Chapter 2
Fundamentals of Reactive Power in AC Power Systems

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Abstract The fundamentals of reactive power in AC power systems are discussed in the second chapter. The chapter presents basic theory of AC circuits including two-ports linear elements, basic equations and definition of powers in AC circuits. The phasor diagrams and power measurement techniques in AC networks are also presented. The chapter also investigates the effects of reactive power as well as power factor compensation in consumers. The end part of the chapter is related to minimum active and reactive absorbed power in linear AC circuits and also non-sinusoidal conditions. All of the parts include some practical examples and case studies. The chapter is closed with a large list of bibliographic references.

2.1 Chapter Overview

The chapter opens with an overview. Starting on the Kirchhoff's laws expressed in terms of symbolic (complex) form the basic theory of AC circuits is summarized in first section of chapter two. The impedance and admittance are used in order to characterize the behavior of two-ports linear elements. Also a review about the analysis methods of AC circuits is absolutely necessary to emphasize the equations systems that are commonly used. Then are introduced the definitions and physical interpretation of powers in AC power systems: the active, reactive and apparent

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power, the power, reactive and deforming factor, the overall harmonic distortion of voltage and current, the harmonic level. Several examples put in evidence the theoretical aspects presented below. General problems about energy and power in AC power systems is intuitive and lead to a better understanding of the definitions of these energetic parameters. Accompanied by presentation of some modern measurement methods of these parameters and calculation examples for studied cases of power systems, all of them are the subjects exposed in the third section of this chapter.

The next section has as a main objective the reader's assimilation of the fundamental problems of reactive power consumption automated management in power systems and of some methods used to limit the negative effects due to a reduced power factor. On this line, there has been elaborated a selection of the equipment for power factor correction based on the analysis of the customer's electric energy quality. There are also presented the notions needed to design some simple systems used to compensate the reactive power for different levels of the installation. Several examples have been studied separately in this section.

The qualitative and quantitative aspects related to the active and reactive power circulation in AC networks are presented in section four of this chapter. In this way the recent Principles of Minimum Absorbed Active and Reactive Power (PMARP) in AC Power Systems are demonstrated and formulated. For AC circuits under sinusoidal and non-sinusoidal conditions the PMARP proves, on one hand, that the active power absorbed by all the resistances of the AC power system is minimum and, on the other hand, that the reactive power absorbed (generated) by all the resistive-inductive (resistive-capacitive) elements is minimum.

Also one demonstrates that (i) this principle is verified by the currents which satisfy Kirchhoff current law (KCL) and nodal method (NM), and (ii) the co-existence (CEAPP) of PMARP and of maximum active power transfer theorem (MPTT). Several examples presented hereinafter demonstrate the PMARP and CEAPP for classical linear and reciprocal AC circuits under sinusoidal and non-sinusoidal signals and prove the originality of the new theoretical concepts introduced by authors. The second chapter is closed with a specific list of bibliographic references.

2.2 Basic Theory of AC Circuits

2.2.1 Two-Port Linear Elements

(i) **Sinusoidal signals—Characterization, symbolic representation**

By definition, a sinusoidal signal is that signal whose time variation is described by an expression of the following form [1–4]

$$x(t) = X_{\max}\sin(\omega t + \varphi) = X\sqrt{2}\sin(\omega t + \varphi) \tag{2.1}$$

In Eq. (2.1) the signals have the following significance:

- $X_{\max}$—is the *amplitude* or the peak value of the sinusoidal signal and it represents the maximum positive value of $x(t)$ variation during one period.
- X—is *the effective* (*root mean square−rms*) value of the sinusoidal signal. Between amplitude and rms value there is a relationship as it can be deduced from (2.1), which is the dependence: $X_{\max} = X\sqrt{2}$. The rms value X is the value indicated by the measuring equipment.
- ω—*angular frequency*. For a given signal, between the angular frequency and its frequency (or period—T) there is the following relationship

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{2.2}$$

- $\alpha = \omega t + \varphi$—represents the phase at a given moment in time (t arbitrary). For $t = 0$ one obtains the initial phase φ of the sinusoidal signal.

To illustrate better the physical significance of these signals we represent graphically the time variation of a sinusoidal signal in Fig. 2.1.

By definition, *the mean value* of a periodic signal is given by:

$$\langle x \rangle = \frac{1}{T}\int\limits_{t_0}^{t_0+T} x(t)\mathrm{d}t = 0 \tag{2.3}$$

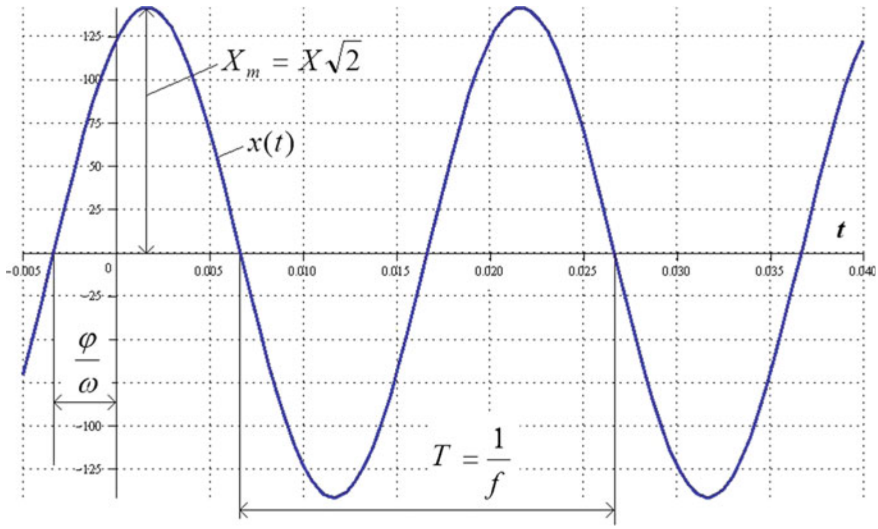


Fig. 2.1 Signals and values characteristic for a sinusoidal variation

From Eq. (2.3) for a sinusoidal signal its corresponding mean value is zero. A zero mean value periodic signal is called *alternative signal*. *The rms value* of a signal is the square root of the mean value of the square of its corresponding variation.

$$X = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\mathrm{d}t} = \frac{X_{\max}}{\sqrt{2}} \tag{2.4}$$

For two signals having the same angular frequency ω one defines the *phase shift* φ as being the difference between the phases corresponding to the two sinusoidal signals—i.e. the difference between their initial phases.

$$\begin{aligned} x_1(t) &= X_1\sqrt{2}\sin(\omega t + \varphi_1) \\ x_2(t) &= X_2\sqrt{2}\sin(\omega t + \varphi_2) \\ \varphi &= (\omega t + \varphi_1) - (\omega t + \varphi_2) = \varphi_1 - \varphi_2 \end{aligned} \tag{2.5}$$

In Fig. 2.2, is presented the phase-shift between two signals having different amplitudes and different initial phases.

(ii) **Complex representation of sinusoidal signals**

For any sinusoidal signal *x*(*t*) of an angular frequency ω, one can bi-univocally associate a complex number $\underline{X}$ called *its complex* or *the complex image of x*(*t*), of

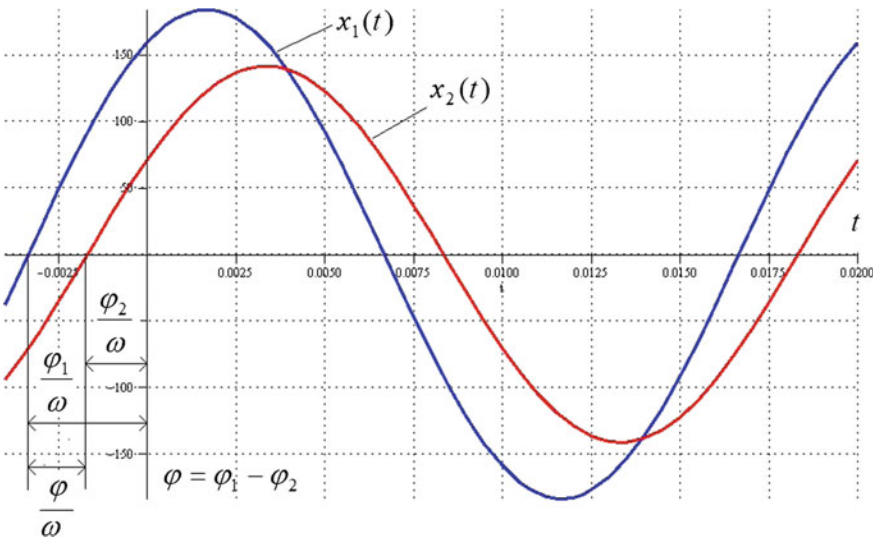


Fig. 2.2 The phase-shift between two sinusoidal signals

modulus equal to its rms value and the argument equal to the initial phase of the sinusoidal signa

$$x(t) = X\sqrt{2}\sin(\omega t + \varphi) \Leftrightarrow \underline{X} = Xe^{j\varphi} = X(\cos\varphi + j\,\sin\varphi) \qquad (2.6)$$

In Eq. (2.6) one denoted by $j = \sqrt{-1}$ the complex number of modulus equal to one and the phase equal to $\pi/2$. This analytical representation used for sinusoidal signals is called *complex representation*. This representation allows also a representation in the complex plan of the sinusoidal signals (Fig. 2.3).

This representation is very useful because it allows the computation of AC sinusoidal electrical circuits easier and allows also a better interpretation of the obtained results.

As a consequence, during the first step, the sinusoidal signals will be expressed using complex numbers. Then, after computation, using in principal the same equivalence theorems and the same solving methods as in DC, one will came back to time domain using bi-univocal properties of complex transformation.

(iii) **Two-ports linear circuits' elements**

Passive circuit elements

In principal, these circuit's elements are represented by: resistor, coil, capacitor and mutually connected coils, each of these elements being characterized by a single constant parameter: the resistance R, the inductivity L, the capacity C, respectively the mutual inductance M, that is a parameter apart from the parameters corresponding to the two magnetic coupled coils.

In Fig. 2.4 are presented the analytical equations characterizing each element. For magnetic coupled coils the sign between the two terms is + if i_1 and i_2 have the same direction with respect to the polarized terminals and the sign is − if i_1 enters the polarized terminal and i_2 exits the polarized terminal, or vice versa. As it is shown in the figure for the direction of the currents and for the terminals position marked in the figure, the sign is positive.

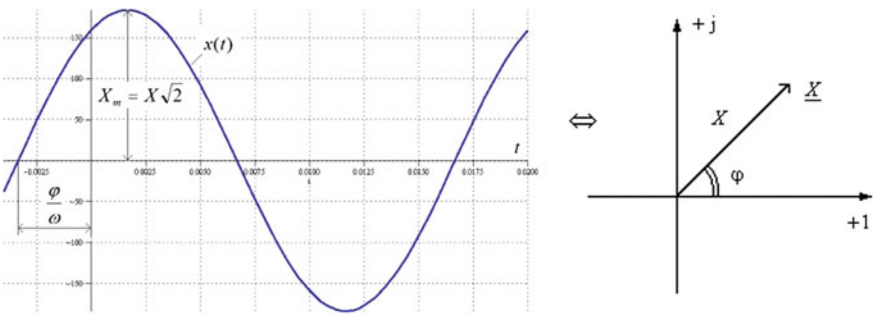
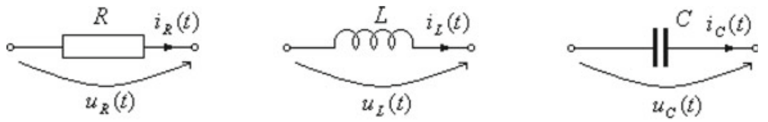


Fig. 2.3 Complex representation of sinusoidal and complex signals



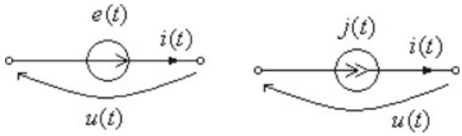
$$u_R(t) = Ri_R(t) \qquad u_L(t) = L\frac{di_L(t)}{dt} \qquad u_c(t) = \frac{1}{C}\int i_C(t)dt$$

$$u_{L_1}(t) = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

$$u_{L_1}(t) = L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt}$$

Fig. 2.4 Passive circuit's elements

Fig. 2.5 Ideal current ad voltage generators



Active circuit elements

These elements are: the ideal voltage generator and the ideal current generator. The ideal voltage generator is characterized by that no matter what the value of the current intensity $i(t)$ is, this gives at its terminals a constant voltage $u(t)$ equal to the value of the voltage generator $e(t)$. The ideal current generator is characterized by that no matter what the value of the voltage $u(t)$ at its terminals is, this gives to the circuits a constant current $i(t)$ equal to the value given by the current generator $j(t)$.

The symbols and the functioning equations corresponding to the ideal voltage generator and to the ideal current generator are presented in Fig. 2.5.

For real voltage and current generators, there is also another component called inner resistance placed in series with the voltage generator and in parallel with the current generator.

(iv) **Complex imittances**

The computation of periodic AC electrical circuits can be done in a systematic way by using the notions of *complex impedance* respectively *complex admittance* named using the common name of *complex imittances*.

To do this, one considers a passive linear two-port system, whose constitutive inductive elements do not present magnetic couplings with the exterior.

The voltage and the current at its terminals have a sinusoidal variation Fig. 2.6.

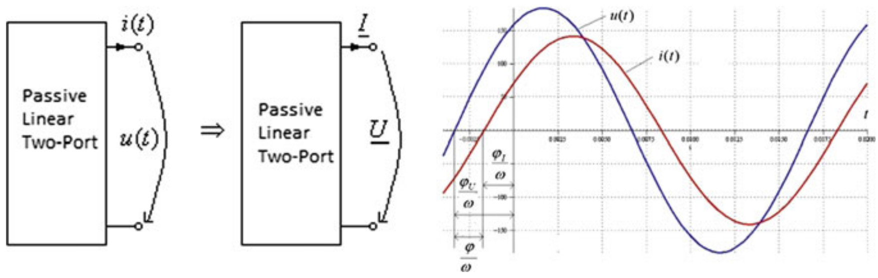


Fig. 2.6 Passive linear two-port system magnetically not coupled to the exterior

The time variation of the voltage and current at two-port system's terminals:

$$\begin{aligned} u(t) &= U\sqrt{2}\sin(\omega t + \varphi_U), \quad \underline{U} = Ue^{j\varphi_U} = U(\cos\varphi_U + j\sin\varphi_U) \\ i(t) &= I\sqrt{2}\sin(\omega t + \varphi_I), \quad \underline{I} = Ie^{j\varphi_I} = I(\cos\varphi_I + j\sin\varphi_I) \end{aligned} \tag{2.7}$$

By definition one calls *the complex impedance* corresponding to the two-port system as being the ratio between the complex images of the voltage at its terminals and the absorbed current (Ohm's law)

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U}{I}e^{j(\varphi_U - \varphi_I)} = Ze^{j\varphi} = Z(\cos\varphi + j\sin\varphi) = R + jX \tag{2.8}$$

The modulus Z [Ω] is called the two-port system's real impedance and its argument $\varphi = \varphi_U - \varphi_I$ is called the two-port system's phase and

$$\begin{aligned} R &= \Re e\{\underline{Z}\} = Z\cos\varphi - \text{two-port system equivalent inner resistance } [\Omega] \\ X &= \Im m\{\underline{Z}\} = Z\sin\varphi - \text{two-port system equivalent inner reactance } [\Omega] \end{aligned} \tag{2.9}$$

Obviously one can determine the relationships

$$Z = \frac{U}{I} = \sqrt{R^2 + X^2} \tag{2.10}$$

By definition Y [S] one calls *complex admittance* of the two-port system the ratio between the complex images of the current and of the voltage at its terminals:

$$\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{I}{U}e^{-j(\varphi_U - \varphi_I)} = Ye^{j\varphi} = Y(\cos\varphi - j\sin\varphi) = G - jB \tag{2.11}$$

In Eq. (2.11) one identifies G—the equivalent conductance and B—the equivalent susceptance as the real, respectively the changed sign coefficient of the imaginary part from $\underline{Y}$.

$$\begin{aligned} G &= \Re e\{\underline{Y}\} = Y\cos\varphi - \text{two-port system equivalent inner conductance}[\mathrm{S}] \\ B &= \Im m\{Y\} = Y\sin\varphi - \text{two-port system equivalent inner susceptance } [\mathrm{S}] \end{aligned} \tag{2.12}$$

As for the impedance case, for the admittance we have the relationships

$$Y = \frac{I}{U} = \sqrt{G^2 + B^2},\ \varphi = \arctan\frac{B}{G} \tag{2.13}$$

We notice that the admittance (real or complex) represents the inverse of the impedance (real or complex). As a consequence, between the parameters shown above one can determine a series of relationships

$$\begin{array}{lll} \underline{Y} = \dfrac{1}{\underline{Z}} & R = \dfrac{G}{Y^2} = GZ^2 & X = \dfrac{B}{Y^2} = BZ^2 \\ Y = \dfrac{1}{Z} & G = \dfrac{R}{Z^2} = RY^2 & B = \dfrac{X}{Z^2} = XY^2 \end{array} \tag{2.14}$$

Taking into account the Eqs. (2.10) and (2.13) we notice that it is possible to build two right triangles generically called the impedances' triangle and the admittances' triangle (Fig. 2.7).

The linear not-coupled with the exterior two-port system should compulsory satisfy the condition

$$\varphi \in \left[-\frac{\pi}{2},\quad \frac{\pi}{2}\right] \tag{2.15}$$

The condition (2.15) is equivalent to $\Re e\{\underline{Z}\} = R \geq 0$. If $\Im m\{\underline{Z}\} > 0$ or $\varphi > 0$ the regime is inductive. In this case we can equate the whole two-port system either in series or in parallel (depending on the way in which one works: either in impedance or in admittance) with a resistor in connection to an inductor.

$$\begin{array}{ll} \text{For series connection} & \text{For parallel connection} \\ R = \Re e\{\underline{Z}\} & G = \Re e\{\underline{Y}\} \\ X_L = \Im m\{\underline{Z}\} \Rightarrow L = \dfrac{X_L}{\omega} & B_L = \Im m\{\underline{Y}\} \Rightarrow L = \dfrac{1}{\omega B_L} \end{array} \tag{2.16}$$

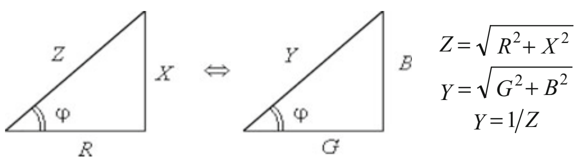


Fig. 2.7 Imittances' triangles

If $\Im m\{\underline{Z}\} < 0$ or $\varphi < 0$ the regime is capacitive. In this case we can equate the whole two-port system either in series or in parallel (depending on the way in which one works: either in impedance or in admittance) with a resistor in connection with a capacitor.

$$\begin{array}{ll} \text{For series connection} & \text{For parallel connection} \\ R = \Re e\{\underline{Z}\} & G = \Re e\{\underline{Y}\} \\ X_C = \Im m\{\underline{Z}\} \Rightarrow C = \dfrac{1}{\omega X_C} & B_C = \Im m\{\underline{Y}\} \Rightarrow C = \dfrac{B_C}{\omega} \end{array} \tag{2.17}$$

2.2.2 *Basic Equations: Kirchhoff's Laws in Complex Representation [2–5]*

For circuits containing resistors, coils, capacitors and ideal sinusoidal electromotive sources (*emf*s), the Kirchhoff's Currents (KCL) and Voltage Laws (KVL) in instantaneous values are given below

$$\sum_{l_k \in n_j} i_k = 0 \tag{2.18}$$

$$\sum_{l_k \in b_i} \left(R_k \cdot i_k + L_k \cdot \frac{di_k}{dt} + M_{kh} \cdot \frac{di_h}{dt} + \frac{1}{C_k}\int i_k dt\right) = \sum_{l_k \in bi} e_k \tag{2.19}$$

where: $j = 1, \ldots, n-1$ are the $n-1$ nodes of the circuit that can be expressed using $n-1$ independent KCL equations, $i = 1, \ldots, l-n+1$ are the $b = l-n+1$ loops of the circuit that can be expressed using $l-n+1$ KVL independent equations, and $k = 1,\ldots, l$ are the l branches of the circuit, eventually coupled with other $h = 1, \ldots, l,\ h \neq k$, branches of the circuit.

Using the properties of the symbolic method, the equations in complex form corresponding to Kirchhoff's laws are obtained

$$\sum_{I_K \in n_j} \underline{I}_k = 0, \quad j = 1, 2, \ldots, n-1 \tag{2.20}$$

$$\sum_{I_K \in b_j} \left(R_k \cdot \underline{I}_k + j\omega L_k \cdot \underline{I}_k + j\omega\, M_{kh} \cdot \underline{I}_h + \frac{1}{j\omega C_k} \cdot \underline{I}_k\right) = \sum_{I_k \in bi} E_k$$
$$\text{with } i = 1, 2, \ldots, l-n+1 \tag{2.21}$$

The solutions $\underline{I}_k$ of the complex Eqs. (2.20) and (2.21) are therefore the sinusoidal particular solutions of the integral-differential Eqs. (2.18) and (2.19).

The resistive, inductive and capacitive voltages from the left side of Eq. (2.21) can be written in complex form having a common representation that is $\underline{Z}.\underline{I}$ with the complex impedance defined according to its corresponding branch. Using this convention, KVL is compactly expressed as follows

$$\sum_{l_K \in b_j} \underline{Z}_k \cdot \underline{I}_k = \sum_{l_k \in b_j} \underline{E}_k \tag{2.22}$$

So, the statements of the two theorems are:

- KCL in complex form: "the sum of the currents' complex representations in a node n_j is zero";
- KVL in complex form: "the sum of the voltages' complex representations computed at the terminals of l_k branches along a loop b_j, is equal to the sum of the complex representation corresponding to the *emf* from the branches belonging to the same loop".

In this way, the Eqs. (2.20) and (2.21) or (2.22) represent—for a circuit in sinusoidal state having n nodes and l branches—the complete Kirchhoff independent equations system with a number of $l = (n–1) + (l–n + 1)$ equations and with l unknowns.

There is a formal analogy between AC (sinusoidal state) and DC circuits, both regarding the DC equations system and the AC circuits' complex equations systems, as well as regarding the signals that characterize and describe the DC circuits functioning and the AC circuits' complex signals. as the following

$$\begin{array}{cc} \text{DC} & \text{AC} \\ I \Leftrightarrow & \underline{I} \\ U \Leftrightarrow & \underline{U} \\ R \Leftrightarrow & \underline{Z} \\ E \Leftrightarrow & \underline{E} \end{array}$$

The only difference between the two functioning states (DC and AC), is represented by circuits with magnetic couplings in sinusoidal state that have no correspondent in DC state. That's why, the computation, analysis methods and the theorems established for DC state can be used without any modification for sinusoidal state circuits with no magnetic couplings.

The algorithm for AC circuit computation using Kirchhoff's laws is:

- compute the circuit's impedances, the complex *emf*s and the complex current sources;
- draw the equivalent complex scheme of the circuit using the corresponding complex;
- express Kirchhoff's equations in complex form and solve the system (either in currents unknowns, or in voltages unknowns);

– determine the corresponding instantaneous values form previously computed complex values.

2.2.3 Definitions of Powers in AC Circuits [1, 6, 7]

To define the powers in sinusoidal periodic regime we consider again the case of the linear, passive and inductive not-coupled with the exterior two-port system (Fig. 2.6).

The instantaneous power—p is defined as the received power at each instance of time at its terminals and is the product between the instantaneous values of the voltage and current, having the following expression

$$p(t) = u(t)i(t) = UI[\cos(\varphi_U - \varphi_I) - \cos(2\omega t + \varphi_U + \varphi_I)] \tag{2.23}$$

As one can notice from Eq. (2.23) the instantaneous power contains two terms: a constant term that characterizes the average power exchange of the two-port system and an alternative term that has the angular frequency twice as much as the frequency of the applied voltage.

Active power—P is by definition the average function of time of the instantaneous power

$$P = \langle p \rangle = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\mathrm{d}t = UI\,\cos(\varphi_U - \varphi_I) = UI\,\cos\varphi \quad [\mathrm{W}] \tag{2.24}$$

Taking into account the Eq. (2.24), the active power is always positive so it is received by the passive linear two-port system.

Taking into consideration the relationships stated for the linear two-port system case, the active power consumed by this one, can be also expressed function of its resistance, respectively its conductance

$$P = RI^2 = GU^2 \tag{2.25}$$

The active power is consumed by the active elements from a circuit (the resistances) its unit measure being the watt (W).

The reactive power—Q received by the two-port system is defined in a similar manner as the active power

$$Q = UI\,\sin\varphi \quad [\mathrm{VAr}] \tag{2.26}$$

This power changes its sign together with the phase shift φ between the voltage and the current, such that it can be both positive and negative, therefore consumed and generated by the two-port system. As in the active power case, the reactive power can be expressed function of reactances or susceptances

$$Q = XI^2 = BU^2 \tag{2.27}$$

The reactive power is "consumed" by the circuit's reactive elements (coils, capacitors and magnetic coupled coils), its unit measure being volt-ampere reactive (VAr).

The apparent power—S is by definition the product between the rms values of voltage and current

$$S = UI \quad [\mathrm{VA}] \tag{2.28}$$

As in the previous cases we can express the apparent power function of the passive linear two-port system's imittances as

$$S = ZI^2 = YU^2 \tag{2.29}$$

The apparent power is an indicator upon the circuit's functioning, being the maximum of the active power for $\varphi = 0$, respectively of the reactive power for $\varphi = \pi/2$. Its corresponding unit measure is (VA). Taking into account the definition procedure of these powers one can also introduce, as for the imittances' case, a triangle corresponding to the three powers: active, reactive ad apparent. In Fig. 2.8 there is represented the powers' triangle as well as the computation relationships for the active and reactive power, function of apparent power.

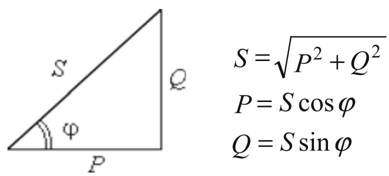
A very important signal from an energetic point of view is *the power factor* (*PF*) defined as the ratio between the two-port system's consumed active power and the apparent power

$$PF \equiv \cos\varphi \equiv \lambda = \frac{P}{S} \in [0 \quad 1] \tag{2.30}$$

A synthesis of the powers defined above is *the complex power* $\underline{S}$, defined as the product between the complex image of the voltage applied to the two-port system and the complex conjugated complex image of the absorbed current

$$\underline{S} = \underline{U}\,\underline{I}^* = Se^{j\varphi} = S(\cos\varphi + j\sin\varphi) = P + jQ \tag{2.31}$$

Fig. 2.8 Powers' triangle



As one can notice the modulus of the complex power is represented by the apparent power, its real part can be identified with the active power and the coefficient of the imaginary part can be identified with the reactive power defined for the two-port system. Equation (2.28) states these remarks.

$$\begin{aligned} |\underline{S}| &= S \\ P &= \Re e\{\underline{S}\} \\ Q &= \Im m\{\underline{S}\} \end{aligned} \tag{2.32}$$

For these reasons, when computing the powers, one proceeds directly to compute the complex power after which one identifies the active and reactive powers by separating its components.

Reactive circuits elements—energy sources (voltage sources, respectively current sources) that generate complex power in the circuit. For the voltage sources' case it is given by the product between the complex image of the voltage at its terminals and the complex conjugated image of the supplied current that circulates the source. For the current source, the complex apparent power is given by the product between the complex image of the voltage at its terminals and the complex conjugated image of the source current. For both sources the relationships are taken with the plus sign if the directions chosen for voltage and the current obeys the generator rule, otherwise the complex powers present minus sign in front of the above expressions (Fig. 2.5).

One should emphasize that the direction of the voltage at the current source terminals should be chosen from the extremity indicated by the arrow at the base. The total complex power for a circuit consists of a sum between all the complex powers corresponding to all energy sources (voltage and current) from that circuit; its real part should be equal to the active power, and its imaginary part should be equal to the reactive power of the circuit.

$$\begin{aligned} \underline{S} &= \sum_{k=1}^{n} \underline{E}_k \underline{I}_k^* + \sum_{l=1}^{n} \underline{U}_k \underline{J}_k^* = P + \mathrm{j}Q \\ P &= \sum_{k=1}^{n} R_k I_k^2 \\ Q &= \sum_{k=1}^{n} \omega L_k I_k^2 - \sum_{k=1}^{n} \frac{1}{\omega C_k} I_k^2 \pm \sum_{k=1}^{n} \sum_{l=1}^{m} 2M_k \Re e\{\underline{I}_k \underline{I}_l^*\} \end{aligned} \tag{2.33}$$

If one computes separately the active, respectively the reactive power, the following identities exist $P = \Re e\{\underline{S}\}$, $Q = \Im m\{\underline{S}\}$, respectively. The power balance mainly validates the computed currents values of a specific AC circuit.

2.2.4 Examples

Example 2.1 Let us consider the passive circuit from Fig. 2.9. The following parameters of the circuit elements are known: $R_3 = 2\,\Omega$, $f = 50\,Hz$, $L_2 = 40/\pi$ mH, $L_5 = 20/\pi$ mH, $L_6 = 60/\pi\,$mH, $C_1 = 5/\pi\,$mF, $C_4 = 10/\pi$ mF, $u_{\mathrm{AB}}(t) = 128 \cdot \sin(\omega t + \pi/4)$ [V].
Find:

(1) the instantaneous voltages at the terminals of each passive element from the given circuit (using the voltage divider rule);
(2) the instantaneous electric currents for each passive element from the given circuit (using the current divider rule).

We build the equivalent scheme in Fig. 2.10a. The passive elements are characterized by impedances, and the source is characterized by a voltage phasor.

In the equivalent scheme we have

$$\underline{Z}_{C_1} = \frac{1}{j\omega \cdot C_1} = -2j, \underline{Z}_{C_4} = \frac{1}{j\omega \cdot C_4} = -j, \underline{Z}_{L_2} = j\omega \cdot L_2 = 4j$$
$$\underline{Z}_{L_5} = j\omega \cdot L_5 = 2j, \underline{Z}_{L_6} = j\omega \cdot L_6 = 6j, \underline{Z}_{R_3} = R_3 = 2$$
$$\underline{U}_{AB} = \frac{128}{\sqrt{2}} \cdot e^{j\cdot\frac{\pi}{4}} = 64\sqrt{2} \cdot \left[\cos\left(\frac{\pi}{4}\right) + j\,\sin\left(\frac{\pi}{4}\right)\right] = 64 \cdot (1 + j)$$

where $\omega = 2\pi f = 100\pi$.
(1) Determination of the electric voltages at the circuit's elements terminals. In order to use the relationships from the voltage divider rule one should have the impedances placed in series. Notice that the impedances $\underline{Z}_{C_4}$ and $\underline{Z}_{L_5}$ are connected in parallel and the equivalent impedance between these two is: $\underline{Z}_{e_1} = \frac{\underline{Z}_{C_4} \cdot \underline{Z}_{L_5}}{\underline{Z}_{C_4} + \underline{Z}_{L_5}}$. It results: $\underline{Z}_{e_1} = -2j$. The impedances $\underline{Z}_{R_3}$ and $\underline{Z}_{e_1}$ are connected in series (Fig. 2.10b), so the equivalent impedance between these two is: $\underline{Z}_{e_2} = \underline{Z}_{R_3} + \underline{Z}_{e_1}$. It results: $\underline{Z}_{e_2} = 2 - 2j$.

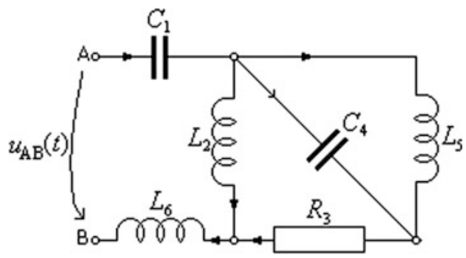


Fig. 2.9 AC circuit with 6 branches

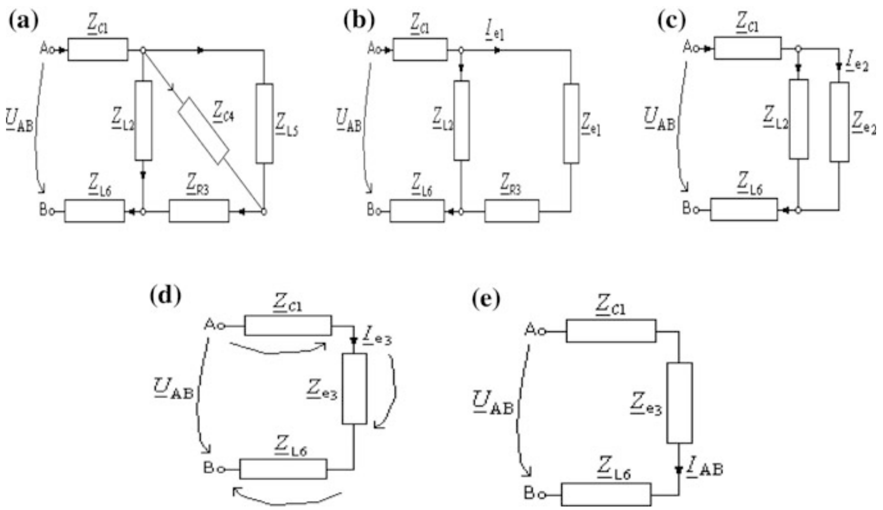


Fig. 2.10 The equivalent AC circuit

Notice the impedances $\underline{Z}_{e_2}$ and $\underline{Z}_{L_2}$ are connected in parallel (Fig. 2.10c), and the equivalent impedance between these two is: $\underline{Z}_{e_3} = \frac{\underline{Z}_{e_2} \cdot \underline{Z}_{L_2}}{\underline{Z}_{e_2} + \underline{Z}_{L_2}}$. It results: $\underline{Z}_{e_3} = 4 \cdot (1 - j)$.

We obtained three impedances in series connection. We can apply the relationships from the voltage divider rule (Fig. 2.10d).

$$\begin{cases} \underline{U}_{C_1} = \dfrac{\underline{Z}_{C_1}}{\underline{Z}_{C_1} + \underline{Z}_{e_3} + \underline{Z}_{L_6}} \cdot \underline{U}_{AB} \\ \underline{U}_{e_3} = \dfrac{\underline{Z}_{e_3}}{\underline{Z}_{C_1} + \underline{Z}_{e_3} + \underline{Z}_{L_6}} \cdot \underline{U}_{AB} \\ \underline{U}_{L_6} = \dfrac{\underline{Z}_{L_6}}{\underline{Z}_{C_1} + \underline{Z}_{e_3} + \underline{Z}_{L_6}} \cdot \underline{U}_{AB} \end{cases}$$

Results:

$$\begin{cases} \underline{U}_{C_1} = 32 \cdot (1 - j) = 32\sqrt{2} \cdot e^{-j\frac{\pi}{4}} \\ \underline{U}_{e_3} = 128 = 128 \cdot e^{j0} \\ \underline{U}_{L_6} = 96 \cdot (-1 + j) = 96\sqrt{2} \cdot e^{j\frac{3\pi}{4}} \end{cases}$$

Because $\underline{U}_{e_3}$ is the voltage from the equivalent impedances' terminals, we'll compute the voltages for each realized equivalence. The impedances $\underline{Z}_{e_2}$ and $\underline{Z}_{L_2}$ are connected in parallel. It results: $\underline{U}_{L_2} = \underline{U}_{e_2} = \underline{U}_{e_3}$. $\underline{U}_{L_2} = 128 = 128 \cdot e^{j0}$.

The impedances $\underline{Z}_{R_3}$ and $\underline{Z}_{e_1}$ are connected in series, so we can use the relationships corresponding to the voltage divider rule

$$\begin{cases} \underline{U}_{R_3} = \frac{\underline{Z}_{R_3}}{\underline{Z}_{R_3} + \underline{Z}_{e_1}} \cdot \underline{U}_{e_2} \\ \underline{U}_{e_1} = \frac{\underline{Z}_{e_1}}{\underline{Z}_{R_3} + \underline{Z}_{e_1}} \cdot \underline{U}_{e_2} \end{cases}$$

It results:

$$\begin{cases} \underline{U}_{R_3} = 64 \cdot (1 + j) = 64\sqrt{2} \cdot e^{j\frac{\pi}{4}} \\ \underline{U}_{e_1} = 64 \cdot (1 - j) \end{cases}$$

The impedances $\underline{Z}_{C_4}$ and $\underline{Z}_{L_5}$ are connected in parallel. It results: $\underline{U}_{e_1} = \underline{U}_{C_4} = \underline{U}_{L_5}$.

We obtain:

$$\begin{cases} \underline{U}_{C_4} = 64 \cdot (1 - j) = 64\sqrt{2} \cdot e^{-j\frac{\pi}{4}} \\ \underline{U}_{L_5} = 64 \cdot (1 - j) = 64\sqrt{2} \cdot e^{-j\frac{\pi}{4}} \end{cases}$$

Therefore, there are determined all the voltages at the terminals corresponding to each passive element in the complex domain. The instantaneous values of the voltages determined above are

$$\begin{cases} u_{C_1}(t) = 64 \cdot \sin\left(\omega t - \frac{\pi}{4}\right) [V]; \quad u_{L_2}(t) = 128\sqrt{2} \cdot \sin(\omega t) [V] \\ u_{R_3}(t) = 128 \cdot \sin\left(\omega t + \frac{\pi}{4}\right) [V]; \quad u_{C_4}(t) = 128 \cdot \sin\left(\omega t - \frac{\pi}{4}\right) [V] \\ u_{L_5}(t) = 128 \cdot \sin\left(\omega t - \frac{\pi}{4}\right) [V]; \quad u_{L_6}(t) = 192 \cdot \sin\left(\omega t + \frac{3\pi}{4}\right) [V] \end{cases}$$

(1) Determination of the currents across the circuit's elements. We should determine the current $\underline{I}_{AB}$ from the Fig. 2.10,e. Because the impedances $\underline{Z}_{C_1}$, $\underline{Z}_{e_3}$ and $\underline{Z}_{L_6}$ are in series connection the current across them is : $\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{C_1} + \underline{Z}_{e_3} + \underline{Z}_{L_6}}$ and it results $\underline{I}_{AB} = \underline{I}_{C_1} = \underline{I}_{e_3} = \underline{I}_{L_6} = 16 \cdot (1 + j)$. So we have

$$\begin{cases} \underline{I}_{C_1} = 16 \cdot (1 + j) = 16\sqrt{2} \cdot e^{j\frac{\pi}{4}} \\ \underline{I}_{L_6} = 16 \cdot (1 + j) = 16\sqrt{2} \cdot e^{j\frac{\pi}{4}} \end{cases}$$

Because the impedances $\underline{Z}_{e_2}$ and $\underline{Z}_{L_2}$ are connected in parallel, we use the relationships from the current divider rule to determine the currents $\underline{I}_{L_2}$ and $\underline{I}_{e_2}$

$$\begin{cases} \underline{I}_{L_2} = \frac{\underline{Y}_{L_2}}{\underline{Y}_{L_2} + \underline{Y}_{e_2}} \cdot \underline{I}_{AB} \\ \underline{I}_{e_2} = \frac{\underline{Y}_{e_2}}{\underline{Y}_{L_2} + \underline{Y}_{e_2}} \cdot \underline{I}_{AB} \end{cases}$$

It results

$$\begin{cases} \underline{I}_{L_2} = 16 \cdot (1 - j) = 16\sqrt{2} \cdot e^{-j\frac{\pi}{4}} \\ \underline{I}_{e_2} = 32j \end{cases}$$

The impedances $\underline{Z}_{R_3}$ and $\underline{Z}_{e_1}$ are connected in series, so it results: $\underline{I}_{e_1} = \underline{I}_{R_3} = \underline{I}_{e_2}$. So: $\underline{I}_{R_3} = 32j = 32 \cdot e^{j\frac{\pi}{2}}$. Because the impedances $\underline{Z}_{C_4}$ and $\underline{Z}_{L_5}$ are connected in parallel, we determine the currents $\underline{I}_{L_5}$ and $\underline{I}_{C_4}$ using the relationships from the current divider rule

$$\begin{cases} \underline{I}_{L_5} = \dfrac{\underline{Y}_{L_5}}{\underline{Y}_{L_5} + \underline{Y}_{C_4}} \cdot \underline{I}_{e_1} \\ \underline{I}_{C_4} = \dfrac{\underline{Y}_{C_4}}{\underline{Y}_{L_5} + \underline{Y}_{C_4}} \cdot \underline{I}_{e_1} \end{cases}$$

We obtain:

$$\begin{cases} \underline{I}_{L_5} = -32j = 32 \cdot e^{-j\cdot\frac{\pi}{2}} \\ \underline{I}_{C_4} = 64j = 64 \cdot e^{j\cdot\frac{\pi}{2}} \end{cases}$$

The instantaneous values of the electric currents for each element are

$$\begin{cases} i_{C_1}(t) = 32 \cdot \sin\left(\omega t + \frac{\pi}{4}\right)\ [A]; \quad i_{L_2}(t) = 32 \cdot \sin\left(\omega t - \frac{\pi}{4}\right)\ [A] \\ i_{R_3}(t) = 32\sqrt{2} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)\ [A]; \quad i_{C_4}(t) = 64\sqrt{2} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)\ [A] \\ i_{L_5}(t) = 32\sqrt{2} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)\ [A]; \quad i_{L_6}(t) = 32 \cdot \sin\left(\omega t + \frac{\pi}{4}\right)\ [A] \end{cases}$$

Example 2.2 In the circuit given in Fig. 2.11 we have: $R_1 = 2\,\Omega$, $\omega L_1 = \frac{1}{\omega C_1} = 3\,\Omega$, $\omega L_2 = 2\Omega$, $\frac{1}{\omega C_3} = 1\,\Omega$, $e_1(t) = 20 \cdot \sin\left(\omega t + \frac{\pi}{4}\right)$ [V], $j_3(t) = 10\sqrt{2} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$ [A].

(1) solve the circuit using Kirchhoff's laws;
(2) solve the circuit using the superposition principle;
(3) power balance.

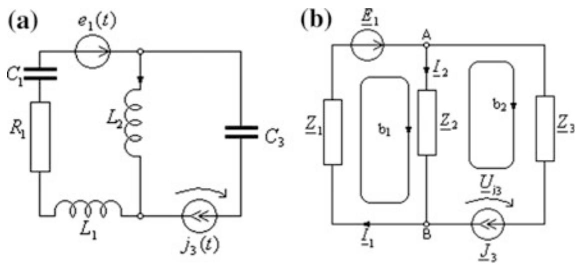


Fig. 2.11 AC circuit with two energy sources

The computation of AC circuits is done using the complex representation. In complex domain, the passive elements are characterized by impedances and the energy sources by phasors. For the passive circuit elements, the impedances are computed as follows

$$\underline{Z}_{R_1} = R_1 = 2,\ \underline{Z}_{L_1} = j\omega \cdot L_1 = 3j,\ \underline{Z}_{C_1} = \frac{1}{j\omega \cdot C_1} = -3j, \underline{Z}_{L_2} = j\omega \cdot L_2 = 2j,$$
$$\underline{Z}_{C_3} = \frac{1}{j\omega \cdot C_3} = -j$$

The voltage source is represented in complex domain by the phasor $\underline{E}_1 = \frac{20}{\sqrt{2}} \cdot e^{j\frac{\pi}{4}} = 10 \cdot (1 + j)$ and the current source is represented in complex domain by the phasor $\underline{J}_3 = \frac{10\sqrt{2}}{\sqrt{2}} \cdot e^{j\frac{\pi}{2}} = 10j$. Grouping all the impedances from a branch in an equivalent impedance one obtains (Fig. 2.11b), in which: $\underline{Z}_1 = \underline{Z}_{R_1} + \underline{Z}_{L_1} + \underline{Z}_{C_1} = 2$, $\underline{Z}_2 = \underline{Z}_{L_1} = 2j$ and $\underline{Z}_3 = \underline{Z}_{C_1} = -j$.

(1) *Kirchhoff's equations method*

The topological elements are: 2 nodes, 3 branches and 2 loops. KCL and KVL are expressed as

$$\begin{cases} KCL(A) : -\underline{I}_1 + \underline{I}_2 + \underline{J}_3 = 0 \\ KVL(b_1) : \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z}_2 \cdot \underline{I}_2 = \underline{E}_1 \\ KVL(b_2) : \underline{Z}_3 \cdot \underline{J}_3 - \underline{Z}_2 \cdot \underline{I}_2 - \underline{U}_{j_3} = 0 \end{cases}$$

We obtain

$$\begin{cases} \underline{I}_1 = 5j \\ \underline{I}_2 = -5j \\ \underline{U}_{j_3} = 0 \end{cases}$$

These values are transformed in time domain

$$i_1(t) = 5\sqrt{2} \cdot \sin\left(\omega t + \frac{\pi}{2}\right) \quad [\mathrm{A}]$$
$$i_2(t) = 5\sqrt{2} \cdot \sin\left(\omega t - \frac{\pi}{2}\right) \quad [\mathrm{A}]$$
$$u_{j_3}(t) = 0 \quad [\mathrm{V}]$$

(2) *Superposition principle*

The initial circuit (in complex domain) contains two ideal energy sources and so we'll have to compute two cases

Case 1

In this circuit the ideal voltage source is reduced to its inner resistance, and the ideal current source is characterized by the phasor $\underline{J}_3$ (Fig. 2.12, a). We determine the currents using the relationships from the current divider rule:

$$\begin{cases} \underline{I}'_1 = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2} \cdot \underline{J}_3 = \frac{\frac{1}{\underline{Z}_1}}{\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}} \cdot \underline{J}_3 \\ \underline{I}'_2 = \frac{\underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2} \cdot (-\underline{J}_3) = \frac{\frac{1}{\underline{Z}_2}}{\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}} .(-\underline{J}_3) \end{cases}$$

We obtain:

$$\begin{cases} \underline{I}'_1 = 5 \cdot (-1 + j) \\ \underline{I}'_2 = 5 \cdot (-1 - j) \end{cases}$$

To determine the voltage $\underline{U}'_{j_3}$ we apply KVL for the loop "b" such that: $(b) : \underline{Z}_3 \cdot \underline{J}_3 - \underline{Z}_2 \cdot \underline{I}'_2 - \underline{U}'_{j_3} = 0$, and results: $\underline{U}'_{j_3} = 10j$.

Case 2

In this circuit the ideal current source is reduced to its inner resistance) and the ideal voltage source is characterized by the phasor $\underline{E}_1$ (Fig. 2.12b). Applying KVL for the loop "b" such that: $(b) : \underline{Z}_1 \cdot \underline{I}''_1 + \underline{Z}_2 \cdot \underline{I}''_2 = \underline{E}_1$. We take into account that $\underline{I}''_1 = \underline{I}''_2$, and results $\underline{I}''_1 = \underline{I}''_2 = 5$. One notices that $\underline{U}''_{j_3} = -\underline{Z}_2 \cdot \underline{I}''_2$. We obtain: $\underline{U}''_{j_3} = -10j$.

The final results are obtained by superposing the results obtained independently in each of the two cases. Taking into account the directions of the determined signals for each case compared to the directions from the initial circuit

$$\begin{cases} \underline{I}_1 = \underline{I}'_1 + \underline{I}''_1 \\ \underline{I}_2 = \underline{I}'_2 + \underline{I}''_2 \\ \underline{U}_{j_3} = \underline{U}'_{j_3} + \underline{U}''_{j_3} \end{cases}$$

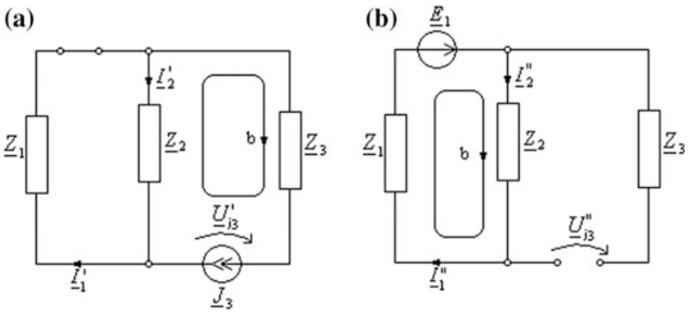


Fig. 2.12 Superposition principle

So, the final results are

$$\begin{cases} \underline{I}_1 = 5j \\ \underline{I}_2 = -5j \\ \underline{U}_{j_3} = 0 \end{cases}$$

The results obtained using the *superposition principle* are identical with the results obtained using the *Kirchhoff's method.*

(3) *Power balance*

The active power consumed in the circuit is: $P_c = R_1 \cdot I_1^2 = 2 \cdot 25 = 50\,\mathrm{W}$. The reactive power consumed in the circuit is computed as follows:

$$\begin{aligned} Q_c &= \mathrm{Im}(\underline{Z}_1) \cdot I_1^2 + \mathrm{Im}(\underline{Z}_2) \cdot I_2^2 + \mathrm{Im}(\underline{Z}_3) \cdot I_3^2 \\ &= 0 + X_{L_2} \cdot 25 - X_{C_3} \cdot 100 = -50 \ \ \mathrm{VAr} \end{aligned}$$

The apparent complex power generated is computed as follows

$$\begin{aligned} \underline{S}_g &= \underline{E}_1 \cdot \underline{I}_1^* + \underline{U}_{j_3} \cdot \underline{J}_3^* = 10 \cdot (1+j) \cdot (-5j) + 0 \cdot (-10j) \\ &= 50 - 50j = P_g + jQ_g \end{aligned}$$

We extract the generated active power as being: $P_g = \mathrm{Re}\left\{\underline{S}_g\right\} = 50\mathrm{W}$. In the same way we proceed for the generated reactive power: $Q_g = \mathrm{Im}\left\{\underline{S}_g\right\} = -50\,\mathrm{VAr}$.

We notice that $P_c = P_g$, $Q_c = Q_g$. Therefore, it is verified the power balance corresponding to the active and reactive powers consumed, respectively generated. Implicitly, the consumed, respectively the complex apparent powers balance is verified: $\underline{S}_c = \underline{S}_g$.

Example 2.3 A mono-phase receiver (AC electric drive shown in Fig. 2.13, has the following nominal data: the voltage U_n = 230 V, f = 50 Hz, the active power P_n = 1 kW and the power factor $\cos\varphi = 00.86$ (inductive). The consumer is connected using two copper conductors having the conductivity $\sigma = 56.87 \cdot 10^6$ S/m = 56.87 m/Ωmm^2 and the section A = 1.5 mm^2. The conductors have the length l = 50 m. Determine the following:

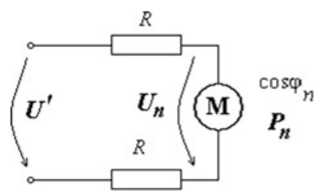


Fig. 2.13 A simple installation for an AC electric drive

(1) The current absorbed by the receiver (the corresponding rms and complex values) under nominal conditions.
(2) The reactive power, Q, the apparent power, S, corresponding to the receiver.
(3) The rms value U' of a voltage source such that at the consumers' terminals one finds its nominal voltage.

(1) From the expression of the active power one can determine the computation relationship for the absorbed current

$$I_n = I = \frac{P_n}{U_n \cos \varphi} = 5.05 \text{ A}$$

The complex value of the current (taking into account the inductive character) can be expressed as follows

$$\underline{I} = Ie^{j\varphi} = I(\cos \varphi - j \sin \varphi) = 4.34 - 2.57j$$
$$\sin \varphi_1 = \sqrt{1 - \cos^2 \varphi_1} = 0.51$$

(2) The reactive power is

$$Q = P \tan \varphi = P \frac{\sqrt{1 - \sin^2 \varphi}}{\cos \varphi} = 593.36 \text{ VAr}$$

The apparent power

$$S = \sqrt{P^2 + Q^2} = \frac{P}{\cos \varphi} = 1162.79 \text{ VA} = 1.16 \text{ kVA}$$

(3) The rms value U' of the voltage needed to power the system, such that at the consumer one has the nominal value, can be determined, using the simplest mode, from the computed power balance. Therefore

$$S' = U'I = \sqrt{(P + 2RI^2)^2 + Q^2} \Rightarrow U' = \frac{\sqrt{(P + 2RI^2)^2 + Q^2}}{I}$$

where

$$R = \frac{l}{\sigma A} = 0.586 \ \Omega$$

As a consequence, the rms value of the voltage at the end of the line is

$$U' = \frac{\sqrt{(P + 2RI^2)^2 + Q^2}}{I} = 235.45 \text{ V}$$

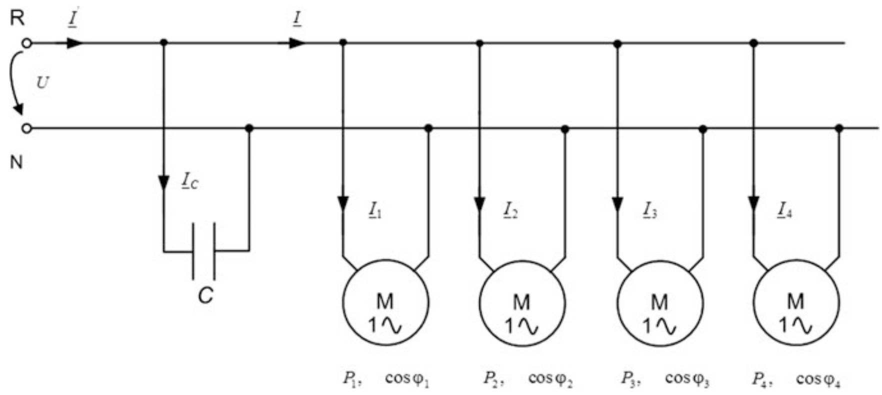


Fig. 2.14 The electric equipment in the mechanical work-place

Example 2.4 Let us consider a mechanical work-place, where there are mono-phase electric drives shown in Fig. 2.14, with the specifications given below (nominal electric power and the power factor (inductive)

$$P_1 = 1\ \text{kW}, \cos\varphi_1 = 0.60, U = 220\ V$$
$$P_2 = 4\ \text{kW}, \cos\varphi_2 = 0.70, U = 220\ V$$
$$P_3 = 5\ \text{kW}, \cos\varphi_3 = 0.76, U = 220\ V$$
$$P_4 = 10\ \text{kW}, \cos\varphi_4 = 0.80, U = 220\ V$$

All drives are powered in parallel with a sinusoidal voltage having the rms value $U = 220\ V$ and the frequency 50 Hz.
Find:

(1) The currents absorbed by the receivers and the total current for its corresponding power line (rms and complex values);
(2) The installation total power factor;
(3) The value of the capacitors' battery that should be placed in parallel with respect to the drives, such that the power factor of the entire installation to be $\cos\varphi' = 0.92$;
(4) The rms value of the current absorbed by the installation after introducing the capacitors' battery;
(5) The current drop and the losses on the power line after introducing the power factor compensation system (given in percentage).

Solution:

(1) From the expressions corresponding to the active power and to the power factor (inductive for all consumers), one can determine the electric currents absorbed by the drives (in complex and rms value), as well as the reactive power absorbed by these ones

$$I_1 = \frac{P_1}{U\cos\varphi_1} = 7.57\ \text{A},\ \underline{I}_1 = I_1 e^{j\varphi_1} = I_1(\cos\varphi_1 - j\sin\varphi_1) = 4.54 - 6.06j$$

$$\sin\varphi_1 = \sqrt{1-\cos^2\varphi_1} = 0.8,\ Q_1 = P_1\tan\varphi_1 = P_1\frac{\sqrt{1-\sin^2\varphi_1}}{\cos\varphi_1} = 1.33\ \text{kVAr}$$

$$I_2 = \frac{P_2}{U\cos\varphi_2} = 25.97\ \text{A},\ \underline{I}_2 = I_2 e^{j\varphi_2} = I_2(\cos\varphi_2 - j\sin\varphi_2) = 18.18 - 18.54j$$

$$\sin\varphi_2 = \sqrt{1-\cos^2\varphi_2} = 0.71,\ Q_2 = P_2\tan\varphi_2 = P_2\frac{\sqrt{1-\sin^2\varphi_2}}{\cos\varphi_2} = 4.08\ \text{kVAr}$$

$$I_3 = \frac{P_3}{U\cos\varphi_3} = 29.90\ \text{A},\ \underline{I}_3 = I_3 e^{j\varphi_3} = I_3(\cos\varphi_3 - j\sin\varphi_3) = 22.72 - 19.43j$$

$$\sin\varphi_3 = \sqrt{1-\cos^2\varphi_3} = 0.64,\ Q_3 = P_3\tan\varphi_3 = P_3\frac{\sqrt{1-\sin^2\varphi_3}}{\cos\varphi_3} = 4.27\ \text{kVAr}$$

$$I_4 = \frac{P_4}{U\cos\varphi_4} = 56.81\ \text{A},\ \underline{I}_4 = I_4 e^{j\varphi_4} = I_4(\cos\varphi_4 - j\sin\varphi_4) = 45.45 - 34.09j$$

$$\sin\varphi_4 = \sqrt{1-\cos^2\varphi_4} = 0.60,\ Q_4 = P_4\tan\varphi_4 = P_4\frac{\sqrt{1-\sin^2\varphi_4}}{\cos\varphi_4} = 7.50\,\text{kVAr}$$

The value of the total current absorbed by the installation is:

$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 + \underline{I}_4 = 90.90 - 78.13j$$

with the rms value

$$I = 119.873\ \text{A}$$

An alternative method for determining the value of the rms current absorbed by the whole installation is used in the power balance formula. Therefore, the rms value of the total apparent absorbed power can be expressed function of the values corresponding to the voltage and to the current

$$S = UI = \sqrt{P^2 + Q^2}$$

where

$$\begin{aligned}
P &= P_1 + P_2 + P_3 + P_4 \\
Q &= Q_1 + Q_2 + Q_3 + Q_4 \\
S &= \sqrt{(20000)^2 + (17189.95)^2} = 26372.23\ \ VA \\
I &= \frac{S}{U} = \frac{26372.23}{220} = 119.873\,A
\end{aligned}$$

(2) The installation power factor is

$$\cos\varphi = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{20000}{236372.23} = 00.758$$
$$\tan\varphi = \frac{Q}{P} = 00.85$$

(3) The value of the capacitors' battery necessary for power factor correction at $\cos\varphi' = 0.92$ can be determined taking into account that the active power remains constant after connecting the capacitors. After connecting the capacitors, the reactive consumed power is

$$Q' = P\tan\varphi'$$

where

$$\tan\varphi' = \frac{\sqrt{1 - \sin^2\varphi'}}{\cos\varphi'} = 0.48$$

On the other hand

$$Q' = P\tan\varphi' = P\tan\varphi + Q_C = P\tan\varphi - \omega C U^2 = 9.68\ \text{kVAr}$$

From where it results

$$C = \frac{P(\tan\varphi - \tan\varphi')}{\omega U^2} = 493.48\ \mu\text{F}$$

(4) The new value of the current absorbed by the installation after connecting the capacitors' battery can be determined by either applying the KCL or one takes into account the active power conservation.

– Applying the KCL, it results

$$\underline{I}' = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 + \underline{I}_4 + \underline{I}_C$$

where

$$\underline{I}_C = j\omega C U = j34.10$$

with rms value $I' = 101.01$ A.

– If we take into account the active power conservation

$$P = UI\cos\varphi$$

respectively,

$$P = UI'\cos\varphi'$$

From where

$$I' = I\frac{\cos\varphi}{\cos\varphi'} = 119.873\frac{0.758}{0.9} = 101.01\ \text{A}$$

(5) The percentage reduction of the current computed on the power line and of the active power losses can be evaluated as follows:

The current percentage reduction

$$\Delta I\% = \frac{I - I'}{I}\cdot 100 = \left(1 - \frac{I'}{I}\right)\cdot 100 = \left(1 - \frac{\cos\varphi}{\cos\varphi'}\right)\cdot 100 = 15.73\ \%$$

The percentage reduction of the active power losses

$$\begin{aligned}\Delta P\% &= \frac{\Delta P - \Delta P'}{\Delta P}\cdot 100 \\ &= \left(1 - \left(\frac{I'}{I}\right)^2\right)\cdot 100 = \left(1 - \left(\frac{\cos\varphi}{\cos\varphi'}\right)^2\right)\cdot 100 = 29.10\ \%\end{aligned}$$

where $\Delta P' = 2R_l\, I'^2$ and $\Delta P = 2R_l\, I^2$, represents the losses on the power line after and respectively before connecting the capacitors' battery, where the resistance corresponding to one power line is denoted by R_l.

2.3 Intuitive Understanding of Powers in AC Power Systems

2.3.1 Energy and Power in AC Power Systems

Let's consider a two-port circuits source (generator) and receptor—shown in Fig. 2.15-having at the terminals the voltage u_b and across it the current i. The *instantaneous power* defined as in Eq. (2.23) at the two-port terminals of the generator and receptor is:

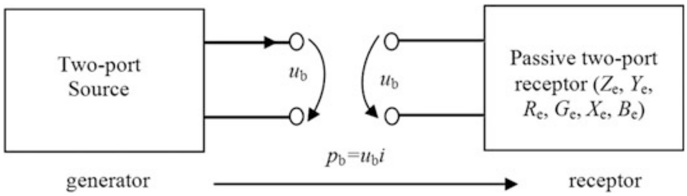


Fig. 2.15 Convention of the directions for defining the power at terminals

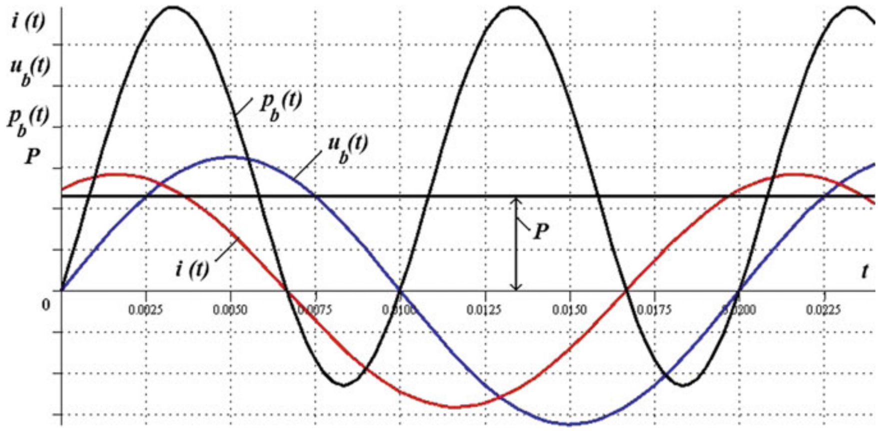


Fig. 2.16 The current and voltage signals of the instantaneous power

$$\begin{aligned} p_b &= u_b i = 2U_b I \, \sin(\omega t + \beta) \sin(\omega t + \gamma) \\ &= U_b I \, \cos\varphi - U_b I \, \cos(2\omega t + 2\beta - \varphi) \\ u_b &= U_b\sqrt{2}\sin(\omega t + \beta), \quad i = I\sqrt{2}\sin(\omega t + \beta - \varphi) \end{aligned}$$

This power is a periodic signal having a constant component and a sinusoidal one, of double frequency (Fig. 2.16).

The energy received (or generated) by a two-port system, in a time $\tau = nT$ or much bigger than a period $\tau \gg T$, is obtained by multiplying the active power with the corresponding time. Indeed, for the energy, we obtain the following successive expressions [2–4, 7]:

$$\begin{aligned} W_\tau &= \int_0^\tau p_b dt = \int_0^{nT} p_b dt = n\int_0^T p_b dt = nTP \\ &= \tau U_b I \, \cos\varphi = \tau R_e I^2 = \tau G_e U^2 \end{aligned} \tag{2.34}$$

where R_e and G_e are the equivalent resistance respectively conductance of the passive two-port receptor. The *active power* P received by a passive two-port receptor is always positive or at least zero (for non-dissipative circuits). The expression of the instantaneous power shows that even if the circuit is passive, so it is a receiver ($P > 0$), there are moments in time when it is negative, so the circuit gives energy. In those time moments, the energy accumulated in the coils' magnetic field or in the capacitors' electric field is partially returned to the power source.

The *apparent power* (2.28) of the considered passive two-port receptor can be expressed also using the equivalent impedance Z_e or admittance Y_e of passive two-ports receptor $S = U_b I = Z_e I^2 = \sqrt{R_e^2 + X_e^2} I^2 = Y_e U_b^2 = \sqrt{G_e^2 + B_e^2} U_b^2$, where X_e and B_e are the equivalent reactance respectively susceptance.

The apparent power does not have the same important energetic significance, as the active power, but it represents an important computation quantity, because it represents the maximum possible value of the active power at different not-variable values of the voltage and current and for variable phase. Because the machine drives, the electric transformers and the electric pieces of equipment are characterized by maximum admitted limit values of the current (such that the losses caused by Joule effect in conductors don't determine excessive heating) and of the voltage (such that the magnetic circuit doesn't saturate and the isolation is not damaged etc.), the apparent power characterizes their functioning limits and is usually written on the nameplate of the respective device [7, 8].

The *power factor* (at receivers) defined in (2.30) as a positive and subunitary ratio between the active power and the apparent power $\lambda = PF = P/S$, then for two-ports receptor in sinusoidal state is always $\lambda = \cos\varphi$. In order that a given apparent power installation to function with a maximum active power, the corresponding power factor should be maximum (or as close as possible to the unit) that is in sinusoidal state the phase-shift to be as small as possible.

The *reactive power* Q defined as in (2.26) is for the considered two-port receptor in sinusoidal state $Q = S\,\sin\varphi = U_b I\,\sin\varphi = X_e I^2 = B_e U_b^2$. The reactive power of the resistive circuit is zero, also the coils consume reactive power, and the capacitors generate reactive power.

The reactive power has been introduced based on the definition relationship built in analogy with the active power expression, parallelism that can be found also in other relationships. The reactive power does not correspond to an average energy contribution at the terminals. However it has a practical significance for the following reasons:

– the power factor (2.30) can be written as

$$\lambda = PF = \frac{P}{S} = \frac{\sqrt{S^2 - P^2}}{S} = \sqrt{1 - \left(\frac{P}{S}\right)^2} \tag{2.35}$$

– the last expression emphasizes the fact that power factor improvement is equivalent to the reactive power (compensation) reduction problem;

- as it will be shown, the reactive power has conservation properties and it will be used for computing the power balance, as if it will correspond to certain specific energy forms (different from the usual energy, that conditions the active power balance);
- the reactive power received by a passive network is proportional to the difference between the average value of the magnetic field energy of the network's coils and the average value of the electric field energy of the network's capacitors.
- the reactive power measures the un-compensation of the internal energy exchange between the magnetic and electric field.

2.3.2 Case Studies

Example 2.5 The instantaneous energetic balance of an *RLC* series circuit where the equation of the voltage of the circuit is $u = u_R + u_L + u_C$. Then by multiplying this equation with i, one obtains the equation corresponding to instantaneous power [6–9]

$$p_b = u_R i + u_L i + u_C i = Ri^2 + Li\frac{di_L}{dt} + Cu_c\frac{du_c}{dt} = Ri^2 + \frac{d(w_e + w_m)}{dt}$$

To simplify the expressions, one chooses the current as phase origin. Then it results

$$u_b = U_b\sqrt{2}\sin(\omega t + \varphi), \quad i = I\sqrt{2}\sin\omega t$$

The voltage can be written as the sum between the active and reactive components

$$u_b = U_b\sqrt{2}\cos\varphi\ \sin\omega t + U_b\sqrt{2}\sin\varphi\ \cos\omega t = u_R + u_X$$

With this decomposition, the instantaneous power can be written as the sum between two components

$$p_b = p_R + p_X = U_b I\ \cos\varphi(1 - \cos 2\omega t) + U_b I\ \sin\varphi\ \sin 2\omega t$$

The first term corresponds to current multiplication with the active component of the voltage and it represents the power developed in the circuit's resistance

$$p_R = u_R i = Ri^2 = RI^2 2\ \sin^2\omega t = UI\ \cos\varphi(1 - \cos 2\omega t)$$

This power is always positive (or zero), and it is called *pulsating* power and it has the average value equal to the active power.

The second term

$$p_X = u_X i = (u_L + u_C i) = \frac{d(w_m + w_e)}{dt} = p_b - p_R = UI \sin\varphi \sin 2\omega t$$

is the instantaneous power resulted from multiplying the current with the reactive component of the voltage and is called *harmonic instantaneous power*, having zero average value and the amplitude equal to the reactive power modulus. The harmonic instantaneous power is equal to the variation speed of the total instantaneous energy (electric w_e and magnetic w_m) of the circuit.

So, *the reactive* power (in modulus) is equal to the amplitude of the variation speed of the energy accumulated by the circuit electromagnetic field.

Another interpretation can be as follows. The reactive power is written in successive forms [10–14] as

$$Q = X_e I^2 = (X_C - X_L) I^2 = L\omega I^2 - \frac{I^2}{C\omega} = 2\omega\left(LI^2 - CU_C^2\right)$$

The magnetic energy average value is

$$\tilde{w}_m = \frac{1}{2T}\int_0^T L\, i^2 dt = \frac{1}{2} L \left\{ \frac{1}{T}\int_0^T i^2 dt \right\} = \frac{1}{2} L I^2$$

and the electric energy average value is

$$w_e = \frac{1}{2T}\int_0^T C\, u_C^2 dt = \frac{1}{2} C \left\{ \frac{1}{T}\int_0^T u_C^2 dt \right\} = \frac{1}{2} C U_C^2$$

Finally, we have the relationship

$$Q = 2\omega(\tilde{w}_m - \tilde{w}_e)$$

The reactive power is proportional to the difference between the average magnetic energy and average electric energy of the circuit. It results the reactive power is zero when $\tilde{w}_m - \tilde{w}_e = 0$. This relationship corresponds to a zero phase-shift $\varphi = 0$ and defines the resonance condition of the circuit.

2.4 Effects of Reactive Power on the Power System Parameters

The irrational and big reactive power consumption, that generates a reduced power factor, presents a series of disadvantages for the electric installation, among which we mention [15]: the increase of active power losses in the passive elements of the installation, the increase of voltage losses, installation oversize and its diminished electric energy transfer capacity.

(a) *Power losses* in the installation's conductors, having the resistance R are given by the following relationship:

$$\Delta P = 3RI^2 = \frac{R}{U^2}S^2 = \frac{RP^2}{U^2\lambda^2}$$

We notice that it varies inverse proportional to the square of the PF at P = cte. and U = cte. Therefore, if the same active power P is transported using different power factors $\lambda_1 < \lambda_2$, then the power losses ΔP_1 and ΔP_2 are inter-dependent according to the relationship: $\Delta P_2 = \Delta P_1 \left(\frac{\lambda_1}{\lambda_2}\right)^2$ from where it results that, by improving the power factor, the power losses diminishes.

(b) *Rms value voltage alternations*

If PF is an inductive one, for the sinusoidal state, it takes place a voltage reduction on the power mains, and if the power factor is a capacitive one, the voltage in the installation increases. In the sinusoidal state, for an inductive power factor, the longitudinal voltage drop $\Delta U \cong U_1 - U_2$ is given by the relationship:

$$\Delta U = RI\,\cos\varphi + XI\,\sin\varphi = \frac{R\cdot P + X\cdot Q}{U_2} = \Delta U_a + \Delta U_r$$

where U_1 is the phase voltage at the power source terminals, U_2 is the phase voltage at the power bars, R and X—the electric resistance and, respectively, the reactance of the line that connects the source to the receiver, P and Q—active power and, respectively, the reactive power transported on a phase of the electric installation, and ΔU_a and ΔU_r—the voltages drops determined by the circulation of the active and, respectively reactive power. As a consequence, it results for Q = 0, that is without a reactive power circulation, one obtains: $\Delta U = \Delta U_{\min} = \Delta U_a$.

(c) *Diminishing the installation's active power loading capacity* due to a reduced power factor.

So, for the same apparent power S_n it corresponds many active powers $P_1 = S_n\lambda_1$, $P_2 = S_n\lambda_2$ function of power factor's value. If $\lambda_2 < \lambda_1$ then we have: $P_2 = \frac{\lambda_2}{\lambda_1}P_1$ from where it results active power reduction $P_2 < P_1$ increasing the reactive power consumption.

(d) *Electric installations oversize* (implies supplementary investment) that functions at a low power factor is explained by that the energy conductors (the electric line) are dimensioned function of the admissible voltage loss and is being verified for heating in a long enough time state. Therefore, if we take into account the admissible voltage losses expression (in a sinusoidal state) $\Delta U_{ad} = \Delta U_a + \Delta U_r$, whose quantity is normalized, then for given P and Q it results

$$\Delta U_r = X \cdot Q / U_n = \text{cte}$$

that leads to

$$\Delta U_a = \Delta U_{ad} - \Delta U_r = \rho \frac{L}{s} \frac{P}{U_n} = \text{cte.} \quad \text{or} \quad s = \rho \frac{L}{\Delta U_a} \frac{P}{U_n}$$

where

L the length of the line;
s the phase conductor's section;
ρ the resistivity of the conducting material;
P the active power flow.

For a given active power, the investment in electric energy sources is inverse proportional to the square of the power factor, and the installed apparent power varies inverse proportional to the power factor.

2.4.1 Investigating the Powers Flow Process in AC Systems

In AC electric installations, characterized from the electric point of view, by a scheme containing active and reactive elements, in general, it takes place a transfer of *active power P* from the source to the receiver, in correlation with the consumers' requirements, as well as a transfer of *reactive power Q* and of *distortion power D*.

An objective energetic characterization of a consumer is done using *the apparent power S* that connects the above mentioned powers [16, 17].

Lately, due to the large-scale use in all domestic and industrial installations of power electronics and due to numerous non-linear receivers, practically we cannot talk about receivers for which the voltage signals and especially the signal of the absorbed current to be pure sinusoids. So, the weight of the distortion power becomes more significant. Therefore, one defines the following: *apparent, active, reactive and distortion power*. For a single-phase consumer, these become

$$\begin{aligned} S &= UI = \sqrt{\sum_{k=0}^{n} U_k^2} \cdot \sqrt{\sum_{k=0}^{n} I_k^2} \\ P &= \sum_{k=0}^{\infty} U_k I_k \cos \varphi_k \\ Q &= \sum_{k=1}^{\infty} U_k I_k \sin \varphi_k \\ D &= \sqrt{S^2 - P^2 - Q^2} \end{aligned} \tag{2.36}$$

where U, I represent the rms value of the voltage respectively of the current and φ_k is the phase shift between the voltage and the current, corresponding to the harmonic of rank k, with $k = 1 \div n$. The unit measures for the four types of powers are given in their definition order: VA, W, VAr and VAd. Most of modern measuring pieces of equipment designed for measuring the electric power and energy can measure instantly each of the 4 powers (energies) previously defined.

The reactive power of the total load of an electric energy consumer has, usually, an inductive character, the load current being phase-shifted behind the voltage; in this case, one considers, conventionally, that the reactive power is positive ($Q_L > 0$) and the receivers represent *reactive power consumers*. For other receiver, the absorbed current is phase-shifted before the voltage; these receivers are considered, conventionally, *reactive power sources*, and the corresponding power in taken in computation with minus sign ($Q_C < 0$). The total reactive power is: $Q = Q_L - Q_C$.

In electric installations, *the Power Factor* (*PF*) defined in (2.30) can be rewritten as:

$$\lambda \equiv PF = \frac{P}{S} = \frac{\sum_{k=0}^{\infty} U_k I_k \cos \varphi_k}{UI} \tag{2.37}$$

In a similar manner, the *Displacement Power Factor* (*DPF*) is defined as the ratio between the active power P_1 and the apparent power S_1, corresponding to the fundamental harmonic ($k = 1$)

$$\lambda_1 \equiv DPF = \frac{P_1}{S_1} = \frac{U_1 I_1 \cos \varphi_1}{U_1 I_1} = \cos \varphi_1 \tag{2.38}$$

We mention that the two definitions for the power factor (*PF* and *DPF*) lead, in general, to different values and the equality $\lambda = \frac{P}{S} = \cos \varphi$ $(PF = DPF)$ is valid only in single-phase circuits and only for pure *sinusoidal state* (ideal case). As a consequence, the interpretation of the power factor function of the phase-shift between the current and the applied voltage signals should be carefully taken into consideration.

To characterize the high order harmonics loading degree corresponding to the voltage wave shape or to the consumer's absorbed current, one can use several indicators, the most used one being the *Total Harmonic Distortion* (*THD*) defined in percentage as the ratio between the distortion residue of the signal and the rms value of the fundamental harmonic. So, for current

$$THD_i = \frac{1}{I_1} \sqrt{\sum_{k=1}^{n} I_k^2} \cdot 100\,[\%] \tag{2.39}$$

where I_k represents the rms value corresponding to harmonic k and I_1 is the one corresponding to the fundamental harmonic. Using the value of the total harmonic distortion, one can easily prove the relationship

$$PF = \frac{\cos \varphi_1}{\sqrt{1 + THD_i^2}} = \frac{DPF}{\sqrt{1 + THD_i^2}} \tag{2.40}$$

valid for the case when the voltage distortion is negligible $THD_u < 5\%$ (frequently enough for low-voltage installations). Therefore, if $DPF = 0.92$ but $THD_i = 30\%$ the value of the power factor is only $PF = 0.88$.

The previous relationships define the *instantaneous power factor* that corresponds to a certain moment from the consumer's installations functioning.

Because the electric charge presents fluctuations, the current legislation recommends the determination *of the weighted mean power factor* based on the consumption of:

– active energy

$$W_a = \int_0^t P \cdot dt$$

– reactive energy

$$W_r = \int_0^t Q \cdot dt \tag{2.41}$$

– distortion energy

$$W_d = \int_0^t Q \cdot dt$$

– for a certain period t (month, year)

$$PF_{med} = \frac{W_a}{\sqrt{W_a^2 + W_r^2 + W_d^2}}$$

From an energetic point of view, this is useful to characterize the consumer's installation and to charge its consumed electric energy. The weighted average power factor can be *natural*—when is determined without taking into consideration the reactive power compensation and *general*—when considering its evaluation one takes into account the losses corresponding to this installation. The value of the general weighted mean power factor from which the reactive energy consumption is no further charged is called *neutral power factor*. This value is being determined using technical-economical computation for minimizing the active power losses and is a local quantity that depends on the position of the consumers in the electric network. At international energetic system level the established value is equal to 0.92 [18, 19].

2.4.2 Reactive Power Consumers

The main reactive power consumers are *the asynchronous motors and the electrical transformers* which consume around 60% respectively 25% from the installation's total reactive power due to producing alternating magnetic fields [20–25]. At industrial consumers' level the weight is around 70% for asynchronous motors and around 20% for transformers. The difference between the reactive power consumptions for asynchronous motors and transformers, at the same active power and the same magnetic stress, comes from the fact that the magnetization reactive power, that constitute the most important component of the reactive power, depends on the volume of the magnetic circuit to which, for asynchronous motors' case, one adds also the volume of the air-gap (not existent for transformers). Another component of the reactive power for the asynchronous motors and for transformers is the reactive power dependent on load, called also dispersion reactive power.
(A) For *asynchronous motors* case, the magnetization reactive power (or the no load power) represents the most part from the motor's reactive power, function of motor's loading and function of the air-gap. Taking into account the exploitation average loading: $\beta < 0.5$ (evaluated using the loading factor as the ratio between the mechanical shaft power P and mechanical nominal power P_n of the motor $\beta = P/P_n$), one can approximate the reactive power of an asynchronous motor as being constant and independent of the load, while the active power depends on the motor' load. Also, the motors' active power remains practically constant for small deviations of the voltage compared to the nominal voltage, while the reactive power depends essentially on the voltage's variation. *Asynchronous motors* functioning with a load factor $\beta < 1$ due to an inappropriate technological exploitation, determines the power factor reduction under its nominal value.

The reactive power Q absorbed by asynchronous motor for any load P, is determined using the relationship [26, 27]

$$Q = Q_0 + Q_d = Q_n\left[\alpha + (1-\alpha)\beta^2\right] = Q_0 + (Q_n - Q_0)\beta^2$$

where $\alpha = Q_0/Q_n$ is the ratio between the reactive power Q_0 for unloaded running $(\beta = 0)$ and the reactive power Q_n absorbed at nominal load $(\beta = 1)$; $Q_d = (1-\alpha)\cdot\beta^2 Q_n$—dispersion reactive power. Considering the definition of PF in sinusoidal state, for a symmetrical loading of the three-phase network, it results

$$\lambda = \cos\varphi = \frac{\beta}{\sqrt{\beta^2 + \left[\alpha + (1-\alpha)\beta^2\right]^2 \tan^2\varphi_n}} \tag{2.42}$$

For values $\beta < 0.5$ the reduction of the *PF* under its nominal value is highly accentuated. If in exploitation, the power voltage of the asynchronous motors increases, it results an increase of the absorbed reactive power, having undesired consequences upon *PF*—Fig. 2.17a. This is due to the increase of the magnetization current in the saturated area. In Fig. 2.17b is presented, as a rough guide, the dependence of the absorbed reactive power function of the nominal one (Q/Q_n) for asynchronous motors of low and high power function of the relative value of the power voltage for different loading factor's values [28, 29].

(B) For *transformers* the absorbed reactive power is computed using the relationship [26, 27, 30, 31]

$$Q = Q_0 + Q_d = \frac{S_n}{100} \cdot \left(i_0 + k_f \cdot \beta^2 \cdot u_{sc}\right)$$

where S_n is the nominal apparent power, i_0—the current for unloaded running (expressed in percentage function of nominal current), k_f—the form factor of the load signal (defined as the ratio between the mean square value and the mean value, of the load current, computed for a given time interval), $\beta = S/S_n$—the transformer load factor, u_{sc}—the voltage short-circuit voltage (expressed in percent).

As for asynchronous motors case, the transformers functioning at a power under the nominal one determines the reduction of *PF*. In real exploitation conditions the

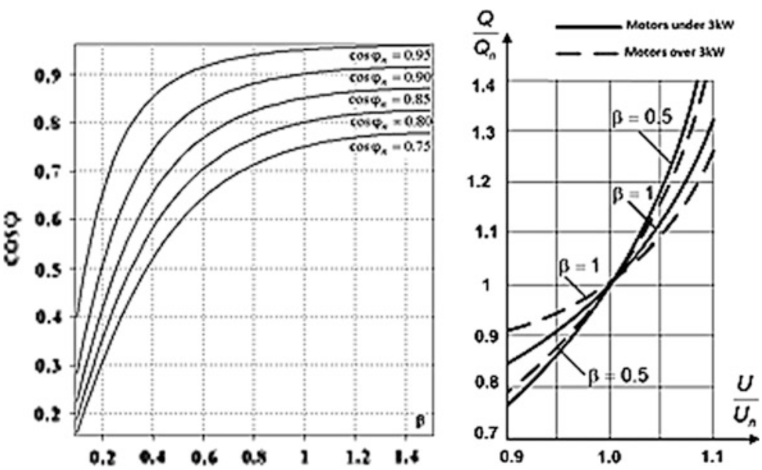


Fig. 2.17 Power factor variation function of asynchronous motor's loading and the reactive power consumption for small and big power motors function of the relative power voltage

transformers' total reactive power can be evaluated to 10% of the nominal power (8% unloaded running power and 2% dispersion power).

(C) *Electric ovens* installations (arc or induction) consume reactive power from the oven's power supply transformer, the oven adjustable autotransformer and the power supply circuit of the oven from the transformer. Therefore, for a three-phase arc oven, the *PF* of the installation is, usually, sufficiently big ($0.8 \div 0.9$) but the absolute reactive power consumption is big, compared to the oven's power (that can reach a value of 80 MW for a capacity of 400 t).

(D) *Electric Distribution Lines* can determine reactive power consumption Q_L that can be computed using the relationship $Q_L = \omega LI^2$ where L is the line's equivalent inductivity, ω—the voltage pulsation on the line, and I—the electric current that circulates the line. On the other hand, the electric lines generate reactive power Q_C due to their capacity C against the ground: $Q_C = \omega CU^2$. For electric lines case, the resulting reactive power can have positive or negative values function of the values of the two components Q_L and $Q_{C,}$ the first one depending on the square of the electric current that circulates the line and the second one depending on the square of the voltage.

(E) *Power Controlled rectifying installations* supplied by a transformer represent also an important reactive power source. The schemes adopted for controlled rectifiers can lead also to a small power factor [32–37]. Lately, the consumer containing power electronic circuits represent, due to their large scale use, the main distortion power source from installations. As a consequence, the power factor of the whole installation containing such receivers is diminished.

(F) A contribution to the reactive power flow is given by *electric discharge lamps* in metallic vapours when they are connected in an uncompensated inductive ballast schemes.

The *PF* monitoring is very important for the producer, the transporter, the distributer, the provider and the final user of electric energy, because it influences the performance characteristics of all these services, it determines the availability capacity of the energy transfer for electro-energetic pieces of equipment and imposes the final electric energy providing costs [29–31].

2.4.3 Power Factor Compensation in AC Power Systems

The means to improve the *PF* are grouped into *natural means and special means*. *Natural means* derive from a rational and correct choice and exploitation of the existing machinery in installations and consists of technical and managerial measures (replacing the low-loaded transformers and motors, nonlinear loads aggregation, reducing the unloaded functioning time of the machineries etc.) and *the special* ones assume the introduction in installation of some pieces of equipment generating reactive power and/or limiting the distorted state (installation of some capacitor banks, of active filters networks conditioners etc.).

As a consequence, *PF* improvement includes, on a first stage, *operations for mitigation the high harmonics' attendance from the wave shapes* of the voltage and the absorbed current, and, on the second stage, *the limitation of reactive power flow*. Any method dedicated to reducing the reactive power absorbed by a consumer is efficient only if the power voltage is practically sinusoidal.

(1) *The use of capacitors banks as a reactive power source*

This way for improving the power factor, even it's highly spread, it's efficient only if the signals corresponding to the current and to the voltage are close to a sinusoid (the contribution of the distorted state is modest). The placement of the capacitors in a low voltage installation represents the compensation procedure that can be [36–38]: global (placement in a single point for the entire installation), by sector (group with sector), local or individual (at each equipment) or a combination between the last two. Generally speaking the ideal compensation is applied at the consumption place and it has the level in concordance with the instantaneous power values, but in practice the choice is decided by economical and technical factors—Fig. 2.18.

(2) *Individual compensation* is applied, firstly, in the big reactive power loads (asynchronous motors, electric ovens etc.) and with continuous functioning, ensuring the reactive power compensation at the consumption place and unloading the remaining of the network from the reactive power flow, with all the advantages deriving from this one—Fig. 2.19.

For individual compensation of *asynchronous motors*, the capacitors bank is usually directly connected to the motors' terminals and the decoupling from the network taking place at the same time as the motor, using the same switching device. To avoid the overcompensation that occur for the no loaded and the auto-excitation (when the motor' breaks), around 90% from the unloaded functioning power is compensated, ensuring a power factor of around 0.9 at normal loading and approx. 0.95 at incomplete loading or unloaded functioning. The reactive power Q_c necessary to compensate three-phase asynchronous motors is determined by the motors' unloaded operating current value I_0 (given in theirs catalogue data)

$$Q_C = Q_0 = 0.9\sqrt{3}U_n I_0 \tag{2.43}$$

For motors having powers bigger than 30 kW one can choose a covering value $Q_C \cong 0.35P_n$ [39].

For power distribution transformers' case, the compensation bank is placed on the low voltage part and very often consists of a fixed step (necessary to compensate the unloaded absorbed reactive power) followed by a step that covers the necessary reactive power at nominal load: the bank's dimensioning takes place using the relationship

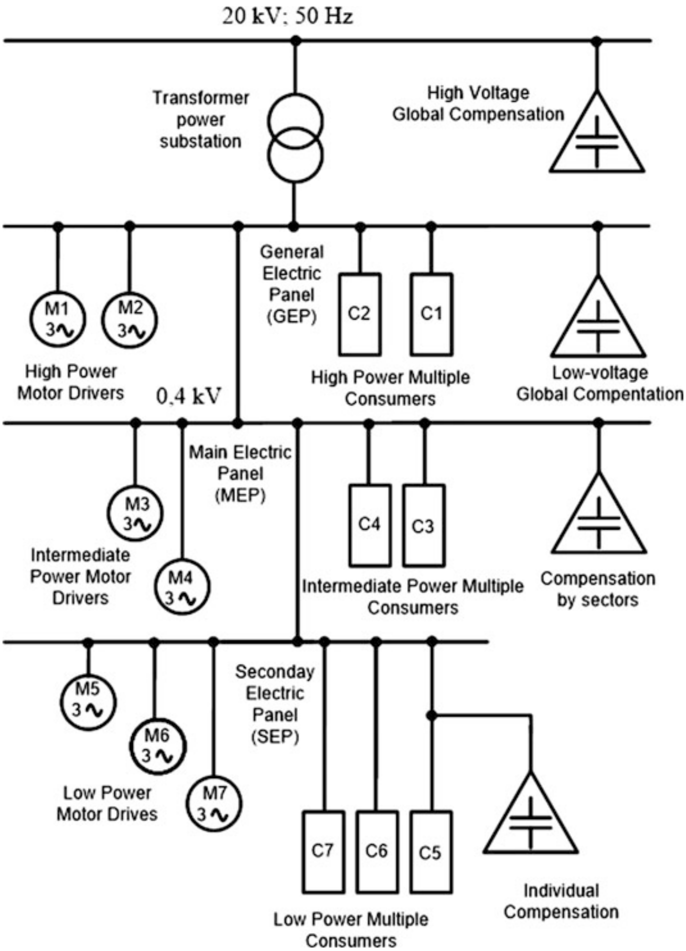


Fig. 2.18 Examples for placing the capacitors bank

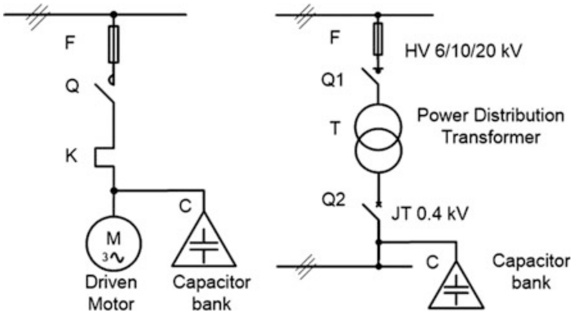


Fig. 2.19 Individual compensation for asynchronous motors and for transformers

$$Q_C \cong Q_0 + Q_d = \frac{i_0[\%]}{100} S_n + \frac{u_{sc}[\%]}{100} S_n \cdot \beta^2 \tag{2.44}$$

where i_0 [%] respectively u_{sc} [%] represent the percent values corresponding to the unloaded running current and to the short circuit voltage of the transformer (given in its catalogue data) and $\beta = S/S_n$ is the load factor rate. Usually, one can consider: $Q_C = (0.1 \div 0.2)S_n + (0.5 \div 0.6)S_n$.

The individual compensation is applied also to illumination lamps' with metallic vapours electric discharge, the capacitors' values is being mentioned by the producer as a function of the lamp's rated power.

(3) *Compensation on receivers' groups* (by sector) is applied when the reactive power consumers are grouped, the capacitors' banks being connected to the mains from the distribution panels corresponding to the receivers' groups. The bank's power and the operating mode are established as a function of the receivers' uncompensated reactive power signal. This compensation mode limits also the reactive power circulation in the network above the setting place.

(4) *Centralized compensation* can be realized by connecting the capacitors bank at the mains (coupling points) from the general electric distribution panel from the transformer power substation. The bank is executed in commutation steps, usually automated, function of the reactive power that should be compensated, corresponding to powered receivers running. The bank can be connected also to the intermediate high voltage of the transformer power substation. For a centralized compensation, intermediate high voltage, the positive technical effects are not present at low voltage installations belonging to the consumer (downstream to the installing place). But, the big consumers powered directly in intermediate high voltage, use the centralized compensation that provides a general power factor bigger than the neutral power factor value such that the electrical energy consumption to be billed only for active energy.

(5) *Mixed compensation* uses all the procedures presented before for reactive power compensation. The solution is applied in steps or, when there are certain conditions specific to the respective consumer.

The power Q_b corresponding to the capacitors bank is determined such that, to a certain given active power P, absorbed under a power factor $\cos\varphi_1$, to obtain an improved power factor $\cos\varphi_2 > \cos\varphi_1$. The reactive power consumed by these capacitors will be: $Q_b = -3\omega C U_c^2$. The receiver consumes a reactive power before adding the bank, $Q_1 = P\mathrm{tg}\varphi_1$. The necessary power from the network after adding the capacitors will be $Q_2 = Q_1 + Q_b$, from where

$$Q_b = Q_1 - Q_2 = P(\tan\varphi_1 - \tan\varphi_2) \tag{2.45}$$

where $Q_2 = P\tan\varphi_2$ represents the reactive power received from the network after introducing the capacitors bank—the active power P remains constant, and $\cos\varphi_2$ is the new power factor that should be realized. As a consequence, the value of the capacity for one capacitor from the triangle bank will be

$$C = \frac{P(\tan \varphi_1 - \tan \varphi_2)}{3\omega U_c^2} \quad (2.46)$$

In Fig. 2.20 the connection possibilities for the capacitors that form the bank in single and three phase installations are presented. If one takes into account the power loss ΔP_C in the capacitors' dielectric the power necessary for the bank becomes

$$Q_b = P \cdot \tan \varphi_1 - (P + \Delta P_C) \cdot \tan \varphi_2 = Q_1 - Q_2 \text{ with } \Delta P_c = \frac{p_d}{100} \cdot Q_b \quad (2.47)$$

where $p_d = 0.25 \div 0.35\%$, represents the losses from the dielectric expressed as a percent function of the capacitors bank power Q_b. These depend on the tangent of the material's losses angle $tg\delta$ after the relationship: $\Delta P_C = \omega C U^2 \tan \delta$ and can be considered (especially in low voltage) negligible.

One can note that for Y bank's connection result capacitors having a capacity three times bigger, because instead of U_c is the voltage $U_f = U_l / \sqrt{3}$. Results $C_\Delta / C_Y = \left(U_f / U_l \right)^2 = 1/3$ and, as a consequence, the power factor compensation is an economical and technical problem, taking into account that, in low voltage, the cost of the capacitors is proportional to their capacity. This is the reason why Δ connection is preferred. The Y assembly is advantageous for intermediate high voltage networks, because nominal voltage of the capacitors is then reduced. One can notice that the maximum voltage for Δ connection capacitors is over 580 V compared to Y connection where the voltage is only 320 V. The capacitors bank power depends on the network voltage variation. So, if the network voltage varies from U_n to U_{n2} the capacitors bank power modifies from Q_b to the value $Q_{b2} = Q_b (U_{n2}/U_n)^2$.

The time variation signal corresponding to the reactive power consumption Q in an electrical installation is, in general, not a linear one. The necessary reactive power depends on the way in which the electric energy is being used in various technological processes, fact emphasised by the load variation. Therefore, the absorbed reactive power can vary slowly or rapidly in very large limits. In this

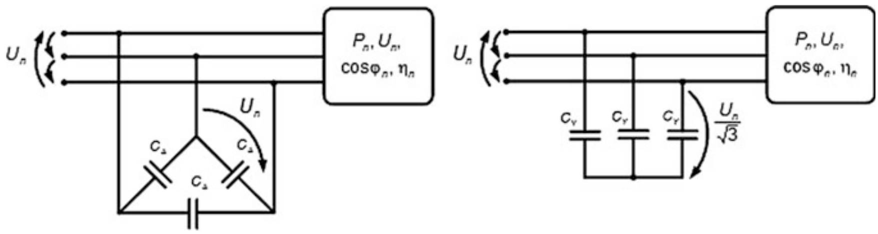


Fig. 2.20 Connections possibilities for capacitors bank

situation, the adoption of some capacitor banks with fix value can determine over and under compensations regimes. To avoid such a major inconvenient, the problem is solved by realizing a capacitor bank from several sections (stages) of fixed power. Each section consists of automatic controlled commutation pieces of equipment, following several criteria, for example, function of the mains voltage, the load current, and the direction of the reactive power change with the energetic system or function of the running time corresponding to different loads—Fig. 2.21.

The selection of the number and of the values for the capacitors bank sections should take into account that the control efficiency increases with the number of steps, making possible to follow very closely the load reactive power signal. On the other hand, an excessive fractioning of the bank becomes at a certain point not viable from economical point of view because it implies the use of a complex commutation apparatuses. Usually, the capacitor banks use a number of 4 ÷ 12 sections having the powers between 5 ÷ 25 kVAr [34, 38, 39].

For the scheme presented in Fig. 2.21, the control block CB establishes the value of the power factor (cos φ) and knowing the capacities corresponding to the bank's sections, connects the elements necessary to realize the prescribed power factor. Exceeding the prescribed power factor leads to disconnect the capacitors bank sections.

Systems' selection for reactive power compensation in non-sinusoidal state. To select a system for power factor correction in an installation where there are present high-order harmonics in the current and voltage waves shape, function of the user's data, one uses the following methods [35–40]:

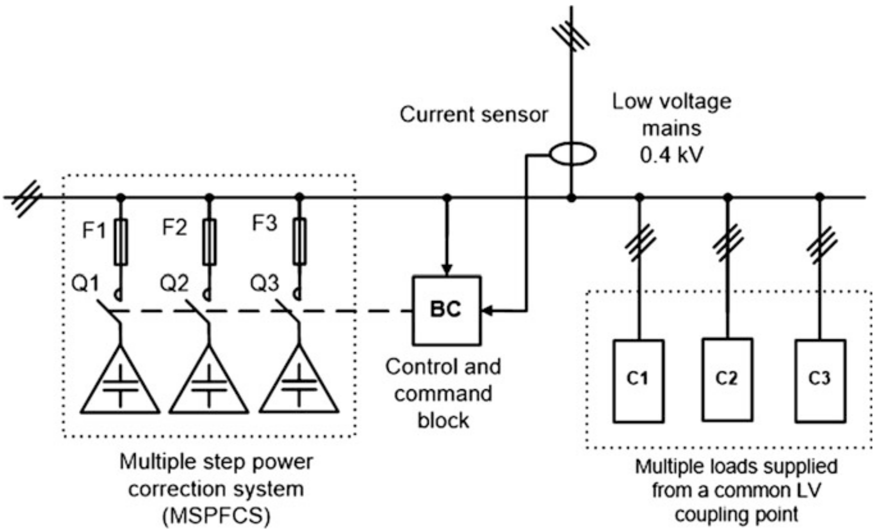


Fig. 2.21 The principle scheme for an automated controlled power factor in an installation

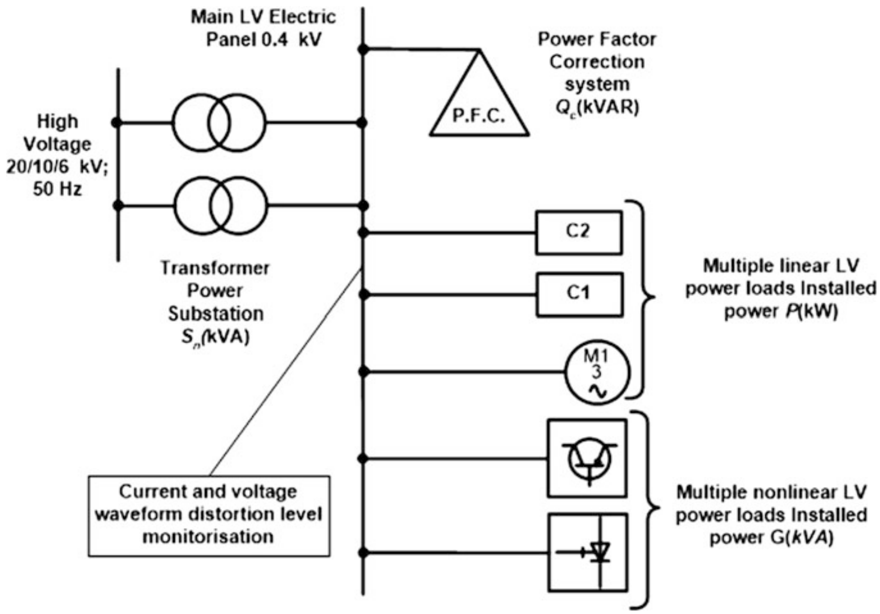


Fig. 2.22 Connecting the power factor correction system in a non-sinusoidal state system

(A) *Distorted installed power procedure*, whose principle is described in Figs. 2.22 and 2.23. The following notations are used:

– G is the sum of apparent powers taken for all pieces of equipment that generate harmonics (power static converters, inverters variable speed drives, etc.) connected to the mains where the capacitors bank is also connected. If, for some equipment the active power is given, to compute the apparent power one should take into consideration the covering PF.
– S_n is the sum of apparent powers taken for all system's power transformers to which the distribution mains belong.

The above method can be applied only for limited voltage and current harmonic distorted level ($THD_u < 5\%$) ($THD_i < 30\%$).
(B) *Voltage and current total harmonic distorted level determination method*, applicable when one knows exactly (by measurements) the values corresponding to THD_u respectively to THD_i. The principle of this method is presented in Fig. 2.24.

The selection methods indicate the use of (depending on the harmonic's distortion level): standards capacitor banks, capacitor banks with increased nominal voltage (usual with 10%), capacitor banks with increased nominal voltage and with detuned reactors or harmonic mitigation filters (active, passive or hybrid).

One should mention that in actual conditions, where the installation's load with voltage or especially current high-order harmonics are very frequent, it is used the

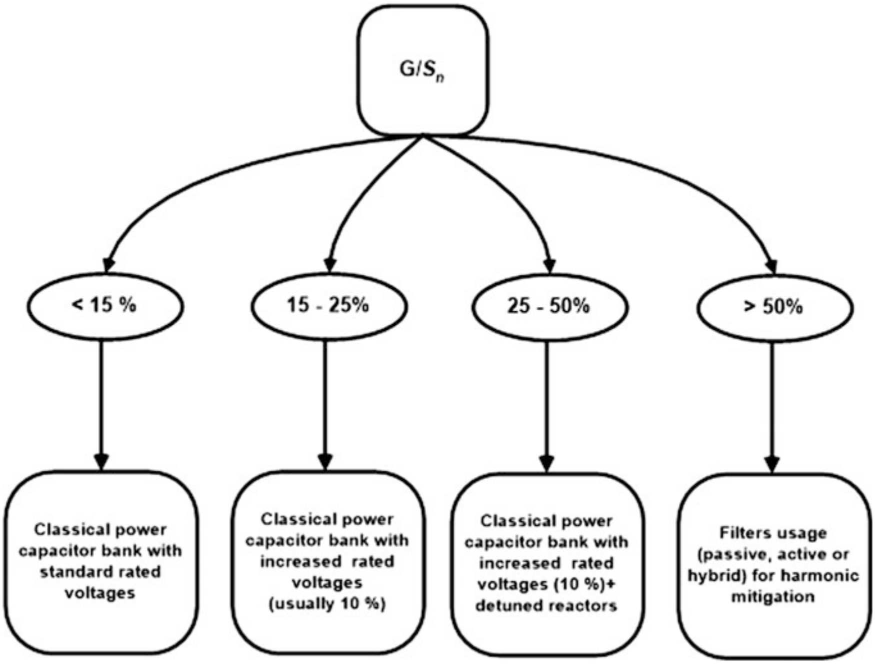


Fig. 2.23 Selecting the reactive power compensation possibility function of installation's nonlinear receivers' weight

solution (cheaper than that with active filters) of capacitor banks placed in series with certain coils also called "detuned reactors" shown in Fig. 2.25 [38–40]. The reactive power corresponding to these detuned reactors is chosen usually as a percent, called detuned reactor factor, of $p = 5$, 7 or 11% from the compensation reactive power of the bank. Therefore, by a correct selection of the detuned reactor factor, one may avoid the resonance on the dominant installation's harmonics and at the fundamental frequency the compensation function is satisfactory maintained. Moreover, the detuned reactors, that have values of mH order, together with the connecting conductors' inductivity, limit the commutation inrush currents in the case of banks from automatic power factor correction system (APFC). Moreover, the detuned reactors supplementary stress the capacitors' dielectric, reason for which these should be dimensioned at voltages at least 10% bigger than the nominal ones.

The reduction of the connecting current value is done by placing in series some very small inductivity coils (a few μH). For low-voltage banks where the harmonics do not have a consistent weight ($THD_{u,i} < 10\%$) also the connecting cable, out of which several turns are coiled, realizes a current limitation. So, only the conductivity of a cable of 10 m is sufficient to limit the current necessary to connect a bank up to 25 kVAr [29–31].

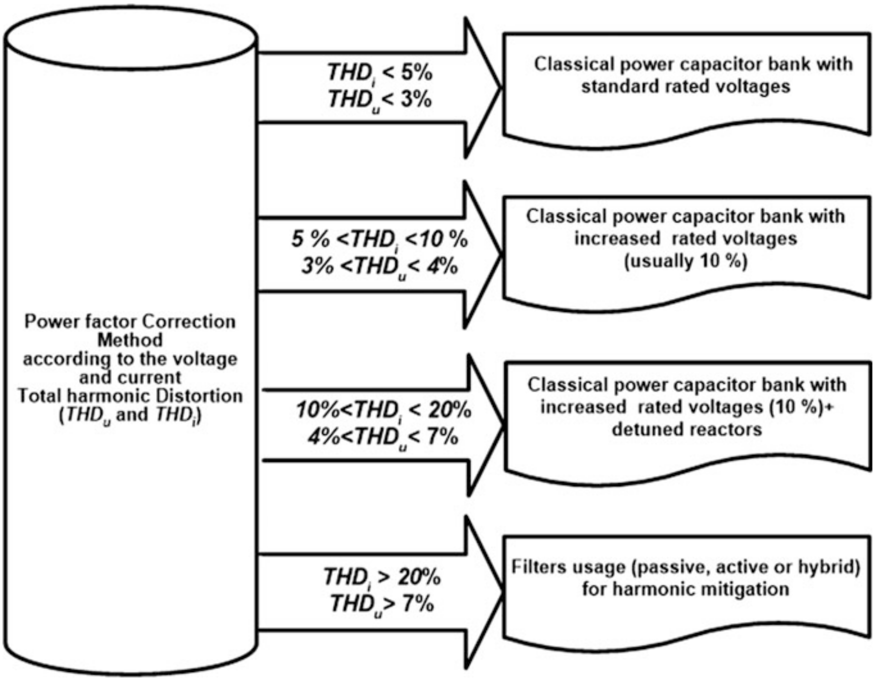
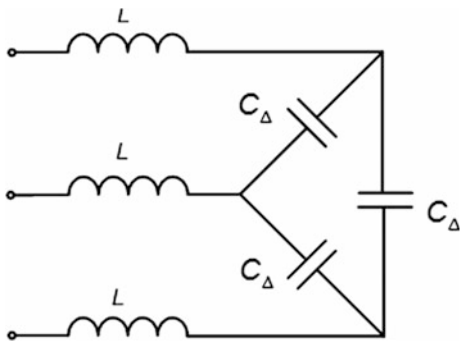


Fig. 2.24 Selection of reactive power compensation functions of installation's nonlinear loads' weight

Fig. 2.25 Connecting the detuned reactors in a Δ connection capacitors bank



These measures should determine the consumers to function with a power factor in exploitation at values very close to the nominal one and finally to lead to an increased average weighted power factor at least up to the neutral power factor value (usually 0.92).

2.4.4 Case Studies

Example 2.6 Let's consider an industrial load that contains linear power consumers (especially asynchronous single and three-phase power motors) for which the electric power quality parameters are presented in Fig. 2.26. The figures are captured by using a Fluke 435—a "class A" power quality analyzer instrument. As one can notice, from the main parameters analysis (current and voltage signals, harmonics distortions etc.), the system allows the connection of a capacitor bank.

Its power is evaluated taking into consideration both the value of the actual measured power factor ($PF_1 = \cos\varphi_1$), as well as that of the desired power factor (mostly is the value of the neutral factor value: $PF_2 = \cos\varphi_2 = 0.92$ or even 0.95). Knowing the active power $P = 223.7\,kW$ and choosing for $\cos\varphi_2 = 0.95$ we can determine the reactive capacitor bank value

$$Q_{bat} = P \cdot (\tan\varphi_1 - \tan\varphi_2) = 223.7 \cdot (0.619 - 0.203) \cong 93.2 \text{ kVAr}$$

The three-phase capacitors used to improve the PF can be placed in Δ or Y connection (Fig. 2.20). The bank's capacity on a phase results from the following relationship:

$$C_\Delta = \frac{Q_C}{\omega \cdot \left(U/\sqrt{3}\right)^2} = \frac{93.2 \cdot 10^3}{2 \cdot \pi \cdot 50 \cdot \left(400/\sqrt{3}\right)^2} = 5.56 \text{ mF}$$
$$C_\Delta = \frac{C_\Delta}{3} = \frac{Q_C}{\omega \cdot U^2} = \frac{93.2 \cdot 10^3}{2 \cdot \pi \cdot 50 \cdot (400)^2} = 1.85 \text{ mF}$$

So, the for the analyzed system case, one recommends an automatic power factor correction system (APFC) (in steps taking into consideration that energy can vary in large limits, function of the number of loads connected at the same time) of power value: $Q_{bat} = 95$ kVAr. A solution of the automatic capacitor bank can be realized in three steps of 30, 30 respectively 55 kVAr. With the new power factor (0.95), the current absorbed by the consumer becomes

$I_2 = I_1 \frac{PF_1}{PF_2} = 402 \cdot \frac{0.86}{0.95} = 363$ A, (the measured current one being $I_1 = 402$ A)

With this value, the losses in a three-phase copper power cable $3 \times$ CYY 2×120 mm^2, can be computed. The resistance on the Cu cable per unit length of section 120 mm^2 is $r_l = 0.154$ mΩ/m. But on each phase two such cables of length $l = 100$ m are used. Thus the total specific resistance is given by the parallel equivalent resistance of the cables

$$r_t = \frac{r_l}{2} = \frac{0.154}{2}\frac{\text{m}\Omega}{\text{m}} = 0.077\,\frac{\text{m}\Omega}{\text{m}}$$
$$R_0 = r_t \cdot l = 0.077 \cdot 100 = 7.7 \text{ m}\Omega$$

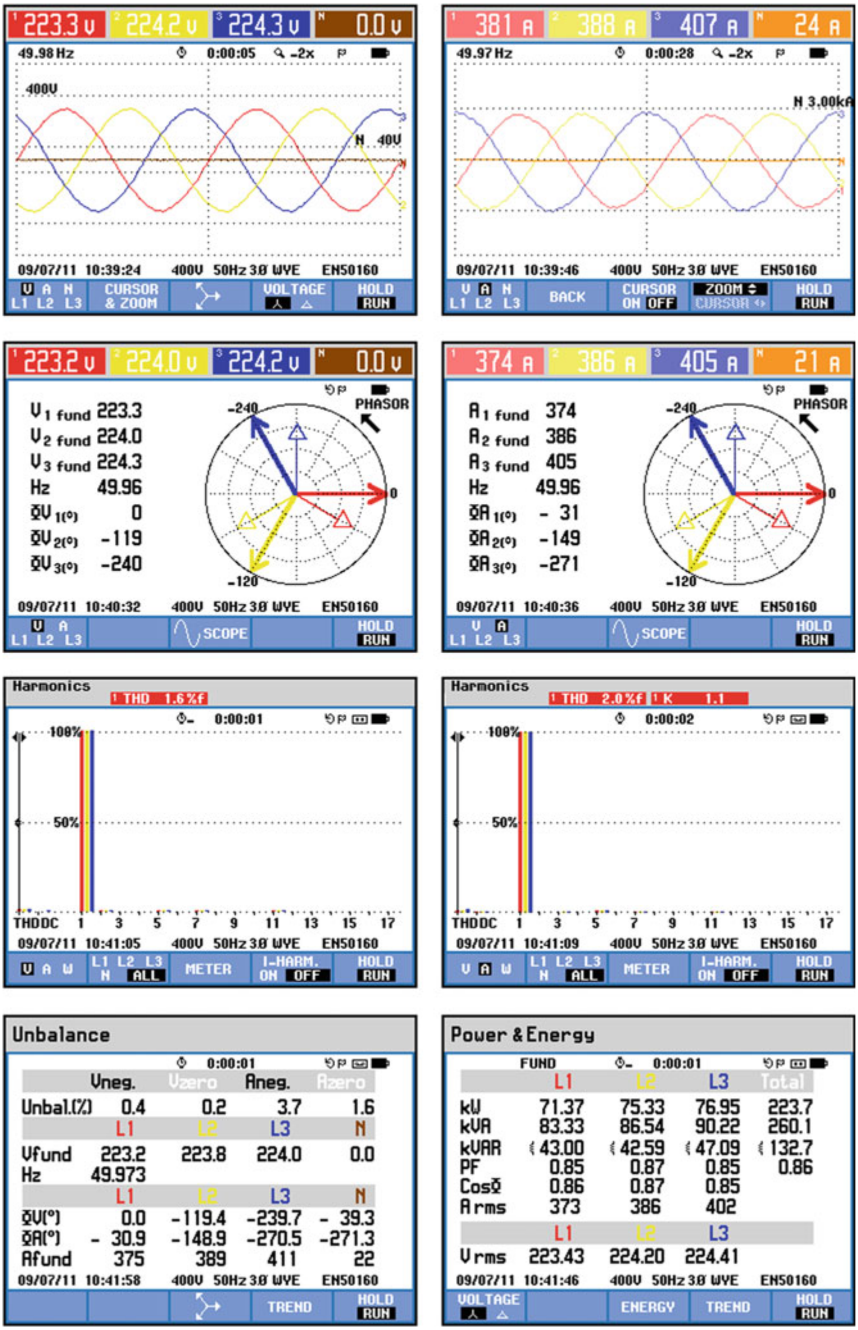


Fig. 2.26 The energy quality parameters of the industrial consumer under investigation

Considering the average working temperature of the cable θ_2 = 42 °C, (resulted eventually from analyzing the thermal image), we will compute the value of the conductors' resistance

$$R_L = R_0(1 + \alpha \cdot (\theta_2 - \theta_1)) = 7.7 \cdot (1 + 0.00339 \cdot (42 - 20)) = 8.27\ m\Omega$$

The power losses before and after compensating the reactive energy flow are:
Before

$$\Delta P_1 = 3 \cdot I_1^2 R_L = 3 \cdot 402^2 \cdot 8.27 \cdot 10^{-3} = 4\ \text{kW}$$

and after

$$\Delta P_2 = 3 \cdot I_2^2 R_L = \Delta P_1 \left(\frac{PF_2}{PF_1}\right)^2 = 4\left(\frac{0,86}{0,95}\right)^2 = 3.27\ \text{kW}$$

Results a consumer's power cable loss

$$\Delta P_1 - \Delta P_2 = 730\ \text{W}$$

or given in percent

$$\varepsilon = \frac{\Delta P_1 - \Delta P_2}{\Delta P_1} = 18.25\%$$

The power cable consumed energy can be correct evaluated if one knows the form factor of the current variation (on the most loaded phase), that is represented in Fig. 2.27.

With this information one will compute the average value I_m and the square average value I_{mp} of the current, and its corresponding form factor, respectively

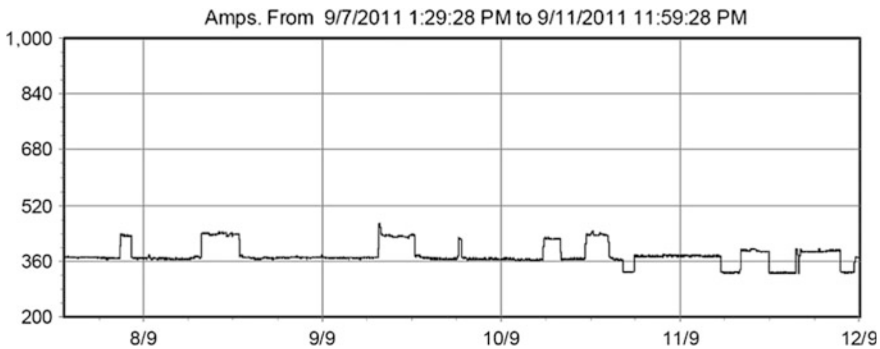


Fig. 2.27 The current variation on the consumer's most loaded phase taken on a monitor interval

$$I_m = 363\,\text{A},\, I_{mp} = \, 368\,\text{A},\, k_f = I_{mp}/I_m = 1.063$$

The energy for a 30 days period (720 h) before and respectively after compensation will be

$$\Delta E_1 = 3 \cdot k_f I_m^2 R_L \tau_f 10^{-3} = 3 \cdot 1.063 \cdot (363)^2 \left(8.27 \cdot 10^{-3}\right) \cdot 720 \cdot 10^{-3} = 2.502\ \text{MWh}$$
$$\Delta E_2 = 3 \cdot k_f \left(I_m \frac{PF_1}{PF_2}\right)^2 R_L \tau_f 10^{-3} = \left(\frac{PF_1}{PF_2}\right)^2 \Delta E_1 = 2.049\ \text{MWh}$$

It results a consumer's power cable losses reduction

$$\Delta E_1 - \Delta E_2 = 453\ \text{kWh}$$

or given in percent

$$\delta = \frac{\Delta E_1 - \Delta E_2}{\Delta E_1} = 18.10\%$$

2.5 New Principle of Minimum Active and Reactive Absorbed Power (PMARP) in AC Power Systems

In different real systems such as mechanics, thermodynamics, Earth climate system, hydrology, scientific studies have been carried out to express the balanced state of the systems in energy and/or power terms [41–50]. These studies are mostly based on the concept that for conservative systems it is possible to define suitable functionals in Hilbert space related to power and/or energy. An important step forward in order to define the functional optimization problems is to calculate their limits. On the other hand the studies cited below report that the steady state in mechanic, thermodynamic, hydrology or climate conservative system is an edge state, generally a minimum state, from the energy or power point of view.

The introduction of "content and co-content" function applied to non-linear resistors represented a milestone in electric circuits theory [51–53]. Authors like Penfield in [54] demonstrate for nonlinear resistive circuits the "variational principle" of "content and co-content" function, respectively Smith in [55], [56] analyses for linear passive circuits the average energy storage, the minimum energy and the dependence between electric and magnetic energy storage to one-port of circuit. Romanian scientists have important contributions to the minimum/maximum theory of power in electric circuits. For example, the application of Hilbert space functionals to power analysis in non-sinusoidal state is introduced by Ionescu in [57], the theorem of minimum power absorbed by resistances in DC circuits is demonstrated by Mocanu in [58], and the analysis and solving of electromagnetic field based on energy functional are defined by Hantila in [59] and Fireteanu in [60].

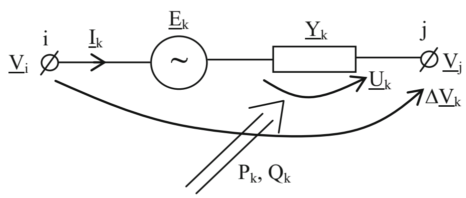
This section refers to new theorems (principles) of minimum active and reactive absorbed power (PMARP) for linear electric circuits in quasi-stationary regime which are based on power functionals and Hilbert space properties such in the recent contributions of authors [61–69]. On the other hand, based on analogy between linear electric and magnetic circuits in quasi-stationary regime, similar principles are described by authors in [70–74]. In this work for AC circuits, including non-sinusoidal regime, is demonstrated a complete set of principles of minimum absorbed active and reactive power, are defined the specific conditions and limits of their applications and finally are provided suggestive examples for each circuit. For all the types of AC circuits the principles demonstration are similar. As a starting point is the definition of power functionals and their expressing depending on the potential of circuit nodes which are considered variable.

In this respect the Kirchhoff voltage law (KVL) and the topology of circuit are used. Then by imposing the conditions for obtaining the minimum point of functionals (when existing), it is demonstrated that these equations are similar with Kirchhoff current law (KCL) and nodal method (NM). Consequently by using the power flow theory, either a complete system of Kirchhoff equations (KVL and KCL) is obtained from which one determines the circuit currents and voltages or a complete system of NM from which one computes the potentials of nodes. Or in other words, KCL equations can be substituted with those arising from PMARP. Likewise in this section is demonstrated the coexistence (CEAPP) of PMARP and maximum power transfer theorem (MPTT) for active power in AC circuits. Therefore PMARP can be considered as part of basic concept of circuit theory.

2.5.1 PMARP for Linear AC Circuit

Let us consider the common branch of a linear and reciprocal AC circuit under sinusoidal signal shown in Fig. 2.28. The circuit contains N nodes and K branches with voltage sources and RLC passive elements. In order to put in evidence the

Fig. 2.28 Representation of an AC circuit branch



power flow between the sources and passive elements, one defines in $\Re^{2(N-1)}$—dimensional Hilbert space, two functionals. The first one is the active power Eq. $\Im_P : \Re^{2(N-1)} \rightarrow \Re$, and the second one is the reactive power Eq. $\Im_Q : \Re^{2(N-1)} \rightarrow \Re$ defined as

$$\Im_P = \mathrm{Re}[\underline{u}^T \underline{i}^*] \tag{2.48}$$

$$\Im_Q = \mathrm{Im}[\underline{u}^T \underline{i}^*] \tag{2.49}$$

where $\underline{u}$ represents the K-dimensional vector in C^K of complex voltages at the branch admittances terminals and $\underline{\boldsymbol{i}}$ represents the K-dimensional vector in C^K of the branch complex currents. In this section the superscript * denotes the conjugate operator. Related to the branch voltages and current baseline direction presented in Fig. 2.28, the power Eqs. (2.48) and (2.49) mean: (i) $\Im_P$ is the *active power absorbed by all the passive elements' conductances (resistances) of the circuit*, respectively (ii) $\Im_Q$ is the *reactive power absorbed* or *generated by all the reactive elements (inductors and capacitors) of the circuit*. By using KCL equation for each k-branch $\Delta\underline{V}_k = \underline{V}_{i,k} - \underline{V}_{j,k} = \underline{U}_k - \underline{E}_k = \frac{\underline{I}_k}{\underline{Y}_k} - \underline{E}_k$, for $k = 1, \ldots, K$ and considering as variables the complex potentials of circuit nodes $\underline{V}_i = x_i + jy_i$, for $i = 1, \ldots, N-1$ $x_i, y_i \in \Re$, then $\Im_P$ and $\Im_Q$ can be expressed as

$$\begin{aligned}\Im_P(x,y) &= \mathrm{Re}[\underline{u}^T \underline{i}^*] = \mathrm{Re}[(\Delta\underline{v} + \underline{e})^T \underline{Y}^* (\Delta\underline{v} + \underline{e})^*] \\ &= \mathrm{Re}[(C\underline{V} + \underline{e})^T \underline{Y}^* (C\underline{V} + \underline{e})^*] \\ &= \sum_{\substack{k=1,K \\ i,j=1,N-1 \\ i \neq j}} G_k[(x_i - x_j + a_k)^2 + (y_i - y_j + b_k)^2]\end{aligned} \tag{2.50}$$

$$\begin{aligned}\Im_Q(x,y) &= \mathrm{Im}[\underline{u}^T \underline{i}^*] = \mathrm{Im}[(\Delta\underline{v} + \underline{e})^T \underline{Y}^* (\Delta\underline{v} + \underline{e})^*] \\ &= \mathrm{Im}[(C\underline{V} + \underline{e})^T \underline{Y}^* (C\underline{V} + \underline{e})^*] \\ &= \sum_{\substack{k=1,K \\ i,j=1,N-1 \\ i \neq j}} B_k[(x_i - x_j + a_k)^2 + (y_i - y_j + b_k)^2]\end{aligned} \tag{2.51}$$

where $\Delta\underline{v}$ is the K-dimensional vector in C^K of branch complex voltages, $\underline{\boldsymbol{e}}$ is the K—dimensional vector in C^K of branch voltage sources expressed as $\underline{E}_k = a_k + jb_k$, where a_k, b_k are considered constant, $\underline{Y} = \mathrm{diag}(\underline{Y}_1, \underline{Y}_2, \ldots, \underline{Y}_K)$ is the $K \times K$—dimensional diagonal matrix of branch admittances, where $\underline{Y}_k = G_k - jB_k$, $k = 1, \ldots, K$ with $G_k > 0$, $B_k > 0$ for inductive branches respectively $B_k \langle 0$ for capacitive branches, $\underline{V}$ is the reduced (N-1)-dimensional vector in C^{N-1} of nodes potentials, and $C = [c_{1,n}]$ is the $K \times (N–1)$-dimensional reduced branch-to-node incidence matrix whose elements $c_{1,n}$ can take the values −1, 0 or 1. The superscript T is used to indicate the transposition.

The next step of PMARP demonstration is to analyze the sign functionals defined in (2.50) and (2.51), and as a consequence of the nature, minimum or maximum, of thereof extreme point.

(α) Evidently if $G_k > 0$, then the active power Eq. is a quadratic form (strictly positive) $\Im_P(x,y)\rangle 0$, and its *extreme point is a minimum*. Therefore one can affirm that the *active power absorbed by all the passive elements (conductances) of the AC circuit is minimum*. By imposing the concurrent conditions $\partial\Im_P/\partial x_i = 0$ and $\partial\Im_P/\partial y_i = 0$, for $i = 1,\ldots,N-1$, then is obtained the minimum point of $\Im_P$. Related to the potential of the i-node of the circuit these derivative conditions are expressed as

$$\begin{aligned}\frac{\partial\Im_P}{\partial x_i} &= \frac{\partial}{\partial x_i}\mathrm{Re}[(C\underline{V}+\underline{e})^T\underline{Y}^*(C\underline{V}+\underline{e})^*] \\ &= 2\sum_{l_k\in n_i} c_{l_k,n_i}G_k(x_i - x_j + a_k) = 0\end{aligned} \tag{2.52}$$

$$\begin{aligned}\frac{\partial\Im_P}{\partial y_i} &= \frac{\partial}{\partial y_i}\mathrm{Re}[(C\underline{V}+\underline{e})^T\underline{Y}^*(C\underline{V}+\underline{e})^*] \\ &= 2\sum_{l_k\in n_i} c_{l_k,n_i}G_k(y_i - y_j + b_k) = 0\end{aligned} \tag{2.53}$$

for $k = 1,\ldots,K$, $i,j = 1,\ldots,N-1, i \neq j$.

(β) The sign of the reactive power Eq. depends on the sign of B_k: (β1) when all the branches of the AC circuit have an equivalent resistive-inductive admittance $B_k > 0$, $k = 1,\ldots,K$, then $\Im_Q(x,y) > 0$, so $\Im_Q$ is strictly positive and its *extreme point is a minimum*. Therefore one can affirm that the *reactive power absorbed by all the reactive elements of AC circuit is minimum*; (β2) when all the branches of AC circuit have an equivalent resistive-capacitive admittance $B_k < 0$, $k = 1,\ldots,K$, then $\Im_Q(x,y) < 0$. In this case reversing the sign of $\Im_Q$ and changing its significance from absorbed in "generated", then $-\Im_Q > 0$ is strictly positive and its *extreme point is a minimum*. Therefore one can affirm that the *reactive power generated by all the reactive elements of the AC circuit is minimum*; (β3) when the branch admittances of the circuit verify the particular case of resonance condition, then $\Im_Q(x,y) = 0$; (β4) when the circuit includes inductive and capacitive branches and is not possible to conclude about the sign of B_k then the extreme of $\Im_Q$ can not be fixed.

Related only to the cases (β1) and (β2) by imposing the concurrent conditions $\partial\Im_Q/\partial x_i = 0$ and $\partial\Im_Q/\partial y_i = 0$, for $i = 1,\ldots,N-1$ then is obtained the minimum point of $\Im_Q$. With respect to the potential of the i-node these derivative conditions are expressed as

$$\frac{\partial\Im_Q}{\partial x_i} = \frac{\partial}{\partial x_i}\mathrm{Im}[(C\underline{V}+\underline{e})^T\underline{Y}^*(C\underline{V}+\underline{e})^*] = 2\sum_{l_k\in n_i} c_{l_k,n_i}B_k(x_i - x_j + a_k) = 0 \tag{2.54}$$

$$\frac{\partial \Im_Q}{\partial y_i} = \frac{\partial}{\partial y_i} \mathrm{Im}[(C\underline{V} + \underline{e})^T \underline{Y}^* (C\underline{V} + \underline{e})^*] = 2 \sum_{l_k \in n_i} c_{l_k, n_i} B_k (y_i - y_j + b_k) = 0 \quad (2.55)$$

Finally, the minimum points of $\Im_P$ and $\Im_Q$ are the solutions of concurrent conditions (2.52)–(2.55) which means the below system with 4(N–1) Eqs.

$$\frac{\partial \Im_P}{\partial x_i} = \sum_{l_k \in n_i} c_{l_k, n_i} G_k (x_i - x_j + a_k) = 0; \quad \frac{\partial \Im_P}{\partial y_i} = \sum_{l_k \in n_i} c_{l_k, n_i} G_k (y_i - y_j + b_k) = 0$$
$$\frac{\partial \Im_Q}{\partial x_i} = \sum_{l_k \in n_i} c_{l_k, n_i} B_k (x_i - x_j + a_k) = 0; \quad \frac{\partial \Im_Q}{\partial y_i} = \sum_{l_k \in n_i} c_{l_k, n_i} B_k (y_i - y_j + b_k) = 0 \quad (2.56)$$

If we calculate the algebraic sum of Eq. (2.56), with the derivatives $\partial \Im_P / \partial y_i$ and $\partial \Im_Q / \partial x_i$ multiplied by j respectively $-j$, we will get

$$\sum_{l_k \in n_i} c_{l_k, n_i} \underline{Y}_k (\underline{V}_i - \underline{V}_j + \underline{E}_k) = 0 \quad (2.57)$$

Equation (2.57) represent for $i, j = 1, \ldots, N-1, i \neq j$ the Eq. of *NM* applied for $N-1$ nodes of the considered AC circuit. Withal by rewriting the Eq. (2.57) we will obtain

$$\sum_{l_k \in n_i} c_{l_k, n_i} \underline{I}_k = 0 \quad (2.58)$$

that represents even the *KCL Eqs*. for N–1 nodes of the considered AC circuit.

Taking into account those shown so far one can formulate the following Principle of Minimum Active and Reactive Power (PMARP): *"In reciprocal and linear AC circuits the condition of minimum active absorbed power in the resistances and minimum reactive absorbed (generated) power in the inductances (capacitances) is consistent with the NM and KCL"*. In more specific terms, "*in reciprocal and linear AC circuits: (a) the branch currents and voltages are distributed such as the absorbed active power and the absorbed (generated) reactive power in the resistive-inductive (resistive-capacitive) branches of the circuit are minimum; (b) PMARP along with KVL constitute the basic forms of linear and independent Eqs. system of AC circuits analysis; (c) PMARP is consistent with NM*".

Also it is possible to analyze together PMARP and MPTT in what concerns the active power flow in AC circuits [75]. First according to MPTT let's consider that certain AC circuit branches—named load—verify the conditions for maximum active power transfer. In terms of PMARP that means the case (β3) when $\Im_Q(x, y) = 0$ and the active power Eq. $\Im_P(x, y)$ has a minimum. For these reasons can be formulated the Co-Existence of Active Power Principles (CEAPP) for linear

AC circuits as: “*in linear and reciprocal AC circuits the branches currents and voltages comply simultaneously PMARP and MPTT when one of the branch admittance verifies the MPTT condition*”.

2.5.2 *PMARP for Linear AC Circuits Under Non-sinusoidal Conditions*

The concepts illustrated above can be extended to the case of AC circuits operating in non-sinusoidal regime. A similar linear circuit with K branches and N nodes is considered but in this case operating under non-sinusoidal conditions with radian frequency ω at the fundamental harmonic. For each branch of the circuit, the voltage and the current are expressed by using the Fourier series

$$u_k(t) = \sum_{h=0}^{H} U_k^{(h)} \sqrt{2} \sin(h\omega t + \varphi_k^{(h)}) \tag{2.59}$$

$$i_k(t) = \sum_{h=0}^{H} I_k^{(h)} \sqrt{2} \sin(h\omega t + \varphi_k^{(h)} - \gamma_k^{(h)}) \tag{2.60}$$

where $k = 1, \ldots, K$ is the number of branches, H is the finite number of harmonics considered, $U_k^{(h)}$ and $I_k^{(h)}$ represent the rms values of the hth harmonic of voltage and current, respectively, while $\varphi_k^{(h)}$ and $\varphi_k^{(h)} - \gamma_k^{(h)}$ represent the phase angle of the hth harmonic of voltage and current, respectively. Considering branch k of the circuit (Fig. 2.29), by using the complex representation for the hth harmonic of voltage and current, with the same conventional circuit representation adopted in Sect. 2.5.1 for AC circuits in sinusoidal regime, we get

$$\underline{\Delta V}_k^{(h)} = \underline{V}_{i,k}^{(h)} - \underline{V}_{j,k}^{(h)} = \underline{U}_k^{(h)} - \underline{E}_k^{(h)} = \frac{\underline{I}_k^{(h)}}{\underline{Y}_k^{(h)}} - \underline{E}_k^{(h)} \tag{2.61}$$

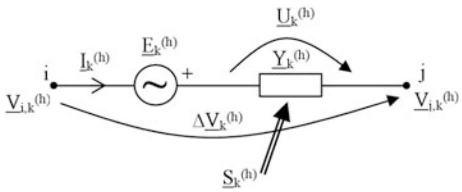


Fig. 2.29 AC circuit under non-sinusoidal conditions

In the Hilbert space we can define, similarly with Eqs. (2.50) and (2.51), the functionals corresponding to the hth harmonic of active and reactive absorbed power by all the K branches. We obtain the Eqs. $\Im_R^{(h)}:\Re^{2(N-1)}\to\Re$ and $\Im_Q^{(h)}:\Re^{2(N-1)}\to\Re$ such that, respectively:

$$\begin{aligned}\Im_R^{(h)}(x_1^{(h)},\ldots,x_{N-1}^{(h)},y_1^{(h)},\ldots,y_{N-1}^{(h)}) &\equiv \mathrm{Re}[\bar{S}^{(h)}]\\ &=\sum_{k=1}^{K}G_k^{(h)}[(x_{i,k}^{(h)}-x_{j,k}^{(h)}+a_{E,k}^{(h)})^2+(y_{i,k}^{(h)}-y_{j,k}^{(h)}+b_{E,k}^{(h)})^2]\end{aligned}\tag{2.62}$$

$$\begin{aligned}\Im_Q^{(h)}(x_1^{(h)},\ldots,x_{N-1}^{(h)},y_1^{(h)},\ldots,y_{N-1}^{(h)}) &\equiv \mathrm{Im}[\bar{S}^{(h)}]\\ &=\sum_{k=1}^{K}B_K^{(h)}[(x_{i,k}^{(h)}-x_{j,k}^{(h)}+a_{E,k}^{(h)})^2+(y_{i,k}^{(h)}-y_{j,k}^{(h)}+b_{E,k}^{(h)})^2]\end{aligned}\tag{2.63}$$

These equations are a function class C^2 in $\Re^{2(N-1)}$. Considering $G_k^{(h)}>0$ for $k=1,\ldots,K$ the real equation of the complex power is always positive defined, that is, $\Im_R^{(h)}(x_1^{(h)},x_2^{(h)},\ldots,x_{N-1}^{(h)},y_1^{(h)},y_2^{(h)},\ldots,y_{N-1}^{(h)})>0$, for all the pairs $(x_i^{(h)},y_i^{(h)})$ with $i=1,\ldots,N-1$ and $h=1,\ldots,H$; then, *the active absorbed power has a minimum*. The reactive power equation can be analyzed, for all the pairs $(x_i^{(h)},y_i^{(h)})$, $i=1,\ldots,N-1$, and for each harmonic $h=1,\ldots,H$ as single-phase AC circuits described in Sect. 2.5.1 for AC circuits. Similarly, such as in case (β1) with resistive-inductive branches, the Eq. (2.63) is *positive defined*, thus the *total reactive power absorbed by the inductive components has a minimum.* Furthermore, likewise case (β2) with resistive-capacitive branches by changing the sign and significance of the imaginary functional, in order to obtain a *positive defined* functional and to conclude that the *total reactive power generated by the capacitive components has a minimum.* For the general case in which reactive components of different nature—inductive and capacitive—coexist at the given harmonic $h=1,\ldots,H$, the sign of Eq. $\Im_Q^{(h)}$ cannot be fixed and the extreme of $\Im_Q^{(h)}$ cannot be stated.

Related only to the cases (β1) and (β2), the minimum point of $\Im_Q$ is obtained from the below 4(*N*–1) Eqs. system that results by imposing to the Eqs. (2.62) and (2.63) the derivative concurrent conditions for extreme

$$\begin{aligned}\frac{\partial\Im_R^{(h)}}{\partial x_i^{(h)}}=0,\quad \frac{\partial\Im_R^{(h)}}{\partial y_i^{(h)}}=0\\ \frac{\partial\Im_Q^{(h)}}{\partial x_i^{(h)}}=0,\quad \frac{\partial\Im_Q^{(h)}}{\partial y_i^{(h)}}=0\end{aligned}\tag{2.64}$$

where $i=1,\ldots,N-1$ and $h=1,\ldots,H$.

By calculating the algebraic sum of the solutions, with the derivatives taken with respect to the imaginary parts multiplied by ($-j$), in the same fashion as seen in AC examples, we obtain the expressions

$$\sum_{l_k \in n_i} c_{l_k,n_i} \underline{Y}_k^{(h)} (\underline{V}_i^{(h)} - \underline{V}_j^{(h)} + \underline{E}_k^{(h)}) = 0 \tag{2.65}$$

$i, j = 1, \ldots, N-1, i \neq j$, $k = 1, \ldots, K$ that represent NM Eqs. for hth harmonic expressed at $N-1$ nodes of the circuit. On the other hand by using in (2.65) Ohm's law for each circuit branch then results

$$\sum_{l_k \in n_i} c_{l_k,n_i} \underline{I}_k^{(h)} = 0 \tag{2.66}$$

which are the KCL equations for the hth harmonic, expressed for $N-1$ nodes of the circuit.

Taking into account those shown so far, one can formulate the following *Principle of Minimum Active and Reactive Power (PMARP)*: "*In reciprocal and linear AC circuits the condition of minimum active absorbed power in the resistances and minimum reactive absorbed (generated) power in the inductances (capacitances) is consistent with the NM and KCL*". In more specific terms, "*in reciprocal and linear AC circuits: (a) the branch currents and voltages are distributed such as the absorbed active power and the absorbed (generated) reactive power in the resistive-inductive (resistive-capacitive) branches of the circuit are minimum; (b) PMARP along with KVL constitute the basic forms of linear and independent equations system of AC circuits analysis; (c) PMARP is consistent with NM*".

Consequently, we may state the *Principle of Minimum Active and Reactive absorbed Power in linear circuits under Non-sinusoidal conditions (PMARPN)*: "*In linear and reciprocal circuits under non-sinusoidal conditions, for each h*th *harmonic, the minimum of the active absorbed power and minimum of the reactive absorbed (generated) power by the inductances (capacitances) is consistent with the NM and KCL*". Another description of the same principle can be stated as follows: "*In the linear and reciprocal circuits under non-sinusoidal periodic signals for each h*th *harmonic: (a) the branch currents and voltages are distributed such as the absorbed active power and the absorbed (generated) reactive power in the resistive-inductive (resistive-capacitive) branches of the circuit are minimum; (b) PMARPN along with KVL constitute the basic forms of linear and independent Eqs. system of AC circuits analysis; (c) PMARPN is consistent with NM*".

It is essential to remark that the same observations regarding the limits of applicability of the minimum power principle indicated in Section for AC circuits hold at every harmonic order. Thus, the minimum power principle applies when no mixture of inductive and capacitive components appears in the circuit at any frequency. Also a similar CEAPP can be stated for each frequency of AC circuits in non-sinusoidal conditions.

The existence of the minimum power condition is verified if, starting from all inductive (or capacitive) reactive components, no resonance conditions is reached in the circuit at any frequency in the observed range, since the reactive nature of the circuit would change above the resonance frequency [76]. This could be the case of filters containing both inductive and capacitive components, whose equivalent admittance exhibits capacitive behavior at relatively low frequency up to the resonance conditions, and inductive behavior at frequencies higher than the one corresponding to the resonance conditions.

2.5.3 Practical Examples

Example 2.7 demonstration of PMARP for the AC circuit shown in Fig. 2.30 with $N = 2$ nodes and $K = 3$ resistive-inductive branches. The admittances contain only resistive and inductive elements, that is, $G_k > 0$ and $B_k > 0$ for $k = 1, 2, 3$, satisfying the conditions imposed in case β1. The real and imaginary parts of nodes potentials are considered variables, while the voltage source is constant. Then, by applying KVL the expressions of the complex conjugated currents of the circuit branches are

$$\begin{aligned}
\underline{V}_1 &= x_1 + jy_1;\ \underline{V}_2 = x_2 + jy_2;\ \underline{E}_1 = a + jb \\
\underline{I}_1^* &= (G_1 + jB_1)[(x_2 - x_1 + a) - j(y_2 - y_1 + b)] \\
\underline{I}_2^* &= (G_2 + jB_2)[(x_1 - x_2) - j(y_1 - y_2)] \\
\underline{I}_3^* &= (G_3 + jB_3)[(x_1 - x_2) - j(y_1 - y_2)]
\end{aligned}$$

The active and reactive absorbed powers of the circuit branches elements expressed by the Eqs. (2.50) and (2.51) are positive defined $\Im_R > 0$, $\Im_Q > 0$ in the forms

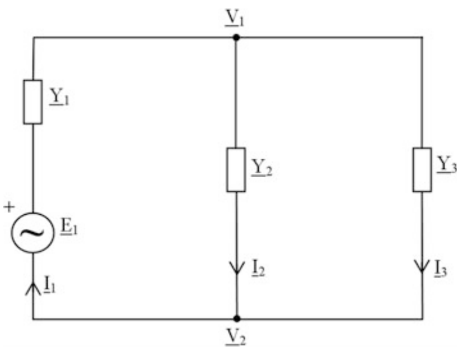


Fig. 2.30 Resistive-inductive AC circuit

$$\begin{aligned}\Im_R &= \Big\{G_1\Big[(x_2-x_1+a)^2+(y_2-y_1+b)^2\Big]+G_2\Big[(x_1-x_2)^2+(y_1-y_2)^2\Big]\\&\quad+G_3\Big[(x_1-x_2)^2+(y_1-y_2)^2\Big]\Big\}\\\Im_Q &= \Big\{B_1\Big[(x_2-x_1+a)^2+(y_2-y_1+b)^2\Big]+B_2\Big[(x_1-x_2)^2+(y_1-y_2)^2\Big]\\&\quad+B_3\Big[(x_1-x_2)^2+(y_1-y_2)^2\Big]\Big\}\end{aligned}$$

Related only to x_1 and y_1 as variables, the minimum of the Eqs. $\Im_R$ and $\Im_Q$ is obtained by imposing the derivative concurrent conditions for extreme as the solution of the 4-Eqs. system

$$\begin{aligned}\frac{\partial\Im_R}{\partial x_1}&=-G_1(x_2-x_1+a)+G_2(x_1-x_2)+G_3(x_1-x_2)=0\\\frac{\partial\Im_R}{\partial y_1}&=-G_1(y_2-y_1+b)+G_2(y_1-y_2)+G_3(y_1-y_2)=0\\\frac{\partial\Im_Q}{\partial x_1}&=-B_1(x_2-x_1+a)+B_2(x_1-x_2)+B_3(x_1-x_2)=0\\\frac{\partial\Im_Q}{\partial y_1}&=-B_1(y_2-y_1+b)+B_2(y_1-y_2)+B_3(y_1-y_2)=0\end{aligned}$$

On the other hand, by summing up the first equation of the system and the third equation multiplied by $(-j)$, and summing up the second equation with the fourth equation multiplied by $(-j)$, we obtain KCL expressed at node 1, with $L^{(1)}=\{1,\,2,\,3\}$

$$-\underline{I}_1^*+\underline{I}_2^*+\underline{I}_3^*=0$$

Withal the system above represents the NM expressed in node 1.

A similar result is obtained if the variables are considered the real part x_2 and imaginary part y_2 of the potential $\underline{V}_2$.

Example 2.8 Demonstration of the PMARP for a resistive-capacitive AC circuit similar to that shown in Fig. 2.30. Unlike the previous circuit, the admittances of this circuit contain only resistive and capacitive elements, that is $G_k>0$ and $B_k<0$ for $k=1,2,3$, satisfying the conditions imposed in case β2. The expressions of the potentials, voltage source, and complex conjugated currents of the circuit are the same as in example 2.7. Also the Eqs. (2.50) and (2.51) have the same relations as in the precedent example, where the real functional is always positive defined $\Im_R>0$ but the imaginary functional is negative $\Im_Q<0$. Then, if changing the sign of the reactive power functional, one achieves "positive" sign for $-\Im_Q$ and its significance is changing in *reactive generated power by all the capacitive elements which have a minimum.* Assuming x_1 and y_1 as variables, the minimum of the equation $\Im_R$, and of the equation $\Im_Q$ with reversed sign, is the solution of the system

$$\frac{\partial \Im_R}{\partial x_1} = -G_1(x_2 - x_1 + a) + G_2(x_1 - x_2) + G_3(x_1 - x_2) = 0$$
$$\frac{\partial \Im_R}{\partial y_1} = -G_1(y_2 - y_1 + b) + G_2(y_1 - y_2) + G_3(y_1 - y_2) = 0$$
$$\frac{\partial(-\Im_Q)}{\partial x_1} = -[-B_1(x_2 - x_1 + a) + B_2(x_1 - x_2) + B_3(x_1 - x_2)] = 0$$
$$\frac{\partial(-\Im_Q)}{\partial y_1} = -[-B_1(y_2 - y_1 + b) + B_2(y_1 - y_2) + B_3(y_1 - y_2)] = 0$$

Similarly as in the previous example, we obtain the KCL and NM expressed at node 1, with $L^{(1)} = \{1, 2, 3\}$

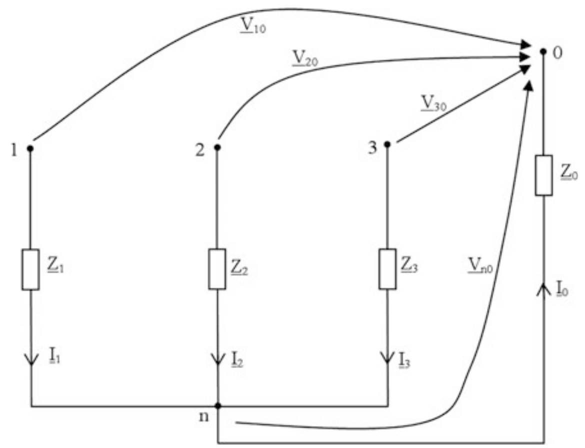
$$-\underline{I}_1^* + \underline{I}_2^* + \underline{I}_3^* = \sum_{k \in L^{(1)}} \underline{I}_k^* = 0$$

Example 2.9 Demonstration of PMARP for a three-phase star circuit. Let us consider the star circuit with neutral point n shown in Fig. 2.31, where the branch impedances are modeled as series-connected resistances and inductances, and can be transformed into equivalent admittances $\underline{Y}_k = 1/\underline{Z}_k = G_k - jB_k$, for $k = 0, 1, 2, 3$. For the sake of representation, the values of $B_k > 0$ for $k = 1,\ 2, 3$ satisfy the conditions imposed in case β1. We start from known three-phase symmetrical voltages of the system

$$\underline{U}_{10} = U; \quad \underline{U}_{20} = -a - jb; \quad \underline{U}_{30} = -a + jb$$

and if we consider the neutral point displacement voltage $\underline{U}_{n0} = x + jy$ as variable, then the Eqs. (2.50) and (2.51) power can be expressed as

Fig. 2.31 Three-phase circuit with star connection



$$\Im_{\mathrm{R}} = \left\{ G_1[(U-x)^2 + y^2] + G_2[(-a-x)^2 + (-b-y)^2] + G_3[(-a-x)^2 + (b-y)^2] + G_0(x^2+y^2) \right\}$$

respectively

$$\Im_Q = \left\{ B_1[(U-x)^2 + y^2] + B_2[(-a-x)^2 + (-b-y)^2] + B_3[(-a-x)^2 + (b-y)^2] + B_0(x^2+y^2) \right\}$$

The minimum of the Eqs. $\Im_R$ and $\Im_Q$ is given by the solution of the system containing 4-Eqs.

$$\frac{\partial \Im_R}{\partial x} = 0, \frac{\partial \Im_R}{\partial y} = 0, \frac{\partial \Im_Q}{\partial x} = 0, \frac{\partial \Im_Q}{\partial y} = 0$$

so that

$$\begin{aligned}
\frac{\partial \Im_R}{\partial x} &= -G_1(U-x) - G_2(-a-x) - G_3(-a-x) + G_0x = 0; \\
\frac{\partial \Im_R}{\partial y} &= G_1y - G_2(-b-y) - G_3(b-y) + G_0y = 0 \\
\frac{\partial \Im_Q}{\partial x} &= -B_1(U-x) - B_2(-a-x) - B_3(-a-x) + B_0x = 0; \\
\frac{\partial \Im_Q}{\partial y} &= B_1y - B_2(-b-y) - B_3(b-y) + B_0y = 0
\end{aligned}$$

From this system, by summing up the first equation and third equation multiplied by $(-j)$, respectively by summing up second equation with fourth equation multiplied by $(-j)$, is obtained

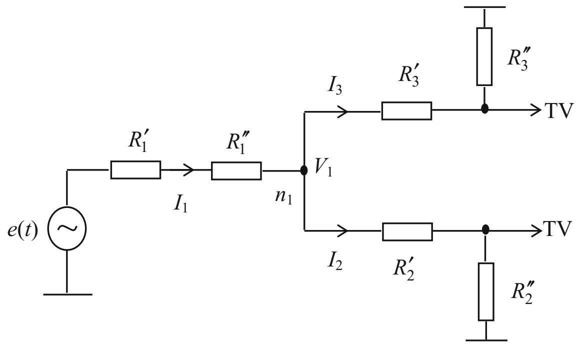
$$x = \frac{(G_1 - jB_1)\,U - (G_2 - jB_2)\,a - (G_3 - jB_3)\,a}{(G_0 - jB_0) + \sum\limits_{k=1}^{3} (G_k - jB_k)}$$

and

$$y = \frac{-(G_2 - jB_2)\,b + (G_3 - jB_3)\,b}{(G_0 - jB_0) + \sum\limits_{k=1}^{3} (G_k - jB_k)}$$

The outcomes of above equations reproduce the well-known relation

Fig. 2.32 The "splitter"



$$\underline{U}_{n0} = x + jy = \frac{\sum_{k=1}^{3} (G_k - jB_k)\underline{U}_{k0}}{(G_0 - jB_0) + \sum_{k=1}^{3} (G_k - jB_k)}$$

that incorporates KCL expressed at node n.

Example 2.10 Let's consider a linear circuit—"splitter"—which is used in TV cables as power amplifiers for signals, shown in Fig. 2.32. The instantaneous values of non-sinusoidal voltage source have the practical form:

$$e(t) = 0.27386 + 0.27386\sqrt{2}\sin(2\pi 40 \cdot 10^6 t) + 0.27386\sqrt{2}\sin(2\pi 900 \cdot 10^6 t)\ \mathrm{V}$$

If one considers the potential V_1 variable, for each harmonic of the voltage source we can calculate the minimum of the absorbed power equations.
(a) The absorbed power equation for DC regime of splitter can be expressed by

$$\Im_R^{(0)} = \frac{1}{2}\left[G_1(-V_1^{(0)} + E^{(0)})^2 + G_2 V_1^{(0)^2} + G_3 V_1^{(0)^2}\right]\rangle 0, \forall V_1^{(0)} \in \Re$$

where $G_1 = \frac{1}{R_1' + R_1''}$, $G_2 = \frac{1}{R_2' + R_2''}$, $G_3 = \frac{1}{R_3' + R_3''}$ do not depend on the frequency. Then, the minimum point of functional is obtained when

$$\frac{\partial \Im_R^{(0)}}{\partial V_1^{(0)}} = -G_1(-V_1^{(0)} + E^{(0)}) + G_2 V_1^{(0)} + G_3 V_1^{(0)}) = 0$$

relation that is identical with KCL expressed in node 1.
(b) If we consider the potential $\underline{V}_1^{(p)} = x + jy^{(p)}$, for the two harmonics, $p_1 = 8 \cdot 10^5$ and $p_2 = 18 \cdot 10^6$, then for the AC regime of the splitter the functional of the complex absorbed power is

$$\Im_R^{(p)} = G_1\left[(-x^{(p)} + E^{(p)})^2 + y^{(p)2}\right] + G_2(x^{(p)2} + y^{(p)2}) + G_3(x^{(p)2} + y^{(p)2})\rangle 0, \forall (x^{(p)}, y^{(p)}) \in \Re$$

Then, the minimum point of the active power Eq. $\Im_R$ is the solution of the system

$$\frac{\partial \Im_R^{(p)}}{\partial x^{(p)}} = -G_1(-x^{(p)} + E^{(p)}) + G_2 x^{(p)} + G_3 x^{(p)} = 0$$
$$\frac{\partial \Im_R^{(p)}}{\partial y^{(p)}} = G_1 y^{(p)} + G_2 y^{(p)} + G_3 y^{(p)} = 0$$

and, if we calculate the algebraic sum of the solutions of the system above, with the second equation multiplied with $-j$, results KCL expressed in node 1

$$-\underline{I}_1^{(p)^*} + \underline{I}_2^{(p)^*} + \underline{I}_3^{(p)^*} = 0$$

The imaginary part of power equation is zero, $\Im_Q^{(p)} = 0$. By using a Multisim measurement procedure, shown in Fig. 2.33a–c, the absorbed active power for different values of the resistances is calculated. The results are shown in Table 2.1. The first line contains the nominal values of conductance, and the second line of the table contains the adapted values of all the resistances, i.e. each resistance absorbs from the circuit the maximum active power. In this case, the value of the absorbed active power is big compared with the other values.

Example 2.11 Demonstration of CEAPP for AC circuit presented in Fig. 2.34, where the numerical values of elements are $\underline{E}_1 = 100$ V, $\underline{Z}_1 = \underline{Z}_2 = 10(1 + j)\,\Omega$. The load impedance $\underline{Z}_L = 5(1 + j)\,\Omega$ was selected so as to verify MPTT.

Assuming that the potential of node 1, $\underline{V}_1 = x_1 + jy_1$ is variable, then the power Eqs. (2.50) and (2.51) are expressed as follows

$$\Im_P = \frac{1}{20}\left[(-x_1 + 100)^2 + (-y_1)^2\right] + \frac{1}{20}\left[(-x_1)^2 + (-y_1)^2\right] + \frac{1}{10}\left[(x_1)^2 + (y_1)^2\right]$$
$$\Im_Q = \frac{1}{20}\left[(-x_1 + 100)^2 + (-y_1)^2\right] + \frac{1}{20}\left[(-x_1)^2 + (-y_1)^2\right] + \frac{1}{10}\left[(x_1)^2 + (y_1)^2\right]$$

By applying the extreme conditions for $\Im_R$ and $\Im_Q$ then results

$$\frac{\partial \Im_P}{\partial x_1} = -\frac{-x_1 + 100}{20} - \frac{-x_1}{20} + \frac{x_1}{10} = 0; \frac{\partial \Im_P}{\partial y_1} = -\frac{-y_1}{20} - \frac{-y_1}{20} + \frac{y_1}{10} \equiv 0$$
$$\frac{\partial \Im_Q}{\partial x_1} = -\frac{-x_1 + 100}{20} - \frac{-x_1}{20} + \frac{x_1}{10} = 0; \frac{\partial \Im_Q}{\partial y_1} = -\frac{-y_1}{20} - \frac{-y_1}{20} + \frac{y_1}{10} \equiv 0$$

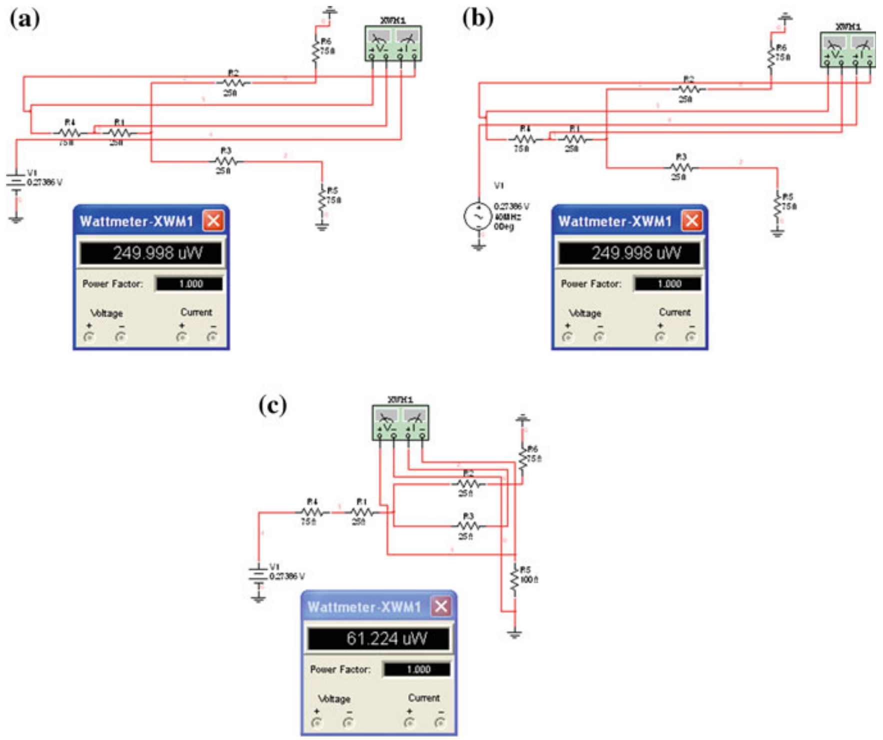
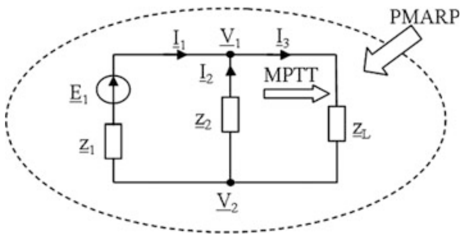


Fig. 2.33 Multisim analysis of power absorbed by, **a** R_1' in DC regime, **b** R_1'' in AC regime, **c** R_2'' in DC regime

Table 2.1 Absorbed active power for different values of the resistances

Values of conductances (S)	Total absorbed power in DC and AC (mW)
$G_1 = G_2 = G_3 = 0.01$	499.593
$G_1 = 0.01$; $G_2 = 0.013$; $G_3 = 0.01$	524.818
$G_1 = 0.01$; $G_2 = 0.02$; $G_3 = 0.025$	1.662
$G_1 = 0.01$; $G_2 = 0$; $G_3 = 0.01$	374.832
$G_1 = 0.01$; $G_2 = 0.04$; $G_3 = 0.01$	624.810
$G_1 = 0.02$; $G_2 = 0.01$; $G_3 = 0.01$	749.664

Fig. 2.34 AC circuit with CEAPP



From the above system by summing up the first equation and the second equation multiplied by (j), and the third equation multiplied by ($-j$) and the fourth equation results the equation of *NM* expressed at node 1

$$-\frac{1-j}{20}[(-x_1+100)+j(-y_1)]-\frac{1-j}{20}[(-x_1)+j(-y_1)]+\frac{1+j}{10}[x_1+jy_1]=0$$

Furthermore, if we rewrite the NM relation we'll obtain KCL at node 1

$$-\underline{I}_1-\underline{I}_2+\underline{I}_3=0$$

For the numerical value $\underline{V}_1 = 25\text{V}$ which results from KVL, the total absorbed active power is $P_{abs,tot}$ = 125 W. This numerical value coincides with the minimum value of active power functional (PMARP) by solving the above system. Also for these numerical values of the circuit elements as consequences of the MPTTT it results: (i) the reactive power is null (resonant conditions, case β3), (ii) the load absorbs maximum active power $P_{L\max}$ = 62.5 W, and (iii) the efficiency of the circuit in what concerns the flow of active power from circuit to load is 50%. Thus the conditions of CEAPP are verified.

2.6 Conclusion

The Principle of Minimum absorbed Power (PMP) is extended and demonstrated from DC to AC circuits (PMARP) under sinusoidal and non-sinusoidal signals. Also from active power flow is proved the co-existence of PMARP and MPTT for AC circuits. In other works as [61–74] for magnetic circuits in stationary and quasi-stationary regime the authors introduced and proved the Principle of Minimum absorbed Energy (PME).

From a theoretical point of view, the introduction of these principles for AC linear and reciprocal circuits in quasi-stationary state is extremely useful in order to formulate their basic system of equations related to the flow of active and reactive power. On the other hand, all these principles are formulated for natural distribution of the potentials, currents and voltages of circuit that verify the fundamental laws KCL and KVL, which approaches the proposed principles to the basic theory of circuits.

The examples presented above that refer to linear and reciprocal AC circuits, prove the introduced theoretical concepts and reveal the direct dependence between the power flow with the classical theory of electric circuits.

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