Preface

This textbook originated from independent collections of lecture notes that slowly evolved and eventually merged together. The first embryonic version was a set of lecture notes on special stochastic processes with an emphasis on percolation models and interacting particle systems. Then came two more manuscripts both focusing on the general theory of stochastic processes developed as lecture notes for two courses on this topic, one for advanced undergraduate students and later its counterpart for graduate students based on measure theory. Since most of the proofs about percolation models, interacting particle systems, and other special stochastic models involve results from martingale theory, Poisson processes, and discrete-time and continuous-time Markov chains, it became natural to merge these lecture notes into the same document.

Material and structure

There are seventeen chapters that, together, form three coherent parts.

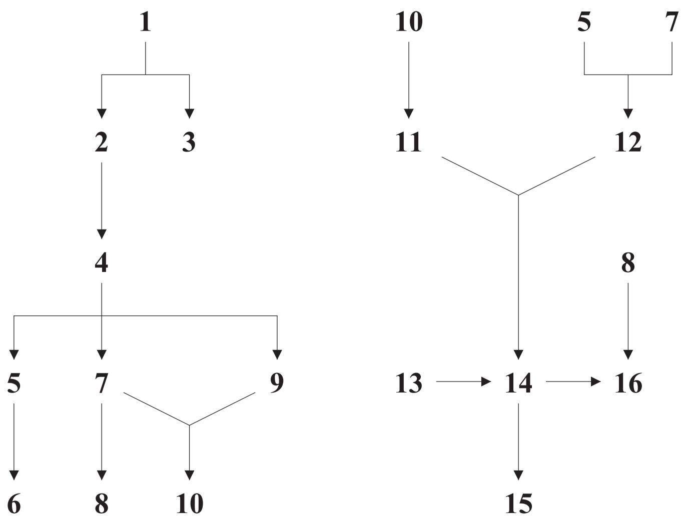
Part I. Probability theory — Chapters 1–3 cover the classical material on probability theory seen at the undergraduate level, from random variables to conditional probability and standard limit theorems. The main objective of this part is to redefine the main probability concepts with an emphasis on their connection to concepts from measure theory. Because the reader is expected to be familiar with basic probability and because the primary focus is not measure theory, the style is intentionally brief, focusing more on intuition rather than the technical details.

Part II. Stochastic processes — Chapters 4–10 cover the classical material on stochastic processes and, in contrast to the first three chapters, are mostly self-contained. This second part presents the main results about martingales, discrete-time Markov chains, Poisson processes, and continuous-time Markov chains. The theory is interspersed with examples of applications, including the popular gambler's ruin chain, branching processes, symmetric random walks on graphs, and birth and death processes, all of which illustrate the main theorems.

Part III. Special models — Chapters 11–17, which are less traditional and more research oriented, focus on models that arise in physics, biology, and sociology with an emphasis on minimal mathematical models that have been used historically to develop new techniques in the field of stochastic processes: the logistic growth process, the Wright–Fisher model, Kingman's coalescent, and some of the simplest percolation models and interacting particle systems. The proofs mostly rely on the subtle combination of results established in the second part and tailored-made arguments. Even though the proofs are model specific and involve a wide variety of techniques, the models covered in this last part form a coherent unit. While going through these seven chapters in the last part, the reader will discover how the models are connected to one another. The end of this part focuses on numerical simulations and summarizes these connections by exploring various algorithms that can be used to construct the models.

For an overview of the main topics mentioned in this textbook organized chronologically from the origin of probability theory in the seventeenth century until the end of the twentieth century along with the names of some of the main contributors, we refer the reader to the timeline at the very end of this textbook.

The logical dependence among the first sixteen chapters is shown in the following directed graph that we have broken down into two connected components to avoid having arrows crossing each other.



Note that Chapter 17 does not appear in the picture even though it gives an overview of the main models presented in this textbook and is therefore related to a number of other chapters. In fact, the reader can go directly to this chapter after reading the definition of the main models because there is no specific need to know about their analysis or the theory that supports them. We also point out that the following seven sections from the part on stochastic processes deal with more specific topics that are not required to understand the rest of the textbook:

Sections 5.3–5.5 from the chapter on martingale theory;
Section 6.4 on the number of individuals in the branching process;
Sections 8.2–8.3 about symmetric random walks;
Section 9.4 on the conditioning property for Poisson processes.

Each chapter starts with a brief overview illuminating the key steps and some additional readings on the topic. Diagrams summarizing the main concepts and results along with their connection are given at the end of the chapters covering the standard material. Exercises, with an emphasis on the classics of probability theory and real-world problems, are given at the end of most of the chapters. Exercises from the first two parts are purely analytical and rely on the techniques covered in the textbook, whereas the exercises in the third part on special models are more research oriented and consist of mixtures of analytical and simulation-based problems.

Prerequisites and course planning

There is a constant balance between mathematical rigor and brevity in order to cover the maximum number of topics in a minimum number of pages, but the reader interested in a specific topic is directed to references providing additional details or results. Even though this textbook is not that long, it seems to be ambitious to teach all the material in only one semester. The first two parts can typically be used to teach a general course on **random processes** for advanced undergraduate and graduate students in applied mathematics. The last part focusing on special stochastic processes can be used to teach a more research oriented **stochastic modeling** course for advanced graduate students in mathematics and applied sciences, while also using the beginning of the book as a refresher. Here are more details on the prerequisites and possible course planning for each of these two parts.

Random processes — The prerequisite for this material is a good knowledge of undergraduate probability: the basics of combinatorics and finite probability, conditional probability, discrete and continuous random variables, properties of expectation, and limit theorems. This is basically an undergraduate version, not relying on measure theory, of the material covered in the first part of this textbook, which is treated for example in *A First Course in Probability* by Sheldon Ross. All the theory in the first ten chapters can be covered in one semester, which should leave some time to also treat some of the examples and exercises. My preference, however, is to skip some of the seven more specific sections listed above in order to more frequently alternate between theory and illustrative practice problems, devoting about half of the lectures to each of these two aspects. The point here is that there are enough examples and exercises so that instructors can adjust the balance between theory and practice to better fit the background and taste of their students.

Stochastic modeling — As previously mentioned, the third part on special processes forms a coherent unit. In particular, skipping any aspect of this part might appear as a missing piece of the puzzle. For a course based on this material, it seems to be more appropriate that the students submit a short research project on the topic of their choice using the material covered in class rather than prepare for a final exam. The prerequisite for the material on special processes is precisely the material covered in the first two parts, with again the exception of the seven more specific sections mentioned above. My suggestion is to cover the prerequisite without treating any of the examples or exercises during the first half of the semester and the part on special processes during the second half of the semester. In my experience, it is also good to give some homework chosen from the exercise sections while covering the prerequisites, but let the students focus on their final research project while lecturing in special processes. In the case when students are already comfortable with the theory of random processes, the instructor can instead go directly to the third part and cover some of the most difficult and more research-oriented exercises.

Finally, while the first two parts have been specifically written for teaching purposes, the reader can also use the last part on special processes as a starting point outside the classroom to learn about a research topic.

Acknowledgements

The material of this textbook has co-evolved in parallel with teaching and the style has largely been shaped by fruitful interactions with students. The final version was submitted while Eric Foxall was working with me on various research projects, and our discussions about probability theory clearly had some beneficial influence on some of the aspects of this textbook. Going backward in time, the first set of lecture notes on special stochastic processes was initiated about eight years ago and is largely the result of my discussions with Claudia Neuhauser and Rick Durrett. Going back a few more years earlier, I am obviously grateful to the very first people who taught me probability theory while I was a graduate student in France, in particular my former advisor Claudio Landim who introduced me to the beautiful field of interacting particle systems. Finally, special thanks to four anonymous reviewers whose comments considerably helped improve the structure and clarity of this textbook, as well as to Donna Chernyk for coordinating the review process.

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