Chapter 2
Descriptions of the Voting Methods to Be Analyzed

Abstract This chapter describes the 18 most well-known voting procedures for electing one out of several candidates. These procedures are divided into three groups:

- A group of five procedures that are vulnerable to non-monotonicity under both fixed and variable electorates: Plurality with Runoff, Alternative Vote, Coombs' Method, Dodgson's Method, Nanson's Method.
- A group of eight procedures that are vulnerable to non-monotonicity under (only) variable electorates: Successive Elimination, Bucklin's Method, Majority Judgment, Copeland's Method, Black's Method, Kemeny's Method, Schwartz's Method, Young's Method.
- A group of five procedures that are invulnerable to any monotonicity failures: Plurality Voting, Approval Voting, Borda's Count, Range Voting, the Minmax Method.

Keywords Voting procedures · Voting methods · Voting rules · Monotonicity failures · Fixed electorates · Variable electorates

2.1 Introduction

Felsenthal (2012) surveyed the susceptibility of 18 voting procedures discussed widely in the literature to various types of voting paradoxes. Only five of these 18 procedures were said to display simple monotonicity failure in fixed electorates: Plurality with Runoff, Successive Elimination, Alternative Vote, Coombs' Method, Dodgson's Method, and Nanson's Method. Thereafter Felsenthal and Tideman

This chapter is largely based on Felsenthal (2012).

D.S. Felsenthal and H. Nurmi, *Monotonicity Failures Afflicting Procedures for Electing a Single Candidate*, SpringerBriefs in Economics,
DOI 10.1007/978-3-319-51061-3_2

(2013, 2014) further analyzed the susceptibility of these five voting procedures to additional types of monotonicity failure under fixed as well as under variable electorates.

Of the remaining 13 voting procedures surveyed by Felsenthal (2012), Felsenthal and Nurmi (2016) showed that eight of them are susceptible to various types of monotonicity failure under variable electorates: Successive Elimination, Bucklin's Method, Majority Judgment, Black's Method, Copeland's Method, Kemeny's Method, Schwartz's Method, and Young's Method.

So of the 18 main voting procedures mentioned in the literature that are designed to elect a single candidate, five are susceptible to types of monotonicity failure under both fixed and variable electorates, eight are susceptible to types of monotonicity failure under (only) variable electorates, and only five methods are not susceptible to any type of monotonicity failure.

In the remainder of this chapter we shall first supply a brief description of each of the five voting procedures that are susceptible to some type of monotonicity failure under both fixed and variable electorates, then we will briefly describe the eight procedures that are susceptible to some type of monotonicity failure under variable electorates and, finally, we will briefly mention the most well-known five voting procedures that are not susceptible to any type of monotonicity failure and explain why this is so.

2.2 Five Voting Methods Susceptible to Types of Monotonicity Failure Under Both Fixed and Variable Electorates

2.2.1 Plurality with Runoff (P-R)

Under the usual version of this method, up to two voting rounds are conducted. In the first round each voter casts one vote for a single candidate. In order to win in the first round a candidate must usually obtain an absolute majority of the votes. If no candidate is declared the winner in the first round then a second round is conducted. In this round only the two candidates who obtained the highest number of votes in the first round participate, and the one who obtains the majority of votes wins. This is a very common method for electing a single candidate and is used, *inter alia*, for electing the President of France. For simplicity, P-R will be treated as operating on a single round of voters' responses, with the responses providing full rankings of the candidates, from which the voters' first choices from available candidates are taken. This simplification would affect the results only if voters using P-R wished to cast second-round ballots that were inconsistent with their votes in the first round.

2.2.2 *Alternative Vote (AV; aka Instant Runoff Voting; Ranked Choice Voting)*

This is the adaptation to the task of electing a single candidate of the multi-winner voting method known as the Single Transferable Vote (STV). STV was proposed independently by Carl George Andrae in Denmark in 1855 and by Thomas Hare in England in 1857. The use of this method for electing a single candidate was first proposed by an American, William Robert Ware, in 1871. It works as follows. All voters submit ballots that rank-order all of the candidates. In the first step one determines whether there exists a candidate who is ranked first by an absolute majority of the voters. If such a candidate exists s/he is declared the winner. If no such candidate exists then, in the second step, the candidate who is ranked first by the smallest number of voters is deleted from all ballots and thereafter one again determines whether there is now a candidate who is ranked first by an absolute majority of the voters. The elimination process continues in this way until a candidate who is ranked first by an absolute majority of the voters is found. The Alternative Vote is used in electing the president of the Republic of Ireland, the Australian House of Representatives, as well as the mayors in some municipal elections in the US. Beginning 2018 the state of Maine will be the first US state to elect by AV its governor, its Congressmen, as well as its state legislators. In May 2010 a referendum was conducted in the UK to decide whether AV should be used for electing the members of the House of Commons; a majority of voters rejected this proposal.

2.2.3 *The Coombs Method* (Cf. Coombs 1964, pp. 397–399; Straffin 1980; Coombs et al. 1984)

This method was proposed by the psychologist Clyde H. Coombs in 1964. It is similar to AV except that the candidate who is eliminated in a given round under the Coombs method is the candidate who is ranked last by the largest number of voters (instead of the candidate who is ranked first by the smallest number of voters as under the AV method).

2.2.4 *The Dodgson Method* (Cf. Black 1958, pp. 222–234; McLean and Urken 1995, pp. 288–297)

This method is named after the Rev. Charles Lutwidge Dodgson, aka Lewis Carroll, who referred to it implicitly in 1876 without explicitly endorsing it. It elects the

Condorcet winner when one exists.[1] If there is no Condorcet winner then the Dodgson method elects that candidate who can be made into a Condorcet winner by the smallest number of transpositions of adjacent candidates in the voters' rankings.

2.2.5 The Nanson Method (Cf. Nanson 1883; McLean and Urken 1995, Chap. 14)

The Nanson method is a recursive elimination method using the Borda method.[2] In the first step one calculates each candidate's Borda score. At the end of the first step the candidates whose Borda scores do not exceed the average Borda score of the candidates in this step are eliminated from all ballots and in the second step a revised Borda score is computed for each uneliminated candidate.[3] The elimination process is continued in this way until one candidate is left. If all of the uneliminated candidates have the same Borda score then one of them is elected according to a pre-determined method for breaking ties. If a Condorcet winner exists then the Nanson method elects him or her.

2.3 Eight Voting Methods Susceptible to Types of Monotonicity Failure Under Variable Electorates

2.3.1 Successive Elimination (Cf. Farquharson 1969)

This procedure is common in parliaments when voting on alternative versions of bills. According to this procedure, voting is conducted in a series of rounds. In each

[1]A Condorcet winner is a candidate who beats all other candidates in head-to-head contests. It is named after Marquis de Condorcet (1785) who proposed that if such a candidate exists s/he ought to be elected. When a Condorcet winner does not exist it is said that the majority method relation is intransitive and contains a top cycle, e.g., when there are three candidates the majority of voters rank candidate a above candidate b, the majority of voters rank candidate b above candidate c, but the majority of voters rank candidate c above candidate a. It is not entirely clear from Condorcet's book which candidate he thought should be elected when no Condorcet winner exists, although Peyton Young (1988) has made a highly plausible conjecture. (Cf. McLean and Urken 1995, Chap. 6). It should be noted that it is possible for a Condorcet winner to exist while the majority method relation among some or all of the remaining candidates is intransitive; it is also possible that neither a Condorcet winner nor a top cycle exist, e.g., when there are four voters—two with preference ordering $a \succ b \succ c$ and two with preference ordering $b \succ a \succ c$.

[2]For a description of the Borda score method see subsection 2.4.3 in the sequel.

[3]Many authors state erroneously that according to the Nanson method one eliminates at the end of each round only the candidate with the lowest Borda score, or only the candidates with below-average Borda scores, rather than all candidates whose Borda scores are equal to, or lower than, the average Borda score. Using the erroneous descriptions of the Nanson elimination process may result in different outcomes than those obtained under the correct Nanson elimination process.

round two alternatives compete; the one obtaining fewer votes is eliminated and the other competes in the next round against one of the alternatives that has not yet been eliminated. The alternative winning in the last round is the ultimate winner. If all voters vote in each round according to their true preference orderings among the alternatives then this procedure is Condorcet consistent.[4]

2.3.2 ***Bucklin's Method*** (Cf. Hoag and Hallett 1926, pp. 485–491; Tideman 2006, p. 203)

This voting system is named after James W. Bucklin of Grand Junction, Colorado, who first promoted it in 1909. In 1913, the US Congress prescribed (in the Federal Reserve Act of 1913, Section 4) that this method be used for electing district directors of each Federal Reserve Bank.

Under Bucklin's method voters rank-order the competing candidates. If there exists a candidate who is ranked first by an absolute majority of the voters s/he is elected. Otherwise for each candidate the number of voters who ranked this candidate in second place are added to the number of voters who ranked him/her first, and if now there exists a candidate supported by a majority of voters s/he is elected. If not, the counting process continues in this way by adding for each candidate his/her third, fourth, … rankings, until a candidate is found who is supported by an absolute majority of the voters. If two or more candidates are found to be supported by a majority of voters in the same counting round then the one supported by the largest majority is elected. However, it is unclear how a tie between two candidates, say a and b, ought to be broken under Bucklin's procedure when both a and b are supported in the same counting round by the same number of voters and this number constitutes a majority of the voters. If one tries to break the tie between a and b in such an eventuality by performing the next counting round in which all other candidates are also allowed to participate, then it is possible that the number of (cumulated) votes of another candidate, c, will exceed that of a and b.[5] So which candidate ought to be elected in this example under Bucklin's procedure? As far as

[4]A voting procedure is said to be 'Condorcet-consistent' if it elects a Condorcet winner when one exists.

[5]To see this, consider the following simple example. Suppose there are 18 voters who must elect one candidate under Bucklin's procedure and whose preference orderings among four candidates, a–d, are as follows: seven voters with preference ordering $a \succ b \succ c \succ d$, eight voters with preference ordering $b \succ a \succ c \succ d$, one voter with preference ordering $d \succ c \succ a \succ b$, and two voters with preference ordering $d \succ c \succ b \succ a$. None of the candidates constitutes the top preference of a majority of the voters. However, both a and b constitute the top or second preference of a majority of voters (15). If one tries to break the tie between a and b by performing the next (third) counting round in which c and d also are allowed to participate, then c will be elected (with 18 votes), but if only a and b are allowed to participate in this counting round then b will be elected (with 17 votes).

we know, Bucklin did not supply an answer to this question. This procedure is not Condorcet consistent (see Felsenthal 2012, p. 54).

2.3.3 *Majority Judgment* (Cf. Balinski and Laraki 2007a, b, 2010)

According to this proposed procedure, the suitability (or level of performance) of every candidate is assessed by every voter and is assigned an ordinal grade (chosen from a pre-specified range) reflecting the candidate's suitability or level of performance in the eyes of the voter. The candidate with the highest median grade is the winner.[6] In case of a tie between the median grades, denoted by α, of two or more leading candidates, one deletes one α grade from each of the tied candidates and then one computes for each of the tied candidates a new value of α, the median grade. If the new α grade of one of the (previously) tied candidates is higher than that of each of the other (previously) tied candidates, then this candidate is the winner. But if there is still a tie between two or more candidates with the new median grade α, then one deletes again one (new) α grade from each of the tied candidates, and the process continues until one finds a candidate whose α grade is higher than that of the others or, if no such candidate is found, one conducts a lottery between the candidates whose α grade is highest.

Although this is debatable, we assume that in single-winner political elections (as distinct, say, from sport competitions), a voter will award a higher grade to candidate x than to another candidate y if s/he prefers that x rather than y will be elected. If one accepts this assumption then the Majority Judgment (MJ) procedure is not Condorcet consistent (see Felsenthal and Machover 2008, p. 330; Felsenthal 2012, p. 59).

2.3.4 *Copeland's Method* (Copeland 1951)

According to this method one performs all paired comparisons. Every candidate x gets one point for every paired comparison with another candidate y in which an absolute majority of the voters prefer x to y, and half a point for every paired comparison in which the number of voters preferring x to y is equal to the number of voters preferring y to x. The candidate obtaining the largest sum of points is the winner.

[6] If the number of voters is even and a candidate's two middle grades are different, then the median is not uniquely defined. Balinski and Laraki take the lower of the two middle grades as the median grade. This asymmetry of the Majority Judgment procedure may create some problems as shown by Felsenthal and Machover (2008, Examples 3.2 and 3.7).

2.3.5 *Black's Method* (Black 1958, p. 66)

According to this (hybrid) method, one first performs all paired comparisons to verify whether a Condorcet winner exists. If such a winner exists, then s/he is elected. Otherwise the winner according to Borda's count is elected.

2.3.6 *Kemeny's Method* (Kemeny 1959; Kemeny and Snell 1960; Young and Levenglick 1978; Young 1988, 1995)

According to this method, too, one first performs all paired comparisons to verify whether a Condorcet winner exists. If such a winner exists, then s/he is elected. Otherwise Kemeny's method (aka the Kemeny-Young rule) specifies that up to $m!$ possible (transitive) social preference orderings should be examined (where m is the number of candidates) so as to determine which of these is the "most likely" true social preference ordering.[7] The selected "most likely" social preference ordering according to this method is the one for which the total number of votes with respect to pairs that are consistent with that ordering is maximized, and the candidate who ought to win is the one at the top of this ordering. Given the voters' various preference orderings, Kemeny's procedure can also be viewed as finding the most likely (or the best predictor, or the best compromise) true social preference ordering, called the *median preference ordering*, i.e., that social preference ordering, S, that minimizes the sum, over all voters i, of the number of pairs of candidates that are ranked oppositely by S and by the ith voter.[8]

[7]Tideman (2006, pp. 187–189) proposes two heuristic procedures that simplify the need to examine all $m!$ preference orderings.

[8]According to Kemeny (1959), the distance between two (individual) preference orderings, R and R', is the number of pairs of candidates (alternatives) on which they differ. For example, if $R = a \succ b \succ c \succ d$ and $R' = d \succ a \succ b \succ c$, then the distance between R and R' is 3, because they agree on three pairs [$(a \succ b)$, $(a \succ c)$, $(b \succ c)$], but differ on the remaining three pairs, i.e., on the preference ordering between a and d, b and d, and between c and d. Similarly, if R'' is $c \succ d \succ a \succ b$, then the distance between R and R'' is 4 and the distance between R' and R'' is 3. According to Kemeny's procedure, the most likely social preference ordering is that R such that the sum of distances of the voters' preference orderings from R is minimized. Because this R has the properties of the median central tendency in statistics it is called the *median preference ordering*. The median preference ordering (but not the *mean preference ordering*, which is that R which minimizes the sum of the squared differences between R and the voters' preference orderings) will be identical to the possible (transitive) social preference ordering W which maximizes the sum of voters that agree with all paired comparisons implied by W.

2.3.7 *Schwartz's Method* (Schwartz 1972, 1986)

According to this method, too, one first performs all paired comparisons to verify whether a Condorcet winner exists. If such a winner exists, then s/he is elected. Otherwise Thomas Schwartz's method is based on the notion that a candidate x deserves to be listed ahead of another candidate y in the social preference ordering if and only if x beats or ties with some candidate that beats y, and x beats or ties with all candidates that y beats or ties with. The Schwartz set (from which the winner should be chosen) is the smallest set of candidates who are unbeatable by candidates outside the set. The Schwartz set is also called GOCHA (Generalized Optimal Choice Axiom). This method, too, is Condorcet consistent.

2.3.8 *Young's Method* (Young 1977)

According to this method, too, one first performs all paired comparisons to verify whether a Condorcet winner exists. If such a winner exists, then s/he is elected. Otherwise one elects that candidate who can become a Condorcet winner (or at least a majority non-loser) by removing the smallest number of voters from the electorate.

2.4 Five Main Procedures that Are not Susceptible to Any Monotonicity Failure

2.4.1 *Plurality (or First Past the Post) Voting Procedure*

This is the most common procedure for electing a single candidate, and is used, *inter alia*, for electing the members of the House of Commons in the UK and the members of the House of Representatives in the US. Under this procedure every voter casts one vote for a single candidate and the candidate obtaining the largest number of votes is elected.

This procedure is not vulnerable to any monotonicity failures for a simple reason: if a candidate, x, is elected by a given electorate—which implies that this candidate obtained the largest number of votes, then this candidate will surely be re-elected if, *ceteris paribus*, some of the original voters who originally did not vote for x will now change their minds and vote for x, or if, *ceteris paribus*, additional voters join the electorate and vote for x.

Similarly, if x was not originally a winner then the fact that, *ceteris paribus*, some voters lowered x in their preference orderings or that additional voters joined the electorate whose bottom preference is x cannot make x the winner because following this change x either loses votes or gains no new votes.

2.4.2 *Approval Voting* (Brams and Fishburn 1978, 1983)

Under this procedure every voter has a number of votes which is equal to the number of competing candidates, and every voter can cast one vote or no vote for every candidate. The candidate obtaining the largest number of votes is elected. So far this procedure has not been used in any public elections but is used by several professional associations and universities in electing their officers.

This procedure (which is but a special version of the Plurality procedure) is not vulnerable to any monotonicity failures for the same reasons that the Plurality procedure is not vulnerable to these failures.

2.4.3 *Borda's Count* (Cf. de Borda 1784; Black 1958; McLean and Urken 1995, pp. 83–89)

This voting procedure was proposed by Jean Charles de Borda in a paper he delivered in 1770 before the French Royal Academy of Sciences entitled 'Memorandum on election by ballot' ('Mémoire sur les elections au scrutin'). According to Borda's procedure each candidate, x, gets no points for each ballot in which s/he is ranked last, 1 point for each ballot in which s/he is ranked second-to-last, and so on, and $m - 1$ points for every ballot in which s/he is ranked first (where m is the number of candidates). The candidate with the largest number of points is elected. Thus if all n voters have strict preference orderings among the m candidates then the total number of points obtained by all the candidates is equal to the number of voters multiplied by the number of paired comparisons, i.e., to $nm(m - 1)/2$.

This procedure is not vulnerable to any monotonicity failures for a simple reason: if a candidate, x, is elected by a given electorate—which implies that this candidate obtained the largest number of points, then this candidate will surely be re-elected if, *ceteris paribus*, some of the voters who originally ranked x lower in their preference orderings will change their minds and rank x higher in their preference orderings, or if, *ceteris paribus*, additional voters join the electorate who will rank x at the top of their preference orderings.

Similarly, if x was not originally a winner then the fact that, *ceteris paribus*, some voters lowered x in their preference orderings or that additional voters joined the electorate whose bottom preference is x cannot increase the number of points obtained by x—thereby x must remain a non-winner.

2.4.4 Range Voting (Smith 2000)

According to this procedure the suitability (or level of performance) of every candidate is assessed by every voter and is assigned a cardinal grade (chosen from a pre-specified range) reflecting the candidate's suitability or level of performance in the eyes of the voter. The candidate with the highest average grade is the winner. This procedure is currently championed by Warren D. Smith (see http://rangevoting.org) and used to elect the winner in various sport competitions.

This procedure, too, is not vulnerable to any monotonicity failures. This is so because if a candidate, x, is elected by a given electorate—it implies that this candidate obtained the highest average grade. So this candidate will surely be re-elected if, *ceteris paribus*, some of the voters who originally awarded x a lower grade will change their minds and award x a higher grade, or if, *ceteris paribus*, additional voters join the electorate who will award x the highest grade.

Similarly, if x was not originally a winner then the fact that, *ceteris paribus*, some voters change their minds and now assign to x the lowest grade or that additional voters join the electorate who assign to x the lowest grade must decrease x's average grade—thereby x must remain a non-winner.

2.4.5 The Minmax Procedure[9]

Although Condorcet specified that the Condorcet winner ought to be elected if one exists, Condorcet did not specify clearly, according to Black (1958, pp. 174–175, 187), which candidate ought to be elected when a Condorcet winner does not exist. Black (1958, p. 175) suggests that "It would be most in accordance with the spirit of Condorcet's … analysis … to discard all candidates except those with the minimum number of majorities against them and then to deem the largest size of minority to be a majority, and so on, until one candidate had only actual or deemed majorities against each of the others." In other words, the procedure attributed by Black to Condorcet when cycles exist in the social preference ordering is a Minmax procedure since it chooses that candidate whose worst loss in the paired comparisons is the least bad. Thus, for each candidate x, one first determines the maximum number of votes against him/her in any pairwise comparison. Let us denote this magnitude by $v(x)$. The set of Minmax winners consists of those candidates whose $v(x)$ value is smallest. When a Condorcet winner, say z, exists in a profile, it is clearly the only alternative in the set of Minmax winners since $v(z) < n/2$, (i.e., the maximum number of votes any other candidate can muster against z is less than half the number of voters), while for every other candidate, y, $v(y) > n/2$ (since each of them is defeated by z—and possibly also by some other candidate).

[9]Young (1977, p. 349) prefers to call this procedure 'The Minimax function'. It is also sometimes called in the literature 'the max-min method'.

The Minmax procedure is also known in the literature as the Simpson-Kramer rule (see Simpson 1969; Kramer 1977). This procedure is, of course, Condorcet consistent.

Although the Minmax procedure is the only procedure of those listed in Sect. 2.4 that is vulnerable to the related No-Show and Twin paradoxes (see Felsenthal 2012), it is invulnerable to any other form of monotonicity failure. This is so because if *x,* the candidate elected originally, is a Condorcet winner, then *x* remains a Condorcet winner under the Minmax procedure if, *ceteris paribus*, some of the voters who originally ranked *x* lower in their preference ordering will now change their minds and rank *x* higher in their preference ordering, or if, *ceteris paribus*, additional voters join the electorate who rank *x* at the top of their preference ordering. And if *x* was not originally a Condorcet winner then the fact that, *ceteris paribus*, some voters raised *x* in their preference orderings or that additional voters joined the electorate whose top preference is *x* must either cause *x* to become a Condorcet winner or further reduce *x*'s worst loss—thereby *x* must remain the Minmax winner. Similarly, if *x* was not originally a winner then the fact that, *ceteris paribus*, some voters lowered *x* in their preference orderings or that additional voters joined the electorate whose bottom preference is *x* must further increase *x*'s worst loss—thereby *x* must remain the Minmax non-winner.

In the next chapter we demonstrate the types of monotonicity failures that may be displayed by each of the five voting procedures listed in Sect. 2.2.

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http://www.springer.com/978-3-319-51060-6

Monotonicity Failures Afflicting Procedures for Electing
a Single Candidate
Felsenthal, D.S.; Nurmi, H.
2017, VIII, 88 p., Softcover
ISBN: 978-3-319-51060-6