

Preface

Three outstanding problems of modern physics are the following:

1. Description of fundamental spacetime structure.
2. Quantum-theoretic description of gravitation.
3. Unification of physical law.

These problems are deeply intertwined. Since Einstein's discovery of general relativity a century ago, gravity has been understood to be *structural* in nature, rather than a "force," in the usual sense of the word, and it is therefore difficult to imagine the development of a successful approach to quantum gravity without simultaneous acquisition of a deeper structural understanding of spacetime. Quantum theory, meanwhile, though expressed and interpreted in a variety of different ways, is believed by most serious theorists to represent a fundamental aspect of nature; hence, any successful unification of physical law is expected to be quantum-theoretic. It seems, then, that two common ingredients necessary for the solution of these three problems are *suitable structural notions*, and a *suitable approach to quantum theory*. This book attempts to help identify and develop these two ingredients, with Part I devoted principally to structure, and Part II devoted principally to quantum theory. The most important aspects of the approach developed here may be summarized in a few short phrases: cause and effect relationships between pairs of events are taken to be the fundamental building blocks of structure; these events are assumed to form a discrete family rather than a continuum; and quantum theory is formulated as an abstraction and adaptation of Feynman's path summation approach. The resulting theory is called *discrete causal theory*.

At a formal level, discrete causal theory models spacetime via structured sets, called *directed sets*, whose elements represent events; these sets are equipped with special binary relations, whose elements represent causal influences between pairs of events. The term "directed set" is used in this book to mean essentially what the term "directed graph" means in graph theory; it does *not* have the more specific meaning involving upper bounds common in order theory and category theory. Ambitious versions of discrete causal theory attempt to treat "particles and fields"

as aspects of the same directed structure, leading to a simple unified picture of nature at the fundamental scale. The theory may also be generalized to admit the possibility of causal relationships between structural components more complicated than individual events. Discrete causal theory has deep historical roots reaching back to the ancient Greeks, and was foreseen to some degree by Riemann, and even by Leibniz. However, Einstein's relativity, the digital information theory of Shannon and others, the path summation approach to quantum theory pioneered by Feynman, and an assortment of modern mathematical tools from algebra, order theory, and graph theory, are all more-or-less necessary to create a natural context for the theory.

The purest version of discrete causal theory presently enjoying an established theoretical niche is *causal set theory*, introduced in 1987 by Bombelli, Lee, Meyer, and Sorkin. Causal set theory is motivated by technical results of Hawking and Malament, called *metric recovery theorems*, which relate the causal, conformal, and metric structures of relativistic spacetime. Rafael Sorkin's well-known phrase, "*order plus number equals geometry*," summarizes the view that causal structure provides enough information, in the discrete context, to describe emergent spacetime. This phrase represents an early version of what I refer to as the *causal metric hypothesis*, which is the more general idea that the properties of the physical universe, including the metric properties of classical spacetime, arise from causal structure at the fundamental scale. Other versions of discrete causal theory exist; perhaps the best known is *causal dynamical triangulations*, invented by Ambjørn, Jurkiewicz, and Loll. This approach takes for granted certain mysterious properties of nature, such as the local four-dimensional structure of spacetime, in an attempt to shortcut to predictive physics. This is a sensible and worthwhile undertaking, and should be considered part of the same broad program as the more general theory developed in this book. Meanwhile, *category-theoretic* and *topos-theoretic* approaches to physics, pioneered by Isham, Raptis, and others, also incorporate some of the structural ingredients of discrete causal theory, although the interpretations assigned to these structures are generally quite different from the interpretations adopted in this book. In addition, there exist related approaches to certain topics in fundamental physics that are "causal but not discrete," such as approaches based on the mathematical apparatus of *domain theory*.

A major attraction of discrete causal theory is that it admits clean axiomatic presentations. For example, causal set theory may be expressed in terms of six axioms: the *binary axiom*, the *measure axiom*, *countability*, *transitivity*, *interval finiteness*, and *irreflexivity*. The first three of these axioms specify the physical interpretation of a causal set, and restrict its cardinality, while the last three encode its essential mathematical structure. In this book, I develop a new, more general version of discrete causal theory, based on a different choice of axioms, and amplified by the incorporation of new conceptual perspectives and technical methods. These innovations are motivated by the realization that the choice to abstain from continuum-based geometry raises the possibility of qualitatively new types of behavior, such as *irreducibility* and *independence* of influences between pairs of events, which are not completely captured by the conventional

order-theoretic machinery of causal set theory. In particular, the transitive binary relations of causal sets are not necessarily optimal for resolving the subtleties of *independent modes of influence*, while the causal set axiom of interval finiteness, sometimes mislabeled as “local finiteness,” permits potentially problematic locally infinite behavior. Interval finiteness also imposes unjustified restrictions on the global structure of classical spacetime.

Under the alternative set of axioms proposed in this book, transitivity is no longer taken for granted, and interval finiteness is replaced with a genuinely local finiteness condition called *star finiteness*. The latter condition encodes the physical idea of *causal local finiteness*, which states that each event possesses only a finite number of direct causes and effects. I refer to a binary relation satisfying these new axioms as a star finite *causal relation*; its transitive closure is called the *transitive relation*, and is analogous to the familiar causal order. The resulting models, which I call *star finite directed sets*, generalize both causal sets and Finkelstein’s *causal nets*, with the exception of a class of locally infinite objects arising in certain idealized constructions in causal set theory. The latter objects have been recognized as physically unrealistic ever since the theory was founded, and were never intended to serve as actual workhorses for quantum gravity, but they remain a useful source of examples for testing new methods and inferring more general behavior. I also adapt, from mathematical sources, a broader interpretation of directed structure, and more generally, multidirected structure, inspired by modern category theory, and by Grothendieck’s scheme-theoretic approach to algebraic geometry. This expansive viewpoint provides an assortment of new methods and insights.

The resulting theory differs significantly from existing versions of discrete causal theory, particularly at the quantum level. A variety of different types of entities more complex than individual spacetime events may be viewed in a unified fashion as elements of “higher-level multidirected sets,” in analogy with Isham’s topos-theoretic approach to quantum gravity, and Sorkin’s *quantum measure theory*. This viewpoint leads to a new background independent quantum theory of spacetime, in which “elements” are classical histories, modeled via directed sets, and “relations” are natural relationships between pairs of classical histories, called *co-relative histories*. For information-theoretic reasons, these natural relationships are subtler than morphisms in the category of directed sets. The resulting multidirected configuration spaces of directed sets, called *kinematic schemes*, provide the basic structural scaffolding for quantum theory, via path summation. Special cases of kinematic schemes appear under different terminology in causal set theory; for example, in Sorkin and Rideout’s theory of *sequential growth dynamics*. A striking property of kinematic schemes is that their higher-level structure is of essentially the same type as the structure of their constituent histories, a phenomenon I refer to as *iteration of structure*. Besides providing an attractive conceptual unification of classical and quantum theory as different levels of hierarchy under the same structural paradigm, this property enables parallel use of the same technical methods in both semiclassical approximations and in the full background independent quantum theory.

These perspectives and methods not only flesh out the new version of discrete causal theory developed in this book, but also enable important improvements in existing versions of the theory, even if one ignores most of the axiomatic and conceptual innovations offered here. For example, systematic use of *relation space* circumvents the generic discrete problem of *permeability of maximal antichains*, thereby removing an important obstruction to foliation-type approaches to dynamics. This leads to the derivation of *causal Schrödinger-type equations*, which describe quantum spacetime dynamics in a very broad context. The theory of *generational dynamics* for acyclic directed sets, which generalizes sequential growth dynamics for causal sets, leads to the definition of natural *entropic phase maps*, which supply specific content to these equations. Possible near-term phenomenological applications of the new theory involve existing searches for deviations from Lorentz invariance and spatiotemporal locality, refinement of causal set-theoretic explanations for the size of the “cosmological constant,” and non-geometric alternatives to the *inflationary hypothesis* in the cosmology of the early universe.

How to read this book. Faced with a 500-page volume, it is natural to wonder if one can absorb the essentials without reading the entire work. Below, I list several abbreviated reading strategies for various classes of readers. The approximate numbers of pages involved are given below each list. Experienced readers know that it is ill-advised to become bogged down during a first reading, and it is often useful to acquire a broad conceptual picture at the outset by reading through multiple chapters without too much concern about the details. One may always delve deeper on a second reading. Chapters 1 and 2 are mostly motivational, since discrete causal theory remains relatively unfamiliar, both to the scientific mainstream and to the general public. These chapters may be skipped entirely, but they are easy to read, and they make the rest of the book easier to understand. Serious readers will likely spend the most time in Chapters 5–7. It would be unfortunate to put serious effort into understanding Part I without seeing the quantum theory developed in Chapter 6!

1. *Physicists or mathematicians familiar with discrete causal topics such as causal sets or causal dynamical triangulations:* read Sections 3.9, 4.4, 4.5, 4.7, 4.8, 4.10, 5.1, 5.4, 5.7, 5.9, 5.10, 6.2, 6.7, 6.9, 6.10, 7.4, 7.5, 7.8, 8.2–8.4. Total pages: about 155.
2. *Physicists or mathematicians unfamiliar with discrete causal theory:* add to the above list Sections 2.8, 3.1–3.5. Total pages: about 180.
3. *Non-technical readers interested in modern science:* read Chapter 1, then read Sections 2.1, 2.2, 2.10, 3.1, 3.2, 3.9, 4.1, 4.8, 4.10, 5.8, 6.1, 6.5, 7.1–7.3, 8.1, 8.8, and 8.10. Total pages: about 150.

All readers should read the single-page abstracts that open each chapter. Students who have not yet seriously studied general relativity should read the first eight sections of Chapter 2, while keeping handy a good relativity text, such as Wald. Sections 3.6 and 3.7 contain important definitions used throughout the book, but these may be skimmed as needed. The index will be a useful resource for many readers.

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