

Contents

Introduction	1
Part I $C_0(X)$ and $B(H)$	
1 Classical physics on a finite phase space	23
1.1 Basic constructions of probability theory	24
1.2 Classical observables and states	26
1.3 Pure states and transition probabilities	31
1.4 The logic of classical mechanics	34
1.5 The GNS-construction for $C(X)$	36
Notes	38
2 Quantum mechanics on a finite-dimensional Hilbert space	39
2.1 Quantum probability theory and the Born rule	40
2.2 Quantum observables and states	43
2.3 Pure states in quantum mechanics	46
2.4 The GNS-construction for matrices	50
2.5 The Born rule from Bohrfication	54
2.6 The Kadison–Singer Problem	57
2.7 Gleason’s Theorem	59
2.8 Proof of Gleason’s Theorem	62
2.9 Effects and Busch’s Theorem	71
2.10 The quantum logic of Birkhoff and von Neumann	75
Notes	80
3 Classical physics on a general phase space	83
3.1 Vector fields and their flows	85
3.2 Poisson brackets and Hamiltonian vector fields	88
3.3 Symmetries of Poisson manifolds	90
3.4 The momentum map	94
Notes	101

4	Quantum physics on a general Hilbert space	103
4.1	The Born rule from Bohrrification (II)	104
4.2	Density operators and normal states	109
4.3	The Kadison–Singer Conjecture	113
4.4	Gleason’s Theorem in arbitrary dimension	119
	Notes	123
5	Symmetry in quantum mechanics	125
5.1	Six basic mathematical structures of quantum mechanics	126
5.2	The case $H = \mathbb{C}^2$	130
5.3	Equivalence between the six symmetry theorems	137
5.4	Proof of Jordan’s Theorem	145
5.5	Proof of Wigner’s Theorem	147
5.6	Some abstract representation theory	151
5.7	Representations of Lie groups and Lie algebras	155
5.8	Irreducible representations of $SU(2)$	158
5.9	Irreducible representations of compact Lie groups	162
5.10	Symmetry groups and projective representations	167
5.11	Position, momentum, and free Hamiltonian	177
5.12	Stone’s Theorem	183
	Notes	187
Part II Between $C_0(X)$ and $B(H)$		
6	Classical models of quantum mechanics	191
6.1	From von Neumann to Kochen–Specker	193
6.2	The Free Will Theorem	202
6.3	Philosophical intermezzo: Free will in the Free Will Theorem	205
6.4	Technical intermezzo: The GHZ-Theorem	210
6.5	Bell’s theorems	213
6.6	The Colbeck–Renner Theorem	221
	Notes	231
7	Limits: Small \hbar	247
7.1	Deformation quantization	250
7.2	Quantization and internal symmetry	253
7.3	Quantization and external symmetry	256
7.4	Intermezzo: The Big Picture	259
7.5	Induced representations and the imprimitivity theorem	262
7.6	Representations of semi-direct products	268
7.7	Quantization and permutation symmetry	275
	Notes	289

8	Limits: large N	293
8.1	Large quantum numbers	294
8.2	Large systems	298
8.3	Quantum de Finetti Theorem	304
8.4	Frequency interpretation of probability and Born rule	310
8.5	Quantum spin systems: Quasi-local C^* -algebras	318
8.6	Quantum spin systems: Bundles of C^* -algebras	323
	Notes	329
9	Symmetry in algebraic quantum theory	333
9.1	Symmetries of C^* -algebras and Hamhalter's Theorem	334
9.2	Unitary implementability of symmetries	344
9.3	Motion in space and in time	346
9.4	Ground states of quantum systems	350
9.5	Ground states and equilibrium states of classical spin systems	352
9.6	Equilibrium (KMS) states of quantum systems	358
	Notes	365
10	Spontaneous Symmetry Breaking	367
10.1	Spontaneous symmetry breaking: The double well	371
10.2	Spontaneous symmetry breaking: The flea	375
10.3	Spontaneous symmetry breaking in quantum spin systems	379
10.4	Spontaneous symmetry breaking for short-range forces	383
10.5	Ground state(s) of the quantum Ising chain	386
10.6	Exact solution of the quantum Ising chain: $N < \infty$	390
10.7	Exact solution of the quantum Ising chain: $N = \infty$	397
10.8	Spontaneous symmetry breaking in mean-field theories	409
10.9	The Goldstone Theorem	416
10.10	The Higgs mechanism	424
	Notes	430
11	The measurement problem	435
11.1	The rise of orthodoxy	436
11.2	The rise of modernity: Swiss approach and Decoherence	440
11.3	Insolubility theorems	445
11.4	The Flea on Schrödinger's Cat	450
	Notes	457
12	Topos theory and quantum logic	459
12.1	C^* -algebras in a topos	461
12.2	The Gelfand spectrum in constructive mathematics	466
12.3	Internal Gelfand spectrum and intuitionistic quantum logic	471
12.4	Internal Gelfand spectrum for arbitrary C^* -algebras	476
12.5	"Daseinisation" and Kochen–Specker Theorem	485
	Notes	493

A	Finite-dimensional Hilbert spaces	495
A.1	Basic definitions	495
A.2	Functionals and the adjoint	497
A.3	Projections	499
A.4	Spectral theory	500
A.5	Positive operators and the trace	507
	Notes	513
B	Basic functional analysis	515
B.1	Completeness	516
B.2	ℓ^p spaces	518
B.3	Banach spaces of continuous functions	522
B.4	Basic measure theory	523
B.5	Measure theory on locally compact Hausdorff spaces	526
B.6	L^p spaces	534
B.7	Morphisms and isomorphisms of Banach spaces	538
B.8	The Hahn–Banach Theorem	541
B.9	Duality	545
B.10	The Krein–Milman Theorem	553
B.11	Choquet’s Theorem	557
B.12	A précis of infinite-dimensional Hilbert space	562
B.13	Operators on infinite-dimensional Hilbert space	568
B.14	Basic spectral theory	577
B.15	The spectral theorem	585
B.16	Abelian $*$ -algebras in $B(H)$	593
B.17	Classification of maximal abelian $*$ -algebras in $B(H)$	601
B.18	Compact operators	608
B.19	Spectral theory for self-adjoint compact operators	611
B.20	The trace	617
B.21	Spectral theory for unbounded self-adjoint operators	625
	Notes	638
C	Operator algebras	645
C.1	Basic definitions and examples	645
C.2	Gelfand isomorphism	648
C.3	Gelfand duality	653
C.4	Gelfand isomorphism and spectral theory	657
C.5	C^* -algebras without unit: general theory	660
C.6	C^* -algebras without unit: commutative case	664
C.7	Positivity in C^* -algebras	668
C.8	Ideals in Banach algebras	671
C.9	Ideals in C^* -algebras	674
C.10	Hilbert C^* -modules and multiplier algebras	677
C.11	Gelfand topology as a frame	685
C.12	The structure of C^* -algebras	691

C.13	Tensor products of Hilbert spaces and C^* -algebras	697
C.14	Inductive limits and infinite tensor products of C^* -algebras	707
C.15	Gelfand isomorphism and Fourier theory	714
C.16	Intermezzo: Lie groupoids	725
C.17	C^* -algebras associated to Lie groupoids	730
C.18	Group C^* -algebras and crossed product algebras	734
C.19	Continuous bundles of C^* -algebras	737
C.20	von Neumann algebras and the σ -weak topology	742
C.21	Projections in von Neumann algebras	746
C.22	The Murray–von Neumann classification of factors	750
C.23	Classification of hyperfinite factors	754
C.24	Other special classes of C^* -algebras	758
C.25	Jordan algebras and (pure) state spaces of C^* -algebras	763
	Notes	768
D	Lattices and logic	777
D.1	Order theory and lattices	777
D.2	Propositional logic	784
D.3	Intuitionistic propositional logic	790
D.4	First-order (predicate) logic	793
D.5	Arithmetic and set theory	797
	Notes	803
E	Category theory and topos theory	805
E.1	Basic definitions	806
E.2	Toposes and functor categories	814
E.3	Subobjects and Heyting algebras in a topos	820
E.4	Internal frames and locales in sheaf toposes	826
E.5	Internal language of a topos	828
	Notes	833
	References	835
	Index	881

Foundations of Quantum Theory
From Classical Concepts to Operator Algebras

Landsman, K.

2017, XXXVI, 861 p. 9 illus., 8 illus. in color., Hardcover

ISBN: 978-3-319-51776-6