A Game-Theoretic Approach to the Analysis of Traffic Assignment

Caixia Li, Sreenatha G. Anavatti, Tapabrata Ray and Hyungbo Shim

Abstract In order to improve the cooperation between traffic management and travellers, traffic assignment is the key component. In terms of the traffic assignment, it can be classified into two models based on the behavior assumption governing route choices: the User Equilibrium (UE) and System Optimum (SO) traffic assignment. By the definition of UE and SO traffic assignment, traffic users usually competitively choose the least cost routes to minimize their own travel cost, while system optimum traffic assignment requires traffic users work cooperatively to minimize overall cost in road network. Thus, the paradox of benefits between UE and SO makes both of them are not practical. Thus, a solution technique needs to be proposed to balance between UE and SO models, which can compromise both sides and give more feasible traffic assignments. In this paper, Stackelberg game theory is introduced to the traffic assignment, which can achieve the trade-off process between traffic management and travellers. Since the traditional traffic assignments have low convergence rates, the gradient projection algorithm is proposed to improve the efficiency of the traffic assignment.

Keywords Traffic management · Traffic assignment · Route choices · Stakelberg game theory

C. Li · S.G. Anavatti (✉) · T. Ray
UNSW Canberra, Campbell, Australia
e-mail: S.Anavatti@adfa.edu.au; agsrenat@adfa.edu.au

C. Li
e-mail: Cara.cxl@hotmail.com

T. Ray
e-mail: T.Ray@adfa.edu.au

H. Shim
Seoul National University, Seoul, Korea
e-mail: hshim@snu.ac.kr

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1 Introduction

In the Wardrop's [1] first principle, the journey times in all routes actually used are equal or less than those which would be experienced by a single vehicle on any unused route. This first principle is often referred to as "user optimal" (UE) model. Under the "user optimal" model, each traveler act competitively and make their own optimal route for their own benefit. According to Wardrop's second principle, at equilibrium, the overall system journey time is minimum. This implies that each user behaves cooperatively in choosing his own route to ensure the most efficient use of the whole system, which is referred to as "System Optimal" (SO) model. Under the UE principle, traffic users competitively choose the least cost route to minimize their own travel cost, while the SO principle requires the traffic users work cooperatively to minimize their overall cost. Thus, the competitive and cooperative interaction between traffic information provider and traffic users can be interpreted as a game.

Game theory is related to several parties with different interests to decide the optimal choice. The benefit to each party not only depends on its own actions, but also on the choices of the other parties. For the UE, it consists of many travellers and each traveller is a game player to compete with each other to find the shortest path for his/her benefit. In terms of SO model, there is only the traffic control centre to control all the travellers on the roads. It is assumed all the travelers can cooperate with each other to get the minimized travel cost of the system. Therefore, users are fully competitive or fully cooperative in the Wardrop's theory.

In reality, both competition and cooperation among users exist in the traffic network. More general situation is proposed by Haurie and Marcotte [2], who present a relationship between non-cooperative Coumot-Nash (CN) and Wardrop's model. It denotes users belonging to a common player are fully cooperative, while different players are fully competitive. For the first situation it approaches to SO model and the latter one corresponds to the UE model. A mixed behavior situation is considered by Harker [3], where some distinct players are controlled by the CN players, while the other users follow the UE principle. Vuren et al. [4] studied the route guidance problem by combining the UE principle and SO principle in the traffic assignment model and different levels of information are incorporated into the model via a multiple user class Stochastic User Equilibrium (SUE). Wie [5] introduced a differential game model of Nash equilibrium on a congested traffic network and applied it to solve the dynamic mixed behaviour traffic network equilibrium problem [6]. The objective of the research is to establish the relationship between the Nash equilibrium and the dynamic user equilibrium. Friesz et al. [7] examined a certain class of dynamic games known as open loop differential Nash games. Kumar and Peeta [8] introduced the strategies to enhance path based static traffic assignment. A Stackelberg routing strategy is combined into the network optimum model by Korilis et al. [9]. A non-cooperative game framework combining the traffic control model with routing is proposed by Altman et al. [10], and more detailed study is done on uniqueness, efficiency and computational method of Nash equilibrium. In addition, multi-class equilibrium models are also studied by La and Anantharam [11], which is analogous to the multiple equilibrium behaviour models. The game theory algorithm is introduced in agent based cooperative decentralized airplane system by Sislak et al. [12].

The Stackelberg strategy safeguards each player against any attempts by the other players to deviate, but in a sequential manner. In other words, once the leader and the follower are specified; then the leader has no better choice than to select a Stackelberg strategy under his own leadership and the follower has no better choice than to react according to his reaction set. The leader has no incentive to cheat, since he knows that his control is continuously monitored by the follower. In this sense the Stackelberg strategy is also an equilibrium point. Thus, the relationship between traffic authority and traffic users can be modelled by the Stackelberg game model. The traffic manager is regarded as a leader and traffic users act as followers and the traffic manager makes this optimal strategy and let the users converge to their respective equilibrium.

In terms of the traffic assignment, the method of successive average (MSA) [13], the method of simple projection (SP) [4], the method of day-to-day swapping [14], and the method of modified alternative direction [15] are the most well documented. Although these algorithms are implemented in the transportation networks, their relative performance is still unclear.

Moreover, a simple optimization algorithm is required in view of the relationship of Stackelberg game between traffic management and drivers. Considering the implementation of the game based strategy between traffic management and traffic users, an efficient solution algorithm is required, since it needs to update route choice advices as often as possible. In recent years, Gradient projection algorithms have been found to outperform the Frank-Wolfe algorithm and the feasibility of applying the gradient projection algorithm to the traffic networks is demonstrated. Thus, a modified gradient projection algorithm is introduced in view of Stackelberg game between traffic management and drivers to mediate their relationship.

The remainder of this paper is organized as follows: Sect. 2 presents the game theory based traffic assignment for the road network. The gradient projection algorithm is introduced to improve the efficiency of traffic assignment in Sect. 3. Section 4 gives a description of the solution methodology for the integrated system. Numerical experiments are implemented in Sect. 5. Concluding remarks are given in Sect. 6.

2 Stackelberg Game Based Modelling

There are two decision variables in the game strategy, where one is a set of path flows caused by the drivers' route choice behaviour, and the other is a set of travel cost information as a result of traffic management strategy. The control variables are set by the information providers to gain their objectives, which can be combined in a mathematical form. Thus, the cooperation and competition relationship between traffic management and traffic users can be expressed as a mathematical minimization problem;

$$min_{\alpha} Z_{\alpha}(\alpha, x^{*}(\alpha)) \tag{1}$$

where α is a vector of traffic information influencing drivers' perception of the travel cost, and $x^{*}(\alpha)$ is a vector of traffic flows as a result of the traffic assignment which is a mathematical formulation of drivers' route choice problem. Traffic flow patterns can be achieved by solving the traffic assignment problem based on the information vector α determined by the above problem.

The traffic assignment representing drivers' route choice behaviour can also be expressed as a minimization problem;

$$\min_{x} Z_{x}\left(\alpha^{*}, x\left(\alpha^{*}\right)\right) \tag{2}$$

The state of equilibrium of $(\alpha^{*}, x(\alpha^{*}))$ can be achieved by bi-level programming problem: For the upper level of Stackelberg game programming, find the optimal guidance indicator α for given link g that minimizes total system cost, for the given link g, drivers can make compromise with information providers on sub-paths to accept the route advices. Thus the total system cost for minimization is as follows:

$$\min_{\alpha_g} Z_U = \sum_{a \in A} C^a(x^a + \dot{x}^a) * (x^a + \dot{x}^a) \tag{3}$$

Subject to:

$$\dot{x}^a = x^a - \sum_k \sum_{\tau} \sum_i \sum_j r_{ijk}^{\tau} * \delta_{ijk}^{\tau ta} \tag{4}$$

where Eq. (4) is the constraint of the minimization process, representing the traffic volume change by previous traffic volume and the selected traffic volume. Equation (3) is the traffic volume $\dot{x}^a$ added to the current traffic flow x^a. r_{ijk}^{τ} is the number of vehicles on the link from previous node *i* to next node *j* at time τ and $\delta_{ijk}^{\tau ta}$ is the 0–1 time dependent link-path incidence variables corresponding to the number of vehicles assigned to each link of specific link *a*.

For the lower level of Stackelberg game programming, find the optimal traffic flow pattern to satisfy the UE condition;

$$minZ_L = \sum_{a \in A} \int_0^{x^a} C^a(w + \dot{x}^a) * \alpha_g dw \tag{5}$$

where C^a is the travel cost on link a with traffic volume x^a.

While the upper level problem is the system optimal traffic assignment, the lower level problem is a process of a user equilibrium traffic assignment problem. In order to improve the efficiency of the problem, the traffic assignment problem can be solved by the gradient projection algorithm, where the main target of the problem is to find the guidance indicator α for sub-paths. The sub-path represents the paths which can be perceived by drivers for a user-optimal redistribution under the system optimal traffic assignment to yield better system travel cost.

3 Gradient Projection Based Traffic Assignment

Gradient projection algorithm has been shown as an efficient algorithm for solving the traffic assignment. The GP algorithm is a path-based flow formulation, which cannot find auxiliary solutions in the link-flow space. The feasible space for the gradient projection algorithm is defined only by the non-negativity, since GP algorithm makes moves to the direction of the minimum of the Newton approximation. The update step can be expressed by the following interactive equation:

$$f_k^{rs}(n+1) = [f_k^{rs}(n) - \alpha(n)D(n)\nabla\dot{Z}(n)]^+ \tag{6}$$

where $\alpha(n)$ is the step size, $D(n)$ denotes a diagonal, positive definite scaling matrix, $\nabla\dot{Z}(n)$ is the gradient of the transformed objective function, and $[.]^+$ is the projection of the argument on the positive axis of the independent variables $f_k^{rs}(n)$. This operation of moving demand conservation constraints from the constraint to the objective function can make projection simpler. In the process of the operation, $f_k^{rs}(n)$ is partitioned into the least cost path flow $f_{\bar{k}_{rs}}^{rs}(n)$ and the non-least cost path flow $f_k^{rs}(n)$ in the path set K_{rs}.

$$f_{\bar{k}_{rs}}^{rs}(n+1) = q_{rs} - \sum\nolimits_{k \in K_{rs}, k \neq \bar{k}_{rs}} f_k^{rs}(n+1) \tag{7}$$

where $\bar{k}_{rs}$ is the least cost path from origin to destination. The optimization problem can be transformed into the following form by substituting the least cost path flow $f_{\bar{K}_{rs}}^{rs}(n)$. Thus, the minimization process is related to find non-least traffic cost with non-least traffic cost flow;

$$min\,\dot{Z}(\dot{f}) \tag{8}$$

Subject to:

$$f_k^{rs} \geq 0, \forall k \in K_{rs}, k \neq \bar{k}_{rs}, r \in R, s \in S \tag{9}$$

where $\dot{f}$ is the set of non-least cost path flows for all origin to destination pairs. The objective value can be improved by moving in the negative gradient direction. The gradient of the transformed objective function is related to the set of non-least cost paths, and a diagonal scaling of the gradient direction can be achieved by the second derivatives of the independent variables.

$$\frac{\partial \dot{Z}}{\partial f_k^{rs}} = \frac{\partial Z}{\partial f_k^{rs}} - \frac{\partial Z}{\partial f_{\bar{k}_{rs}}^{rs}}, \forall k \in K_{rs}, k \neq \bar{k}_{rs}, r \in R, s \in S \tag{10}$$

where Z is the original objective function including both the least cost and non-least cost path. Each component of the gradient becomes the difference between the first derivative cost of a non-least cost path and the least cost path, where the first derivative of Z related to any path is link traversal cost based on the current traffic flow information.

$$\frac{\partial Z}{\partial f_k^{rs}} = \sum\nolimits_{a \in A} c^a(x^a)\delta_k^{rsa} \tag{11}$$

$$\frac{\partial Z}{\partial f_k^{rs}} = \sum\nolimits_{a \in A} c^a(x^a)\delta_{\bar{k}_{rs}}^{rsa} \tag{12}$$

Thus, the diagonals of the second derivatives of the transformed objective function are the differentiation of the gradients,

$$\frac{\partial^2 \dot{Z}}{\partial f_k^{rs2}} = \sum\nolimits_{a \in A} c^{a'}(x^a)(\delta_k^{rsa} - \delta_{\bar{k}_{rs}}^{rsa})^2 \tag{13}$$

where $c^{a'}(x^a)$ denotes the first derivative of the link traversal time.

Let d_k^{rs} and $d_{\bar{k}_{rs}}^{rs}$ be the first derivative costs of path k and the least cost path $\bar{k}_{rs}$ of the origin to destination pair, the iterative flow update can be expressed as follows:

$$f_k^{rs}(n+1) = \max\{0, f_k^{rs}(n) - \frac{\alpha(n)}{s_k^{rs}(n)}[d_k^{rs}(n) - d_{\bar{k}_{rs}}^{rs}]\} \tag{14}$$

where $\alpha(n)$ denotes a scalar modifier. Once all the non-least cost paths are updated, the traffic flow on the least cost path is appropriately updated so that the demand is conserved.

4 Solution Methodology for the Stackelberg Game Based Traffic Assignment

In order to improve the efficiency of the game based traffic assignment, the Gradient Projection method is implemented. In the principle of the GP method, the flow f_k^{rs} is partitioned into the least cost path flow $f_{\bar{K}_{rs}}^{rs}(n)$ and the non-least cost path flow $f_k^{rs}(n)$ in the path set K_{rs}. Substituting the partition of the least cost path flow and non-least cost path, the path flow f_k^{rs} in the game based traffic assignment is partitioned into user equilibrium least cost path flows and system optimal least cost path flows.

For the different objectives of the UE model and SO model, the least cost path flows on the UE model and SO model corresponding to the same O-D (origin to destination) pair are different. Let the traffic flow on the least cost path between UE model and SO model be the least cost path flow $f_{\bar{K}_{rs}}^{rs}(n)$ and the other one be the non-least cost path flows $f_k^{rs}(n)$. Moreover, the paths with traffic cost between the two least cost of the UE model and SO model are defined as the sub-paths and stored as the path set K_{rs}. Substituting the least cost path flow $f^{rs}_{K_{rs}}(n)$ given by Eq. (12) for each O/D pair into the objective function, the optimization problem can also be formalized by the form of Eq. (13). Then, the modified GP method can be implemented in the traditional implementation process.

Thus, the Stackelberg game based integrated system can be improved by the Gradient Projection method to achieve efficient traffic assignment. The solution algorithm, as shown in Fig. 1 gives the framework for the Stackelberg game based traffic assignment, can be described as follows:

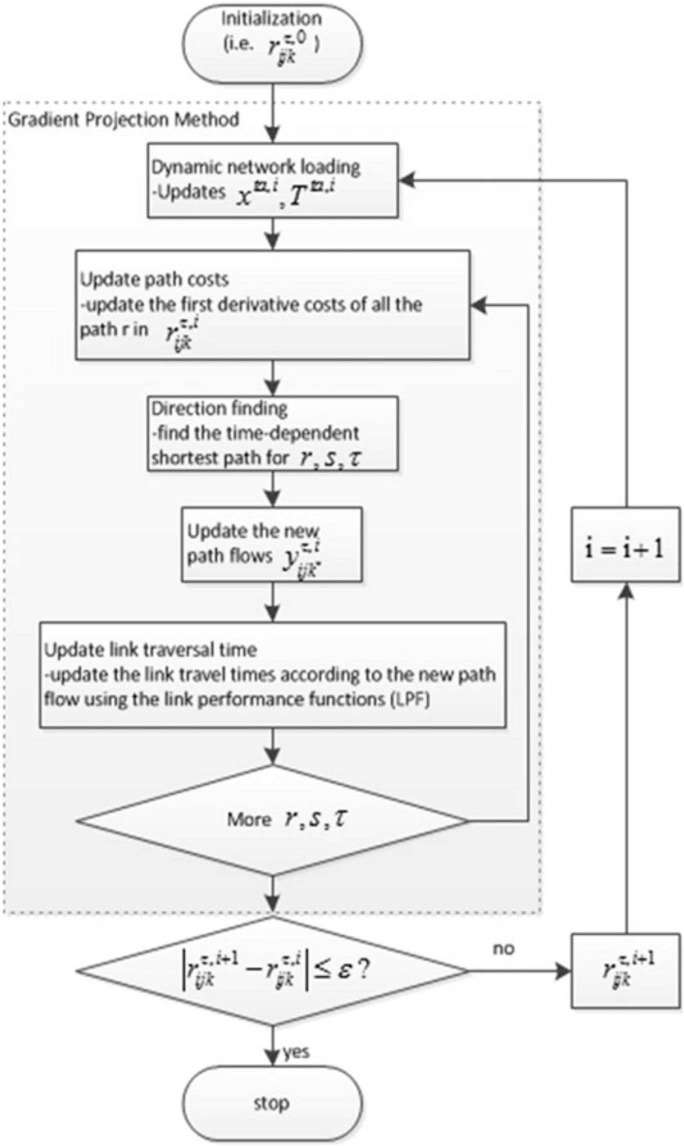


Fig. 1 The algorithmic framework for the Stackelberg game based traffic assignment

(1) Pre-assignment
(2) Find sub-paths associated with traffic management and traffic users
(3) Update optimal guidance indicator value to satisfy system optimal traffic assignment
(4) Update sub-paths, indicator, and total cost
(5) Stop, if no more sub-paths, otherwise go to step 2.

5 Numerical Experiment

The game based gradient projection algorithm is implemented in the Sydney road network which consists of 287 nodes, 592 directed edges with positive demands. Figure 2 shows the traffic road network and real-time traffic flow information in Sydney, where the red line denotes more than 800 vehicles/lane/hour on the road, orange line denotes more than 600 vehicles/lane/hour on the road, and green line denotes less than 600 vehicles/lane/hour on the road. Results generated from various experiments form the comparisons of system performance under time-dependent game based gradient projection, user equilibrium and system optimal traffic assignment, which can present clear qualitative and quantitative differentiations between the game based gradient projection, user equilibrium and system optimal solutions.

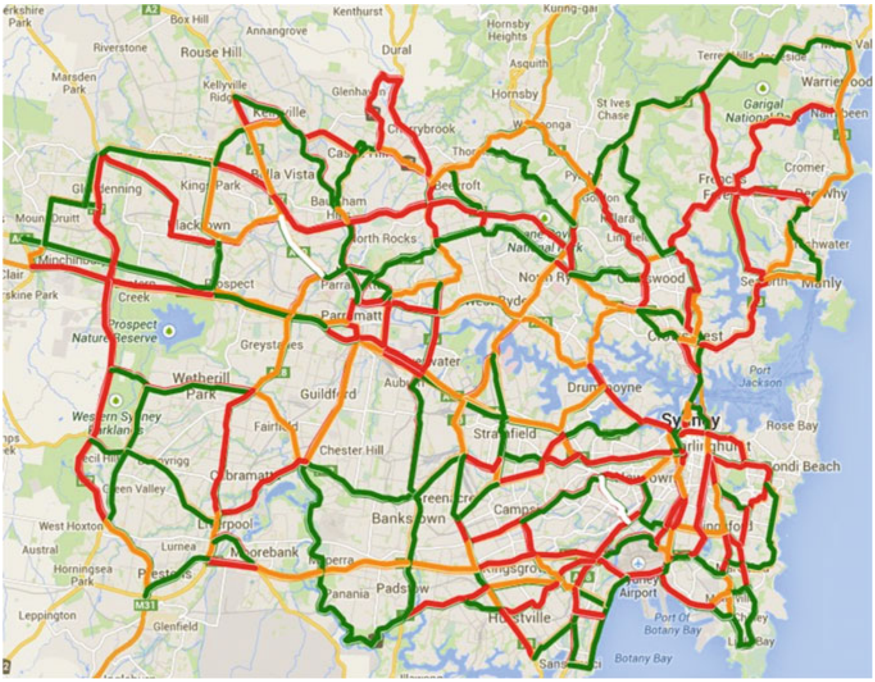


Fig. 2 The traffic road network and real-time traffic flow information in Sydney

5.1 Convergence and Computation Performance of the Gradient Projection Method

In order to manifest the effectiveness of gradient projection algorithm, the gradient projection algorithm is compared with Frank-Wolfe algorithm and the method of successive averages (MSA) algorithm regarding convergence and computation performance. In term of the Frank-Wolfe algorithm [13], it is one of the promising algorithms for traffic assignment, since it can make full use of the network structure of the road networks. The search directions of the Frank-Wolfe algorithm usually tend to be perpendicular to the steepest descent directions of objective function as the iteration proceeds. For the method of successive averages algorithm [14], it is one of the most widely used solution methods in simulation-based dynamic traffic assignment. This method relies on predetermined step sizes without requiring derivative information, so that it can obviate the need to solve one-dimensional line search problems for finding the optimal move size.

Figure 3 shows the Solution convergence of the Gradient Projection method Compared with the Frank-Wolfe algorithm. Both the Frank-Wolfe (FW) algorithm and the gradient projection (GP) algorithm are initialized with zero flows on all the links in the road network. In the FW model, it uses the all-at-once flow update, where the total link-flow pattern are adjusted after the traffic demands from all O/D pairs are assigned to the network, while the GP model updates the flow pattern one O/D at-a-time, that is, the total link-flow pattern is revised after the assignment of an O/D pair before continuing to the next O/D pair. Since different traffic flow patterns are updated, the convergence rates are also different. As shown in Fig. 3, the GP converges faster than FW. Typically, the 5th or 6th iteration in GP corresponds to the 10th iteration in FW. Actually, FW slowly approaches to the minimum solution, and the objective value of FW in the 100th iteration is exactly the same as the 10th iteration. However, the GP can quickly approach to the minimum solution.

Fig. 3 Solution convergence

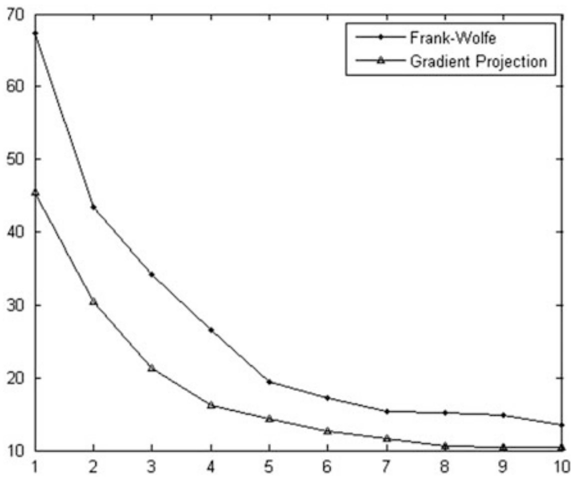


Table 1 Computation time and number of iterations to convergence for various size of road network

–	10 nodes, 42 links, 170 O/Ds	36 nodes, 92 links, 670 O/Ds	100 nodes, 375 links, 5670 O/Ds	287 nodes, 592 links, 51670 O/Ds
–	Iteration/time (s)	Iteration/time (s)	Iteration/time (s)	Iteration/time (s)
FW	43/17.63	56/25.46	161/58.76	268/95.56
MSA	31/19.92	59/23.12	139/57.6	336/113.21
GP	6/2.41	9/4.37	13/11.39	18/45.23

In order to test the computation of the Gradient Projection method, the GP method is tested on various sizes of grid networks. Table 1 shows the performance associated with computation time and number of iterations for the Frank Wolfe algorithm, the method of successive averages (MSA) and the Gradient Projection algorithm tested on various sizes of road network, ranging from 10 to 592 nodes. From the comparison results, it shows that GP takes much less iterations and less computation time than the other two algorithms to reach the same objective value in all the situations. As the size of the road network increases, the ratio of computation time comparing with FW and MSA decreases. However, it is still more efficient than the conventional FW and MSA methods. Moreover, it suggests that the GP method in the decomposed networks of smaller size can achieve significant benefit in computation time.

5.2 *Travel Performance Comparison with SO and UE*

Since the implementation of the game based traffic assignment is based on the interaction process between UE and SO traffic assignment, in order to manifest the travel performance benefits of the game based gradient projection algorithm, the game based traffic assignment is compared with the user equilibrium and system optimal traffic assignment. The system optimal traffic assignment is a time-dependent path based traffic assignment, which can provide system optimal traffic assignment for traffic managements.

The user equilibrium traffic assignment is based on the route choices selection process, which can generate traffic assignment considering divers' preferences. The game based traffic assignment is the coordinated strategy between system optimum and user equilibrium traffic assignment, which is used to compromise the benefits between traffic managements and travellers.

The game based traffic assignment is implemented as follows: Firstly, an independent UE and SO traffic assignment are modelled respectively; Secondly, based on the UE and SO traffic assignment, the Stackelberg game based traffic assignment improved by the gradient projection method is compared with the individual traffic assignment. The traffic assignments are tested under different network congestion levels, achieved by different network loading levels. The network loading factor

Table 2 Loading factors and the corresponding number of generated vehicles for the numerical experiments

Loading factor	Number of generated vehicles
1.0	19346
1.4	27168
1.8	34887
2.0	38762
2.2	42631

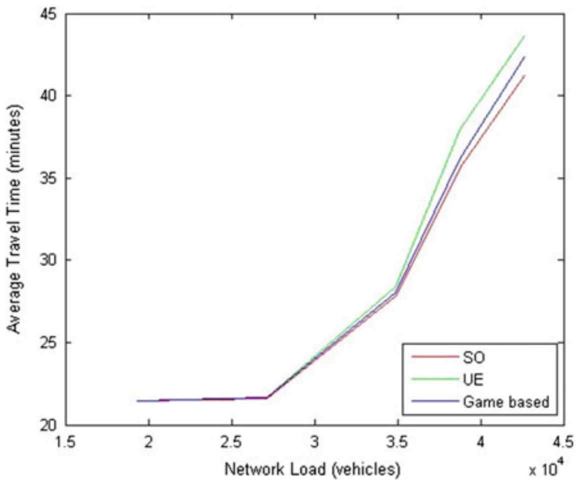
denotes the ratio of the total number of vehicles generated in the road network during the traffic assignment periods compared to a given reference number (19346 vehicles over 15 min period represent a loading factor of 1.0). In the numerical experiments, five loading factors are considered, namely, 1.0, 1.4, 1.8, 2.0 and 2.2. Table 2 shows the number of vehicles generated for each loading factor.

Table 3 shows the system performance under the time dependent SO, UE and Stackelberg game based traffic assignment associated with different loading factors. Based on the principle of the traffic assignment, when the road network is relatively uncongested (at low loading levels), the average travel time of vehicles in the road network is relatively close. As the loading factor is increased, congestion can be caused and the average travel time increases with the loading factor. The results show that the average travel time is significantly increased with the increase of the loading factors, while the average travel distance makes limited variation under the various loading levels. It indicates that the greater traffic congestion is the primary cause of the higher system travel time instead of the travel routes. Moreover, the

Table 3 Comparison results of SO, UE and GAME based traffic assignment associated with various loading factors

Loading factor	Av. travel time (min)	Total travel time (h)	Av. travel distance (km)	Total travel distance (km)	Av. speed (kmph)
1.0	21.46	6759.419	15.43	290508.8	44.44073
1.4	21.62	9599.536	15.58	414777.4	44.53774
1.8	27.83	15971.75	16.73	573659.5	37.36899
2.0	35.64	22744.63	17.46	663784.5	30.69394
2.2	41.24	28991.71	18.43	764689.3	28.11377
1.0	21.46	6759.079	15.43	290435.8	44.39473
1.4	21.65	9613.12	15.53	413774.4	44.31926
1.8	28.36	16279.92	16.49	565286.6	36.18717
2.0	37.94	24230.50	17.24	655256.9	28.56410
2.2	43.65	30704.05	18.37	762131.5	26.55086
1.0	21.46	6759.209	15.43	290474.8	44.43034
1.4	21.63	9603.854	15.56	414600.1	44.45197
1.8	28.04	16093.65	16.51	565984.4	36.61781
2.0	36.17	23086.82	17.33	658745.5	30.03728
2.2	41.31	29751.75	18.41	763836.7	27.39725

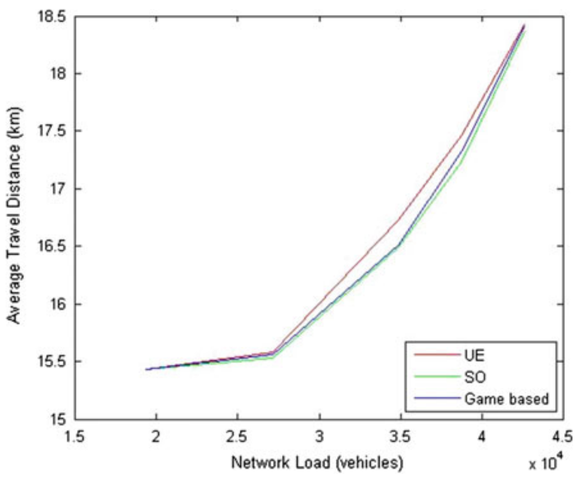
Fig. 4 Average travel time comparison between UE, SO and game based traffic assignment



average travel distance increases with the loading level increased, suggesting an increase number of drivers assigned to longer travel routes. In addition, the average travel distances under UE are less than average travel distances under SO for various loading factors, indicating some drivers are assigned to longer routes under SO in order to reduce traffic congestion to achieve system optimal benefit.

For different objectives of the UE and SO models, Stackelberg game based traffic assignment is proposed to balance the benefit between UE and SO in order to relieve traffic congestion and reduce travel cost. The results under the Stackelberg game based traffic assignment show that the game based traffic assignment makes compromise between UE and SO. Figures 4 and 5 compare the average travel time and average travel distance under UE, SO and Stackelberg game based traffic

Fig. 5 Average travel distance comparison between UE, SO and game based traffic assignment



assignment. The results show that limited variation in both average travel time and average travel distance is caused by the game based traffic assignment at the lower loading levels.

The reason is that the traffic is not congested at lower loading levels, and travel costs as a result of the UE and SO traffic assignment are identical to each other, thus, limited variation is caused by the game based model. With the increase of the loading factors, the average travel time approaches the SO model, indicating a number of vehicles assigned to SO model to relieve traffic congestion on the roads at the expense of the increase of average travel distance.

6 Conclusion

In this paper, a Stackelberg game based traffic assignment is applied to deal with the cooperation and competition relationship between UE and SO. While the traffic management requires the traffic users cooperatively to achieve SO traffic assignment, traffic users competitively make route choices based on UE traffic assignment. Thus, the cooperation and competition relationship between traffic management and traffic users can be modelled by the game theory model, which can balance the benefit between traffic managements and travellers. Comparing with the Nash game theory, the Stackelberg game theory with the leader and follower player can better model the relationship between traffic authorities and traffic users. Moreover, the Gradient Projection algorithm is introduced to improve the efficiency of the game based traffic assignment.

The integrated system is implemented in the Sydney road network, and the introduced gradient projection algorithm can improve convergence rates and computation time comparing with Frank-Wolfe algorithm and MSA method. The system performance results show that route choices given by the game based redistribution can compromise between traffic management and traffic users to avoid congested routes and reduce travel time in the road network.

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