Chapter 2
Robust Cooperative Resource Allocation

This chapter covers cooperative resource allocation in wireless networks. We discuss how this can be achieved by focusing on two specific cases. The first case is robust transmit power allocation for secondary users (SUs) in cognitive radio networks where a single uplink channel is shared between SUs and primary users (PUs) and channel gains between SUs and primary base stations are uncertain. We also consider robust transmit power allocation for device-to-device communication between SUs where PUs' multiple uplink channels are shared with SUs. In both cases, we assume worst-case uncertainty in channel gains, and show that under some conditions on the uncertainty region, the computational complexity of obtaining robust solutions can be the same as that of obtaining nominal nonrobust solutions. We also discuss algorithms to trade off between throughput reduction and robustness and present a scheme to reduce signaling in robust solutions. Our approach is applicable to other cases where the objective is the maximization of social utility, where users are cooperative.

2.1 Introduction

Robust solutions are desirable in the sense that optimal performance can be guaranteed even when exact parameter values are not available. However, robustness is achieved by considering additional constraints involving uncertain parameters, which complicates the problem. For practical reasons and so far, the preferred approach in the literature for introducing robustness has been to assume the worst-case bounded uncertainty in parameter values.

In general, there are two approaches to solving optimization problems: the centralized approach and the distributed approach. In some instances, centralized schemes are nonscalable and require measurements and parameter values pertaining to all users, that is, they need extensive message passing. In contrast, distributed

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schemes are scalable and, to the extent possible, utilize locally available measurements and information, that is, minimal message passing; but their performance in general is below that of a centralized scheme. To improve the performance of distributed schemes, decomposition algorithms have been developed, but they require additional message passing (as compared with conventional distributed algorithms) between distributed nodes to guarantee convergence. However, introducing robustness in decomposition algorithms increases the amount of additional message passing because of the coupling between optimization variables.

It is evident from the preceding statements that robust algorithms with less computational complexity and reduced signaling that can also trade off between performance and robustness are very desirable. Our objective in this chapter is to present such algorithms. We begin by formulating robust problems in the same manner that nominal (i.e., nonrobust) problems are formulated, but with additional constraints to include robustness requirements by using protection functions. This will enable us to simplify the robust problem for those cases where uncertainty is confined to a specific region, where, for example, the general definition of norm or the polyhedron model can be used. We also analyze the overhead of obtaining robust solutions and devise appropriate schemes to reduce such overhead to the extent possible.

We study the aforementioned issues by focusing on underlay cognitive radio networks (CRNs), where the secondary users (SUs) keep their interference on the primary users (PUs) below a given interference threshold (IT) [1–5]. In this way, we consider two scenarios in CRNs: (1) a single-channel scenario in which the SUs share one uplink channel with the PUs and (2) a multichannel scenario in which the SU pairs utilize device-to-device (D2D) communication. In both cases, the SUs cooperate with each other to maximize their total throughput, but the PUs are not obliged to provide their measurements and system information (including channel gains) to the SUs. Obtaining such information by the SUs is difficult and may entail uncertainty, which in turn may either violate the PUs' IT or result in inadequate SU signal-to-interference-plus-noise ratios (SINRs) [6], both because the SUs' allocated transmit power levels are inaccurate (higher in the former, lower in the latter), leading to undesirable fluctuations in the SUs' social utility value. Such concerns can be alleviated by applying robust schemes.

As indicated earlier, we formulate robust problems by considering additional constraints to include robustness requirements by utilizing protection functions. However, protection functions in many cases are nonlinear, which significantly increases the computational complexity of obtaining robust solutions. We will show that when uncertainty is confined to some specific uncertainty regions, nonlinear protection functions can be replaced either by their safe linear approximations or by a more tractable formulation. In this way, the complexity of solving the reformulated robust problem is significantly reduced, and pseudo water-filling formulations can be obtained in some cases.

We will also apply sensitivity analysis to show that throughput is reduced when uncertainty is increased [7], and we will demonstrate how to trade off between throughput and robustness via the D-norm approach and the chance constraint

approach (introduced in Chapter 1, Section 1.3.3.1) [8, 9]. However, assuming less uncertainty makes the robust solution less resilient and increases the probability of violating the constraints, which may not be desirable for either SUs or PUs.

Moreover, we will show that in decomposition-based distributed robust algorithms for single-channel CRNs, it is possible to achieve convergence by infrequent message passing [8]. This is important because convergence of the conventional decomposition-based distributed schemes for both nominal and robust problems requires excessive message passing (which is not desirable).

We will conclude this chapter by providing a brief review of existing works on robust cooperative optimization and show that, although the development of schemes for reducing complexity, trading off between throughput and robustness, and reducing the amount of message passing in distributed robust algorithms is very desirable and beneficial, such schemes have not been extensively studied in the literature.

2.2 Single-Channel Cellular Cognitive Radio Networks

Consider a cellular CRN with one uplink channel, where the coverage area of a cognitive cell partially overlaps with the coverage areas of neighboring primary cells sharing the same frequency band. We assume that each user communicates with its own base station. The system model for this case is shown in Fig. 2.1. The set of PUs' base stations (PBSs) is $\mathcal{Q} = \{1, \ldots, Q\}$, and the set of SUs serviced by the secondary base station (SBS) is $\mathcal{S} = \{1, \ldots, S\}$. The channel gain between SU s and its SBS is h_s, and the interference channel gain between SU s and PBS q is g_{sq}. Cooperation between the SBS and PBSs is not assumed, but the SBS knows the predefined IT of PBSs. The SINR of SU s at its SBS is

$$\gamma_s = \frac{p_s h_s}{f_{\mathcal{Q}} + \sum_{m \in \mathcal{S},\, m \neq s,} p_m h_m + \sigma^2} \qquad \forall\, s \in \mathcal{S}, \tag{2.1}$$

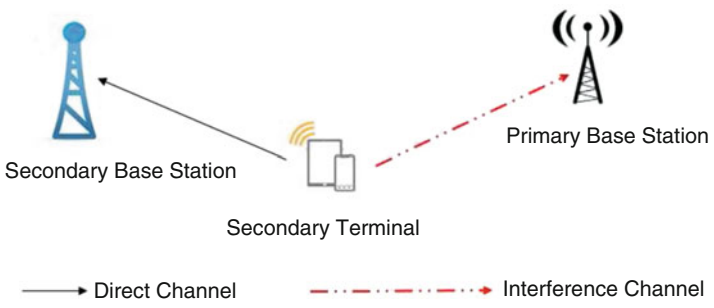


Fig. 2.1 Typical underlay CRN

where $p_s \in [p_s^{\min}, p_s^{\max}]$ is the transmit power of SU s, $f_{\mathscr{Q}}$ is the aggregate interference of PUs on the SBS, $\sum_{m\neq s,\, m\in\mathscr{S}} p_m h_m$ is the interference caused by other SUs on the SBS, and σ^2 is the noise power.

The resource allocation problem is

$$\max_{p_s^{\min}\le p_s\le p_s^{\max}} \quad \sum_{s\in\mathscr{S}} v_s(\gamma_s), \tag{2.2}$$

$$\text{subject to} \begin{cases} C_1 : \hat{\gamma}_s \le \gamma_s & \forall\, s\in\mathscr{S}, \\ C_2 : \left(\sum_{s\in\mathscr{S}} g_{sq} p_s\right) \le IT_q & \forall\, q\in\mathscr{Q}, \end{cases}$$

where C_1 represents the minimum required SINRs of SUs, C_2 is on the IT for PU q, and $v_s(\gamma_s)$ is the utility function of SU s. In general, (2.2) is nonconvex in p_s, but when two conditions are simultaneously satisfied—(a) $v_s(\gamma_s)$ is concave, strictly increasing, and twice continuously differentiable over $(0,\infty)$; and (b) $-\gamma_s v_s''(\gamma_s)/v_s'(\gamma_s) \ge 1$; problem (2.2) can be transformed into a convex optimization problem via geometric programming (GP) and logarithmic transformations [7, 10–12]. A utility function that simultaneously satisfies both conditions (a) and (b) is

$$v_s(\gamma_s) = w_s \ln \gamma_s, \tag{2.3}$$

where $w_s > 0$ is a per-user coefficient [10]. Note that by properly choosing w_s for each s, the utility function (2.3) can lead to proportionate fairness among SUs [10, 13, 14]. In [13, 15], it is shown that when the utility function (2.3) is applied, GP can be used to transform (2.2) into

$$\max_{p_s^{\min}\le p_s\le p_s^{\max}} \quad \sum_{s\in\mathscr{S}} v_s(h_s p_s \varrho_s^{-1}), \tag{2.4}$$

$$\text{subject to} \begin{cases} C_1 : \varrho_s \hat{\gamma}_s \le h_s p_s, \\ C_2 : \left(\sum_{s\in\mathscr{S}} g_{sq} p_q \le IT_q\right) & \forall q\in\mathscr{Q}, \\ C_3 : \left(f_{\mathscr{Q}} + \sum_{m\in\mathscr{S},\, m\neq s,} p_m h_m + \sigma^2\right) \le \varrho_s, \end{cases}$$

where ϱ_s is an auxiliary variable corresponding to the interference plus noise for SU s. One can use the logarithmic transformations $p_s = e^{y_s}$ and $\varrho_s = e^{z_s}$ to convert (2.4) into a convex optimization problem [15, 16].

2.2.1 Robust Problem

In (2.4), the two parameters related to PBSs or PUs are $\mathbf{g}_q = [g_{1q}, \ldots, g_{Sq}]$ and $f_{\mathscr{Q}}$, the exact values of which are not easy to obtain by SUs because PBSs are not obliged to provide any information to SUs. To formulate the robust counterpart of (2.4), consider an uncertainty set for each uncertain parameter, containing the distances between the actual (uncertain) and the nominal (exact) values, as follows:

- The actual (uncertain) and the exact (nominal) channel gain vectors between the SUs and PBS q are $\mathbf{g}_q$ and $\bar{\mathbf{g}}_q$, whose elements for SU s are g_{sq} and $\bar{g}_{sq}$, respectively. The uncertainty set for channel gains between SUs and PBS q is

$$\mathscr{R}_{\mathbf{g}_q} = \{\mathbf{g}_q \| \|\mathbf{g}_q - \bar{\mathbf{g}}_q\| \leq \varepsilon_{\mathbf{g}_q}\}, \quad \forall q \in \mathscr{Q}, \tag{2.5}$$

  where $\varepsilon_{\mathbf{g}_q}$ is the upper bound on the uncertainty set for the channel gains, and $\|\mathbf{x}\|$ is the general norm.
- The actual (uncertain) and exact (nominal) interference levels caused by PUs on the SBS are $f_{\mathscr{Q}}$ and $\bar{f}_{\mathscr{Q}}$. The uncertainty set for interference levels is

$$\mathscr{R}_{f_{\mathscr{Q}}} = \{f_{\mathscr{Q}} \| \|\bar{f}_{\mathscr{Q}} - f_{\mathscr{Q}}\| \leq \varepsilon_{f_{\mathscr{Q}}}\}, \tag{2.6}$$

  where $\varepsilon_{f_{\mathscr{Q}}}$ is the upper bound on the uncertainty set of PUs' interference on the SBS.

Both g_{sq} and $f_{\mathscr{Q}}$ are present in the linear constraints C_2 and C_3 in (2.4). To deal with such uncertainties, we use the worst-case robust optimization for affine and convex constraints [17, 18], where the solution to (2.4) is robust if for any realization of $\mathbf{g}_s \in \mathscr{R}_{\mathbf{g}_q}$ and $f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}$ the optimal solution satisfies C_2 and C_3. Consequently, the robust counterpart of (2.4) is

$$\max_{p_s^{\min} \leq p_s \leq p_s^{\max}} \quad \sum_{s \in \mathscr{S}} v_s(h_s p_s \varrho_s^{-1}), \tag{2.7}$$

$$\text{subject to} \begin{cases} C_1 : \varrho_s \hat{\gamma}_s \leq h_s p_s \\ C_2 : \left(\sum_{s \in \mathscr{S}} g_{sq} p_s\right) \leq I T_q \qquad \forall\, q \in \mathscr{Q}, \\ C_3 : (f_{\mathscr{Q}} + \sum_{m \in \mathscr{S},\ m \neq s} p_m h_m + \sigma^2) \leq \varrho_s \\ C_4 : \mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}, \qquad f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}, \qquad \forall\, q \in \mathscr{Q}. \end{cases}$$

In the nominal problem (2.4), uncertainty is not considered, that is, the actual values are assumed to be exact, whereas in (2.7), the constraint C_4 assumes the worst-case uncertainty. In what follows, we state the conditions under which (2.7) is convex.

Proposition 2.1. *When $\mathscr{R}_{\mathbf{g}_q}$ and $\mathscr{R}_{f_{\mathscr{Q}}}$ are compact and convex sets, (2.7) is convex.*

Proof. This is true because C_2 and C_3 are satisfied when $\max_{\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}} \sum_{s \in \mathscr{S}} g_{sq} p_s \leq I T_q$ and $\max_{f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}} (f_{\mathscr{Q}} + \sum_{m \in \mathscr{S},\ m \neq s} p_m h_m + \sigma^2) \leq \varrho_s$. We rewrite these conditions as $(\sum_{s \in \mathscr{S}} \bar{g}_{sq} p_s + \max_{\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}} \sum_{s \in \mathscr{S}} (g_{sq} - \bar{g}_{sq}) p_s) \leq I T_q$ and $(\bar{f}_{\mathscr{P}} + \max_{f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}} (f_{\mathscr{Q}} - \bar{f}_{\mathscr{P}}) + \sum_{m \in \mathscr{S},\ m \neq s} p_m h_m + \sigma^2) \leq \varrho_s$, respectively. Since the `max` function over a convex set is a convex function (Section 3.2.4 in [19]), C_2 and C_3 [and, consequently, (2.7)] are convex.

The two terms $\Delta_q = \max_{\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}} \sum_{s\in\mathscr{S}} (g_{sq} - \overline{g}_{sq}) p_s$ and $\Delta_{f_\mathscr{Q}} = \max_{f_\mathscr{Q} \in \mathscr{R}_{f_\mathscr{Q}}} (f_\mathscr{Q} - \overline{f}_\mathscr{Q})$ are called the protection values against variations in channel gains and interference levels, respectively [20]. Using the preceding terms, the robust problem is written

$$\max_{p_s^{\min} \le p_s \le p_s^{\max}} \quad \sum_{s\in\mathscr{S}} v_s(h_s p_s \varrho_s^{-1}), \tag{2.8}$$

$$\text{subject to} \begin{cases} C_1 : q_s \hat{\gamma}_s \le h_s p_s, \\ C_2 : \left(\sum_{s\in\mathscr{S}} \overline{g}_{sq} p_s + \Delta_q\right) \le I T_q \qquad \forall q \in \mathscr{Q}, \\ C_3 : \left(\overline{f}_\mathscr{Q} + \sum_{\forall m\in\mathscr{S},\ m\neq s} p_m h_m + \Delta_{f_\mathscr{Q}} + \sigma^2\right) \le \varrho_s. \end{cases}$$

Note that constraint C_4 in (2.7) is omitted in (2.8) by applying the protection values. However, since the protection values involve nonlinearity due to the use of `max`, the computational complexity of solving (2.7) is high. In what follows, we use the bounds on the uncertainty regions to avoid nonlinearity and reduce the computational complexity.

Proposition 2.2. *When the uncertainty set is stated in terms of the general norm, the protection values are*

$$\Delta_{f_\mathscr{Q}} = \varepsilon_{f_\mathscr{Q}} \|f_\mathscr{Q}\|^* \quad \textit{and} \quad \Delta_q = \varepsilon_{\mathbf{g}_q} \|\mathbf{p}\|^*,$$

where $\mathbf{p} = [p_1, \ldots, p_S]$, *and* $\|\mathbf{x}\|^*$ *is the dual norm of* $\|\mathbf{x}\|$ *(Definition 1 in [20]).*

Proof. See Appendix 1.

Proposition 2.1 establishes the conditions for the convexity of the robust power allocation problem. In Proposition 2.2, note that for a linear norm with order $a \ge 2$, that is, $\|\mathbf{x}\|_a = \sqrt[a]{\sum |x|^a}$, the dual norm is a linear norm with order $b = 1 + \frac{1}{a-1}$. In this way, the nonlinear function `max` in Δ_q and $\Delta_{f_\mathscr{Q}}$ in constraints C_2 and C_3 of (2.8) is avoided. Consequently, the robust power allocation problem is changed to the standard form of convex optimization with deterministic constraints, which can be solved very efficiently. For instance, with $a = 2$ (the ellipsoid norm), we have

$$\Delta_{f_\mathscr{Q}} = \varepsilon_{f_\mathscr{Q}} \|f_\mathscr{Q}\|_2 \quad \text{and} \quad \Delta_q = \varepsilon_{\mathbf{g}_q} \|\mathbf{p}\|_2,$$

and (2.8) becomes

$$\max_{p_s^{\min} \le p_s \le p_s^{\max}} \quad \sum_{s\in\mathscr{S}} v_s(h_s p_s \varrho_s^{-1}), \tag{2.9}$$

$$\text{subject to} \begin{cases} C_1 : q_s \hat{\gamma}_s \le h_s p_s, \\ C_2 : (\sum_{s\in\mathscr{S}} \overline{g}_{sq} p_s + \varepsilon_{\mathbf{g}_q} \sqrt{\sum_{s\in\mathscr{S}} p_s^2}) \le IT_q \qquad \forall q \in \mathscr{Q}, \\ C_3 : (\overline{f}_{\mathscr{P}} + \sum_{m\in\mathscr{S},\ m\neq s} p_m h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2) \le \varrho_s. \end{cases}$$

Note that, as with (2.4), there are only two optimization variables, p_s and ϱ_s, in (2.9). Using the logarithmic transformations $p_s = e^{y_s}$ and $\varrho_s = e^{z_s}$, the convexity of problem (2.9) is established (Appendix 2). In the sequel, we will solve the robust problem (2.9) via its Lagrange function and show that the computational complexities of solving the nominal problem and its robust counterpart are the same.

2.2.1.1 Iterative Algorithm for Solving Nominal and Robust Problems

Considering the convexity of the nominal problem (2.2) and its robust counterpart (2.9), their respective optimal solutions $\mathbf{p}^* = [p_1^*, \ldots, p_S^*]$ and $\widetilde{\mathbf{p}}^* = [\tilde{p}_1^*, \ldots, \tilde{p}_S^*]$ can be obtained using analytical and numerical methods such as the interior point method and the Lagrange function [19] and the available software such as CVX [21], respectively. The computational complexity of solving the nominal problem is the same as that of its robust counterpart, that is, obtaining the robust solution does not entail additional calculations in this setup. As an example, in what follows, we use the Lagrange function to solve (2.9):

$$\begin{aligned} &L(\lambda_s, \mu_s, \nu_q, e^{z_s}, e^{y_s}) \\ &= -\sum_{s\in\mathscr{S}} \upsilon_s \left(h_s e^{y_s - z_s}\right) + \sum_{s\in\mathscr{S}} \lambda_s \left(\frac{e^{z_s}\hat{\gamma}_s}{h_s e^{y_s}} - 1\right) \\ &\quad + \sum_{q\in\mathscr{Q}} \nu_q \left(\frac{\overline{g}_{sq} e^{y_s} + \varepsilon_{\mathbf{g}_q} \sqrt{\sum_{s\in\mathscr{S}} e^{2y_s}}}{IT_q} - 1\right) \\ &\quad + \sum_{s\in\mathscr{S}} \mu_s \left(e^{-z_s} \left(\overline{f}_{\mathscr{P}} + \sum_{m\in\mathscr{S},\ m\neq s} e^{y_m} h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2\right) - 1\right), \end{aligned} \tag{2.10}$$

where λ_s, ν_q, and μ_s are the Lagrange multipliers for C_1, C_2, and C_3 in (2.9), respectively. This Lagrange function can be solved by applying a gradient-based algorithm to derive the primal and dual variables of the optimization problem. The gradient-based updates of variables and multipliers are

$$y_s(t+1) = \left[y_s(t) - \beta_{y_s} \times \partial L/\partial y_s\right]_{y_s^{\min}}^{y_s^{\max}}, \tag{2.11}$$

$$z_s(t+1) = \left[z_s(t) - \beta_{z_s} \times \partial L/\partial z_s\right]^{+}, \tag{2.12}$$

$$\lambda_s(t+1) = \left[\lambda_s(t) + \beta_{\lambda_s} \times \left(\frac{e^{z_s - y_s}\hat{\gamma}_s}{h_s} - 1\right)\right]^{+}, \tag{2.13}$$

$$\mu_s(t+1) = \left[\mu_s(t) + \beta_{\mu_s} \times \left(e^{-z_s}\left(\bar{f}_{\mathscr{Q}} + \sum_{m\in\mathscr{S},\ m\neq s} e^{y_m} h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2\right) - 1\right)\right]^+, \tag{2.14}$$

$$v_q(t+1) = \left[v_q(t) + \beta_{v_q} \times \left(\frac{\bar{g}_{sq} e^{y_s(t)} + \varepsilon_{\mathbf{g}_q}\sqrt{\sum_{s\in\mathscr{S}} e^{2y_s(t)}}}{IT_q} - 1\right)\right]^+, \tag{2.15}$$

where β_{y_s}, β_{z_s}, β_{λ_s}, β_{μ_s}, and β_{v_q} are small step sizes for the Lagrange multipliers, and $x^+ = \max\{0, x\}$. The partial derivatives of the Lagrange dual function with respect to y_s and z_s are

$$\begin{aligned}\partial L/\partial y_s = &-v_s'\left(h_s e^{y_s - z_s}\right) h_s e^{y_s - z_s} - \lambda_s \frac{e^{z_s}\hat{\gamma}_s}{h_s e^{y_s}} \\ &+ \sum_{q\in\mathscr{Q}} \frac{v_q}{IT_q}\left(\bar{g}_{sq} e^{y_s} + \varepsilon_{\mathbf{g}_q} \frac{e^{2y_s}}{\sqrt{\sum_{s\in\mathscr{S}} e^{2y_s}}}\right) + e^{y_s} \sum_{m\in\mathscr{S},\ m\neq s} \mu_m h_m e^{-z_m}\end{aligned} \tag{2.16}$$

and

$$\begin{aligned}\partial L/\partial z_s = &\ v_s'\left(h_s e^{y_s - z_s}\right) h_s e^{y_s - z_s} + \lambda_s \frac{e^{z_s}\hat{\gamma}_s}{h_s e^{y_s}} \\ &-\mu_s e^{-z_s}\left(\bar{f}_{\mathscr{P}} + \sum_{m\in\mathscr{S},\ m\neq s} e^{y_m} h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2\right).\end{aligned} \tag{2.17}$$

Iterations in (2.11)–(2.17) lead to $\widetilde{\mathbf{p}}^*$. The same approach can be used to obtain $\mathbf{p}^*$ by setting the protection function to zero. Note that the computational complexity of solving the nominal problem is the same as that of solving its robust counterpart.

2.2.1.2 Reduced Throughput in Robust Solution

Tobtain the robust solution's throughput, we compare the feasibility region of (2.9) with that of the nominal problem. The uncertainties in C_2 and C_3 in (2.9) affect the SUs' transmit power levels in opposite directions. On the one hand, because of uncertainty in $\mathbf{g}_q$, the SUs' transmit power levels should be reduced to satisfy the PUs' IT. On the other hand, because of the uncertainty in $f_{\mathscr{Q}}$, the SUs' transmit power levels should be increased to improve the SUs' SINRs at the SBS, which increases interference. In either case, the transmit power allocation problem may become infeasible, meaning that a transmit power vector that simultaneously satisfies the constraints in (2.9) may not exist. To obtain the feasibility conditions, we use the notion of feasibility region, defined as follows:

Definition 2.1. The feasibility region, feasibility set, search space, or solution space of an optimization problem is the set of all possible points (sets of variables' values) that can satisfy all constraints in the optimization problem, including inequalities and equalities for integer and continuous variables. The optimal solution corresponds to a subset within the feasibility region whose performance is superior to those of other points in this region. The feasibility region for problem (1.1) in Chapter 1, denoted by $\mathbf{x}$, is obtained by solving the following problem [19].

$$\text{Find} \quad \mathbf{x} \tag{2.18}$$

$$\text{subject to} \begin{cases} f_y(\mathbf{x}, \mathbf{c}_y) = 0, & \forall y \in \mathscr{Y}, \\ f_z(\mathbf{x}, \mathbf{c}_z) \leq 0, & \forall z \in \mathscr{Z}. \end{cases}$$

Note the following points concerning the feasibility region:

- Some constraints in resource allocation problems may not be satisfied for some parameter values, meaning that the problem may be infeasible (the feasibility set may be empty) [13, 22];
- When the feasibility region of the nominal problem contains the feasibility region of its robust counterpart, the performance of the nominal problem is higher than that of its robust counterpart [19, 23].

In what follows, we use the preceding points to compare the performance of the nominal problem and its robust counterpart in terms of the total throughput of a CRN. We begin by comparing the feasibility regions of (2.4) and (2.9). Consider $\upsilon = [\frac{\hat{\gamma}_1 n_0}{h_1}, \ldots, \frac{\hat{\gamma}_S n_0}{h_S}]^{\mathrm{T}}$ and $n_0 = \overline{f}_{\mathscr{Q}} + \sigma^2$; denote the $S \times Q$ matrix of channel gains between SUs and PBSs by $\mathbf{G}$, whose elements are $[G]_{sq} = g_{sq}$, and the vector of ITs for PBSs by $\mathbf{IT} = [IT_1, \ldots, IT_Q]$. The upper bounds on the SUs' transmit power levels are $\mathbf{p}^{\max} = [p_1^{\max}, \ldots, p_S^{\max}]^{\mathrm{T}}$, and the normalized gain matrix of SUs is $\mathbf{F}$, whose elements are

$$F_{sm} = \begin{cases} 0 & \text{if} \quad s = m, \\ \hat{\gamma}_s h_m / h_s & \text{if} \quad s \neq m. \end{cases}$$

In what follows, Proposition 2.3 states the feasibility conditions of (2.4) and (2.9), where $\rho(\mathbf{F})$ is the spectral radius of $\mathbf{F}$:

Proposition 2.3. *Problem (2.4) is feasible [24] if and only if*

(1) $\rho(\mathbf{F}) < 1$,
(2) $\mathbf{b} \leq \mathbf{p}^{\max}$,
(3) $\mathbf{b}^{\mathrm{T}}\mathbf{G} \leq \mathbf{IT}$,

where $\mathbf{b} = (\mathbf{I} - \mathbf{F})^{-1}\upsilon$ *[8].*

The robust problem (2.9) is feasible [7, 22] if and only if

(4) $\rho(\mathbf{F}) < 1$,
(5) $\mathbf{b}_{\Delta} \leq \mathbf{p}^{\max}$,
(6) $\mathbf{b}_{\Delta}^{T}\overline{\mathbf{G}} + \boldsymbol{\varepsilon}\|\mathbf{b}_{\Delta}\|_2 \leq \mathbf{IT}$,

where $\mathbf{b}_{\Delta} = (\mathbf{I} - \mathbf{F})^{-1}(\upsilon + \boldsymbol{\Delta})$, $\boldsymbol{\Delta} = [\frac{\hat{\gamma}_1\varepsilon_{f_{\mathscr{Q}}}}{h_1}, \ldots, \frac{\hat{\gamma}_S\varepsilon_{f_{\mathscr{Q}}}}{h_S}]^T$, $\overline{\mathbf{G}}$ *is the exact channel gain matrix between SUs and PBSs, whose elements are* $[\overline{G}]_{sq} = \overline{g}_{sq}$ *and* $\boldsymbol{\varepsilon} = [\varepsilon_{\mathbf{g}_1}, \ldots, \varepsilon_{\mathbf{g}_Q}]$.

Proof. See Appendix 3.

Conditions (1)–(3) and (4)–(6) in Proposition 2.3 correspond to the feasibility regions of (2.4) and (2.9), respectively. Comparing conditions (2) and (4) reveals that the feasibility region of (2.9) depends on its protection function, and from condition (5) we see that increasing the value of $\varepsilon_{f_{\mathscr{Q}}}$ tightens condition (5) as compared to (2). Also, comparing conditions (3) and (6) reveals that increasing the values of $\varepsilon_{\mathbf{g}_q}$ and $\varepsilon_{f_{\mathscr{Q}}}$ shrinks the feasibility region of Eq. (2.9) as compared to that of Eq. (2.4). Consequently, the SUs' total throughput for the robust solution is less than that for the nominal solution [8].

To illustrate the relationship between the uncertainty and feasibility regions, let us consider a simple example in which a CRN consists of one PBS and one SU, and $\overline{f}_{\mathscr{P}} = 2\sigma^2$. The impact of $\varepsilon_{\mathbf{g}}$ on the feasibility set of (2.9) is shown in Fig. 2.2. In this setup, since we only have one SU and one PBS, for simplicity the indices are omitted from $\hat{\gamma}_s, p_s, IT_q, \overline{g}_{sq}$, and other variables, as well as in Fig. 2.2. Now, without considering C_2 and when $\varepsilon_{f_{\mathscr{Q}}} = 0$, the SU reaches its $\hat{\gamma}$ for $0.5 \times p^{\max}$. As can be seen in Fig. 2.2, by increasing $IT/\overline{g}$ and $\varepsilon_{\mathbf{g}}$, constraint C_2 shrinks the feasibility set of (2.9) until the transmit power that satisfies C_2 and $\boldsymbol{\varepsilon}$ falls bellow $0.5 \times p^{\max}$, meaning that for those values of $IT/\overline{g}$ and $\varepsilon_{\mathbf{g}}$ that correspond to the transmit power levels below $0.5 \times p^{\max}$ there is no transmit power for the SU to satisfy both C_1 and C_2 in (2.9), that is, the robust power allocation problem is infeasible when the SU's transmit power is less than $0.5 \times p^{\max}$.

Next, we study the feasibility region for the SU's transmit power when two parameters are uncertain. Let us start by assuming $\varepsilon_{f_{\mathscr{Q}}} > 0$ when C_2 is not considered. The required transmit power for reaching $\hat{\gamma}$ is

$$p = 0.5 \times \left(1 + \frac{1}{3}\varepsilon_{f_{\mathscr{Q}}}\right) p^{\max}. \tag{2.19}$$

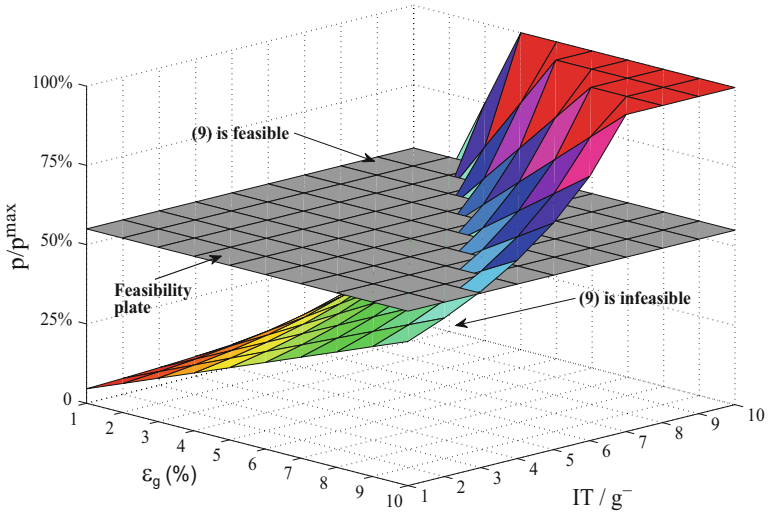


Fig. 2.2 Feasibility set of (2.9) versus $\varepsilon_{\mathbf{g}}$ and $IT/\overline{g}$

When both C_1 and C_2 are considered, $\varepsilon_{f_{\mathscr{Q}}} = 90\%$, $IT/\overline{g} = 9$, and $\varepsilon_{\mathbf{g}} = 4\%$, from (2.19), the SU should transmit at $0.65 \times p^{\max}$ to reach $\hat{\gamma}$. However, from Fig. 2.2, when $IT/\overline{g} = 9$ and $\varepsilon_{\mathbf{g}} = 4\%$, the SU's transmit power should be less than $0.6 \times p^{\max}$ to satisfy the IT. Due to this contradiction, the problem is infeasible. From the preceding considerations, when both $\varepsilon_{f_{\mathscr{Q}}}$ and $\varepsilon_{\mathbf{g}}$ have nonzero values, they affect the SU's transmit power in opposite directions, which may lead to the infeasibility of the robust problem, whereas the nominal problem is feasible.

Note also that increasing $\varepsilon_{\mathbf{g}}$ for any given value of $IT/\overline{g}$ causes (2.9) to be infeasible, and the value of $\varepsilon_{\mathbf{g}}$ that makes the robust problem infeasible depends on the value of $IT/\overline{g}$. In a tight IT, for example, when $IT/\overline{g} < 2$, the value of $\varepsilon_{\mathbf{g}}$ is smaller compared to that for $IT/\overline{g} \geq 2$.

Throughput reduction in the worst-case approach is reported in [8] for a CRN with two partially overlapping cells, one for the PUs and one for the SUs, as shown in Fig. 2.3a, b, where the index q is dropped for simplicity and $\varepsilon_{\mathbf{g}}$ and ε_f are expressed in percentages as $\varepsilon_{\mathbf{g}} = \frac{\|\mathbf{g}-\overline{\mathbf{g}}\|_2}{\|\overline{\mathbf{g}}\|_2}$ and $\varepsilon_{f_{\mathscr{Q}}} = \varepsilon_f = \frac{\|f_{\mathscr{Q}}-\bar{f}_{\mathscr{Q}}\|_2}{\|\bar{f}_{\mathscr{Q}}\|_2}$. The simulation scenario for these two figures are as follows. The PBS is located at the center of a circular cell with a radius of 2 km, and the SBS is located 0.5 km from the PBS and has a 1 km radius. There are three active SUs at $d = [150, 200, 350]$ m from the SBS and $D = [550, 300, 400]$ m from the PBS. The transmit power for each SU is within $[0.001, 1]$ W, $\hat{\gamma}_s = 40$ dBm, $w_s = 1$ for all SUs, $\sigma^2 = -110$ dBm, and $\bar{f}_{\mathscr{Q}} = 2 \times \sigma^2$. The channel gains are $h_s = \frac{k}{d_s^4}$ and $g_{sq} = \frac{k}{D_{sq}^4}$ with $k = 0.09$. Note that expanding the uncertainty region (i.e., increasing $\varepsilon_{\mathbf{g}}$ and ε_f) reduces the robust throughput.

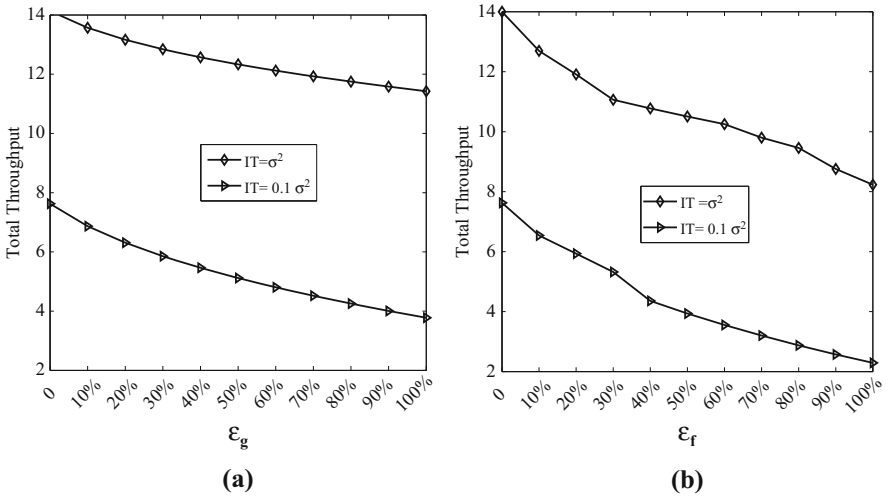


Fig. 2.3 Total throughput of SUs versus (**a**) $\varepsilon_{\mathbf{g}}$ and (**b**) $\varepsilon_{f_{\mathscr{Q}}} = \varepsilon_f$

As stated in Proposition 2.3 and shown in Fig. 2.2, satisfying the PUs' IT while considering uncertainties in $\mathbf{g}_q$ shrinks the feasibility set of (2.8) compared to that of (2.4) and forces the SUs to reduce their transmit power levels, which in turn reduces the SUs' social utility when Proposition 2.3 holds. This is not desirable from the SUs' point of view and calls for a trade-off between increasing the SUs' total throughput and preserving the IT for all instances of channel uncertainties. In practice, the uncertainty does not always correspond to its worst case, and moderation of the worst-case approach is important and desirable to increase the SUs' throughput [17, 25]. This can be achieved via a probabilistic approach in which the uncertainty set is chosen in such a way that the probability of violating the IT is kept below a predefined level, and the SUs' total throughput is kept close to the optimal value of the nonrobust case.

Introducing robustness reduces the SUs' total throughput, measured by

$$d_{\Delta} = \|v^* - v_{\Delta}^*\|_2, \tag{2.20}$$

where v^* and v_{Δ}^* are the SUs' optimal total throughput values for (2.4) and (2.8), respectively. In the following lemma, we will show how d_{Δ} can be obtained.

Lemma 2.1. *Let* $\boldsymbol{\nu}^* = \{\nu_1^*, \ldots, \nu_Q^*\}$ *and* $\boldsymbol{\mu}^* = \{\mu_1^*, \ldots, \mu_S^*\}$ *be the optimal values of Lagrange multipliers for* C_2 *and* C_3 *in (2.4), respectively. For all values of* Δ_q *and* $\Delta_{f_{\mathscr{Q}}}$ *in Proposition 2.2 we have [7, 26]*

$$d_\Delta \approx \sum_{q\in\mathscr{Q}} \nu_q^* \Delta_q + \sum_{s\in\mathscr{S}} \mu_s^* \Delta_{f_{\mathscr{Q}}}. \tag{2.21}$$

Proof. See Appendix 4.

By adjusting Δ_q, one can control d_Δ. To do so, we apply the D-norm approach [25], where the uncertainty in each channel gain is $|\hat{g}_{sq}| \le \varepsilon_{sq}$ with a symmetric (e.g., uniform) distribution, that is,

$$g_{sq} \in [\overline{g}_{sq} - \hat{g}_{sq}, \overline{g}_{sq} + \hat{g}_{sq}], \quad \forall\, s \in \mathscr{S}, \quad \forall\, q \in \mathscr{Q}. \tag{2.22}$$

For a nonnegative integer $\Gamma_q \le |\mathscr{S}|$, the protection value [25] is

$$\Delta_q = \max_{e_q\in\mathscr{S},\, |e_q|=\Gamma_q,\, |\hat{g}_{sq}|\le\varepsilon_{sq}} \sum_{s\in e_q} \hat{g}_{sq} p_s, \qquad \forall\, q \in \mathscr{Q}, \tag{2.23}$$

where e_q is the subset of all SUs that affect the protection value in the D-norm approach. The size of e_q is Γ_q, and from (2.23) the value of Γ_q determines the total number of uncertain parameters in the protection value. Note that the values of g_{sq} for all s and q are uncertain, but only those values that pertain to the set e_q are considered in the protection function of the D-norm approach. By adjusting Γ_q, a trade-off is made between robustness and throughput. When $\Gamma_q = 0$, no protection is considered, and increasing Γ_q means more protection, that is, increasing the number of uncertain parameters results in a higher protection value. Now, the robust counterpart of (2.4) is

$$\max_{p_s^{\min}\le p_s\le p_s^{\max}} \sum_{s\in\mathscr{S}} v_s(h_s p_s \varrho_s^{-1}), \tag{2.24}$$

$$\text{subject to} \begin{cases} C_1 : \varrho_s \hat{\gamma}_s \le h_s p_s, & \\ C_2 : \left(\sum_{s\in\mathscr{S}} \overline{g}_{sq} p_s + \Delta_q\right) \le I T_q, & \forall\, q \in \mathscr{Q}, \\ C_3 : \left(\overline{f}_{\mathscr{Q}} + \sum_{m\in\mathscr{S},\, m\ne s} p_m h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2\right) \le \varrho_s, & \forall\, s \in \mathscr{S}, \end{cases}$$

where $\Delta_q = \max_{e_q\in\mathscr{S},\, |e_q|=\Gamma_q,\, \hat{g}_{sq}\le\varepsilon_{sq}} \sum_{s\in e_q} \hat{g}_{sq} p_s$. Since the protection value in the D-norm approach is a special form of the norm function [20], the convexity of (2.24) is preserved. Now the question is how to expand the feasibility region, which would increase the SUs' total throughput in the D-norm approach. To answer this question, we state the feasibility conditions of (2.24) in the following proposition.

Proposition 2.4. *The robust problem (2.24) is feasible for* $\varepsilon_q = \varepsilon_{sq}, \forall\ s \in \mathscr{S}$ *when*

(1) $\rho(\mathbf{F}) < 1$,
(2) $\mathbf{b}_\Delta \leq \mathbf{p}^{\max}$,
(3) $\mathbf{b}_\Delta^T \overline{\mathbf{G}} + \boldsymbol{\varepsilon} \times \max(\|\mathbf{b}_\Delta\|_\infty, \|\mathbf{b}_\Delta\|_1 / \Gamma_q) \leq \mathbf{IT}$,

where $\mathbf{b}_\Delta = (\mathbf{I} - \mathbf{F})^{-1}(\upsilon + \boldsymbol{\Delta})$, $\boldsymbol{\Delta} = [\frac{\hat{\gamma}_1 \delta}{\bar{h}_1}, \ldots, \frac{\hat{\gamma}_S \delta}{\bar{h}_S}]^T$, $\boldsymbol{\varepsilon} = [\varepsilon_1, \ldots, \varepsilon_Q]$, *and* $\|\mathbf{b}_\Delta\|_1 = \sum_{s=1}^{S} \|(b_\Delta)_s\|$, *in which* $(b_\Delta)_s$ *is the sth element of* $\mathbf{b}_\Delta$.

Proof. See Appendix 5.

From Proposition 2.4 we note that Γ_q adjusts the constraints that affect the feasibility region. In what follows, we focus on the conditions that expand the feasibility set in the D-norm approach, leading to a higher total throughput for the SUs.

Lemma 2.2. *When the uncertainty boundaries in the ellipsoid uncertainty model and the D-norm uncertainty model are the same, the optimal total throughput of (2.7) for the D-norm uncertainty model is higher than or equal to the optimal total throughput for the ellipsoid uncertainty model if*

$$\Gamma_q \leq \sqrt{|\mathscr{S}|}, \quad \forall q \in \mathscr{Q}, \tag{2.25}$$

where $|\mathscr{S}|$ *is the size of* $\mathscr{S}$.

Proof. See Appendix 6.

Reducing the protection value means that the PUs' interference constraint may not be satisfied for all instances of uncertainties. From C_2 in (2.24), the probability of violating the IT for a given protection value in the D-norm approach can be derived via (1.18) in Chapter 1.

By fixing the probability of violating C_2 to δ_q, the lower bound of Γ_q can be obtained from (1.18) in Chapter 1, as shown in Fig. 1.3 of Chapter 1. Note that reducing δ_q increases the value of Γ_q, and vice versa. From (1.18) in Chapter 1, when the number of SUs increases, the SBS needs a higher value of Γ_q to keep the violation probability acceptable. In addition, unlike (2.7), which satisfies the PBSs' interference constraint for all instances of uncertainties, by using the D-norm in (2.24), the interference constraint is probabilistically satisfied. The SBS can calculate the value of Γ_q via (1.18) in Chapter 1 for any value of the violation probability for PBSs and use it to obtain the protection value. In practice, the value of δ_q can be set by the regulatory authority to trade off between the throughput of SUs and the violation probability of PUs.

2.2.1.3 Distributed Robust Solutions and Extra Message Passing

Using the Lagrange function (2.10) for solving (2.9) has three features that are very useful for developing distributed algorithms:

- Each user's utility function only depends on that user's primal variables;
- Dual variables can be divided into local dual variables for each SU (i.e., μ_s and λ_s) and global dual variables of all SUs (i.e., ν_q);
- Protection functions are simple and linear, and each user can calculate its own protection value for the corresponding constraint by using a broadcasted scaler.

Based on the preceding points, we now present a distributed algorithm for solving (2.9) using the gradient-based updates of primal and dual variables (2.11)–(2.17). We make the following assumptions:

- Each SU knows its normalized SINR at the SBS and receives its $\varepsilon_{\mathbf{g}_q}$ and $\varepsilon_{f_{\mathscr{Q}}}$;
- Each SU needs the two global scalar variables $b_1(t) = \sum_{s\in\mathscr{S}} \mu_s h_s e^{-z_s(t)}$ and $b_2(t) = \sqrt{\sum_{s\in\mathscr{S}} e^{2y_s(t)}}$ to update its primal and dual local variables.

The global dual variable ν_q is updated at the SBS because we need to know all values of the SUs' transmit power levels, channel gains g_{sq}, and IT_q. The distributed Algorithm 1 for power control based on the Lagrange dual decomposition is as follows:

Distributed Algorithm 1
Initialization: At $t = 0$, initialize all primal values with small positive random values and Lagrange multipliers with positive random values.
At SBS:
For $t = 1, 2, \ldots$, the values of $b_1(t)$, $b_2(t)$, and $\nu_q(t)$ are updated using the SUs' parameter values and broadcasted to all SUs.
At each SU s:
For $t = 1, 2, \ldots$:

1. Each SU s receives $b_1(t)$, $b_2(t)$ and $\nu_q(t)$ from the SBS.
2. Each SU s locally calculates its $z_s(t)$ and $y_s(t)$ using (2.11)–(2.12) and updates its Lagrange multipliers using (2.13)–(2.14).
3. Each SU s transmits $\mu_s(t)h_s e^{-z_s(t)}$ and $y_s(t)$ to the SBS.

Since (2.9) is a convex problem, the distributed Algorithm 1 converges to the optimal solution when the conditions in Proposition 2.3 hold and the step size is sufficiently small (Theorem 3.5 in [16]) [10, 16, 23, 27]. The distribution of calculations among SUs is a key advantage of distributed algorithms compared to centralized schemes. However, the distributed Algorithm 1 requires message passing in each iteration between the SUs and the SBS, as well as one additional variable passing for (2.9), that is, b_2, compared to (2.4). If message passing can be done at a slower rate without affecting the convergence and optimality of the allocated transmit power, the algorithm's signaling overhead is reduced.

2.2.1.4 Infrequent Message Passing

To reduce signaling in the robust distributed algorithm, we now present an algorithm with infrequent message passing. Let D denote the time difference between two successive message passings for $b_1(t)$, $b_2(t)$, and $v_q(t)$. The algorithm with infrequent message passing is as follows:

Distributed Algorithm 2
Initialization: At $t = 0$, initialize the primal values with small positive random values and the Lagrange multipliers with positive random positive values.
At SBS:

1. For $\xi = 0, 1, \ldots,$ at $t = \xi D + \tau$ for $0 \leq \tau < D$, the value of $v_q(t)$ in (2.15) is updated using $y_s(\xi D)$ and $\mu_s(\xi D) h_s e^{-z_s(\xi D)}$ received from all users.
2. At $t = D, 2D, \ldots$:
   - The values of $b_1(t)$ and $b_2(t)$ are updated using $\mu_s(\xi D) e^{-z_s(nD)}$ received from all users;
   - The values of $y_s(\xi D)$ and $v_q(t)$ for all s and q are broadcasted to SUs.

At each SU:

1. For $\xi = 0, 1, \ldots$ at $t = \xi D + \tau$ for $0 \leq \tau < D$, each SU s locally calculates its $z_s(t)$, $y_s(t)$, and Lagrange multipliers using $b_1(\xi D)$, $b_2(\xi D)$, and $v_q(\xi D)$;
2. At $t = D, 2D, \ldots$, each SU s transmits $\mu_s(t) h_s e^{-z_s(t)}$ and $y_s(t)$ to its SBS.

Obviously, a higher value for D leads to less message passing but may cause nonconvergence of the distributed algorithm. Hence, we need to find the maximum value of D for which the distributed Algorithm 2 converges to the optimal solution of (2.9).

Lemma 2.3. *Let Λ denote the total number of dual and primal variables of the Lagrange dual function (2.10), and let $\boldsymbol{\Theta}$ denote the vector of all step sizes of the gradient algorithm in (2.11)–(2.15). The distributed Algorithm 2 converges to the optimal value of (2.9) when*

$$D \leq \left(\frac{1}{\|\boldsymbol{\Theta}\|_\infty} - 1 \right) \times \frac{1}{\Lambda - 1}, \tag{2.26}$$

where $\|\boldsymbol{\Theta}\|_\infty$ is the maximum absolute value of the elements of $\boldsymbol{\Theta}$.

Proof. See Appendix 7.

The distributed Algorithms 1 and 2 can be slightly modified to solve (2.24) via the D-norm approach in the following manner. In the modified Algorithm 1, instead of b_2, the vector $\mathbf{b}_2 = [\widetilde{b}_1, \ldots, \widetilde{b}_Q]$, in which $\widetilde{b}_q$ is the protection value Δ_q in (2.23), is calculated by the SBS and broadcasted to all SUs. Since (2.24) is convex, the modified Algorithm 1 converges to the optimal solution when the conditions in Proposition 2.4 hold and the step size is sufficiently small. Algorithm 2 can also be slightly modified to reduce the signaling by way of infrequent message passing in the D-norm approach. The modified Algorithm 2 also converges to the optimal solution when the conditions in Proposition 2.4 hold and the upper bound on D in Lemma 2.3 is satisfied.

Convergence of ν and the transmit power in the distributed Algorithms 1 and 2 are shown in Figs. 2.4(a) and 2.4(b), respectively, where both $\varepsilon_{f_{\mathscr{Q}}}$ and $\varepsilon_{\mathbf{g}}$ are 10 % and $IT = \sigma^2$. Other simulation parameters are the same as those for Figs. 2.3(a) and 2.3(b). Figure 2.4(a) shows that Algorithm 1 converges relatively fast, whereas Fig. 2.4(b) shows that Algorithm 2 cannot converge to its optimal value with a large step size, for example, 0.5, and a large delay, for example, $D = 20$. By reducing the step size to 0.01 and delay to $D = 5$, Algorithm 2 converges but may take an order of magnitude more iterations as compared to Algorithm 1, which is in line with Lemma 2.3.

2.3 Multi-channel Cognitive Radio Networks

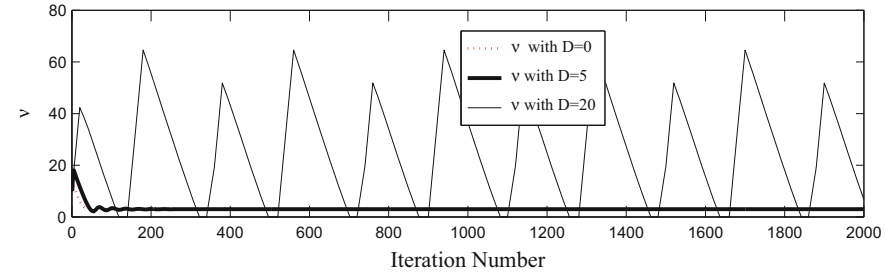
Now we consider a multi-channel CRN with D2D communications between the secondary transmitters and receivers in the presence of PBSs, as shown in Fig. 2.5. Each D2D secondary transmitter and its receiver are considered as one SU pair. The set of all D2D SU pairs is $\mathscr{S} = \{1, 2, \ldots, S\}$, the set of PBSs is $\mathscr{Q} = \{1, 2, \ldots, Q\}$, and the set of uplink channels shared between SU pairs and PBSs is $\mathscr{K} = \{1, 2, \ldots, K\}$. For simplicity, we limit our consideration to D2D communications only and neglect cellular communications between SUs and the SBS.

We assume that the SBS solves the network utility maximization (NUM) problem for SU pairs subject to the IT of PBSs and the power constraint of SUs. The NUM's objective function is the sum of utility values of all SU pairs. The utility of SU s is its total throughput over all K channels, defined as

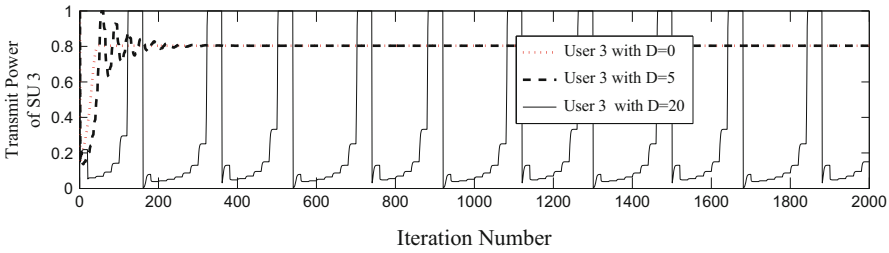
$$v_s(\gamma_s^k) = \sum_{k=1}^{K} \log(1 + \gamma_s^k), \quad \forall\, s \in \mathscr{S}, \tag{2.27}$$

where γ_s^k is the SINR of SU s defined as

$$\gamma_s^k = \frac{p_s^k h_{ss}^k}{\sigma^2 + f_{\mathscr{Q}}^k + \sum_{m=1, m\neq s}^{S} p_m^k h_{ms}^k}, \quad \forall\, s \in \mathscr{S}, \tag{2.28}$$



(a)



(b)

Fig. 2.4 Convergence of Algorithms 1 and 2 for the transmit power of SU 3 (**b**) and ν (**a**). In Algorithm 1, $D = 0$ and all step sizes are 0.5. In Algorithm 2, when $D = 5$, all step sizes are 0.1; and when $D = 20$, all step sizes are 0.5

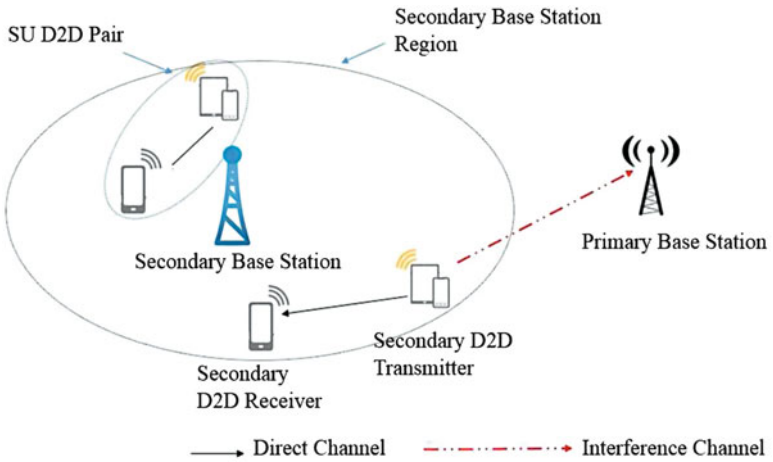


Fig. 2.5 A CRN with D2D SU pairs

where p_s^k is the transmit power of SU s in channel k, h_{ms}^k is the channel gain between transmitter SU m to receiver SU s in channel k, and $\sigma^2 + f_{\mathscr{Q}}^k$ is additive white Gaussian noise (AWGN) power plus interference caused by PUs to SU s in channel k. Now, the NUM is

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \sum_{s=1}^{S} v_s(\gamma_s^k), \tag{2.29}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p_s^k \le p_s^{\max}, & \forall\ s \in \mathscr{S}, \\ C_2 : \mathbf{p}_s \ldots \mathbf{g}_{sq}^{\mathrm{T}} \le IT_{sq}, & \forall\ s \in \mathscr{S}, \quad \forall\ q \in \mathscr{Q}, \end{cases}$$

where $\mathbf{p}_s = [p_s^1, \ldots p_s^K]$ and $p_s^{\max}$ are the transmit power vector of SU s over all channels and the maximum transmit power of SU s, respectively. Also, $\mathbf{g}_{sq} = [g_{sq}^1, \ldots, g_{sq}^K]$ is the channel gain vector between SU s and PU q over all channels, and IT_{sq} is the IT of SU s on PU q. Even without considering C_2, (2.29) is nonconvex and NP-hard [28]. In order to solve this class of problems, one may use their Lagrange dual function [29]. The duality gap between the optimal solution and its upper bound obtained by the Lagrange dual function approaches zero as the number of channels tends to infinity [29]. Hence, the solution obtained by the Lagrange dual function can be considered as close to the globally optimal solution for large values of K.

When the constraint on the IT of PBSs is relaxed, the Lagrange dual function of (2.29) is

$$D(\mathbf{p}, \boldsymbol{\omega}) = \max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \min_{\omega_{sq}} \left\{ \sum_{s=1}^{S}\sum_{k=1}^{K} \log(1+\gamma_s^k) - \sum_{s=1}^{S}\sum_{q=1}^{Q} \omega_{sq}\mathbf{p}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}} + \sum_{s=1}^{S}\sum_{q=1}^{Q} \omega_{sq} IT_{sq} \right\}, \tag{2.30}$$

$$\text{subject to} \quad C_1 : \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \quad \forall s \in \mathscr{S},$$

where $\omega_{sq} \ge 0$ is the Lagrange multiplier for constraint C_2 for SU s and PBS q. To solve the preceding problem, one can use the following iterative formula:

$$\omega_{sq}^{r+1} = \left[\omega_{sq}^r + \kappa(\mathbf{p}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}} - IT_{sq})\right]^+, \quad \forall\ s \in \mathscr{S}, \quad \forall\ k \in \mathscr{K}, \tag{2.31}$$

where κ is a small step size in the iterative formula, and r is the iteration number. By solving the following subproblem, the optimal $\mathbf{p}_s$ can be iteratively obtained [29]:

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \left(\sum_{s=1}^{S}\sum_{k=1}^{K} \log(1+\gamma_s^k) - \sum_{s=1}^{S}\sum_{q=1}^{Q} \omega_{sq}^r \mathbf{p}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}} \right), \tag{2.32}$$

$$\text{subject to} \quad C_1 : \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \quad \forall\ s \in \mathscr{S}.$$

2.3.1 Robust Problems

To introduce robust interference control in (2.29), we treat $\mathbf{g}_{sq}$ as an uncertain parameter that can be modeled by $\mathbf{g}_{sq} = \overline{\mathbf{g}}_{sq} + \hat{\mathbf{g}}_{sq}$, where $\overline{\mathbf{g}}_{sq}$ and $\hat{\mathbf{g}}_{sq}$ are the exact value and the error in the parameter value, respectively. For each pair of SU s and PBS q, the uncertain gain is $\mathbf{g}_{sq} = (\overline{\mathbf{g}}_{sq} + \hat{\mathbf{g}}_{sq}) \in \mathscr{R}_{\mathbf{g}_{sq}}$, where $\mathscr{R}_{\mathbf{g}_{sq}}$ is the uncertainty region for $\mathbf{g}_{sq}$. Now, the robust counterpart of (2.29) is

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \sum_{s=1}^{S} v_s(\gamma_s^k), \tag{2.33}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p_s^k \le p_s^{\max}, & \forall\ s \in \mathscr{S}, \\ C_2 : \mathbf{p}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}} \le IT_{sq}, & \forall\ s \in \mathscr{S}, \quad \forall\ q \in \mathscr{Q}, \\ C_3 : \mathbf{g}_{sq} \in \mathscr{R}_{\mathbf{g}_{sq}}, & \forall\ s \in \mathscr{S}, \quad \forall\ q \in \mathscr{Q}. \end{cases}$$

In what follows, we will show how to solve (2.33) using the polyhedron model and the general norm introduced in Chapter 1, Section 1.3.2.1. For the polyhedron model, the uncertainty region for the pair SU s and PU q is

$$\mathscr{R}_{\mathbf{g}_{sq}} = \{\mathbf{g}_{sq} | \mathbf{M}_{sq} \cdot \mathbf{g}_{sq}^{\mathrm{T}} \le \boldsymbol{\varepsilon}_{\mathbf{g}_{sq}}\},$$

where $\mathbf{M}_{sq}$ is the $R^{K\times K}$ weight matrix for $\mathbf{g}_{sq}$, and $\boldsymbol{\varepsilon}_{\mathbf{g}_{sq}}$ is the $R^{K\times 1}$ vector. Next, we transform C_2 in (2.33) into a linear constraint [18]. When C_2 holds for a given fixed value of $[\mathbf{p}_1, \ldots, \mathbf{p}_S] = [\tilde{\mathbf{p}}_1, \ldots, \tilde{\mathbf{p}}_S]$, the value of IT_{sq} must be higher than the optimal objective value of the following optimization problem for each SU [9, 18]:

$$\max_{\mathbf{g}_{sq}} \tilde{\mathbf{p}}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}}, \tag{2.34}$$

$$\text{subject to} \quad \mathbf{M}_{sq} \cdot \mathbf{g}_{sq}^{\mathrm{T}} \le \boldsymbol{\varepsilon}_{\mathbf{g}_{sq}} \quad \forall\ s \in \mathscr{S}, \quad \forall\ q \in \mathscr{Q}.$$

Since (2.34) is feasible, its optimal objective value, denoted by Δ_{sq}^*, can be obtained via its Lagrange dual function [18, 25],

$$\min_{\mathbf{y}_{sq} \ge \mathbf{0}} \quad \boldsymbol{\varepsilon}_{\mathbf{g}_{sq}} \cdot \mathbf{y}_{sq}^{\mathrm{T}},$$

$$\text{subject to} \quad \tilde{\mathbf{p}}_s^{\mathrm{T}} \le \mathbf{M}_{sq} \cdot \mathbf{y}_{sq}^{\mathrm{T}}, \quad \forall\ s \in \mathscr{S}, \quad \forall\ q \in \mathscr{Q}, \tag{2.35}$$

where $\mathbf{y}_{sq} = [y_{sq}^1, \ldots, y_{sq}^K]$ is the qth Lagrange multiplier for (2.34), and $\mathbf{0}$ is the zero vector with the same size as $\mathbf{y}_{sq}$. A feasible solution to (2.35), denoted by $\mathbf{y}_{sq}^*$, satisfies $\boldsymbol{\varepsilon}_{\mathbf{g}_{sq}} \cdot \mathbf{y}_{sq}^{*\mathrm{T}} \le IT_{qs}$ and also satisfies $\Delta_{sq}^* \le \mathbf{c}_{sq} \cdot \mathbf{y}_{sq}^{*\mathrm{T}} \le IT_{sq}$ for all uncertainty in $\mathbf{g}_{sq}$. In this case, we can replace C_2 in (2.33) with the following constraints:

$$\boldsymbol{\varepsilon}_{\mathbf{g}_{sq}} \cdot \mathbf{y}_{sq}^{\mathrm{T}} \le IT_{sq}, \qquad \mathbf{p}_s^{\mathrm{T}} \le \mathbf{M}_{sq} \cdot \mathbf{y}_{sq}^{\mathrm{T}}, \qquad \mathbf{y}_{sq} \ge \mathbf{0}, \quad \forall\ q \in \mathscr{Q}, \quad \forall\ s \in \mathscr{S}. \tag{2.36}$$

In this way, one new constraint and one new optimization variable are added for each SU and PBS in each channel for solving (2.33), as compared to (2.29). An iterative approach that uses the nominal problem's dual function (2.30) can also be applied to (2.33), except that (2.31) is replaced by

$$\widetilde{\omega}_{sq}^{r+1} = \left[\omega_{sq}^{r} + \kappa_1(\boldsymbol{\varepsilon}_{\mathbf{g}_{sq}} \cdot \mathbf{y}_{sq}^{\mathrm{T}} - IT_q)\right]^{+}, \quad \forall\, s \in \mathscr{S}, \quad \forall\, k \in \mathscr{K}, \tag{2.37}$$

and

$$\widehat{\omega}_{sq}^{r+1} = \left[\widehat{\omega}_{sq}^{r} + \kappa_2\left(\mathbf{p}_s^{\mathrm{T}} - (\mathbf{M}_{sq} \cdot \mathbf{y}_{sq}^{\mathrm{T}})\right)\right]^{+}, \quad \forall\, s \in \mathscr{S}, \quad \forall\, k \in \mathscr{K}, \tag{2.38}$$

where κ_1 and κ_2 are small step sizes, r is the iteration number, and $\widetilde{\omega}_{sq} \leq 0$ and $\widehat{\omega}_{sq} \leq 0$ are the Lagrange multipliers for the linear constraints in (2.36).

When all primal and dual variables are iteratively updated to solve (2.29) and (2.33), for each new constraint or each new variable added to (2.29) to introduce robustness, there is one additional iterative formula. Additional calculations for each new iterative formula are not significant due to the fact that it is a linear function. Hence, we only consider the number of new iterative formulas to obtain the order of additional calculations. The order of additional calculations for solving (2.33) as compared to (2.29) is $O(2{\times}S{\times}K{\times}Q)$. Hence, the order of additional calculations for solving (2.33) as compared to (2.29) is linearly increased by increasing the number of SUs for the polyhedron model, meaning that the problem is tractable.

For the general norm, consider the uncertainty region as

$$\mathscr{R}_{\mathbf{g}_{sq}} = \{\mathbf{g}_{sq} \parallel \mathbf{M}_{sq} \cdot (\mathbf{g}_{sq} - \overline{\mathbf{g}}_{sq})^{\mathrm{T}} \parallel \leq \boldsymbol{\varepsilon}_{\mathbf{g}_{sq}}\},$$

where $\mathbf{M}_{sq}$ is an invertible $R^{K\times K}$ weight matrix of channel gains between the transmitter of SU s and the receiver of PBS q, and $\boldsymbol{\varepsilon}_{\mathbf{g}_{sq}}$ is the bound on the uncertainty region. Now, using the protection function, (2.33) is equivalent to

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \sum_{s=1}^{S} v_s(\gamma_s^k), \tag{2.39}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p_s^k \leq p_s^{\max}, & \forall\, s \in \mathscr{S}, \\ C_2 : \left(\mathbf{p}_s \cdot \overline{\mathbf{g}}_{sq}^{\mathrm{T}} + \Delta_{sq}(\mathbf{p}_s)\right) \leq IT_{sq}, & \forall\, q \in \mathscr{Q}, \end{cases}$$

where $\Delta_{sq}(\mathbf{p}_s) = \max_{\mathbf{g}_{sq}\in\mathscr{R}_{\mathbf{g}_{sq}}} \mathbf{p}_s \cdot (\mathbf{g}_{sq} - \overline{\mathbf{g}}_{sq})$ is the protection function for the pair SU s and PU q. Additional calculations in (2.33) or (2.39) compared to (2.29) are due to uncertainty in C_2 and C_3 for each SU. When C_2 and C_3 in (2.33) can be replaced by deterministic functions, or their corresponding protection function in (2.39) can be replaced by a set of linear constraints, the problem becomes tractable [9].

For the linear norm with order a, we have $\Delta_{sq}(\mathbf{p}_s) = \varepsilon_{\mathbf{g}_{sq}} \left(\sum_{k=1}^{K} (\mathbf{m}_{sq}^k \cdot \mathbf{p}_s^{\mathrm{T}})^b \right)^{\frac{1}{b}}$, where $\mathbf{m}_{sq}^k$ is the k^{th} row of $\mathbf{M}_{sq}^{-1}$, and $b = 1 + 1/(1-a)$. In this case, since there is no additional optimization variable or constraint, the optimal solution to (2.39) can be obtained with no significant additional calculations as compared to (2.29).

Feasibility of Nominal Problem and Its Robust Counterpart: Note that (2.29) and (2.33) [or (2.39)] are always feasible even for a large number of SUs in a CRN. Let the set of transmit power levels of SU s in (2.29) and (2.33) be

$$\mathscr{A}_s = \left\{ \mathbf{p}_s | \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \qquad \mathbf{p}_s \cdot \mathbf{g}_{sq}^{\mathrm{T}} \le IT_{sq}, \ \forall\, q \in \mathscr{Q} \right\}$$

and

$$\widetilde{\mathscr{A}}_s = \left\{ \mathbf{p}_s | \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \qquad \left(\mathbf{p}_s \cdot \bar{\mathbf{g}}_{sq}^{\mathrm{T}} + \Delta_{sq}(\mathbf{p}_s) \right) \le IT_{sq} \right\},$$

respectively, where $\Delta_{sq}(\mathbf{p}_s) = \max_{\mathbf{g}_{sq} \in \mathscr{R}_{\mathbf{g}_{sq}}} \mathbf{p}_s \cdot (\mathbf{g}_{sq} - \bar{\mathbf{g}}_{sq})$ for all $q \in \mathscr{Q}$. The sets $\mathscr{A}_s$ and $\widetilde{\mathscr{A}}_s$ are nonempty due to the fact that at least one power vector $\mathbf{p}_s = \mathbf{0}$ exists that satisfies the constraints on $\mathscr{A}_s$ and $\widetilde{\mathscr{A}}_s$. In addition, the constraints on each SU only depend on that SU's transmit power and are decoupled from other SUs' constraints. Consequently, the feasibility set of (2.29) (i.e., $\mathscr{A} = \bigcup_{s=1}^{S} \mathscr{A}_s$) and the feasibility set of (2.33) (i.e., $\widetilde{\mathscr{A}} = \bigcup_{s=1}^{S} \widetilde{\mathscr{A}}_s$) are nonempty, meaning that (2.29) and (2.33) [or its equivalent (2.39)] are feasible [19]. Note also that the transmit power of SU s can be zero (i.e., no transmission), depending on the size of the uncertainty region, the values of IT_{sq}, and the channel gains $\mathbf{g}_{sq}$. Uncertainty in $\mathbf{g}_{sq}$ causes the feasibility set of (2.33) to be smaller than that of (2.29). When quality of service (QoS) is considered in the constraints of the optimization problem, for example, C_1 in (2.2), the feasibility region for the nominal problem (and for its robust counterpart) depends on the channel gains between users, system information, and each user's transmit power limitation [22], as already indicated in Proposition 3. In such a case, the problem may be infeasible, as in Fig. 2.2, where the minimum required SINR for each SU may not be attained. On the other hand, when QoS is not considered in the constraints, the feasibility region can be obtained in a straightforward manner.

Now, the question is whether the robust solution can be obtained in a manner similar to how the nominal solution was obtained. To answer this question, we focus on a single SU scenario and, for notational simplicity, drop the index s in variables. The nominal problem (2.29) is transformed into

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}^k}}\right), \tag{2.40}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, \quad p^k \ge 0, & \forall\, k \in \mathscr{K}, \\ C_2 : \mathbf{p} \cdot \mathbf{g}_q^{\mathrm{T}} \le IT_q, & \forall\, q \in \mathscr{Q}. \end{cases}$$

Since (2.40) is convex, its solution obtained via the Lagrange dual function [30] is

$$p^k = \left[\frac{1}{\lambda + \sum_{q=1}^{Q} \nu_q g_q^k} - \frac{\sigma^2 + f_{\mathscr{Q}}^k}{h^k}\right]^+, \tag{2.41}$$

where λ and ν_q are nonnegative Lagrange multipliers associated with C_1 and C_2 in (2.40), and $x^+ = \max\{0, x\}$. The optimal transmit power and Lagrange multipliers should satisfy the following Karush–Kuhn–Tucker (KKT) conditions [19]:

$$\lambda\left(\sum_{k=1}^{K} p^k - p^{\max}\right) = 0, \quad \text{and} \quad \nu_q(\mathbf{p} \cdot \mathbf{g}_q^{\mathrm{T}} - IT_q) = 0, \quad \forall\, q \in \mathscr{Q}. \tag{2.42}$$

We now present a pseudo water-filling algorithm for the robust problem. Consider the uncertainty region for the polyhedron model as

$$\mathscr{R}_{\mathbf{g}_q} = \{\mathbf{g}_q | \mathbf{M}_q \cdot \mathbf{g}_q^{\mathrm{T}} \le \boldsymbol{\varepsilon}_{\mathbf{g}_q}\}, \tag{2.43}$$

where $\mathbf{M}_q$ is the weight matrix for $\mathbf{g}_q$, and $\boldsymbol{\varepsilon}_{\mathbf{g}_q} = [\varepsilon_{\mathbf{g}_q}^1, \ldots, \varepsilon_{\mathbf{g}_q}^K]$ is an $R^{K\times 1}$ vector that represents the weighted maximum deviation of $\mathbf{g}_q$ from $\overline{\mathbf{g}}_q$. In this case, the robust counterpart of (2.40) is

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.44}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, & p^k \ge 0, \quad \forall\, k \in \mathscr{K}, \\ C_2 : \max_{(\mathbf{g}_q | \mathbf{M}_q \cdot \mathbf{g}_q^{\mathrm{T}} \le \boldsymbol{\varepsilon}_{\mathbf{g}_q})} \left(\mathbf{p} \cdot (\mathbf{g}_q^{\mathrm{T}} - \overline{\mathbf{g}}_q^{\mathrm{T}})\right) \le IT_q, & \forall\, q \in \mathscr{Q}. \end{cases}$$

From (2.36), C_2 can be replaced by

$$\boldsymbol{\varepsilon}_{\mathbf{g}_q} \cdot \mathbf{y}_q^{\mathrm{T}} \le IT_q, \qquad \mathbf{p}^{\mathrm{T}} \le \mathbf{M}_q \cdot \mathbf{y}_q^{\mathrm{T}}, \qquad \mathbf{y}_q \ge \mathbf{0}, \quad \forall\, q \in \mathscr{Q}.$$

Now, when the uncertainty region is (2.43), problem (2.44) is equivalent to

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.45}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, & \\ C_2 : \sum_{k=1}^{K} \varepsilon_{\mathbf{g}_q}^k y_q^k \le I T_q, & \forall\ q \in \mathscr{Q}, \\ C_3 : p^k \le \sum_{i=1}^{K} [M_q^{\mathrm{T}}]^{ki} y_q^i, \quad \mathbf{y}_q \ge \mathbf{0}, & \forall\ q \in \mathscr{Q}, \end{cases}$$

where $[M_q^{\mathrm{T}}]^{ki}$ is the element of $\mathbf{M}_q^{\mathrm{T}}$ in its k^{th} row and i^{th} column. Since (2.45) is a convex optimization problem with linear constraints, it can be solved with fewer calculations than (2.44) with the same uncertainty region (2.43).

Proposition 2.5. *When* $\sum_{i=1}^{K} \mu_q^i [M_q^T]^{ki} = \hat{v}_q \varepsilon_{\mathbf{g}_q}^k$ *for all* q *and* k*, the optimal solution to* (2.45) *is*

$$\tilde{p}^{*k} = \left[\frac{1}{\lambda + \sum_{q=1}^{Q} \mu_q^k} - \frac{\sigma^2 + f_{\mathscr{Q}}^k}{h^k}\right]^+, \tag{2.46}$$

where λ*,* $\hat{v}_q$*, and* μ_q^k *are nonnegative Lagrange multipliers for* C_1*,* C_2*, and* C_3 *in* (2.45)*, respectively.*

Proof. See Appendix 8.

From Proposition 2.5, the robust problem can be transformed into an optimization problem with linear constraints whose solution is obtained by a pseudo water-filling formula. In an orthogonal frequency division multiple access (OFDMA) system, when the frequency and time interleaver with sufficient interleaving depth is applied, fading can be considered uncorrelated across channels [31]. In this case, we can assume that uncertainty in $\mathbf{g}_q$ can be modeled by independent and identically distributed (i.i.d.) random variables [31]. Consequently, $\mathbf{M}_q$ becomes diagonal, and the power allocation problem (2.45) becomes

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.47}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, & \\ C_2 : \sum_{k=1}^{K} \varepsilon_{\mathbf{g}_q}^k y_q^k \le I T_q, & \forall\ q \in \mathscr{Q}, \\ C_3 : p^k \le [M_q^{\mathrm{T}}]^{kk} y_q^k, \quad \mathbf{y}_q \ge \mathbf{0}, & \forall\ q \in \mathscr{Q}. \end{cases}$$

At the optimal point, from the KKT conditions we have $[M_q^{\mathrm{T}}]^{kk} y_q^k = p^k$, and the allocated transmit power (2.46) is simplified to the following pseudo water-filling formula:

$$\tilde{p}^{*k} = \left[\frac{1}{\lambda + \sum_{q=1}^{Q} \frac{\hat{v}_q \varepsilon_{\mathbf{g}_q}^k}{[M_q^{\mathrm{T}}]^{kk}}} - \frac{\sigma^2 + f_{\mathscr{Q}}^k}{h^k} \right]^+, \tag{2.48}$$

where λ and $\hat{v}_q$ are the Lagrange multipliers for C_1 and C_2 in (2.47), respectively. The robust transmit power is obtained by (2.48) in a straightforward manner.

The robust counterpart of (2.40) can be written using the protection function. When $\mathscr{R}_{\mathbf{g}_q}$ is a compact set, the robust counterpart of (2.40) is

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.49}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, & p^k \ge 0, \quad \forall k \in \mathscr{K}, \\ C_2 : \left(\mathbf{p} \cdot \bar{\mathbf{g}}_q^{\mathrm{T}} + \Delta_q(\mathbf{p})\right) \le IT_q, & \forall q \in \mathscr{Q}, \end{cases}$$

where $\Delta_q(\mathbf{p}) = \max_{\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}} \mathbf{p} \cdot (\mathbf{g}_q - \bar{\mathbf{g}}_q)^{\mathrm{T}}$ is the protection function in C_2, whose value (the protection value) depends on the uncertain parameters [18, 20]. For the general norm, e.g., for the ellipsoid norm, the protection function can be replaced by bounds on the uncertainty region and transmit power, which are more tractable. In this way, uncertainty is removed from the optimization variables, resulting in less complexity.

In light of the preceding discussion, we now study the throughput reduction in the worst-case robust optimization approach. In doing so, we consider $d_\Delta = v^* - v_\Delta^*$, where v^* and v_Δ^* are the optimal throughput values for (2.40) and (2.49), respectively. Note that the difference between (2.40) and (2.49) is in their protection function in C_2, which we use to derive the relationship between d_Δ and the protection function for sensitivity analysis. Similar to Lemma 2.1, one can show that [9]

$$d_\Delta \approx \sum_{q=1}^{Q} v_q^* \Delta_q(\mathbf{p}). \tag{2.50}$$

From (2.50), the value of d_Δ depends on the size of the uncertainty region and on the interference constraint in (2.40). The approximation (2.50) gives us a quantitative measure of the impact of C_2 and $\Delta_q(\mathbf{p})$ on the throughput in (2.49). Let $\mathbf{p}^*$ be the optimal transmit power in (2.40). When $\mathbf{p}^* \cdot \mathbf{g}_q^{\mathrm{T}} < IT_q$ for all q, constraint C_2 is not satisfied and the protection function does not affect the optimal

solution to (2.49), and $d_\Delta \approx 0$. On the other hand, when $\mathbf{p}^* \cdot \mathbf{g}_q^{\mathrm{T}} = IT_q$, the value of ν_q^* indicates the impact of the protection function on the optimal solution to (2.49).

To demonstrate the relationship between λ, ν, and d_Δ, we simulate a CRN with one PBS and one SU pair for D2D communication. In this case, for notational simplicity, we drop indices s and q in the formulas. The number of channels used by the SU is 64, $\sigma^2 + f_{\mathscr{Q}}^k$ is -70 dB, and $p^{\max}$ is 10 dBm. The gain in each channel k is $h^k = \frac{\beta}{D^\alpha}$ and $g^k = \frac{\beta}{d^\alpha}$, where $\alpha = 4$, β is a frequency-dependent coefficient, and D and d are distances from the transmitting SU to its receiving SU and from the transmitting SU to the receiving PBS, respectively. To consider both constraints C_1 and C_2, we take $D = d = 500$ m. This is because when $D \ll d$, the value of $\mathbf{g}_q$ is very small, and C_2 in (2.40) can be neglected. Also, when $d \ll D$, the value of $\mathbf{g}_q$ is very high and C_1 in (2.40) can be ignored. In other words, when the values of D and d are very disparate, only one of the two constraints in (2.40) is applicable, and the problem is much simpler. When the values of D and d are close to one another, both C_1 and C_2 in (2.40) should be satisfied, and we can study the impacts of other parameters, for example, $p^{\max}$ and IT, on the allocated robust transmit power to the SU. In simulations, the IT for the PBS's receiver is proportional to its receiving noise level, and the SU's throughput is measured in bits per symbol per Hertz.

We now study the dependence of d_Δ on IT and $p^{\max}$ in (2.40). To do so, and as an example, we assume that the normalized error in channel gain is $\frac{\mathbf{g}-\bar{\mathbf{g}}}{\bar{\mathbf{g}}} = 0.5$, that is, the error in the channel gain is not more than 50 % of its exact value. In Fig. 2.6, the average values of λ^* [i.e., the optimal Lagrange multiplier for C_1 in (2.40)], the average values of $\boldsymbol{\nu}^*$, and the average values of d_Δ are shown. Note the following three distinct regions for different values of $IT/p^{\max}$ in Fig. 2.6:

1. Region I: The value of IT is much higher than $p^{\max}$, that is, $\frac{p^{\max}}{IT} \ll 1$, which means that in (2.40), C_2 can be ignored, but C_1 should be satisfied. In other words, $\lambda^* > 0$ and $\boldsymbol{\nu}^* = 0$.
2. Region II: The interference constraint is moderate, that is, $\frac{p^{\max}}{IT} \approx 1$, which means that in (2.40), both C_1 and C_2 must be considered.

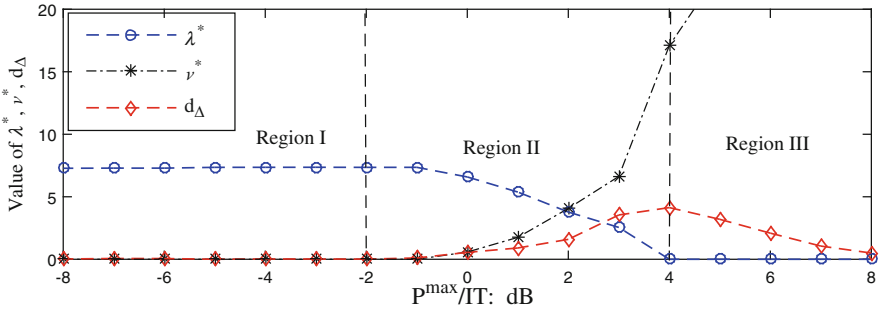


Fig. 2.6 Values of λ^* and ν^* in (2.40) and d_Δ versus $\frac{p^{\max}}{IT}$

3. Region III: The value of IT is much less than $p^{\max}$, that is, $\frac{p^{\max}}{IT} \gg 1$, which means that in (2.40), C_2 should be satisfied but C_1 can be ignored. In other words, $\lambda^* = 0$ and $\boldsymbol{\nu}^* > 0$.

In Region I, where C_2 is not relevant, we have $\nu^* = 0$, meaning that $d_\Delta = 0$ and $v^* = v_\Delta^*$. However, in Regions II and III, we have $\nu^* \neq 0$, and consequently $d_\Delta \neq 0$. Therefore, $v_\Delta^* < v^*$. Implementing the foregoing considerations in an underlay CRN has the following consequences. In Region I, uncertainty in $\mathbf{g}$ and the corresponding trade-off do not affect the SU's throughput. But in Regions II and III, a trade-off algorithm is desirable to increase the SU's throughput. The impact of robustness on the SU's throughput in Regions II and III depends on the values of $\mathbf{h}$ and $\mathbf{g}$, as well as on the values of the trade-off parameters. Note that for a fixed $p^{\max}$ in Region III, increasing the ratio $\frac{p^{\max}}{IT}$ is equivalent to reducing IT, which reduces the SU's allocated transmit power. For very small values of the allocated transmit power, the difference d_Δ in the achieved throughput in the nominal and robust approaches diminishes.

2.3.1.1 Social Utility of Robust Solutions Versus Uncertainty Levels

Now the question concerns the impact of uncertainty levels on the robust solution's social utility. In Fig. 2.7, the polyhedron approach is simulated for D2D communications in a multiple-SU single-PBS scenario with multiple OFDMA channels

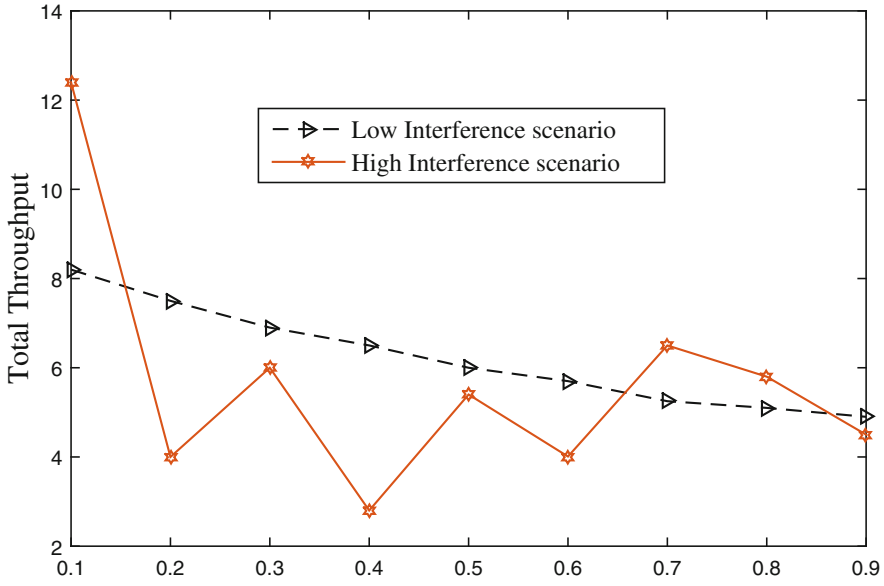


Fig. 2.7 Throughput values for polyhedron model for low- and high-interference cases versus ξ

when interference between SUs is low and when it is high. Simulation parameters are $K = 64$, $Q = 1$, $S = 2$, $D_{ss} \in [450, 550]$ m (distance between the transmitting and receiving SU s), and $d_{sq} \in [450, 500]$ m (distance between the transmitting SU s and receiving PBS q). In this setup, $D_{ss} \approx d_{sq}$, and hence both C_1 and C_2 in (2.29) are applicable. The distance between the transmitting SU s and the receiving SU m, denoted by D_{sm}, is $D_{sm} \in [800, 950]$ m for the low-interference case and $D_{sm} \in [300, 250]$ m for the high-interference case. We assume $\hat{g}_{sq}^k = \xi \overline{g}_{sq}^k$ for all $s \in \mathscr{S}$ and $q \in \mathscr{Q}$ and $k \in \mathscr{K}$.

Note that in the low-interference case, increasing ξ results in monotonically reducing the SUs' total throughput (social utility). This is not the case for the high-interference case, where introducing robustness may reduce each SU's transmit power, and, consequently, the interference of each SU on the other SU may be reduced. Hence, it is possible that the throughput of each SU may increase with increasing ξ due to the reduced interference between SUs. In other words, in a high-interference scenario, it may be possible to increase the total throughput by increasing uncertainty when the SUs reduce their transmit power levels to satisfy the constraints.

2.3.2 Trade-Off Algorithms

In what follows, we discuss the trade-off algorithms for the D-norm and chance-constrained approaches. We begin by considering a CRN with one SU and then extend our formulations for multiple SUs.

2.3.2.1 Trade-Off in D-Norm Approach

The protection value for the D-Norm approach is

$$\Delta_q = \max_{\mathscr{L}_q} \sum_{k \in \mathscr{L}_q} \hat{g}_q^k p^k, \tag{2.51}$$

where $\mathscr{L}_q$ is any subset of channels of size Γ_q. The number of channels in $\mathscr{L}_q$ determines the protection value, meaning that a smaller Γ_q results in a smaller d_Δ. For $\Gamma_q = 0$, the protection value vanishes, and $d_\Delta = 0$. In contrast, for $\Gamma_q = K$, all channels are taken into account, and d_Δ is maximized. By adjusting Γ_q, one can control the cost of robustness from the SU's point of view.

For the D-norm approach, as with the polyhedron uncertainty region, a linear protection function can be obtained [9], resulting in fewer calculations. From Proposition 2.1 in [25], for a given $\mathbf{p} = \tilde{\mathbf{p}}$, the protection function (2.51) for the constraint on the interference on PBS q is

$$\max_{0\le l_q^k\le 1} \sum_{k=1}^{K} \hat{g}_q^k l_q^k \tilde{p}^k \tag{2.52}$$

$$\text{subject to} \sum_{k=1}^{K} l_q^k \le \Gamma_q,$$

where l_q^k is an auxiliary variable that indicates whether the corresponding error $\hat{g}_q^k$ is considered in the protection function of the D-norm approach, and $\tilde{p}^k$ is the kth element of $\tilde{\mathbf{p}}$. Note that l_q^k is by nature a binary variable, but to simplify the problem we take it as a continuous variable, which will eventually be converted into a binary variable. Let $\tilde{\Delta}_q^*$ be the optimal protection value of (2.52), which can be obtained from its dual function [25]

$$\min_{z_q,\boldsymbol{\chi}_q} \left(\Gamma_q z_q + \sum_{k=1}^{K} \chi_q \right), \tag{2.53}$$

$$\text{subject to} \begin{cases} \hat{g}_q^k x_q^k \le (z_q + \chi_q), \\ \tilde{p}^k \le x_q^k \qquad\qquad \forall\, k \in \mathscr{K}, \end{cases}$$

where z_q, $\boldsymbol{\chi}_q = [\chi_q^1, \ldots, \chi_q^K]$, and x_q^k are positive variables. Since (2.52) is feasible and bounded for all $0 \le \Gamma_q \le K$, its dual, that is, (2.53), is also feasible and bounded, and by strong duality, their optimal protection values coincide. Feasible solutions to (2.53), denoted by z_q^* and $\boldsymbol{\chi}_q^* = [\chi_q^{*1}, \ldots, \chi_q^{*K}]$, satisfy $(\Gamma_q z_q^* + \sum_{k=1}^{K} \chi_q^{*k}) \le IT_q - \mathbf{p}.\bar{\mathbf{g}}_q^{\mathrm{T}}$ and also satisfy $(\tilde{\Delta}_q^* \le \Gamma_q z_q^* + \sum_{k=1}^{K} \chi_q^{*k}) \le IT_q - \mathbf{p}.\bar{\mathbf{g}}_q^{\mathrm{T}}$. Therefore, C_2 in (2.49) can be replaced by the following constraints:

$$\left(\mathbf{p}.\bar{\mathbf{g}}_q^{\mathrm{T}} + \Gamma_q z_q + \sum_{k=1}^{K} q_q^k \right) \le IT_q, \quad \hat{g}_q^k x_q^k \le \left(z_q + q_q^k\right), \quad p^k \le x_q^k, \qquad \forall\, k \in \mathscr{K}.$$

Consequently, the robust optimization problem is

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.54}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, \\ C_2 : \left(\sum_{k=1}^{K} p^k \bar{g}_q^k + z_q \Gamma_q + \sum_{k=1}^{K} \chi_q \right) \le IT_q, \\ C_3 : \hat{g}_q^k x_q^k \le (z_q + \chi_q), \\ C_4 : p^k \le x_q^k, \\ C_5 : 0 \le x_q^k, \quad 0 \le z_q, \quad 0 \le \chi_q^k, \quad \forall\, k \in \mathscr{K}, \quad \forall\, q \in \mathscr{Q}. \end{cases}$$

Since all the constraints in (2.54) are affine functions, a pseudo water-filling formulation can be developed for obtaining p^k using their corresponding Lagrange dual functions. When $\hat{g}_q^k \tilde{\mu}_q^k = w_q^k$, $\tilde{\nu}_q \Gamma_q = \sum_{k=1}^{K} \tilde{\mu}_q^k$, and $\tilde{\mu}_q^k = \tilde{\nu}_q$, as shown in [9, 19], the optimal solution to (2.54) is

$$\tilde{p}^{*k} = \left[\frac{1}{\lambda + \sum_{q=1}^{Q} (\tilde{\nu}_q \bar{g}_q^k + w_q^k)} - \frac{\sigma^2 + f_{\mathscr{Q}}^k}{h^k} \right]^+,$$

where λ, $\tilde{\nu}_q$, $\tilde{\mu}_q^k$, and w_q^k are the nonnegative Lagrange multipliers for C_1, C_2, C_3, and C_4 in (2.54), respectively.

2.3.2.2 Trade-off in Chance-Constrained Approach

When A_1 and A_2 in Section 1.3.3.1, Chapter 1 hold, the robust counterpart of (2.40) in the chance-constrained approach is

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log \left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k} \right), \tag{2.55}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \le p^{\max}, \quad p^k \ge 0, & \forall\, k \in \mathscr{K}, \\ C_2 : \Pr\left(\mathbf{p} \cdot \mathbf{g}_q^{\mathrm{T}} \ge I T_q\right) \le \delta_q, & \forall\, q \in \mathscr{Q}, \\ C_3 : \mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}, & \forall\, q \in \mathscr{Q}. \end{cases}$$

Again, adjusting δ_q in (2.55) results in a trade-off between robustness and optimality. By reducing δ_q, the system becomes more robust against uncertainty, and by increasing δ_q, the SU's throughput is increased. In the chance-constrained approach [17], C_2 in (2.55) can be replaced by its convex safe approximation. To do so, we rewrite the SU's interference on PBS q as

$$\mathbf{p} \cdot \mathbf{g}_q^{\mathrm{T}} = \sum_{k=1}^{K} p^k \bar{g}_q^k + \sum_{k=1}^{K} \zeta_q^k p^k \hat{g}_q^k,$$

where $\zeta_q^k = \frac{g_q^k - \bar{g}_q^k}{\hat{g}_q^k}$ for each $q \in \mathscr{Q}$ is known to be in the range $[-1, 1]$. For uncorrelated fading channels [31], all the values of ζ_q^k are independent of each other and their probability distribution function (pdf)is $\mathscr{D}(q)$ (Section 1.3.3.1 in Chapter 1). Hence, C_2 in (2.55) can be replaced by the Bernstein approximations of chance constraints [17], that is,

$$\left(\sum_{k=1}^{K} p^k \bar{g}_q^k + \sum_{k=1}^{K} \eta_{\mathscr{D}(q)}^{+} p^k \hat{g}_q^k + \sqrt{2 \ln \delta_q^{-1}} \left(\sum_{k=1}^{K} \varphi_{\mathscr{D}(q)}^2 (p^k \hat{g}_q^k)^2 \right)^{\frac{1}{2}} \right) \le I T_q, \quad \forall\, q \in \mathscr{Q}, \tag{2.56}$$

or

$$\left(\sum_{k=1}^{K} p^k \overline{g}_q^k + \sum_{k=1}^{K} \eta_{\mathscr{D}(q)}^{+} p^k \hat{g}_q^k + \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} \max_{\forall\, k \in \mathscr{K}} p^k \hat{g}_q^k\right) \leq IT_q, \quad \forall\, q \in \mathscr{Q}, \tag{2.57}$$

where $-1 \leq \eta_{\mathscr{D}(q)}^{+} \leq +1$ and $\sigma_{\mathscr{D}(q)}^2 \geq 0$, which depend on $\mathscr{D}(q)$, are used for safe approximations of the chance constraints. For a specific $\mathscr{D}(q)$, these parameters have fixed values [17], as shown in Table 1.2 in Chapter 1. Note that when $\mathbf{p}$ satisfies (2.56) and (2.57), it certainly satisfies C_2 and C_3 in (2.55). Hence, from (2.56) we have

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.58}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \leq p^{\max}, \\ C_2 : \left(\sum_{k=1}^{K} p^k \overline{g}_q^k + \sum_{k=1}^{K} \eta_{\mathscr{D}(q)}^{+} p^k \hat{g}_q^k + \sqrt{2 \ln \delta_q^{-1}} \sqrt{\sum_{k=1}^{K} \varphi_{\mathscr{D}(q)}^2 \left(p^k \hat{g}_q^k\right)^2}\right) \leq IT_q. \end{cases}$$

Note that (2.58) can be rewritten as a conic quadratic programming problem, which can be solved with few calculations.

We use (2.57) to rewrite (2.55) as

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.59}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \leq p^{\max}, \\ C_2 : \left(\sum_{k=1}^{K} p^k \overline{g}_s^k + \sum_{k=1}^{K} \eta_{\mathscr{D}(q)}^{+} p^k \hat{g}_q^k + \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} \max_{\forall k \in \mathscr{K}} p^k \hat{g}_q^k\right) \leq IT_q, \end{cases}$$

which is equivalent to

$$\max_{\mathbf{p}} \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right), \tag{2.60}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p^k \leq p^{\max}, \\ C_2 : \left(\sum_{k=1}^{K} p^k \overline{g}_q^k + \sum_{k=1}^{K} \eta_{\mathscr{D}(q)}^{+} p^k \hat{g}_q^k + \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} u_q\right) \leq IT_q, \quad \forall\, q \in \mathscr{Q}, \\ C_3 : p^k \hat{g}_q^k \leq u_q, \qquad\qquad \forall\, q \in \mathscr{Q}, \quad \forall\, k \in \mathscr{K}, \end{cases}$$

where $u_q > 0$ is an auxiliary variable representing the maximum error in the estimated interference on PU q in all channels. Utilizing the Lagrange dual function, the optimal solution to (2.60) is

$$\tilde{p}^{*k} = \left[\frac{1}{\lambda + \sum_{q=1}^{Q} \left(\overline{\nu}_q (\overline{g}_q^k + \eta^+_{\mathscr{D}(q)} \hat{g}_q^k) + \overline{\mu}_q^k \hat{g}_q^k \right)} - \frac{\sigma^2 + f^k_{\mathscr{Q}}}{h^k} \right]^+, \tag{2.61}$$

where λ, $\overline{\nu}_q$, and $\overline{\mu}_q^k$ are the nonnegative Lagrange multipliers for C_1, C_2, and C_3, respectively, which satisfy

$$\lambda \left(\sum_{k=1}^{K} \tilde{p}^{*k} - p^{\max} \right) = 0,$$

$$\overline{\nu}_q \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} = \sum_{k=1}^{K} \overline{\mu}_q^k,$$

$$\overline{\mu}_q^k (\tilde{p}^{*k} \hat{g}_q^k - u_q) = 0,$$

$$\overline{\nu}_q \left(\sum_{k=1}^{K} \tilde{p}^{*k} \overline{g}_q^k + \sum_{k=1}^{K} \eta^+_{\mathscr{D}(q)} \tilde{p}^{*k} \hat{g}_q^k + \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} u_q - IT_q \right) = 0.$$

In (2.59), the protection function is

$$\Delta_q = \sum_{k=1}^{K} \eta^+_{\mathscr{D}(q)} p^k \hat{g}_q^k + \varphi_{\mathscr{D}(q)} \sqrt{2K \ln \delta_q^{-1}} \max_{\forall\, k \in \mathscr{K}} p^k \hat{g}_q^k. \tag{2.62}$$

Sensitivity Analysis of d_Δ: Protection functions in the D-norm and chance-constrained approaches depend on Γ_q and δ_q, respectively. In what follows, we study the sensitivity of d_Δ on Γ_q in the D-norm approach and on δ_q in the chance-constrained approach by differentiating d_Δ with respect to Γ_q and δ_q.

- D-Norm Approach: Since $\Delta_q = z_q \Gamma_q + \sum_{q=1}^{Q} q_q^k$, from (2.50) we have

$$d_\Delta \approx \sum_{q=1}^{Q} \nu_q^* \left(z_q \Gamma_q + \sum_{k=1}^{K} q_q^k \right), \tag{2.63}$$

  which means that d_Δ directly relates to Γ_q. Hence, for a given value of IT_q and in a given channel uncertainty region, increasing the value of Γ_q proportionally reduces the SU's throughput, and vice versa. This is shown in Fig. 2.8 for a single SU and a single PBS, where $\hat{g}^k = \xi \overline{g}^k$ and $IT = -3\,\text{dB}$. Note that increasing Γ_q and expanding the uncertainty region ξ

(continued)

proportionally reduces the SU's throughput, as expected from (2.50). The simulation setup for Fig. 2.8 is the same as for Fig. 2.6.

- Chance-Constrained Approach: In this case, the protection function is

$$\Delta_q = \sum_{k=1}^{K} \eta^{+}_{\mathscr{D}(q)} p^k \hat{g}^k_q + \sqrt{2\ln \delta_q^{-1}} \sqrt{\sum_{k=1}^{K} \varphi^2_{\mathscr{D}(q)} (p^k \hat{g}^k_q)^2}, \tag{2.64}$$

and we have

$$d_\Delta \approx \sum_{q=1}^{Q} v^*_q \left(\sum_{k=1}^{K} \eta^{+}_{\mathscr{D}(q)} p^k \hat{g}^k_q \right) + \sqrt{2\ln \delta_q^{-1}} \sqrt{\sum_{k=1}^{K} \varphi^2_{\mathscr{D}(q)} (p^k \hat{g}^k_q)^2}. \tag{2.65}$$

Based on the preceding expressions, the sensitivity of d_Δ to δ_q is

$$S_{\delta_q}(d_\Delta) = \frac{\partial d_\Delta}{\partial \delta_q} = \frac{1}{\delta_q \sqrt{2\ln \delta_q^{-1}}}. \tag{2.66}$$

Figure 2.9 shows that for a given IT_q and a channel uncertainty region, when $\delta_q < 0.2$, d_Δ is very sensitive to δ_q. This is not the case for higher values of δ_q. The same is true of (2.59). The sensitivity of d_Δ to δ_q is shown in Fig. 2.10, where for $\delta \geq 0.2$, the SUs' throughput does not increase very much.

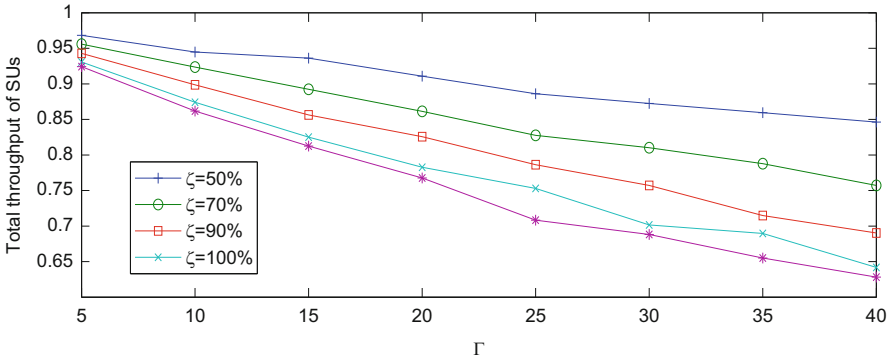


Fig. 2.8 Throughput in D-norm approach versus ξ and $\Gamma = \Gamma_q$ for all $q \in \mathscr{Q}$ and for $IT = \frac{1}{2} p^{\max}$ dB

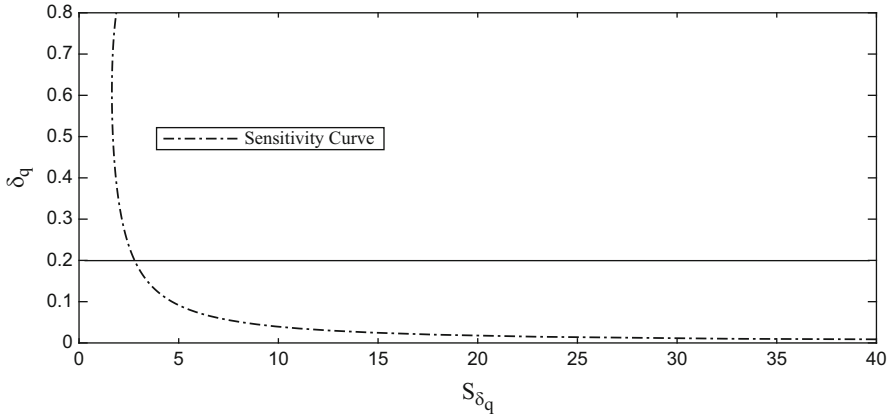


Fig. 2.9 δ_q versus sensitivity of d_Δ in (2.65)

Choosing a Trade-Off Approach: Now we consider the criteria for choosing a trade-off approach for a given scenario. We begin by noting that the computational cost in both choices, namely, in the trade-off algorithm for the D-norm approach and in the trade-off algorithms for the two safe approximations in the chance-constrained approach, is moderate and not a determining factor. However, the *protection value* plays a major role in the choice of trade-off algorithm. An algorithm with less protection has a smaller d_Δ and a higher throughput. Less protection also means a higher probability of violating the constraints. One can keep the violation probability below a given threshold in the D-norm approach, as shown by (1.18) in Chapter 1, and in both safe approximations in the chance-constrained approach. In practice, however, the violation probability may be even less than the given threshold, as reported in [9]. Hence, when high throughput is important, a trade-off algorithm with less protection should be chosen, and when it is important to guarantee the violation probability, a trade-off algorithm with higher protection should be selected. In [9], it is shown that in a single-SU single-PBS CRN, the protection value for (2.56) is higher than those for (2.51) and (2.57), which means (2.56) is preferred by the PBS but is disadvantageous for the SU. The opposite is true of (2.57), where the SU enjoys its highest throughput at the expense of minimal protection for the PBS. While the throughput in the D-norm approach is close to that of (2.57), the probability of violating the IT in the former is considerably lower than that in the latter because the former has higher protection. Hence, the trade-off algorithm in the D-norm approach can be a better choice from the perspectives of both the SU and the PBS. In a similar manner, one could argue which trade-off algorithm is a better choice for other cases.

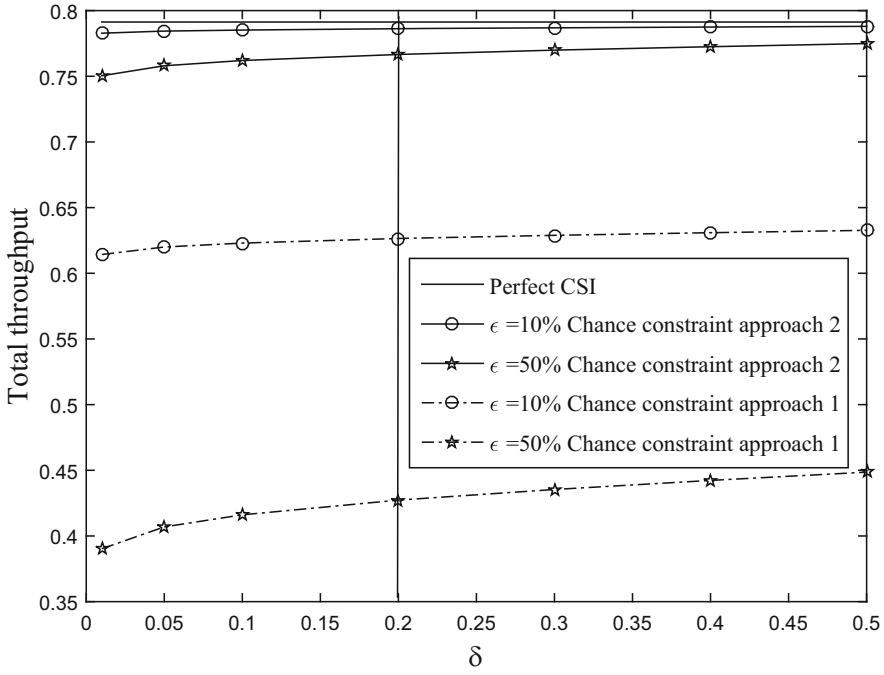


Fig. 2.10 Throughput in chance-constrained approach versus $\delta = \delta_q$ and $\varepsilon = \varepsilon_{\mathbf{g}_q}$ for all $q \in \mathscr{Q}$ for $IT = \frac{1}{2}p^{\max}$

Note that the linear constraint for the D-norm approach in (2.54) and for the chance-constrained approach in (2.60) for the single-SU scenario can also be applied for multiple SUs in (2.39). For instance, in the D-norm approach, (2.39) is transformed into

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \sum_{s=1}^{S}\sum_{k=1}^{K} \log\left(1+\frac{p_s^k h_{ss}^k}{\sigma^2+f_{\mathscr{Q}}^k}\right), \tag{2.67}$$

$$\text{subject to} \begin{cases} C_1 : \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \\ C_2 : \sum_{k=1}^{K} \left(p_s^k \overline{g}_{sq}^k + z_{sq}\Gamma_{sq} + \sum_{k=1}^{K} \chi_{sq}\right) \le IT_{sq}, \\ C_3 : \hat{g}_{sq}^k x_{sq}^k \le (z_{sq} + \chi_{sq}), \\ C_4 : p_s^k \le x_{sq}^k, \\ C_5 : 0 \le x_{sq}^k, \quad 0 \le z_{sq}, \quad 0 \le \chi_{sq}^k, \quad \forall\, k \in \mathscr{K}, \quad \forall\, s \in \mathscr{S}, \quad \forall\, q \in \mathscr{Q}, \end{cases}$$

where Γ_{sq} is the D-norm parameter for protecting PBS q from SU s, and χ_{sq} and x_{sq}^k are nonnegative parameters for converting the D-norm's protection function into linear constraints.

Similarly, in the chance-constrained approach, (2.39) is transformed into

$$\max_{[\mathbf{p}_1,\ldots,\mathbf{p}_S]} \sum_{s=1}^{S}\sum_{k=1}^{K} \log\left(1+\frac{p_s^k h_{ss}^k}{\sigma^2+f_{\mathscr{Q}}^k}\right), \tag{2.68}$$

$$\text{subject to} \begin{cases} C_1: \sum_{k=1}^{K} p_s^k \le p_s^{\max}, \\ C_2: \left(\sum_{k=1}^{K} p_s^k \bar{g}_{sq}^k + \sum_{k=1}^{K}\left(\eta^{+}_{\mathscr{D}(sq)} p_s^k \hat{g}_{sq}^k\right) + \varphi_{\mathscr{D}(sq)}\sqrt{2K\ln \delta_{sq}^{-1}}\, u_{sq}\right) \le IT_{sq}, \ \forall\, q \in \mathscr{Q}, \\ C_3: p_s^k \hat{g}_{sq}^k \le u_{sq}, \qquad \forall\, s \in \mathscr{S}, \quad \forall\, q \in \mathscr{Q}, \quad \forall\, k \in \mathscr{K}, \end{cases}$$

where $u_{sq} > 0$ is an auxiliary variable representing the maximum error in the estimated interference by SU s on PU q in all channels, δ_{sq} is the violation probability for the IT of SU s on PBS q, and $\mathscr{D}(sq)$ is the pdf for the uncertainty in channel gain $\mathbf{g}_{sq}$ between SU s and PBS q (defined in Table 1.2, Chapter 1).

The preceding discussion shows that, by using linear constraints instead of protection functions, the order of additional calculations for introducing robustness in the D-norm approach and in the chance-constrained approach is $O(Q\times(4\times Q\times K+Q))$ and $O(S\times(Q+K\times Q))$, respectively, compared to (2.29) [9], meaning that the computational complexity of obtaining the nominal and robust solutions are similar. To obtain the order of computational complexity associated with solving (2.60) or (2.67), all primal and dual variables are iteratively updated, much like the iterative algorithm for solving (2.30). Therefore, for each new constraint or each new variable in (2.60) or (2.67), there is one additional iterative formula compared to (2.29).

So far, we have shown that replacing nonlinear protection functions in the constraints of robust problems with linear functions can reduce the computational complexity associated with solving robust problems. We have also shown that introducing robustness entails throughput reduction, and sensitivity analysis was used to demonstrate that it is possible to trade off between throughput and robustness.

2.4 Overview of Other Works on Robust Cooperative Resource Allocation

In the preceding sections, we presented robust cooperative schemes for optimal transmit power allocation in wireless networks and explained how to trade off between total throughput and robustness. In what follows, we provide an overview of other works on this topic, highlighted in Table 2.1, and show that while we covered trade-off algorithms for complexity, throughput, and signaling, in general they have not been extensively addressed in the literature.

In [32], the worst-case ergodic resource allocation in the uplink of an underlay CRN is studied, where the objective is to maximize an SU's total throughput subject to PUs' interference threshold. In this work, the uncertainty in channel

Table 2.1 Overview of other existing works on robust cooperative resource allocation

References	Complexity	Throughput	Signaling	Trade-off algorithms	
				D-norm	Chance-constrained
[32]	√	√	–	–	√
[33]	√	√	–	–	√
[34]	√	√	–	–	–
[35]	√	√	√	–	√
[36]	√	√	–	–	√
[37]	√	√	–	–	–
[38]	√	√	–	–	–

gains between SU transmitters and SU receivers and in channel gains between SU transmitters and PU receivers are considered. To reduce the computational complexity, in [32] a middle way between the worst-case and Bayesian approaches is applied for robustness–throughput trade-off via a chance constraint, and the impact of the uncertainty region's bound on SUs' total throughput is investigated via simulations only (no analytical study).

In [33, 34], physical layer security in the presence of eavesdroppers is studied, where the impact of expanding the uncertainty region on the throughput is investigated, and reducing a problem's computational complexity is discussed. In [33], the robustness–throughput trade-off via a chance constraint is considered, but no such trade-off algorithm is presented in [34]. In [35], to maximize power efficiency while maintaining the required signal-to-interference ratio (SIR) for each user in interference-limited wireless networks with uncertain parameters, the problem is formulated via the chance-constrained approach. It is also shown that the proposed approach maintains the convexity and tractability of the problem by solving it in a distributed manner by the Lagrange dual decomposition method. An infrequent message passing algorithm is also proposed to reduce the signaling in the proposed robust scheme.

In [36], worst-case robust optimization is applied in relay-assisted two-hop D2D communications in LTE-A cellular networks, where the objective is to maximize the network's throughput in shared channel environments. The constraints are the bounded transmit power, the interference on relays and receiving users, and user QoS requirements. Uncertainty in channel gains, stated by the linear norm, is assumed to be within the uncertainty region. It is shown that the robust problem is convex and a gradient-based dual decomposition algorithm can be applied to allocate transmit power in a distributed manner. However, [36] is silent on additional message passing in the robust distributed algorithm, but the chance-constrained approach is applied to ameliorate the throughput reduction in the robust scheme by trading off between throughput and robustness.

In [37], a robust resource allocation scheme is proposed for a cooperative CRN in which a set of relays assist SUs, and strict SUs' ITs on PUs are set. Assuming that channel gains are uncertain, the robust resource allocation problem is formulated to

maximize the SUs' sum rate while satisfying the interference constraints on PUs via both the probabilistic and worst-case approaches. It is shown that the original robust optimization problem can be approximated and transformed into a convex deterministic form, and a closed-form solution for the SUs' transmit power levels is derived.

In [38], optimal robust resource allocation in the downlink of renewable-powered HetNets is studied, where channel gains are uncertain due to noise. In this work, robust energy management and transmit beam-forming are proposed to minimize worst-case energy transactions subject to a worst-case guarantee for each user's QoS. The authors use semi-definite programming (SDP) relaxation to transform the problem into a convex approximation, which is solved by applying the gradient method in a distributed manner. In this way, computational complexity is reduced, but the paper is silent on additional message passing in the robust solution. In addition, using simulations, it is shown that expanding the uncertainty region increases the robust transmit power levels.

As can be seen in Table 2.1, while the impact of uncertainty in parameter values on the total throughput or on transmit power levels has been extensively studied, the performance gap between the robust and nominal solutions similar to (2.21) in Lemma 2.1 in this chapter has not been considered in the literature. Note that deriving a mathematical expression for the aforementioned performance gap enables one to see how each constraint in the robust problem affects the performance of the robust solution. Moreover, as shown in Table 2.1, although the D-norm approach is easy to implement and has some interesting aspects, as mentioned in Section 3.2.2, it has not been a very popular topic. Nevertheless, utilizing (1.18) in Chapter 1 for the D-norm approach to make a connection between the violation probability and the protection value seems to be an interesting topic.

2.5 Concluding Remarks

In this chapter, we showed how network-centric robust resource allocation problems in wireless networks with cooperative users can be solved via the dual decomposition approach; in addition, we discussed how the costs of robustness can be mitigated. In what follows, we summarize our conclusions.

- **Computational Complexity**: We demonstrated how to simply robust problems and reduce their computational complexity to the same order of the nominal problem by utilizing protection functions; and showed that for the general norm and for the polyhedron uncertainty region, the robust counterpart problem is convex. We also obtained the optimal solution for some specific cases by way of utilizing a pseudo water-filling formula, whose computational complexity is not significant.

- **Throughput Reduction**: We derived mathematical formulations for a given amount of throughput reduction in robust solutions when the nominal problem is convex. We utilized sensitivity analysis to demonstrate that when the uncertainty region is shrank, the throughput gap between the nominal and robust solutions can be reduced. Also, we showed the application of the D-norm and chance-constrained approaches to trading off between robustness and throughput reduction in different scenarios and compared them by way of sensitivity analysis on the size of the protection function for each approach. We also explained the criteria for choosing a trade-off approach for each specific scenario.
- **Extra Message Passing**: In solving cooperative distributed resource allocation problems, the drawback of applying dual decomposition techniques is the need for extra message passing. To reduce this additional signaling, we proposed a distributed scheme with infrequent message passing and derived the conditions for its convergence to the optimal solution of the original distributed algorithm.

Appendices

Appendix 1: Proof of Proposition 2.2

Recall that the dual norm of $\|\mathbf{x}\|$ is $\|\mathbf{x}\|^* \triangleq \max_{\|\mathbf{s}\|\leq 1} \mathbf{s}^{\mathrm{T}}\mathbf{x}$. For the uncertainty set in $\mathscr{R}_{\mathbf{g}_{sq}}$, we use $\mathbf{y}_q = \frac{\bar{\mathbf{g}}_q - \mathbf{g}_q}{\varepsilon_{\mathbf{g}_q}}$ to rewrite the uncertainty set as

$$\mathscr{R}_{\mathbf{g}_q} = \{\mathbf{y}_q | \|\bar{\mathbf{y}}_q - \mathbf{y}_q\| \leq 1\} \qquad \forall\, q \in \mathscr{Q}. \tag{2.69}$$

We also rewrite the protection value as

$$\max_{\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}} \sum_{s\in\mathscr{S}} \left((g_{sq} - \bar{g}_{sq})p_s\right) = \varepsilon_{\mathbf{g}_q} \max_{\mathbf{y}_q \in \mathscr{R}_{\mathbf{g}_q}} \sum_{s\in\mathscr{S}} \left((y_{sq} - \bar{y}_{sq})p_s\right), \tag{2.70}$$

which corresponds to the dual norm. The same is true of the uncertainty set pertaining to PU interference on the SBS [20].

Appendix 2: Convexity of (9)

By logarithmic transformation of the optimization variable in (2.9), the upper and lower bounds on the transmit power of each SU, as well as C_1 and C_3, are changed to

$$\begin{aligned} p_s^{\min} e^{-y_s} &\leq 1, \\ (p_s^{\max})^{-1} e^{y_s} &\leq 1, \\ \hat{\gamma}_s h_s^{-1} e^{z_s - y_s} &\leq 1, \end{aligned}$$

$$e^{-z_s}(f_{\mathscr{Q}} + \sum_{m \neq s} e^{y_m} h_m + \sigma^2) \leq 1.$$

The left-hand side of the preceding constraints are the affine nonnegative sum of exponential variables and constitute convex functions ([10] and Section 3.2 in [19]). Also, the interference constraint C_2 in (2.9) is changed to

$$\left(\sum_{s \in \mathscr{S}} \overline{g}_{sq} e^{y_s} + \varepsilon_{\mathbf{g}_q} \sqrt{\sum_{s \in \mathscr{S}} e^{2y_s}} \right) \leq IT_q, \quad \forall\, q \in \mathscr{P}.$$

The left-hand side of the preceding expression is the linear norm of the vector of exponential variables with order 2, which is convex (Section 3.2.4 in [19]). Note that (2.4) is a concave function under logarithmic transformation [10, 12]. Hence, (2.9) is convex.

Appendix 3: Proof of Proposition 2.3

Problem (2.9) is feasible when there is at least one transmit power vector that simultaneously satisfies all constraints. The feasibility of problem (2.7) can be written (Section 2.1.3 in [16]) as

$$\min_{1 \leq \varsigma,\ p_s^{\min} \leq p_s \leq p_s^{\max}} \varsigma, \tag{2.71}$$

$$\text{subject to} \begin{cases} C_1 : \chi_{1n}(z_s, y_s) = e^{z_s - y_s} \frac{\hat{\gamma}_s}{h_s} \leq \varsigma, \\ C_2 : \chi_{2n}(z_s, y_s) = \frac{1}{IT_q} (\sum_{s \in \mathscr{S}} g_{sq} e^{y_s}) \leq \varsigma \qquad \forall\, q \in \mathscr{Q}, \\ C_3 : \chi_{3n}(z_s, y_s) = e^{-z_s} (f_{\mathscr{Q}} + \sum_{m \neq s,\ m \in \mathscr{S}} e^{y_m} h_m + \sigma^2) \leq \varsigma, \\ C_4 : \mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}, \quad f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}, \end{cases}$$

where ς is an auxiliary variable representing the upper bound on all constraints. By solving (2.71) and obtaining $(\varsigma^*, \mathbf{p}^* = [p_1^*, \ldots, p_S^*])$, when $\varsigma^* = 1$, the set of posynomial constraints in C_1, C_2, and C_3 are feasible, and the corresponding transmit power vector of all SUs, that is, $\mathbf{p}^* = [p_1^*, \ldots, p_S^*]$ is the feasibility set of (2.7). Otherwise, the set of posynomial constraints and, consequently, (2.7) are infeasible [16]. Based on the preceding expressions, to obtain the feasibility conditions of (2.7), we set $\varsigma = 1$ and obtain $\mathbf{p}^* = [p_1^*, \ldots, p_S^*]$. In doing so, we divide problem (2.71) into two subproblems, one for C_1, C_3, and $f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}$, and the other for C_2 and $\mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}$.

Sub-problem 1. This problem is

$$\min_{1\le\varsigma,\ p_s^{\min}\le p_s\le p_s^{\max}} \varsigma, \tag{2.72}$$

$$\text{subject to} \begin{cases} C_1 : e^{z_s - y_s}\frac{\hat{\gamma}_s}{h_s} \le \varsigma, \\ C_3 : \left(e^{-z_s}(f_{\mathscr{Q}} + \sum_{m\in\mathscr{S},\ m\neq s} e^{y_m}h_m + \sigma^2)\right) \le \varsigma, \\ C_4 : f_{\mathscr{Q}} \in \mathscr{R}_{f_{\mathscr{Q}}}. \end{cases}$$

If we use Δ_F instead of the uncertainty set in (2.7) and consider $\varsigma = 1$, this problem becomes a standard transmit power allocation problem [22] with a protection value. The solution to (2.72) satisfies

$$\upsilon + \boldsymbol{\Delta} \le (\mathbf{I} - \mathbf{F})\mathbf{p}, \tag{2.73}$$

where $\mathbf{p}$ is the transmit power vector of all SUs. This inequality is satisfied [22] when

$$\rho(\mathbf{F}) < 1, \tag{2.74}$$

where ρ is the spectral radius of $\mathbf{F}$, minimized for $\mathbf{b}_\Delta = (\upsilon + \boldsymbol{\Delta})(\mathbf{I}-\mathbf{F})^{-1}$. Also, the transmit power vector obtained in (2.73) should be below its upper bound for each user, that is,

$$\left((\upsilon + \boldsymbol{\Delta})(\mathbf{I} - \mathbf{F})^{-1}\right) \le \mathbf{p}^{\max}. \tag{2.75}$$

Sub-problem 2. For C_2, we obtain the feasibility condition by solving

$$\min_{1\le\varsigma,\ p_s^{\min}\le p_s\le p_s^{\max}} \varsigma, \tag{2.76}$$

$$\text{subject to} \begin{cases} C_2 : \frac{1}{IT_q}\left(\sum_{s\in\mathscr{S}} g_{sq}e^{y_s}\right) \le \varsigma \qquad \forall q \in \mathscr{Q}, \\ C_4 : \mathbf{g}_q \in \mathscr{R}_{\mathbf{g}_q}, \end{cases}$$

which is equivalent to solving the following linear programming problem:

$$\left(\overline{\mathbf{g}}_q\mathbf{p} + \varepsilon_{\mathbf{g}_q}\max_{\|\mathbf{r}\|\le 1}\mathbf{r}\mathbf{p}\right) \le IT_q, \tag{2.77}$$

where $\mathbf{r} = (\mathbf{g}_q - \overline{\mathbf{g}}_q)/\varepsilon_{\mathbf{g}_q}$. Note that the second term in (2.77) is the dual norm [19]. For (2.9), the dual norm of a linear norm with order 2 is a linear norm with order 2. Hence,

$$\left(\overline{\mathbf{g}}_q\mathbf{p} + \varepsilon_{\mathbf{g}_q}\|\mathbf{p}\|_2\right) \le IT_q. \tag{2.78}$$

When (2.74), (2.75), and (2.78) hold for $\mathbf{b}_\Delta$, (2.9) is feasible. Thus, the feasibility conditions for (2.9) are items **(4)**–**(6)** in Proposition 2.3.

Appendix 4: Proof of Lemma 2.1

Note that (2.8) is a perturbed version of (2.4) with protection values in C_2 and C_3. To obtain the relationship between d_Δ and the protection value, we perform a local sensitivity analysis of the optimization problem by perturbing its constraints [7, 19, 26]. Let

$$v^*(\mathbf{a},\mathbf{b}) = \inf\Bigg\{ -\max_{p_s^{\min}\le p_s\le p_s^{\max}} \sum_{s\in\mathscr{S}} v_s(h_s p_s \varrho_s^{-1})| \,,$$
$$\varrho_s\hat{\gamma}_s \le h_s p_s, \left(\sum_{s\in\mathscr{S}} g_{sq}p_s + \Delta_q\right) \le IT_q,$$
$$\left(f_{\mathscr{Q}} + \sum_{m\in\mathscr{S},\, m\neq s} p_s h_s + \Delta_{f_{\mathscr{Q}}} + \sigma^2\right) \le q_s\Bigg\},$$

where $\mathbf{a}$ is a vector whose qth element is Δ_q, and $\mathbf{b}$ is a vector whose elements are all equal to $\Delta_{f_{\mathscr{Q}}}$. When Δ_q and $\Delta_{f_{\mathscr{Q}}}$ are small, $v^*(\mathbf{a},\mathbf{b})$ is differentiable with respect to the perturbation vectors $\mathbf{a}$ and $\mathbf{b}$ [26]. Using a Taylor series, we write

$$v^*(\mathbf{a},\mathbf{b}) = v^*(\mathbf{0},\mathbf{0}) + \sum_{q\in\mathscr{P}} a_q \frac{\partial v^*(\mathbf{0},\mathbf{b})}{\partial a_q} + \sum_{s\in\mathscr{S}} b_s \frac{\partial v^*(\mathbf{a},\mathbf{0})}{\partial b_s} + o, \tag{2.79}$$

where $v^*(\mathbf{0},\mathbf{0})$ is the social utility for (2.4), and $\mathbf{0}$ is the all zero vector. Note that $v^*(\mathbf{a},\mathbf{b})$ and $v^*(\mathbf{0},\mathbf{0})$ are equal to v_Δ^* and v^*, respectively. Since (2.4) is convex, from the sensitivity analysis in [26] we have $\frac{\partial v^*(\mathbf{0},\mathbf{b})}{\partial a_q} \approx -\nu_q^*$ and $\frac{\partial v^*(\mathbf{a},\mathbf{0})}{\partial b_s} \approx -\mu_s^*$ for all $q\in\mathscr{Q}$ and $s\in\mathscr{S}$. Hence,

$$v_\Delta^* - v^* \approx -\sum_{q\in\mathscr{Q}} \mu_q^*\Delta_q - \sum_{s\in\mathscr{S}} \mu_s^*\Delta_{f_{\mathscr{Q}}}. \tag{2.80}$$

Since ν_q^* and μ_s^* are nonnegative optimum Lagrange multipliers, the SUs' social utility is reduced compared to when the exact channel gains are known.

Appendix 5: Proof of Proposition 2.4

Problem (2.24) is feasible (Section 2.1.3 in [16]) when the following optimization problem is feasible:

$$\min_{1\le\varsigma,\ p_s^{\min}\le p_s\le p_s^{\max}} \varsigma, \tag{2.81}$$

$$\text{subject to} \begin{cases} C_1 : e^{y_s - z_s} \frac{\hat{\gamma}_s}{h_s} \leq \varsigma, \\ C_2 : \left(\frac{e^{-z_s}}{IT_q} (\sum_{s \in \mathscr{S}} \bar{g}_{sq} e^{y_s} + \Delta_q) \right) \leq \varsigma \quad \forall\, q \in \mathscr{Q}, \\ C_3 : \left(e^{-z_s} (\bar{f}_{\mathscr{P}} + \sum_{m \in \mathscr{S},\, m \neq s} e^{y_m}_m h_m + \Delta_{f_{\mathscr{Q}}} + \sigma^2) \right) \leq \varsigma, \end{cases}$$

where $\Delta_q = \max_{e_q \in \mathscr{S}, |\hat{g}_{sq}| \leq \varepsilon_{sq}, |e_q| = \Gamma_q} \sum_{s \in e_q} \hat{g}_{sq} e^{y_s}$. As in Appendix 3, the feasibility of this problem can be established by dividing it into two subproblems. The first subproblem is similar to subproblem 1 in Appendix 3. Also, subproblem 2 in Appendix 3 can be utilized for the D-norm approach, where (2.77) is changed to

$$\bar{\mathbf{g}}_q \mathbf{p} + \varepsilon_q \|\mathbf{p}\|^*_{\Gamma_q} \leq IT_q, \tag{2.82}$$

where $\|\mathbf{p}\|_{\Gamma_q} = \max_{e_q \in \mathscr{S}, |e_q| = \Gamma_q} \sum_{k \in e_q} p^k$, and $\|\mathbf{p}\|^*_{\Gamma_q}$ is the dual norm of $\|\mathbf{p}\|_{\Gamma_q}$, defined [20] by

$$\|\mathbf{p}\|^*_{\Gamma_q} = \max \left(\|\mathbf{p}\|_\infty, \frac{\|\mathbf{p}\|_1}{\Gamma_q} \right). \tag{2.83}$$

From the preceding expressions, when (2.74), (2.75), and (2.82) hold for $\mathbf{b}_\Delta$, (2.24) is feasible. Thus, the feasibility conditions for (2.24) are items **(1)–(3)** in Proposition 2.4.

Appendix 6: Proof of Lemma 2.2

Part 1: Since $\left(\min\{1, \frac{\Gamma_q}{\sqrt{|\mathscr{S}|}}\} \|\mathbf{x}\|_2 \right) \leq \max \left(\|\mathbf{x}\|_\infty, \frac{\|\mathbf{x}\|_1}{\Gamma_q} \right)$, the feasibility set of (2.24) is greater than that of (2.9), provided that (2.25) holds. Therefore, there is a transmit power vector for (2.24), denoted by $\mathbf{p}^{\text{D-norm}}$, that satisfies all constraints in (2.9), and

$$\mathbf{p}^{\text{D-norm}} \geq \mathbf{p}^{\text{Ellipsoid}}, \tag{2.84}$$

where $\mathbf{p}^{\text{Ellipsoid}}$ is the allocated transmit power vector for (2.9) in the ellipsoid uncertainty set, meaning that the ellipsoid's feasibility set for (2.9), that is, $\mathscr{F}_{\text{Ellipsoid}}$, is a subset of the D-norm feasibility set for (2.23), that is, $\mathscr{F}_{\text{D-norm}}$.

Part 2: Let the optimal social utility of (2.24) subject to $\mathscr{F}_{\text{D-norm}}$ be $v_{\text{D-norm}}$ and the optimal social utility of (2.9) subject to $\mathscr{F}_{\text{Ellipsoid}}$ be $v_{\text{Ellipsoid}}$. We assume

$$\begin{aligned} &\exists \quad \mathbf{p} \in \mathscr{F}_{\text{Ellipsoid}}, \\ &v_{\text{Ellipsoid}}(\mathbf{p}) > v_{\text{D-norm}}(\mathbf{p}'), \qquad \forall\, \mathbf{p}' \in \mathscr{F}_{\text{D-norm}}. \end{aligned} \tag{2.85}$$

Since $\mathscr{F}_{\text{Ellipsoid}} \subseteq \mathscr{F}_{\text{D-norm}}$, we have $\mathbf{p} \in \mathscr{F}_{\text{D-norm}}$, and from the assumption in (2.85) and convexity of the robust transmit power allocation for the general norm, the value of $v_{\text{Ellipsoid}}(\mathbf{p})$ is also the optimal solution to (2.24) subject to $\mathscr{F}_{\text{D-norm}}$, that is, $v_{\text{Ellipsoid}}(\mathbf{p}) = v_{\text{D-norm}}(\mathbf{p})$. However, from the assumption in (2.85) we have $v_{\text{Ellipsoid}}(\mathbf{p}) > v_{\text{Ellipsoid}}(\mathbf{p})$. This contradiction implies that our assumption was wrong, and thus $v_{\text{D-norm}} \geq v_{\text{Ellipsoid}}$. The preceding point generally holds for any form of convex optimization problems. For example, the social utility of (2.9) is always equal to or less than that of (2.4) due to the fact that the feasible set of (2.9) is smaller than that of (2.4).

Appendix 7: Proof of Lemma 2.3

Let $\boldsymbol{\omega}_i$ be an iterative vector with n_i steps that can be obtained from (2.11)–(2.15) as

$$\boldsymbol{\omega}_i = \begin{cases} [y_s, z_s, \lambda_s, \mu_s] & \text{if} \quad i \in \mathscr{S}, \\ \nu_q & \text{if} \quad i \in \mathscr{P}. \end{cases}$$

Algorithm 1 can be decomposed into $\underline{\Pi} = \sum_{i \in \mathscr{S}+\mathscr{Q}} \Pi_i$ block components $\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_{S+Q}$. Note that in Algorithm 1, $\Pi_i = 4$ for $i \in \mathscr{S}$ and $\Pi_i = 1$ for $i \in \mathscr{Q}$. In addition, for the iterative primal and dual variables obtained from (2.11)–(2.15), the following points hold:

- The Lagrange dual function $-L(\lambda_s, \mu_s, \nu_q, e^{z_s}, e^{y_s})$ is positive.
- The Lagrange dual function is Lipschitz continuous. Since p_s is upper bounded for all users, all Lagrange multipliers are upper bounded as well, and the Lagrange dual function is continuously differentiable.
- From (2.11)–(2.15), Algorithm 1 is a gradient projection algorithm [10]. Hence, Lemma 5.1 in Section 7.5 of [23] holds for Algorithm 1, and Assumptions 5.1 and 5.2 in Section 7 of [23] also hold. In addition, Algorithm 2 is a partially asynchronous implementation of Algorithm 1 with maximum delay D (Section 7.1 in [23]). Hence, there is a step size β_0 such that when all step sizes are smaller than β_0, Algorithm 2 converges to the optimal point of Algorithm 1. The value of β_0 depends on D and N (Propositions 5.1 and 5.3, Section 7 in [23]), that is,

$$\beta_0 \leq \frac{1}{1 + D + N \times D}. \tag{2.86}$$

From the preceding expression, (2.26) can be obtained with some rearrangements. Note that the preceding conditions are also valid for the D-norm approach. Hence, the same constraint (2.26) can be applied for infrequent message passing in the D-norm approach.

Appendix 8: Proof of Proposition 2.5

The Lagrange dual function associated with (2.45) is

$$L(\mathbf{p}, \mathbf{y}_q, \lambda, \hat{\nu}_q, \mu_q^k) = \sum_{k=1}^{K} \log\left(1 + \frac{p^k h^k}{\sigma^2 + f_{\mathscr{Q}}^k}\right) - \lambda\left(\sum_{k=1}^{K} p^k - p^{\max}\right)$$
$$- \sum_{q=1}^{Q} \hat{\nu}_q \left(\sum_{k=1}^{K} \varepsilon_{\mathbf{g}_q}^k y_q^k - IT_q\right) - \sum_{q=1}^{Q}\sum_{k=1}^{K} \mu_q^k \left(p^k - \sum_{i=1}^{K} [M_q^{\mathrm{T}}]^{ki} y_q^i\right),$$

where λ, $\hat{\nu}_q$, and μ_q^k are the dual variables corresponding to C_1, C_2, and C_3 in (2.45), respectively. For the convex optimization problem considered here, the duality gap is zero, and solving the dual problem is equivalent to solving the original problem [19]. The optimal transmit power vectors $\tilde{\mathbf{p}}^* = [\tilde{p}^{*1}, \ldots, \tilde{p}^{*K}]$ and $\mathbf{y}_q$ can be obtained from the saddle point of the following optimization problem:

$$\max_{\mathbf{p},\, \mathbf{y}_q} \min_{\lambda,\, \hat{\nu}_q,\, \mu_q^k} L(\mathbf{p}, \mathbf{y}_q, \lambda, \hat{\nu}_q, \mu_q^k). \tag{2.87}$$

For any given values of λ, $\hat{\nu}_q$, and μ_q^k, differentiating (2.87) with respect to p^k and y_q^k and setting the derivatives to zero leads to the maximization of (2.87), that is,

$$\frac{\partial L}{\partial p^k} = 0 \quad \Rightarrow \quad \tilde{p}^{*k} = \left[\frac{1}{\lambda + \sum_{q\in\mathscr{Q}} \mu_q^k} - \frac{\sigma^2 + f_{\mathscr{Q}}^k}{h^k}\right]^+,$$

$$\frac{\partial L}{\partial y_q^k} = 0 \quad \Rightarrow \quad \sum_{i=1}^{K} \mu_q^i [M_q^{\mathrm{T}}]^{ki} = \hat{\nu}_q \varepsilon_{\mathbf{g}_q}^k.$$

Therefore, the optimal transmit power, that is, $\tilde{\mathbf{p}}^*$, is obtained by (2.46). Also, since each optimal solution to a convex problem satisfies the KKT conditions [19], the values of λ, $\hat{\nu}_q$, and $\tilde{p}^{*k}$ are related via $\lambda \times (\sum_{k=1}^{K} \tilde{p}^{*k} - p^{\max}) = 0$, $\hat{\nu}_q \times (\sum_{k=1}^{K} \varepsilon_{\mathbf{g}_q}^k y_q^k - IT_q) = 0$, and $\mu_q^k \times (\tilde{p}^{*k} - \sum_{i=1}^{K} [M_q^{\mathrm{T}}]^{ki} y_q^i) = 0$, respectively, $\forall q \in \mathscr{Q}$, $\forall k \in \mathscr{K}$.

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