Preface

This book has evolved from courses on the finite element method that have been taught as part of several graduate programmes at the University of Oxford. These courses have all focused on the practical application of the finite element method. A typical course would begin with a specific class of differential equations being written down. The course would then provide students with all the detail that is required to develop a computational implementation of the finite element method for this class of differential equations. This approach requires the following topics to be addressed: (i) derivation of the weak formulation of a differential equation; (ii) discretisation of the domain on which the differential equation is defined into elements; (iii) specification of suitable basis functions; (iv) derivation of a system of algebraic equations that is derived from the finite element formulation of the differential equation; (v) solution of this system of algebraic equations; and (perhaps most importantly) (vi) the practical implementation of all steps. This book is written in the spirit in which these courses have been taught. Mathematical rigour is certainly needed when discussing some of these topics, such as the use of Sobolev spaces when deriving the weak formulation of a differential equation. The focus, however, is on the practical implementation of the finite element method for a given differential equation.

This book begins with a brief overview of the finite element method in Chap. 1. Chapters 2–6 then focus on the application of the finite element method to ordinary differential equations. In Chap. 2, we begin with a very simple example differential equation. Although simple, this example allows illustration of the key ideas that underpin the finite element method. We then consider general, linear ordinary differential equations in Chap. 3, providing a more rigorous discussion than in Chap. 2. The material in Chap. 3 focuses exclusively on finite element solutions that are a linear approximation to the true solution on each element. These concepts are extended in Chap. 4 to explain how a higher order polynomial approximation may be used on each element. We then explain how nonlinear ordinary differential equations are handled by the finite element method in Chap. 5, and how systems of ordinary differential equations are handled in Chap. 6. In all of these chapters, we carefully explain how the values of the solution at each node of the computational mesh satisfy a system of algebraic equations. We explain how these algebraic equations may be assembled, and how they may be solved. Each chapter contains an exemplar computational implementation in the form of MATLAB functions.

Chapters 7–12 focus on the application of the finite element method to elliptic partial differential equations, and these chapters take a very similar approach to that used in earlier chapters. We begin with a simple example in Chap. 7, before considering more general linear elliptic equations in Chap. 8. In both of these chapters, we partition the domain on which the differential equation is defined into triangular elements. In Chap. 9, we explain how the material presented in Chap. 8 may be modified to allow the use of a mesh that arises from partitioning the domain into quadrilateral elements. Higher order polynomial approximations to the solution are considered in Chap. 10, followed by nonlinear partial differential equations in Chap. 11 and systems of elliptic equations in Chap. 12. Exemplar computational implementations of the finite element method are given in several of these chapters. Finally, in Chap. 13, we explain how the finite element method may be applied to parabolic partial differential equations.

Every application of the finite element method in this book requires the solution of a system of algebraic equations. Appendix A provides a brief summary of computational techniques that are available for solving both linear and nonlinear systems of algebraic equations. This summary covers both direct and iterative methods for solving linear systems, preconditioners that accelerate the convergence of iterative methods for solving linear systems, and iterative methods for solving nonlinear systems.

My understanding and appreciation of the finite element method has been substantially enhanced by discussions with colleagues too numerous to mention. Special mention must, however, be made of both the Computational Biology Group in the Department of Computer Science at the University of Oxford, and the Numerical Analysis Group in the Mathematical Institute at the University of Oxford. The finite element research interests of these two groups cover a wide spectrum, addressing a range of both practical and theoretical issues. I am very fortunate to have been a member of both of these research groups for periods of time over the last two decades.

Above all, I would like to thank my family for their support during the writing of this book.

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