticity model is the description of the plastic strain rate by the rate of stress inside the yield surface which cannot be described by the conventional plasticity model assuming the yield surface enclosing a purely elastic domain. However, the defect of the conventional plasticity model cannot be solved endlessly by the cyclic kinematic hardening models incorporating the small yield surface enclosing a purely elastic domain. In addition, the mechanism for the development of plastic strain rate is substantially different from the mechanism for the development of anisotropy such as the kinematic hardening. It can be concluded that only the extended subloading surface model falling within the framework of the latter category possesses the generality and the mathematical structure capable of describing the cyclic loading behavior in wide classes of elastoplastic materials including metals and soils. In addition, the friction phenomenon of solids can be described rigorously by the friction model based on the concept of the subloading surface as will be described in Chap. 18.

In Chap. 9, the formulation of the extended subloading surface model is described in detail, in which the elastic core, i.e., the similarity center of the normal yield and the subloading surfaces, translates with a plastic deformation. Therein, the inelastic strain rate attributable to the stress rate tangential to the subloading surface is incorporated, which is indispensable for the accurate prediction of non-proportional loading behavior and the plastic instability phenomena. In addition, the cyclic stagnation of the isotropic hardening in metals is incorporated. In Chaps. 10 and 11, constitutive equations based on the extended subloading surface

model are shown for metals and soils. Their validities are verified by the comparisons with various test data containing the cyclic loading.

In Chap. 12, the exact finite strain theory based on the multiplicative decomposition of deformation gradient is formulated, in which the extended Hashiguchi (subloading surface) model is incorporated, although only the initial subloading surface model was incorporated in the book of Hashiguchi and Yamakawa (2012). The author aims at formulating the constitutive equation possessing the generality and the universality to be inherited eternally, while any unconventional model, i.e., cyclic plasticity model other than the subloading surface model, has not been extended to the multiplicative finite strain theory up to the present.

In Chap. 13, the history of the development of the viscoplastic constitutive equation for describing rate-dependent deformation induced for the stress level over the yield surface is reviewed first. Then, the pertinent viscoplastic constitutive equation is described, in which the concept of the subloading surface is incorporated into the overstress model. It is applicable to the prediction of rate-dependent deformation behavior in the general rate ranging from quasi-static to impact loads, while the deformation behavior under the impact load cannot be described by the past overstress models. On the other hand, it is revealed that the creep model contains impertinence for the description of a quasi-static deformation behavior, although it has been studied widely. Further, the subloading-overstress model is extended to the multiplicative exact deformation theory.

In Chap. 14, the constitutive equation with the damage phenomenon based on the subloading surface model is described. The softening behavior is often observed, for which the description of a smooth transition from the elastic to the plastic state is required and thus, the subloading surface concept should be introduced inevitably. In Chap. 15, the plasticity for the phase-transformation phenomenon based on the Hashiguchi (subloading surface) model is described briefly.

Special issues related to elastoplastic deformation behavior are discussed in Chaps. 16 and 17. Chapter 16 specifically examines corotational rate tensors, the necessity of which is suggested in Chap. 4. Mechanical features of corotational tensors with various spins are examined comparing their simple shear deformation characteristics. The pertinence of the plastic spin is particularly explained. Chapter 17 opens with a mechanical interpretation for the localization of deformation inducing a shear band. Then, the approaches to the prediction of shear band inception condition, the inclination/thickness of shear band, and the eigenvalue analysis and the gradient theory are explained. The smeared model, i.e., the shear band-embedded model for the practical finite element analysis, is also described.

Chapter 18 addresses the prediction of friction phenomena between solid bodies. All bodies except those floating in a vacuum contact with other bodies so that the friction phenomena occur between their contact surfaces. Pertinent analyses, not only of the deformation behavior of bodies, but also of friction behavior on the contact surface, are necessary for the analyses of boundary-value problems. A constitutive equation of friction is formulated in the similar form to the elastoplastic constitutive equation by incorporating the concept of the subloading surface, which is called the subloading-friction model. It is capable of describing the

transition from a static to a kinetic friction attributable to plastic softening and the recovery of the static friction attributable to creep hardening. The anisotropy based on the orthotropy and the rotation of sliding-yield surface is incorporated. The stick-slip phenomenon is analyzed by incorporating the subloading-friction model. Their validities are shown by comparison with various test data.

In Chap. 19, the crystal plasticity model based on the Hashiguchi (subloading surface) model is described, which would be physically pertinent and possess the high efficiency in numerical analyses. In contrast, note that the creep model without a yield surface is widely incorporated into the crystal plasticity analyses in order to avoid the numerical difficulty, although it is physically quite irrelevant.

The FEM analysis based on elastoplastic constitutive equations described in the former chapters requires pertinent numerical method. In Chap. 20, the return-mapping and the consistent (algorithmic) tangent modulus tensor are explained, which provides the calculation in a high accuracy and efficiency.

Finally, in Chap. 21, the formulations of elastoplastic constitutive equation from the second law of thermodynamics are commented and then it is suggested that the formulations are irrational and merely the prerequisite logic and thus, we have to formulate constitutive equation without falling into the thermodynamic formalism.

The distinguishable features and importance of this book are the comprehensive descriptions of fundamental concepts and formulations including the objectivity, the objective derivative of tensor function, the associated flow rule, the loading criterion, the continuity and smoothness conditions, and their physical interpretations in addition to the circumstantial explanations on wide classes of reversible/irreversible constitutive equations for monotonic, cyclic and non-proportional loading behavior, rate-dependent deformation behavior, and friction behavior of solids.

Most of the theories described in this book fall within the framework of the hypoelastic-based plasticity for the finite deformation under the infinitesimal elastic deformation, and the finite strain theory based on the multiplicative decomposition of deformation gradient is explained concisely. This book is the elaborated compilation of the former books: “*Elastoplasticity Theory*” by Hashiguchi (Springer, 2013) for the hypoelastic-based plasticity and “*Introduction to Finite Strain Theory for Continuum Elasto-Plasticity*” by Hashiguchi and Yamakawa (John-Wiley, 2012) for the multiplicative elastoplasticity (exact hyperelastic-based plasticity). It is recommendable for the readers to read also the latter book for the numerical calculation and the finite element method.

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