

Preface

The aim of this graduate-level textbook is to show how the balance laws for theories of rods and strings can be applied in a systematic manner to develop models for a wide range of problems. My goal for the reader is to enable them to formulate models for problems involving one-dimensional continua such as rods and strings that apply to many interesting applications ranging from the human spine, to columns, and plant stems. The formulation of the models presented in the book presumes an introductory graduate-level familiarity with continuum mechanics and intermediate engineering dynamics. The analyses of the models that are discussed in the book require an elementary knowledge of the solutions to ordinary differential equations. The balance laws discussed in the text include the material momentum balance law and, depending on the problem, this law can serve to provide conservation laws and differential equations of motion. The applications include steadily moving strings, ribbon-like models for DNA, an elastica arm scale, and a series of problems with chains that are subject to shocks and impact. These examples have been chosen to be tractable and interesting, and hopefully they will serve as an inspiration for readers to explore the wealth of recent applications of rod theory to problems ranging from tendril perversion in morning glories to DNA supercoiling.

I have always greatly admired the beauty and elegance of continuum mechanics as it is presented in graduate-level courses. Among the highlights that still resonate nearly 30 years after I first saw them are the manner in which restrictions on constitutive relations are established using invariance requirements under superposed rigid body motions, the method of describing deformation gradients using curvilinear coordinate systems, and Rivlin's exact solutions for deformations of isotropic incompressible elastic materials. However, translating this wealth of material to problems was challenging for me. During my graduate studies, I worked on nonlinear dynamics of elastic strings and found it frustrating that I struggled to apply the material I had learned in courses on continuum mechanics to establish the equations of motion for an application with an elastic string. Because I knew that I would spend months analyzing the resulting equations of motion, I found the lack of confidence in my ability to derive equations of motion to be quite disconcerting.

Subsequently, while I was a junior faculty member at the University of California, Berkeley, I had the opportunity to delve into the works of Green and Naghdi on theories of deformable media. Although I experienced a steep learning curve, their seminal works enabled me to bridge the gap between my understanding of continuum mechanics and the development of feasible models in the form of partial and ordinary differential equations for a range of systems that can be modeled using theories of elastic rods and strings. Substantial portions of this book are summaries of the work that has resulted from these early explorations on the application of rod theories. While Green and Naghdi's works influenced the writing of every single page of this book, I have also benefitted greatly from the works of Antman, Gurtin, Maugin, and many others that are referenced in these pages. I am fortunate to work in a field where the standards for excellence in writing include Antman's *Nonlinear Problems of Elasticity* [12] and Love's *A Treatise on the Mathematical Theory of Elasticity* [213].

The opening chapters of this book are designed to enable a reader to explore how the theory of a deformable elastic string or elastic rod can be formulated for classes of problems. I place great emphasis on examining how the balance laws interplay with constitutive relations to form a set of governing equations. In contrast to other texts, I show how a balance of material momentum can play a key role in forming the equations of motion for a variety of well-known systems. I have found that most of the expertise needed to manipulate the balance laws so as to constitute a closed system of equations of motion can be understood by considering examples using the theory of the string. Consequently, discussions and applications of the theory of inextensible strings and elastic strings form the first part of the book. The second part of the book is devoted to rod theories. I start with a planar theory due to Euler and present several classic and new applications of this wonderful theory. To discuss nonplanar problems, Kirchhoff's theory of an elastic rod is needed. This theory is a nonplanar extension of Euler's work and involves an inextensible, unshearable elastic rod. After discussing both recent and classic applications of Kirchhoff's theory, a more elaborate theory of an extensible, shearable elastic rod is discussed. This theory has been championed by the works of Antman dating to the early 1970s and is ideally suited to many biophysical applications. The book concludes with a discussion of Green and Naghdi's rod theory. This theory subsumes the aforementioned rod theories and can accommodate shear, extension, flexural, and torsional modes of deformation. For the reader's convenience, two supplemental chapters are incorporated into the text. The first of these chapters is a review of continuum mechanics and the second is an equally rapid review of variational methods.

Several colleagues and students have generously given of their time and reviewed drafts of this book during its lengthy gestation period. Despite my best efforts, I doubt if all possible typographical errors or inconsistencies have been removed from the published version. If you find such errors or inconsistencies in this text, I would be most grateful if you could bring them to my attention by sending me an email at oreilly@berkeley.edu. A list of errata for the book will become available on my homepage in due course. Finally, part of the pleasure of writing a book is describing the works of others. On the other hand, it is challenging to decide which works

to exclude in an effort to produce a book of manageable scope and size. I hope the works I have chosen to discuss form an interesting and engaging blend for the reader and apologize in advance to the many researchers whose works I was not able to include in these pages.

Berkeley, CA, USA
September 2016

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Modeling Nonlinear Problems in the Mechanics of
Strings and Rods

The Role of the Balance Laws

O'Reilly, O.

2017, XX, 425 p. 147 illus., 137 illus. in color.,

Hardcover

ISBN: 978-3-319-50596-1