Chapter 2
Modelling Systems with Faults

This book is intended as a guide to synthesis methods of residual generators for various fault detection problems. A main nontrivial task to be fulfilled before employing any of these methods is the development of adequate models which fit into the envisaged design methodologies. Our interest lies primarily in developing synthesis methods for the class of linear time-invariant (LTI) plant models, for which the resulting residual generators are themselves LTI systems or filters. In spite of this apparent limitation, a wide class of fault detection problems can be addressed either by appropriate adjustments of the underlying models or by suitable reformulations of the design objectives.

In this chapter, we address the basic aspects of modelling systems with faults. Two basic approaches are described to model systems with faults. The first approach involves models with *additive faults*, where the faults are explicitly defined as fictive inputs which act on the system similarly to the unknown external disturbance inputs. The main advantage of this modelling approach is that, by avoiding the explicit modelling of different fault modes, a single model can be used to account for many possible physical faults. For example, models with additive faults are widely used to describe systems with various types of actuator and sensor faults. Models with *multiplicative faults* often describe systems with *parametric faults* (i.e., abnormal variations of some model parameters).

The second approach is based on *physical fault models*, where to each fault mode corresponds a dedicated model, which is usually derived by adjusting appropriately the non-faulty system model. For example, parametric faults can alternatively be modelled using physical models by setting the model parameters to some abnormal values. In other cases, physical fault models can be derived by removing some of system control inputs in the case of total loss of control or defining new disturbance inputs to account for specific fault effects.

The physical fault modelling approach typically involves *multiple models*, representing a collection of individual models where each model corresponds to a specific fault situation. The multiple-model based approach for fault modelling is well suited for certain fault tolerant control applications, where the detection of the "right" fault model automatically triggers the reconfiguration of the controller. Another

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important application field is the multiple-model adaptive control, where switching among several controllers is done after "recognizing" the best matching model at each time moment.

In this chapter, we first review the main types of faults and then introduce the input–output and state-space forms of LTI plant models with additive faults. These models represent the basis of all synthesis approaches presented in this book. The underlying fault-free models often include fictive noise inputs, which account for the effects of parametric uncertainties in the matrices of the state-space representations. We describe two approaches to arrive at such models starting from linear parameter-varying models and multiple LTI models. Finally, we present physical fault models described by a collection of LTI models, which form the basis of model detection based approaches.

2.1 Types of Faults

A typical setup for the modelling of a system with faults is presented in Fig. 2.1.

The main system variables are the control inputs u, the unknown disturbance inputs d, the noise inputs w, and the output measurements y. The control inputs u are assumed to be known (measurable) and, in general, can have arbitrary bounded variations. The output y and control input u are the only measurable signals which therefore can be used for fault monitoring purposes. The disturbance inputs d and noise inputs w are non-measurable "unknown" input signals, which act adversely on the system performance. For example, the unknown disturbance inputs d may represent physical disturbance inputs, as for example, wind turbulence acting on an aircraft or external loads acting on a plant. Typical noise inputs are sensor noise signals as well as process input noise. However, fictive noise inputs can also account for the cumulative effects of unmodelled system dynamics or for the effects of parametric uncertainties. In general, there is no clear-cut separation between disturbances and noise, and therefore, the appropriate definition of the disturbance and noise inputs is a challenging aspect when modelling systems for solving fault detection problems.

We define a *fault* as any unexpected variation of some physical parameters or variables of a plant causing an unacceptable violation of certain specification limits

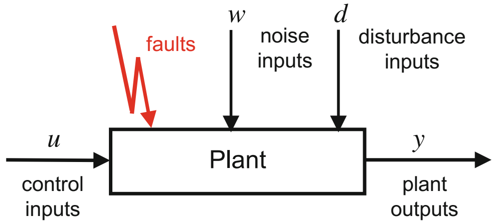

Fig. 2.1 Plant model with faults

for normal operation. For example, a change in the characteristics of an actuator leading to a loss of efficiency or even to a complete breakdown is termed an *actuator fault*, while erroneous measurements (e.g., corrupted by bias or drift) obtained with a defective sensor are caused by a *sensor fault*. The malfunction of an internal component (e.g., leakage, shortcut, etc.) is often assimilated with a *parametric fault*. A *failure* designates a permanent interruption of the plant operation (e.g., complete breakdown) and may be caused by one or more faults.

Besides this physical classification, faults are often classified on basis of their perceived effects. *Additive faults* are fictive inputs acting independently on the plant outputs. These inputs are zero in a fault-free situation and nonzero if a fault occurs. *Multiplicative faults* produce effects on the plant outputs which depend on the magnitude of some internal signals or known inputs. Sensor faults and several types of actuator faults are usually considered as additive faults, while parametric faults are considered as multiplicative faults.

Faults can be also differentiated on the basis of their behaviour over time. *Intermittent faults* are short duration malfunctions, which can still induce long-lasting effects. For example, actuator saturations caused by excessive loads fall often in this category of faults. *Persistent faults* have a long-range time evolution, and often manifest as slowly evolving incipient faults or abrupt changes, with permanent character, of physical parameters or system structure.

2.2 Plant Models with Additive Faults

The synthesis methods presented in this book primarily deal with LTI systems described by input–output relations of the form

$$\mathbf{y}(\lambda) = G_u(\lambda)\mathbf{u}(\lambda) + G_d(\lambda)\mathbf{d}(\lambda) + G_w(\lambda)\mathbf{w}(\lambda) + G_f(\lambda)\mathbf{f}(\lambda), \tag{2.1}$$

where $\mathbf{y}(\lambda)$, $\mathbf{u}(\lambda)$, $\mathbf{d}(\lambda)$, $\mathbf{w}(\lambda)$, and $\mathbf{f}(\lambda)$, with boldface notation, denote the Laplace-transformed (in the continuous-time case) or Z-transformed (in the discrete-time case) time-dependent vectors, namely, the p-dimensional system output vector $y(t)$, m_u-dimensional control input vector $u(t)$, m_d-dimensional disturbance vector $d(t)$, m_w-dimensional noise vector $w(t)$ and m_f-dimensional fault vector $f(t)$, respectively. $G_u(\lambda)$, $G_d(\lambda)$, $G_w(\lambda)$ and $G_f(\lambda)$ are the *transfer function matrices* (TFMs) from the control inputs u, disturbance inputs d, noise inputs w, and fault inputs f to the outputs y, respectively. According to the system type, $\lambda = s$, the complex variable in the Laplace-transform in the case of a continuous-time system or $\lambda = z$, the complex variable in the Z-transform in the case of a discrete-time system. For most of practical applications, the TFMs $G_u(\lambda)$, $G_d(\lambda)$, $G_w(\lambda)$, and $G_f(\lambda)$ are proper rational matrices. However, for complete generality of our problem settings, we will allow that these TFMs are general improper rational matrices for which we will not *a priori* assume any further properties (e.g., stability, full rank, etc.).

Remark 2.1 Throughout this book, the main difference between the disturbance input $d(t)$ and noise input $w(t)$ arises from the formulation of the fault monitoring goals. In this respect, when synthesizing devices to serve for fault diagnosis purposes, we will generally target the *exact* decoupling of the effects of disturbance inputs. Since generally the exact decoupling of effects of noise inputs is not achievable, we will simultaneously try to attenuate their effects, to achieve an *approximate* decoupling. Consequently, we will try to solve synthesis problems exactly or approximately, in accordance with the absence or presence of noise inputs in the underlying plant model, respectively. □

An equivalent *descriptor* state-space realization of the input–output model (2.1) has the form

$$\begin{aligned} E\lambda x(t) &= Ax(t) + B_u u(t) + B_d d(t) + B_w w(t) + B_f f(t)\,, \\ y(t) &= Cx(t) + D_u u(t) + D_d d(t) + D_w w(t) + D_f f(t)\,, \end{aligned} \tag{2.2}$$

with the n-dimensional state vector $x(t)$, where $\lambda x(t) = \dot{x}(t)$ or $\lambda x(t) = x(t+1)$ depending on the type of the system, continuous- or discrete-time, respectively. In general, the square matrix E can be singular, but we will assume that the linear pencil $A - \lambda E$ is regular. For systems with proper TFMs in (2.1), we can always choose a *standard* state-space realization where $E = I$. In general, we can also assume that the representation (2.2) is minimal, that is, the pair $(A - \lambda E, C)$ is *observable* and the pair $(A - \lambda E, [\, B_u \; B_d \; B_w \; B_f \,])$ is *controllable*. The corresponding TFMs of the model in (2.1) are

$$\begin{aligned} G_u(\lambda) &= C(\lambda E - A)^{-1}B_u + D_u, \\ G_d(\lambda) &= C(\lambda E - A)^{-1}B_d + D_d, \\ G_w(\lambda) &= C(\lambda E - A)^{-1}B_w + D_w, \\ G_f(\lambda) &= C(\lambda E - A)^{-1}B_f + D_f \end{aligned} \tag{2.3}$$

or in an equivalent notation

$$\left[\, G_u(\lambda) \; G_d(\lambda) \; G_w(\lambda) \; G_f(\lambda) \,\right] := \left[\begin{array}{c|cccc} A - \lambda E & B_u & B_d & B_w & B_f \\ \hline C & D_u & D_d & D_w & D_f \end{array}\right].$$

Remark 2.2 Although the overall model (2.2) can always be chosen minimal (i.e., controllable and observable), the state-space realizations of individual channels of the input–output model (2.1) may not be minimal. For example, the pair $(A - \lambda E, B_u)$ (which is part of the state-space realization of $G_u(\lambda)$) may be uncontrollable and even not stabilizable. In spite of this apparent deficiency, the chosen form (2.2) of the system model is instrumental for the development of all computational procedures presented in this book. □

An important class of models with additive faults arises when defining the fault signals for two main categories of faults, namely, actuator and sensor faults. Modelling actuator faults can be done by replacing $u(t)$ by a perturbed input $u(t) + S_a f_a(t)$, where $f_a(t)$ is the actuator fault signal and S_a is a fault distribution matrix. S_a is usually a full column rank matrix formed from distinct columns of an identity matrix of appropriate order. Thus, the corresponding fault-to-output TFM is defined as $G_f(\lambda) := G_u(\lambda)S_a$. Similarly, sensor faults can be modelled by replacing $y(t)$ by $y(t) + S_s f_s(t)$, where $f_s(t)$ is the sensor fault signal and S_s is an appropriate fault distribution matrix. The corresponding TFM is simply $G_f(\lambda) := S_s$. In the case when both actuator and sensor faults are present, then for the fault signal $f(t) := \begin{bmatrix} f_a(t) \\ f_s(t) \end{bmatrix}$, the corresponding fault-to-output TFM is

$$G_f(\lambda) := \begin{bmatrix} G_u(\lambda)S_a & S_s \end{bmatrix} . \tag{2.4}$$

In general, the matrices S_a and S_s can be chosen to also ensure a certain uniform range of magnitudes of the expected fault signals via appropriate scaling of fault inputs. The corresponding state-space realization (2.2) is obtained with $B_f := [\, B_u S_a \;\; 0 \,]$ and $D_f := [\, D_u S_a \;\; S_s \,]$. An important aspect of this approach is that the resulting models with additive faults can simultaneously cover several categories of actuator and sensor faults.

Example 2.1 Flight actuators with faults are often modelled as continuous-time LTI models, whose transfer-function representation is

$$\mathbf{y}(s) = G_u(s)\mathbf{u}(s) + G_f(s)\mathbf{f}(s) ,$$

where $u(t)$ and $y(t)$ are respectively, the commanded and achieved surface positions and $f(t)$ is a fault signal. For an input (actuator) fault we can take $G_f(s) = G_u(s)$, while for an output (sensor) fault $G_f(s) = 1$. If both types of faults are present, then $f(t)$ is a two-dimensional vector and $G_f(s) = [\, G_u(s) \; 1 \,]$. First- or second-order actuator models are frequently used for fault detection purposes, where the effects of the load (e.g., air resistance) are included in the actuator parameters. A first-order actuator model has the transfer function

$$G_u(s) = \frac{k}{s+k} ,$$

where k is a constant gain. A second-order actuator model can have the transfer function

$$G_u(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} ,$$

where ζ is the damping ratio and ω is the natural oscillation frequency. These simple additive faulty system models are suitable to serve for monitoring several categories of actuator faults which can be considered as additive faults, as—for example, jamming, runaway, oscillatory failure, or certain types of loss of efficiency. ♢

The underlying plant models (i.e., without noise and fault inputs) often represent linearizations of nonlinear dynamic plant models in specific operation points and for fixed values of plant parameters. To cope with variabilities in operating conditions and plant parameters, alternative representations are often used, which cover a whole family of linearized models. To use such models for solving fault diagnosis problems by employing the linear system techniques described in this book, we have to convert them into LTI state-space representations with additional noise or fault inputs, where the noise inputs account for the effects of existing variabilities in operating points and parameters, or for extreme parameter variations due to parametric faults. In what follows, we show for two classes of plant models, namely state-space models with parameter-dependent matrices and families of linearized state-space models (i.e., multiple LTI models), how they can be recast as standard LTI models with additional noise or fault inputs.

2.2.1 Models with Parametric Uncertainties

The fault-free LTI model without noise inputs which underlies the additive fault model of the form (2.1) or (2.2) often represents an approximation via linearization of a nonlinear system model in a certain nominal operating point for a particular combination of some model parameter values. Therefore, the validity of approximations by linearized models is often restricted to small variations around some nominal operating points and parameter values. To extend the range of validity of linear models, so-called *linear parameter-varying* (LPV) models have been introduced, where the dependence of (time-varying) operating conditions and parameters is explicitly reflected in the model (i.e., in the matrices of the state-space model). LPV models are therefore useful to represent nonlinear systems in terms of a family of linear models. The existing explicit parametric dependence can be exploited in various ways both in robust synthesis methods as well as in robustness analysis. In this section, we describe a useful technique to recast LPV models with parametric uncertainties into LTI models with fictitious noise inputs, which account for the effects of parametric variations. These models can thus serve to arrive at additive fault models of the general form (2.1) or (2.2).

There exist various techniques to determine LPV models. These techniques encompass: (1) the symbolic manipulation of the nonlinear model equations leading to so-called quasi-LPV models, where the matrices of the state-space model depend on a parameter vector, whose components include both plant parameters but also components of the state or output vectors of the nonlinear model; (2) direct parameter estimation using special global identification experiments; or (3) interpolation of a set of local models (e.g., obtained via linearizations) using regression-based parameter fitting techniques. We will not further discuss various existing techniques, but note that this research field is still very active as documented by a rapidly increasing amount of the literature dedicated to this topic.

Let ρ be a time-varying parameter vector and consider a state-space realization of the fault-free system in the LPV form

$$\begin{aligned} E(\rho)\lambda x(t) &= A(\rho)x(t) + B_u(\rho)u(t) + B_d(\rho)d(t)\,, \\ y(t) &= C(\rho)x(t) + D_u(\rho)u(t) + D_d(\rho)d(t)\,. \end{aligned} \tag{2.5}$$

Consider the parameter-dependent matrix

$$S(\rho) := \begin{bmatrix} E(\rho) & A(\rho) & B_u(\rho) & B_d(\rho) \\ 0 & C(\rho) & D_u(\rho) & D_d(\rho) \end{bmatrix} \tag{2.6}$$

and express $S(\rho)$ in the form

$$S(\rho) = S^{(0)} + \Delta_S \Gamma_S(\rho) , \tag{2.7}$$

where $S^{(0)}$ is the (nominal) value of $S(\lambda)$ defined for a constant value $\rho = \rho_0$ as

$$S^{(0)} := S(\rho_0) = \begin{bmatrix} E^{(0)} & A^{(0)} & B_u^{(0)} & B_d^{(0)} \\ 0 & C^{(0)} & D_u^{(0)} & D_d^{(0)} \end{bmatrix}, \tag{2.8}$$

$\Gamma_S(\rho)$ satisfies $\Gamma_S(\rho_0) = 0$, and Δ_S is a constant matrix. The system (2.5) can be alternatively expressed in the form

$$\begin{aligned} E^{(0)}\lambda x(t) &= A^{(0)}x(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + \Delta x(t, \rho), \\ y(t) &= C^{(0)}x(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + \Delta y(t, \rho), \end{aligned} \tag{2.9}$$

where $\Delta x(t, \rho)$ and $\Delta y(t, \rho)$ can be interpreted as input and output noise terms and are given by

$$\begin{bmatrix} \Delta x(t, \rho) \\ \Delta y(t, \rho) \end{bmatrix} := \Delta_S \Gamma_S(\rho) \begin{bmatrix} -\lambda x(t) \\ x(t) \\ u(t) \\ d(t) \end{bmatrix} . \tag{2.10}$$

If we denote with $\mathcal{R}(\cdot)$ the range (or image) of a matrix, then we have for all values of ρ

$$\begin{bmatrix} \Delta x(t, \rho) \\ \Delta y(t, \rho) \end{bmatrix} \in \mathcal{R}(\Delta_S) .$$

We can define a LTI model of the form

$$\begin{aligned} E^{(0)}\lambda x(t) &= A^{(0)}x(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + B_w^{(0)}w(t), \\ y(t) &= C^{(0)}x(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + D_w^{(0)}w(t) , \end{aligned}$$

to replace the LPV model (2.5), provided we can determine the two matrices $B_w^{(0)}$ and $D_w^{(0)}$ to satisfy the range condition

$$\mathcal{R}\left(\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix}\right) = \mathcal{R}(\Delta_S) ,$$

where $w(t)$ is a fictitious "noise" signal, whose dimension m_w is equal to the column dimension of $B_w^{(0)}$ and $D_w^{(0)}$. Such a LTI model can be considered an "exact" (thus not a conservative) replacement of the LPV model (2.5).

The determination of $B_w^{(0)}$ and $D_w^{(0)}$ can be done from the following singular value decomposition (SVD)

$$\Delta_S = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 & V_2 \end{bmatrix}^T = U_1 \Sigma V_1^T ,$$

where $\begin{bmatrix} U_1 & U_2 \end{bmatrix}$ and $\begin{bmatrix} V_1 & V_2 \end{bmatrix}$ are orthogonal matrices, and Σ is a nonsingular diagonal matrix with the decreasingly ordered nonzero singular values on its diagonal. From linear algebra we know that the columns of U_1 form an orthogonal basis for the range of Δ_S. Therefore, we can choose $\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix} = U_1$ or $\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix} = U_1 \Sigma$. The latter choice includes different scalings of noise inputs.

The representation of $S(\rho)$ in the form (2.7) can be easily obtained for LPV models whose matrices depend rationally on the components of ρ. For such models, $S(\rho)$ can always be expressed using an upper *linear fractional transformation*[1] (LFT) based equivalent representation

$$S(\rho) = LFT_u(M, \Delta), \tag{2.11}$$

where $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ is a certain constant matrix with M_{11} square and $\Delta = \Delta(\rho)$ is a diagonal matrix depending on the components of ρ such that $\Delta(\rho_0) = 0$. Straightforward algorithms are available to obtain the above representation. The above LFT-based representation of $S(\rho)$ allows to immediately obtain $S^{(0)} = M_{22}$, $\Delta_S = M_{21}$ and $\Gamma_S(\rho) = \Delta(I - \Delta M_{11})^{-1} M_{12}$.

Example 2.2 We consider an LPV model with a standard state-space realization (2.5) with $E = I$, $B_d = 0$, $D_d = 0$ and

$$A(\rho_1, \rho_2) = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.5(1+\rho_1) & 0.6(1+\rho_2) \\ 0 & -0.6(1+\rho_2) & -0.5(1+\rho_1) \end{bmatrix}, \quad B_u = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \quad D_u = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

In the expression of $A(\rho_1, \rho_2)$, ρ_1 and ρ_2 are uncertainties in the real and imaginary parts of the two complex conjugate eigenvalues $\lambda_{1,2} = -0.5 \pm j0.6$ of the nominal value $A^{(0)} = A(0, 0)$.

We can recast the effects of uncertain parameters ρ_1 and ρ_2 as fictitious noise inputs. Since all system matrices, excepting the state matrix $A(\rho_1, \rho_2)$ are constant, we only have to represent $A(\rho_1, \rho_2)$ as

$$A(\rho_1, \rho_2) = A^{(0)} + \Delta_A \Gamma_A(\rho) ,$$

with

$$A^{(0)} = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.5 & 0.6 \\ 0 & -0.6 & -0.5 \end{bmatrix}$$

and Δ_A and $\Gamma_A(\rho)$ given by

[1]An upper LFT for a partitioned matrix $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$ and a compatible Δ is defined as $LFT_u(M, \Delta) := M_{22} + M_{21}\Delta(I - \Delta M_{11})^{-1} M_{12}$.

$$\Delta_A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \Gamma_A(\rho) = \begin{bmatrix} 0 & -0.6\rho_2 & -0.5\rho_1 \\ 0 & -0.5\rho_1 & 0.6\rho_2 \end{bmatrix}. \tag{2.12}$$

Thus, the noise terms in (2.9) are $\Delta x(t,\rho) = \Delta_A \Gamma_A(\rho) x(t)$ and $\Delta y(t,\rho) = 0$ and therefore, Δ_S has the reduced form

$$\Delta_S = \begin{bmatrix} \Delta_A \\ 0 \end{bmatrix}, \tag{2.13}$$

which can be used for range computation. For this simple LPV model, we can define the noise vector as $w(t) := \Gamma_A(\rho)x(t)$ and the corresponding noise matrices result as

$$B_w^{(0)} := \Delta_A, \quad D_w^{(0)} = 0\,.$$

The resulting equivalent LTI model with noise inputs is

$$\begin{aligned} \dot{x}(t) &= A^{(0)}x(t) + B_u u(t) + B_w^{(0)} w(t), \\ y(t) &= Cx(t) + D_u u(t) + D_w^{(0)} w(t). \end{aligned} \tag{2.14}$$

Using the LFT-based representation $A(\rho) = LFT_u(M, \Delta)$ with $\Delta = \operatorname{diag}(\rho_1, \rho_1, \rho_2, \rho_2)$ and

$$M = \left[\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right] = \left[\begin{array}{cccc|cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.8 & 0 & 0 & 1 & 1 \\ -0.5 & 0 & 0.6 & 0 & 0 & 1 & 0 & 0 & -0.5 & 0.6 & 1 & 0 \\ 0 & -0.5 & 0 & -0.6 & 0 & 0 & 1 & 0 & -0.6 & -0.5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array}\right],$$

we can check that $\mathcal{R}(M_{21}) = \mathcal{R}(\Delta_S)$, with Δ_S given in (2.13). Thus, we obtain the same $B_w^{(0)}$ (though with permuted columns) and $D_w^{(0)}$ as above.

The MATLAB script **Ex2_2** in Listing 2.1 generates the matrices $B_w^{(0)}$ and $D_w^{(0)}$ of the LTI model (2.14) from the LPV model considered in this example. ♢

Listing 2.1 Script **Ex2_2** to convert the LPV model of Example 2.2 to LTI form

```
% Uses the Robust Control Toolbox

% define ρ1 and ρ2 as uncertain parameters
r1=ureal('rho1',0); r2=ureal('rho2',0);

% define E, A(ρ1,ρ2), Bu, C, Du
n = 3; mu = 2; p = 2;        % enter dimensions
E = eye(n);
A = [ -.8 0 0;
      0 -0.5*(1+r1) 0.6*(1+r2);
      0 -0.6*(1+r2) -0.5*(1+r1) ];
Bu = [ 1 1; 1 0; 0 1]; C = [0 1 1;1 1 0]; Du = zeros(p,mu);

% build S(ρ)
S = [ E A Bu; zeros(p,n) C Du];

% compute the elements of LFT-based representation
[M,Delta] = lftdata(S);
```

```
nd = size(Delta,1);                     % size of Δ

% computes orthogonal basis for the range of M21
U1 = orth(M(nd+1:end,1:nd));            % U1 directly from SVD

% compute Bw and Dw, and define the number of noise inputs mw
Bw = U1(1:n,:); Dw = U1(n+1:end,:); mw = size(U1,2);
```

2.2.2 Models with Parametric Faults

Multiplicative faults are frequently used as synonym for parametric faults. Let $\rho(t)$ be a time-varying parameter vector and consider a state-space realization of the system in the LPV form (2.5). The components of ρ consist of those system parameters, whose extreme variations represent the parametric faults for the given plant. In this section, we discuss the conversion of LPV models of the form (2.5) into equivalent LTI models with additive faults of the form (2.2). This conversion can be done using similar techniques as those described in Sect. 2.2.1.

The parameter-dependent matrix $S(\rho)$ in (2.6) can be expressed in the form $S(\rho) = S^{(0)} + \Delta_S \Gamma_S(\rho)$, where $S^{(0)}$ is the fault-free (nominal) value of $S(\lambda)$ defined for a (constant) normal value $\rho = \rho_0$ as in (2.8); $\Gamma_S(\rho)$ satisfies $\Gamma_S(\rho_0) = 0$ and therefore is zero in the fault-free case; and Δ_S is a constant matrix. The system (2.5) can be expressed in the alternative form (2.9), where, $\Delta x(t,\rho)$ and $\Delta y(t,\rho)$ in (2.10) can be now interpreted as fault input terms. These terms defined in (2.10) depend on the magnitudes of state and input vectors, which justifies the characterization of parametric faults as multiplicative faults. The fault input terms are therefore zero if the system state, the state derivative and the system inputs are zero, and they are also zero if the plant operates in its normal condition corresponding to $\rho = \rho_0$.

Let $B_f^{(0)}$ and $D_f^{(0)}$ be two matrices which satisfy the range condition

$$\mathcal{R}\left(\begin{bmatrix} B_f^{(0)} \\ D_f^{(0)} \end{bmatrix}\right) = \mathcal{R}\left(\Delta_S\right) .$$

Since for all values of ρ we have

$$\begin{bmatrix} \Delta x(t,\rho) \\ \Delta y(t,\rho) \end{bmatrix} \in \mathcal{R}\left(\Delta_S\right) ,$$

we can define a fictitious "fault" signal $f(t)$, whose dimension m_f is equal to the column dimension of $B_f^{(0)}$ and $D_f^{(0)}$, and build the equivalent LTI model of the form

$$\begin{aligned} E^{(0)}\lambda x(t) &= A^{(0)}x(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + B_f^{(0)}f(t), \\ y(t) &= C^{(0)}x(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + D_f^{(0)}f(t). \end{aligned} \tag{2.15}$$

Remark 2.3 In general, the parameter vector ρ can be split in two components, $\rho = [\,\rho_1^T\ \ \rho_2^T\,]^T$, where ρ_1 includes those model parameters which are susceptible to parametric faults, while ρ_2 is the part of parameters which have to be exclusively handled as uncertainties. This separation should be also reflected in the equivalent LTI model (2.15), where, besides the additive fault input terms $B_f^{(0)} f(t)$ and $D_f^{(0)} f(t)$, which correspond to the parametric faults in ρ_1, noise input terms $B_w^{(0)} w(t)$ and $D_w^{(0)} w(t)$ have to be added, respectively, which correspond to the parametric uncertainties in ρ_2. Such a model can be easily determined, if the parametric matrix $S(\rho)$ in (2.6) exhibits an additive separability property with respect to the two components of ρ of the form $S(\rho) = S_1(\rho_1) + S_2(\rho_2)$. In this case, the fault and noise input matrices can be generated separately for each term of $S(\rho)$, employing the approaches described in this and previous sections. An important class of parametric models for which this condition is fulfilled is formed by LPV models with affine dependence of system matrices of parameters. □

Although the LTI model (2.15) can be considered an "exact" (thus not a conservative) replacement of the LPV model (2.5), still it hides the complex dependence of the fault input terms on the system parameters and variables exhibited in (2.10). For example, a direct correspondence between the components of ρ and the components of f is not explicitly provided, which could make the fault isolation task difficult. By taking into account the explicit dependence of the fault input terms of the current values of the state and input vectors, a more detailed (structured) representation of parametric faults is possible, which, however, leads to time-varying matrices in the fault input channel of the additive fault model (2.15) (i.e., using time-varying matrices $B_f(t)$ and $D_f(t)$ instead the constant matrices $B_f^{(0)}$ and $B_f^{(0)}$, respectively).

For some particular LPV models, as—for example, those with the system matrices having affine dependence on the components of ρ, it is possible to exploit this feature to obtain a direct correspondence between the components of the additive fault vector $f(t)$ and the components of the parameter vector ρ. To show this, let assume that ρ has k components and the LPV system matrices have the following affine representations

$$E(\rho) = E^{(0)} + \sum_{i=1}^{k} E^{(i)} \rho_i, \quad A(\rho) = A^{(0)} + \sum_{i=1}^{k} A^{(i)} \rho_i,$$

$$B_u(\rho) = B_u^{(0)} + \sum_{i=1}^{k} B_u^{(i)} \rho_i, \quad B_d(\rho) = B_d^{(0)} + \sum_{i=1}^{k} B_d^{(i)} \rho_i,$$

$$C(\rho) = C^{(0)} + \sum_{i=1}^{k} C^{(i)} \rho_i, \quad D_u(\rho) = D_u^{(0)} + \sum_{i=1}^{k} D_u^{(i)} \rho_i, \quad D_d(\rho) = D_d^{(0)} + \sum_{i=1}^{k} D_d^{(i)} \rho_i\,.$$

This allows to express $S(\rho)$ in (2.6) as $S(\rho) = S^{(0)} + \sum_{i=1}^{k} S^{(i)} \rho_i$, with

$$S^{(i)} := \left[\begin{array}{cccc} E^{(i)} & A^{(i)} & B_u^{(i)} & B_d^{(i)} \\ \hline 0 & C^{(i)} & D_u^{(i)} & D_d^{(i)} \end{array}\right] =: \left[\begin{array}{c} S_x^{(i)} \\ \hline S_y^{(i)} \end{array}\right] \tag{2.16}$$

for $i = 0, 1, \ldots, k$. The fault input terms in (2.9) can be now expressed as

$$\begin{bmatrix} \Delta x(t,\rho) \\ \Delta y(t,\rho) \end{bmatrix} := \left(\sum_{i=1}^{k} S^{(i)}\rho_i\right) z(t) = \left[\, S^{(1)}z(t) \;\cdots\; S^{(k)}z(t) \,\right]\rho,$$

where $z(t) := \left[\, -\lambda x^T(t) \;\, x^T(t) \;\, u^T(t) \;\, d^T(t) \,\right]^T$. Using the partitioning of $S^{(i)}$ in (2.16), we can express the fault input terms as

$$\Delta x(t,\rho) = B_f(t)\rho(t), \qquad \Delta y(t,\rho) = B_f(t)\rho(t),$$

where $B_f(t) = \left[\, S_x^{(1)}z(t) \;\cdots\; S_x^{(k)}z(t) \,\right]$ and $D_f(t) = \left[\, S_y^{(1)}z(t) \;\cdots\; S_y^{(k)}z(t) \,\right]$. By defining $f(t) := \rho(t)$, we obtain the equivalent additive fault model

$$\begin{aligned} E^{(0)}\lambda x(t) &= A^{(0)}x(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + B_f(t)f(t)\,, \\ y(t) &= C^{(0)}x(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + D_f(t)f(t)\,, \end{aligned} \tag{2.17}$$

with the fault input channel containing time-varying matrices with a special structure. Although the synthesis methods presented in this book are mainly intended for LTI models with additive faults, still some of these methods can be extended to handle models of the form (2.17) (see Remark 7.4).

The following example illustrates the model conversion techniques presented in this section on the basis of the model considered in Example 2.2.

Example 2.3 Consider the same model as that used in Example 2.2, where only the state matrix $A(\rho_1, \rho_2)$ depends on parameters. Large variations of these parameters are considered parametric faults. We can convert this LPV model to a LTI model with additive faults, using the calculations already done in Example 2.2. The resulting LTI model with additive fault inputs can be set up as

$$\begin{aligned} \dot{x}(t) &= A^{(0)}x(t) + B_u u(t) + B_f^{(0)}f(t)\,, \\ y(t) &= Cx(t) + D_u u(t)\,, \end{aligned} \tag{2.18}$$

with $B_f^{(0)} := \Delta_A$, where $A(\rho_1, \rho_2) = A^{(0)} + \Delta_A \Gamma_A(\rho)$, and Δ_A and $\Gamma_A(\rho)$ are given in (2.12). The equivalent fault input is defined as $f(t) := \Gamma_A(\rho)x(t)$.

For the fault vector $f(t)$, we can use a more structured representation using the alternative affine representation of $A(\rho)$ as $A(\rho) = A^{(0)} + A^{(1)}\rho_1 + A^{(2)}\rho_2$, which leads to

$$A(\rho)x(t) = A^{(0)}x(t) + \left[\, A^{(1)}x(t) \;\, A^{(2)}x(t) \,\right]\rho\,.$$

It follows that with $f(t) := \rho(t)$, we obtain a time-varying input matrix $B_f(t) = \left[\, A^{(1)}x(t) \;\, A^{(2)}x(t) \,\right]$ to replace $B_f^{(0)}$ in (2.18). ♢

2.2.3 *Multiple Linear Models*

A frequent situation which occurs in practical applications is that we only have at our disposal N plant models (i.e., a multiple model) of the form

$$\begin{aligned} E^{(i)}\lambda x^{(i)}(t) &= A^{(i)}x^{(i)}(t) + B_u^{(i)}u(t) + B_d^{(i)}d(t), \\ y^{(i)}(t) &= C^{(i)}x^{(i)}(t) + D_u^{(i)}u(t) + D_d^{(i)}d(t), \end{aligned} \tag{2.19}$$

where, for $i = 1, \ldots, N$, $x^{(i)}(t) \in \mathbb{R}^n$ and $y^{(i)}(t) \in \mathbb{R}^p$ are the state vector and output vector of the i-th system, respectively. For simplicity, we assume that in all models, the dimensions of the state, output and input vectors are the same. Typically, (2.19) describes a family of linearized models for N relevant combinations of plant operating points and plant parameters. In what follows, we describe a simple method to recast such a multiple model into a unique LTI model with additional fictitious noise inputs, which account for the effects of variations in operating points and parameters. The resulting LTI model can then serve to build models with additive faults of the form (2.1) or (2.2).

The matrices of each component model can be expressed in the form

$$E^{(i)} = E^{(0)} + \Delta_E^{(i)}, \quad A^{(i)} = A^{(0)} + \Delta_A^{(i)}, \quad B_u^{(i)} = B_u^{(0)} + \Delta_{B_u}^{(i)}, \quad , \ldots$$

where $E^{(0)}$, $A^{(0)}$, $B_u^{(0)}$, $\ldots$ are some nominal values (or simply the mean values of the corresponding matrices), while $\Delta_E^{(i)}$, $\Delta_A^{(i)}$, $\Delta_{B_u}^{(i)}$, $\ldots$ are the deviations from the nominal (or mean) values. If we denote

$$\Delta_S^{(i)} := \begin{bmatrix} -\Delta_E^{(i)} & \Delta_A^{(i)} & \Delta_{B_u}^{(i)} & \Delta_{B_d}^{(i)} \\ 0 & \Delta_C^{(i)} & \Delta_{D_u}^{(i)} & \Delta_{D_d}^{(i)} \end{bmatrix},$$

then each model of the form (2.19) can be equivalently represented in the form

$$\begin{aligned} E^{(0)}\lambda x^{(i)}(t) &= A^{(0)}x^{(i)}(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + \Delta_x^{(i)}(t), \\ y^{(i)}(t) &= C^{(0)}x^{(i)}(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + \Delta_y^{(i)}(t), \end{aligned} \tag{2.20}$$

where $\Delta_x^{(i)}(t)$ and $\Delta_y^{(i)}(t)$ are the noise terms specific to the i-th model, given by

$$\begin{bmatrix} \Delta_x^{(i)}(t) \\ \Delta_y^{(i)}(t) \end{bmatrix} := \Delta_S^{(i)} \begin{bmatrix} \lambda x^{(i)}(t) \\ x^{(i)}(t) \\ u(t) \\ d(t) \end{bmatrix}.$$

Therefore, for each component model we have

$$\begin{bmatrix} \Delta_x^{(i)}(t) \\ \Delta_y^{(i)}(t) \end{bmatrix} \in \mathcal{R}\big(\Delta_S^{(i)}\big) .$$

We can try to define a unique model of the form

$$\begin{aligned} E^{(0)}\lambda x(t) &= A^{(0)}x(t) + B_u^{(0)}u(t) + B_d^{(0)}d(t) + B_w^{(0)}w(t), \\ y(t) &= C^{(0)}x(t) + D_u^{(0)}u(t) + D_d^{(0)}d(t) + D_w^{(0)}w(t) \end{aligned}$$

to approximate the collection of N models in (2.20), provided the two matrices $B_w^{(0)}$ and $D_w^{(0)}$ satisfy the range condition

$$\mathcal{R}\left(\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix}\right) = \mathcal{R}\big(\Delta_S^{(1)}\big) \cup \cdots \cup \mathcal{R}\big(\Delta_S^{(N)}\big) = \mathcal{R}\left(\begin{bmatrix} \Delta_S^{(1)} & \cdots & \Delta_S^{(N)} \end{bmatrix}\right)$$

and $w(t)$ is a fictitious "noise" signal which formally matches the column dimension of $B_w^{(0)}$ and $D_w^{(0)}$. Such an unique model is certainly conservative, because it includes in a single model the effects of *all* possible parametric variations. Nevertheless, the degree of conservatism may be acceptable in practice, because often the component models share common structural features which are reflected in the matrices $\Delta_S^{(i)}$ (e.g., constant rank, fixed zero entries, etc.).

The determination of $B_w^{(0)}$ and $D_w^{(0)}$ can be done (as in the preceding section) from the SVD

$$\Delta_S := \begin{bmatrix} \Delta_S^{(1)} & \cdots & \Delta_S^{(N)} \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 & V_2 \end{bmatrix}^T = U_1 \Sigma V_1^T , \tag{2.21}$$

where $\begin{bmatrix} U_1 & U_2 \end{bmatrix}$ and $\begin{bmatrix} V_1 & V_2 \end{bmatrix}$ are orthogonal matrices, and Σ is a diagonal matrix with the decreasingly ordered nonzero singular values on its diagonal. Therefore, we can choose $\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix} = U_1$ or $\begin{bmatrix} B_w^{(0)} \\ D_w^{(0)} \end{bmatrix} = U_1\Sigma$. The latter choice includes different scalings of noise inputs. Often, we can even use instead U_1, only a few of its leading columns which correspond to the most significant singular values.

Remark 2.4 The determination of U_1 in the SVD (2.21) involves the computation of the SVD of the potentially large matrix Δ_S with $n+p$ rows and $N(2n+m_u+m_d)$ columns. This computation may require a tremendous computational effort for large N or large n if Δ_S is explicitly formed. Fortunately, this can be avoided by a suitable preprocessing of Δ_S. The proposed computational approach below leads to significant saving in the computational effort if $N(2n + m_u + m_d) \gg n + p$ (which is usually the case). Let Q be an orthogonal matrix which compresses the columns of Δ_S to a $(n+p) \times (n+p)$ matrix R_S (upper triangular) according to the following orthogonal RQ-decomposition of Δ_S as

$$\Delta_S = \begin{bmatrix} R_S & 0 \end{bmatrix} Q .$$

Then, the SVDs of the large matrix Δ_S and of the compressed matrix R_S provide the same U_1 matrix as basis for $\mathcal{R}(\Delta_S)$. Since the computation of the right transformation matrix $\left[\, V_1 \;\, V_2 \,\right]$ is not necessary, it is possible to determine R_S without determining explicitly Q. Moreover, we can compute R_S even without the need to form Δ_S explicitly, using the following recursion based on successive low-dimensional RQ-decompositions

$$\left[\, R_i \;\, 0 \,\right] Q_i = \left[\, R_{i-1} \;\, \Delta_S^{(i)} \,\right], \quad i = 1, \ldots, N \,,$$

where $R_0 = 0_{(n+p)\times(n+p)}$. Here, each R_i is an $(n+p) \times (n+p)$ (upper triangular) matrix and Q_i is an orthogonal matrix of order $3n+p+m_u+m_d$, which need not be computed. At the end we set $R_S = R_N$ and the SVD of R_S provides the orthogonal basis matrix U_1 of $\mathcal{R}(S_\Delta)$ from the SVD (2.21). $\square$

Example 2.4 We consider once again the LPV system with a standard state-space realization (2.5) used in Example 2.2. Consider a set of parameter values $(\rho_1^{(i)}, \rho_2^{(i)})$, for $i = 1, \ldots, N$. For each value $(\rho_1^{(i)}, \rho_2^{(i)})$ we define

$$A^{(i)} := A(\rho_1^{(i)}, \rho_2^{(i)}) = A^{(0)} + \Delta_A^{(i)} \,,$$

where $A^{(0)} = A(0, 0)$ is the nominal value of $A(\rho_1, \rho_2)$

$$A^{(0)} = \begin{bmatrix} -0.8 & 0 & 0 \\ 0 & -0.5 & 0.6 \\ 0 & -0.6 & -0.5 \end{bmatrix}$$

and $\Delta_A^{(i)}$ is given by

$$\Delta_A^{(i)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.5\rho_1^{(i)} & 0.6\rho_2^{(i)} \\ 0 & -0.6\rho_2^{(i)} & 0.5\rho_1^{(i)} \end{bmatrix}.$$

With the reduced $\Delta_S^{(i)}$ defined as

$$\Delta_S^{(i)} := \begin{bmatrix} \Delta_A^{(i)} \\ 0 \end{bmatrix},$$

we have that $\mathcal{R}(\Delta_S) = \mathcal{R}(\Delta_S^{(i)})$ for all simultaneously nonzero $\rho_1^{(i)}$ and $\rho_2^{(i)}$. For convenience, we take $\rho_1^{(1)} = \sqrt{0.5}$ and $\rho_2^{(1)} = \sqrt{0.5}$ and we only compute the SVD of the nonzero part $\Delta_A^{(1)}$ as

$$\Delta_A^{(1)} = \left[\, U_1 \,|\, U_2 \,\right] \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \left[\, V_1 \,|\, V_2 \,\right]^T = \left[\begin{array}{cc|c} 0 & 0 & 1 \\ -0.7071 & -0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \end{array}\right] \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} \left[\begin{array}{cc|c} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]^T.$$

By defining the noise vector as $w(t) := \Sigma V_2^T x(t)$ and the corresponding matrices

$$B_w^{(0)} := \begin{bmatrix} 0 & 0 \\ -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}, \quad D_w^{(0)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

we arrive to a LTI model similar to (2.14). It is easy to see that although $B_w^{(0)}$ in the Examples 2.2 and 2.4 are different, their ranges are the same. $\lozenge$

2.3 Physical Fault Models

For physically modelled faults, each fault mode leads to a distinct model. Assume that we have N LTI models describing the fault-free and faulty systems, and for $i = 1, \ldots, N$ the i-th model is specified in the input–output form

$$\mathbf{y}^{(i)}(\lambda) = G_u^{(i)}(\lambda)\mathbf{u}^{(i)}(\lambda) + G_d^{(i)}(\lambda)\mathbf{d}^{(i)}(\lambda) + G_w^{(i)}(\lambda)\mathbf{w}^{(i)}(\lambda), \tag{2.22}$$

where $y^{(i)}(t) \in \mathbb{R}^{p^{(i)}}$ is the output vector of the i-th system with control input $u^{(i)}(t) \in \mathbb{R}^{m_u^{(i)}}$, disturbance input $d^{(i)}(t) \in \mathbb{R}^{m_d^{(i)}}$ and noise input $w^{(i)}(t) \in \mathbb{R}^{m_w^{(i)}}$, respectively, and where $G_u^{(i)}(\lambda)$, $G_d^{(i)}(\lambda)$ and $G_w^{(i)}(\lambda)$ are the TFMs from the corresponding plant inputs to outputs. The significance of disturbance and noise inputs, and the basic difference between them, have already been discussed in Sect. 2.1. The state-space realizations corresponding to the multiple model (2.22) are for $i = 1, \ldots, N$ of the form

$$\begin{aligned} E^{(i)}\lambda x^{(i)}(t) &= A^{(i)}x^{(i)}(t) + B_u^{(i)}u^{(i)}(t) + B_d^{(i)}d^{(i)}(t) + B_w^{(i)}w^{(i)}(t)\,, \\ y^{(i)}(t) &= C^{(i)}x^{(i)}(t) + D_u^{(i)}u^{(i)}(t) + D_d^{(i)}d^{(i)}(t) + D_w^{(i)}w^{(i)}(t)\,, \end{aligned} \tag{2.23}$$

where $x^{(i)}(t) \in \mathbb{R}^{n^{(i)}}$ is the state vector of the i-th system and, generally, can have different dimensions for different systems.

The multiple-model description represents a very general way to describe plant models with various faults. For example, extreme variations of parameters representing the so-called parametric faults, can be easily described by multiple models. Let ρ be a parameter vector, which includes a set of model parameters whose extreme values characterize the different fault cases. We assume that the system model depending on ρ has the form

$$\mathbf{y}(\lambda) = G_u(\lambda, \rho)\mathbf{u}(\lambda) + G_d(\lambda, \rho)\mathbf{d}(\lambda) + G_w(\lambda, \rho)\mathbf{w}(\lambda)\,. \tag{2.24}$$

Let $\rho^{(i)}, i = 1, \ldots, N$ be a set of values, which characterize both the normal operation as well as the fault cases. Then, the multiple model (2.22) for $i = 1, \ldots, N$ can be defined for $u^{(i)} = u$, $d^{(i)} = d$ and $w^{(i)} = w$ as

$$G_u^{(i)}(\lambda) := G_u(\lambda, \rho^{(i)}), \quad G_d^{(i)}(\lambda) := G_d(\lambda, \rho^{(i)}), \quad G_w^{(i)}(\lambda) := G_w(\lambda, \rho^{(i)})\,. \tag{2.25}$$

Similarly, if the state-space realization of the system model has the LPV form

$$\begin{aligned} E(\rho)\lambda x(t) &= A(\rho)x(t) + B_u(\rho)u(t) + B_d(\rho)d(t) + B_w(\rho)w(t)\,, \\ y(t) &= C(\rho)x(t) + D_u(\rho)u(t) + D_d(\rho)d(t) + D_w(\rho)w(t)\,, \end{aligned} \tag{2.26}$$

then a multiple model of the form (2.23) for $i = 1, \ldots, N$ can be defined with

$$E^{(i)} = E\big(\rho^{(i)}\big), \quad A^{(i)} = A\big(\rho^{(i)}\big), \quad \ldots \tag{2.27}$$

and $x^{(i)} = x$, $u^{(i)} = u$, $d^{(i)} = d$ and $w^{(i)} = w$.

As an example, consider the modelling of a category of loss of efficiency actuator faults for a system of the form

$$\mathbf{y}(\lambda) = G_u(\lambda)\mathbf{u}(\lambda) + G_d(\lambda)\mathbf{d}(\lambda)\,,$$

without noise input. The loss of efficiency of the i-th actuator, can be modelled by defining

$$G_u^{(i)}(\lambda) := G_u(\lambda)F_a^{(i)}, \qquad G_d^{(i)}(\lambda) := G_d(\lambda), \tag{2.28}$$

where $F_a^{(i)}$ is a diagonal matrix with unit diagonal entries excepting the i-th diagonal entry which is set to a nonnegative subunitary value. Several values for $F_a^{(i)}$ can be employed to cope with different degrees of failures of a single actuator. A complete failure of the i-th actuator can be easily modelled by setting the i-th diagonal element to zero.

Different categories of sensor faults (e.g., bias, drift, frozen value) can be modelled by adding fictive disturbances which act on the respective outputs. For example, for a fault in the i-th output sensor we can define the fault model with

$$G_u^{(i)}(\lambda) = F_s^{(i)}G_u(\lambda), \qquad G_d^{(i)}(\lambda) = [\, F_s^{(i)}G_d(\lambda),\ e_i\,],$$

where $F_s^{(i)}$ is chosen a diagonal matrix (similar to $F_a^{(i)}$) to account for the i-th sensor fault and e_i is the i-th column of the identity matrix. Simultaneous sensor faults can be also easily modelled in this way.

The multiple-model approach to fault modelling offers a simple framework to model faults, by associating a distinct model to each fault or combination of several simultaneous faults. Occasionally, this involves defining additional disturbance inputs which account for the effects of modelled faults. However, employing this approach for complex systems with many potential actuator and sensor faults can easily lead to a large number of models. The situation can be even worse if additionally parametric faults can occur. Since the number of required models increases exponentially with the number of faults, the applicability of this modelling framework is restricted to system with a relatively small number of faults. Often, the single-fault-at-time assumption is used to limit the number of considered faults and thus of the associated models.

Example 2.5 Flight actuators can be often modelled as first-order parameter-dependent linear continuous-time models, whose transfer-function representation is (assuming constant parameter)

$$\mathbf{y}(s) = G_u(s,k)\mathbf{u}(s)\,,$$

with

$$G_u(s,k) = \frac{k}{s+k}\,.$$

The input $u(t)$ is usually the demanded position (e.g., angle) of the attached control surface and the output $y(t)$ is the actual surface position. Here, k is the effective actuator gain, which, in general, depends on flight parameters such as the current weight-balance, current flight conditions, as well as the current deflection of the attached control surface. High accuracy models may also cover the dependence of the effects of air resistance on the associated control surface.

Several categories of actuator faults are best modelled as parametric faults and described by several special values of the gain parameter k. Assume that the normal operation of a flight actuator is well approximated by a LTI model with $G_u^{(1)}(s) := G_u(s, k_0)$, where k_0 is a mean value of k in normal operation (e.g., a typical value may be $k_0 = 14$). The actuator disconnection fault due to a broken rod between the actuator and corresponding control surface is considered a severe fault (although very improbable). Physically, this fault is equivalent with the lack of any air resistance and hinge moments, because the actuator rod can move practically without encountering any resistance from the control surface. Therefore, this fault can be modelled by a LTI model with $G_u^{(2)}(s) := G_u(s, k_{max})$, where k_{max} is the highest achievable gain (e.g., a typical value satisfies $k_{max} > 50$). On the opposite side, highly deflected control surfaces produce large air resistance and therefore, the actuators are susceptible to intermittent position saturations. This intermittent fault is called *stall load* and represents a sudden change of gain to its lowest value k_{min} (e.g., a typical value may be $k_{min} = 0.01$). Thus, this fault can be modelled by a LTI with $G_u^{(3)}(s) := G_u(s, k_{min})$. Finally, a sluggish behaviour of the actuator can be associated with a second type of loss-of-effectiveness fault and can be modelled as a LTI with $G_u^{(4)}(s) := G_u(s, \gamma k_0)$, where $0 < \gamma < 1$ is a parameter which indicates the degradation of the actuator dynamics (e.g., $\gamma = 0.5$ for a 50% sluggishness). Several values of γ can be used to characterize different degradation levels. ◊

2.4 Notes and References

Background material on input–output representations via TFMs is given in Sect. 9.1. The structural properties of descriptor system representations are discussed, for example, in [23]. See also Sect. 9.2 for background material on descriptor systems.

The LTI model with additive faults, control, disturbance and noise inputs has been already used in the textbook of Gertler [48], while other authors as Chen and Patton [20] and more recently Ding [26] employ LTI models with faults without making difference between disturbance and noise inputs, when solving fault detection problems. However, the distinction between disturbance and noise inputs is the basis of the synthesis methods presented in this book and allows the exploitation of all existing structural features related to the unknown inputs acting on the system. The presence or absence of noise inputs in the underlying LTI synthesis models determines a direct correspondence with the synthesis methods labelled as "approximate" or "exact", respectively. This systematics has been introduced in a recent survey of synthesis methods [151].

Several methods to recast uncertain models into models with noise inputs are described by Chen and Patton [20]. For the derivation of LPV models, there are many approaches proposed in the literature (see for example the special issue of the IEEE Transactions on Control Systems Technology [79]). The determination of a high fidelity first-order LPV flight-actuator model approximation has been described in [152].

The use of multiple models is a standard way to address robust synthesis problems in the presence of parametric uncertainties. Several applications of multiple-model based approaches are presented in [89]. The use of multiple models for fault detection and fault tolerant control has been proposed by Maybeck [84] and by Boškovic and Mehra [16].

Techniques for handling models with parametric (multiplicative) faults are described by Gertler in [48]. The special case of handling affine LPV models is considered by Ding in [26].

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