

Preface

This volume is a collection of chapters dedicated to Professor Q. I. Rahman who passed away on July 21, 2013, in Montreal, Canada. Professor Rahman was a leading mathematician whose research spanned several areas of approximation theory and classical analysis, including complex analysis. Professor Rahman was viewed as a world expert in the analytic theory of polynomials and entire functions of exponential type by his collaborators and many other colleagues.

We invited outstanding mathematicians, friends, and collaborators of Professor Rahman to submit chapters to be included in this volume. This collection contains original research articles and comprehensive survey contributions by 36 mathematicians from 18 countries. All the chapters were refereed. We hope that the chapters will interest graduate students and researchers in analysis and approximation theory.

Professor Walter K. Hayman, F.R.S., who was one of Professor Rahman's teachers, has prepared the monograph's Foreword. We are extremely grateful to him for the time and effort that he has devoted to the writing of this piece.

The chapters of the monograph are grouped by four themes which reflect some of Professor Rahman's areas of research. The first theme is *Polynomials*. It includes inequalities for polynomials and rational functions, orthogonal polynomials and location of zeros, and comprises the chapters entitled "On the L_2 Markov Inequality with Laguerre Weight", "Markov-Type Inequalities for Products of Müntz Polynomials Revisited", "On Bernstein-Type Inequalities for the Polar Derivative of a Polynomial", "On Two Inequalities for Polynomials in the Unit Disk", "Inequalities for Integral Norms of Polynomials via Multipliers", "Some Rational Inequalities Inspired by Rahman's Research", "On an Asymptotic Equality for Reproducing Kernels and Sums of Squares of Orthonormal Polynomials" and "Two Walsh-Type Theorems for the Solutions of Multi-Affine Symmetric Polynomials". The second theme is *Inequalities and Extremal Problems*, where functions other than polynomials are considered. This theme consists of chapters entitled "Vector Inequalities for a Projection in Hilbert Spaces and Applications", "A Half-Discrete Hardy-Hilbert-Type Inequality with a Best Possible Constant Factor Related to the Hurwitz Zeta Function", "Quantum Integral Inequalities for Generalized Convex Functions", "Quantum Integral Inequalities for Generalized Preinvex Functions"

and “On the Bohr Inequality”. The third theme is *Approximation of Functions*, the approximants being polynomials, rational functions and other types of functions; see Chapters entitled “Bernstein-Type Polynomials on Several Intervals”, “Best Approximation by Logarithmically Concave Classes of Functions”, “Local Approximation Using Hermite Functions”, “Approximating the Riemann Zeta and Related Functions”, “Overconvergence of Rational Approximants of Meromorphic Functions” and “Approximation by Bernstein-Faber-Walsh and Szász-Mirakjan-Faber-Walsh Operators in Multiply Connected Compact Sets of \mathbb{C} ”. The last theme is *Quadrature, Cubature and Applications*. It comprises three chapters, including a posthumous article of Professor Rahman co-authored by one of the editors of this book. This theme includes chapters entitled “Summation Formulas of Euler-Maclaurin and Abel-Plana: Old and New Results and Applications”, “A New Approach to Positivity and Monotonicity for the Trapezoidal Method and Related Quadrature Methods” and “A Unified and General Framework for Enriching Finite Element Approximations”.

In the first chapter, the authors Nikolov and Shadrin have considered L_2 Markov inequality with Laguerre weight over a semi-infinite interval of the real line. They have also obtained an asymptotic value of the constant in their inequality.

In the chapter by Erdélyi, new Markov-type inequalities for products of Müntz polynomials have been proved. These results extend some of the earlier contributions of the author and answer some questions posed by Thomas Bloom.

Govil and Kumar in their survey article mention in a chronological manner Bernstein-type inequalities for polar derivatives of a polynomial. This chapter provides a comprehensive account of results on polar derivatives.

Fournier and Ruscheweyh consider two very different generalizations and refinements of Bernstein’s inequality for polynomials that have been obtained more than 30 years ago. Here, the authors show that one of these inequalities implies the other. They also study the cases of equality.

Pritsker considers a wide range of polynomial inequalities for norms defined by contour and area integrals over the unit disk in the complex plane. He has also proved inequalities using the Schur-Szegő composition.

The chapter by Li, Mohapatra, and Ranasinghe is concerned with some rational inequalities inspired by Rahman’s research. The results include Bernstein-type inequalities for rational functions with prescribed poles and prescribed zeros.

Ignjatovic and Lubinsky investigate an asymptotic equality for reproducing kernels and sums of squares of orthonormal polynomials. These results are motivated by the recent work of Ignjatovic on orthonormal polynomials associated with a symmetric measure with unbounded support and satisfying a recurrence relation. The authors have studied the case of even exponential weights and weights on a finite interval.

In the chapter by B. Sendov and H. Sendov, the authors have considered two Walsh-type theorems for the solution of multi-affine symmetric polynomials. These results can be considered as extensions of the Grace-Walsh-Szegő coincidence theorem.

In the chapter by Dragomir, some vector inequalities related to those of Schwarz and Buzano are established. Also, inequalities involving the numerical range and the numerical radius for two bounded operators are obtained.

Rassias and Yang have used methods of weight functions to obtain a half-discrete Hardy-Hilbert-type inequality with a best constant related to the Hurwitz-Zeta function. Equivalent forms, normed operator expressions, their reverses, and some special cases are also considered.

In the chapter by M. Noor, K. Noor, and Awan, the authors have considered and generalized convex functions involving two arbitrary functions and established some new quantum integral inequalities for the generalized convex functions. Besides, several special cases of interest have been mentioned as corollaries.

M. Noor, Rassias, K. Noor, and Awan have considered quantum integral inequalities involving generalized preinvex functions. They give an account of quantum integral inequalities and in a certain limiting case use these inequalities to obtain many well-known results as special cases. The contents of this chapter are related to that of the previous chapter.

In the chapter by Abu Muhanna, Ali, and Ponnusamy, the Bohr inequality is considered. This survey article considers recent advances and generalizations of the Bohr inequality in the unit disk of the complex plane. Among other things, they have discussed the Bohr radius for harmonic and starlike logharmonic mappings in the unit disk.

In the chapter by Szabados, Bernstein-type polynomials for a set J_s of s finitely many intervals have been considered. On such sets, approximating operators resembling Bernstein polynomials have been defined, and their interpolation properties and rate of convergence are obtained.

The chapter by Dryanov contains results on best approximation by a class of logarithmically concave functions. Exact values of best approximations are found for two specific cases.

The chapter by Mhaskar considers local approximation using Hermite functions. He develops a wavelet-like representation in $L_p(R)$ where the local behavior of the terms characterizes the local smoothness of the target function. He gives new proofs for the localization of certain kernels as well as for the Markov-Bernstein inequality.

Stenger considers a function G which has the same zeros as the well-known Riemann zeta function in the critical strip. For studying its behavior for intermediate values of z , he uses Fourier series and derives an asymptotic approximation for large values of z .

The chapter by Blatt deals with overconvergence of rational approximants of meromorphic functions. It contains results on the degree of convergence and distribution of zeros of the rational approximants. In addition, well-known results on polynomial approximation of holomorphic functions are generalized.

The chapter by Gal is concerned with approximation by Bernstein-Faber-Walsh and Szász-Mirakjan-Faber-Walsh operators in multiply connected compact sets of the complex plane. These results are generalizations of earlier results of the author on q -Bernstein-Faber polynomials and Szász-Faber-type operators in simply

connected compact sets in C . This study leads to a conjecture concerning the use of truncated classical Szász-Mirakjan operators in weighted approximation.

Milovanović discusses old and new results on summation formulas of Euler-Maclaurin and Abel-Plana. He has shown connections between Euler-Maclaurin formula and basic quadrature rules of Newton-Cotes-type as well as the composite Gauss-Legendre rule and its Lobato modifications. Summation formulas such as the midpoint summation formula, the Binet formula, and the Lindelöf formula are also extended and analyzed.

The chapter by Rahman and Schmeisser provides a new approach to positivity and monotonicity for quadrature methods. In all of the known results, sign conditions on some derivatives of the given function are required. The authors propose a new approach based on Fourier analysis and the theory of positive definite functions. This method makes it possible to describe much wider classes of functions for which positivity and monotonicity occur. Their results include the trapezoidal method on a compact interval and also on the whole real line.

The chapter by Guessab and Zaim is devoted to a unified and general framework for enriching finite element approximations through the use of additional enrichment functions. They prove a general theorem that characterizes the existence of an enriched finite element approximation. They also show that their method can be used to obtain a new class of enriched nonconforming finite elements in any dimension. For concrete constructions, the authors employ new families of multivariate trapezoidal, midpoint, and Simpson-type cubature formulas.

It is a pleasure to express our gratitude to all the authors and referees without whose contributions this volume would not have been possible. We would like to thank Ziqin Feng, Dmitry Glotov, and Eze Nwaze for their help in compiling and formatting some of the chapters in this volume. Finally, our thanks are due to the publisher for support and careful handling of this volume.

Auburn, AL, USA
Orlando, FL, USA
Tuskegee, AL, USA
Erlangen, Germany

N.K. Govil
Ram Mohapatra
M.A. Qazi
G. Schmeisser

Progress in Approximation Theory and Applicable
Complex Analysis

In Memory of Q.I. Rahman

Govil, N.K.; Mohapatra, R.N.; Qazi, M.; Schmeisser, G.
(Eds.)

2017, XXXIV, 519 p. 24 illus., 14 illus. in color.,

Hardcover

ISBN: 978-3-319-49240-7