ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

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Preface

Imaging of the Earth's surface by spaceborne synthetic aperture radars (SAR) may be adversely affected by the ionosphere, as the temporal dispersion of radio waves in the ionospheric plasma gives rise to distortions of signals emitted and received by the radar antenna. Those distortions lead to a mismatch between the actual received signal and its assumed form used in the signal processing algorithm. This, in turn, causes a deterioration of the image. The latter may appear particularly noticeable if the carrier frequency of the radar is not very high, which is the case important for a variety of applications (Chapter 3).

The main purpose of this book is to present the most recent developments in the mathematical analysis of ionospheric distortions of the SAR images, as well as to explore the appropriate strategies for their mitigation. As a prerequisite to addressing these topics, the book also discusses the radar ambiguity theory as it applies to synthetic aperture imaging and the propagation of radio waves through the ionospheric plasma, including the anisotropic and turbulent cases. In addition, it covers a host of related subjects, such as the mathematical modeling of extended radar targets (as opposed to point-wise targets) and the scattering of radio waves off those targets, as well as the theoretical analysis of the start-stop approximation, which is used routinely in SAR signal processing but often without proper justification.

From a mathematical standpoint, SAR imaging represents a class of inverse scattering problems in which the aperture of the imaging instrument is synthesized over multiple positions of one and the same antenna at different moments of time. In the Introduction (Chapter 1), we provide a rationale for the study of transionospheric SAR and give a general overview of the problems to be discussed in the rest of the monograph. In particular, we qualitatively explain the mechanism of ionospheric distortions in SAR images, the approach we propose for their mitigation, and the associated difficulties/issues that need to be addressed. We also present a bibliographical review of the relevant publications in both mathematical and engineering literature.

In Chapter 2, we examine the conventional SAR imaging with no account of the Earth's ionosphere yet. In particular, we emphasize that the SAR data collection primarily relies on the phase information of the waves (as opposed to amplitude). We also identify a specific SAR scenario that we subsequently focus on, namely, the monostatic stripmap imaging, as it is most relevant for spaceborne applications analyzed later in the book. Then, we introduce the assumptions that we need in order to build a consistent mathematical model for SAR imaging and present a detailed analysis of the key notions and concepts of the SAR ambiguity theory. Those notions and concepts include the incident and received field, the interrogating waveforms, the matched filter, the synthetic aperture which is based on multiple interrogating pulses emitted and received as the radar antenna moves, the generalized ambiguity function (GAF) which defines the mapping between the imaged quantity and the image, the factorized form of the GAF, and the azimuthal and range resolution. The matched filter and the summation over the synthetic aperture are the two key components of the signal processing algorithm that render inversion of the radar data (received field) in SAR imaging.

One important assumption needed in Chapter 2 for constructing the SAR ambiguity theory is the start-stop approximation. It is a standard tool for data processing in synthetic aperture imaging. Under this approximation, the antenna is considered motionless when each interrogating pulse is emitted and the scattered response is received, after which the antenna moves to the next sending/receiving position along its trajectory. The start-stop approximation considerably simplifies the mathematical analysis of SAR imaging; hence, we employ it in Chapter 2. However, using this approximation for a given imaging scenario may require mathematical justification. We provide this justification in Chapter 6, while in Chapters 3, 4, and 5, we focus on the main subject of the book, which is the effect of the ionosphere on spaceborne SAR imaging.

Another principal requirement for building the SAR ambiguity theory in Chapter 2 is linearity of the scattering on the target. This requirement is indeed of key significance, because it makes the inverse scattering problem linear and thus amenable to solution. Traditionally, it is met by linearizing the scattering. The linearization is attained by assuming that the scattering is weak and employing the first Born approximation. Accordingly, our exposition of the material in Chapters 2 through 5 relies on the first Born approximation. Yet we indicate in Chapter 2 that the use of the first Born approximation may entail some inherent contradictions. The remedying of those contradictions is also deferred until after the ionospheric discussion; see Chapter 7.

Chapters 3, 4, and 5 are central for the study of ionospheric effects on SAR. In Chapter 3, we conduct a thorough theoretical analysis of transionospheric SAR imaging and accurately quantify the distortions of images in the case where the interrogating field is scalar while both the medium, i.e., the ionosphere, and the target are isotropic. We show that the distortions can be attributed to the mismatch between the interrogating field received by the radar antenna and the matched filter used for signal processing. In doing so, we model the ionosphere as cold plasma and pay specific attention to the analysis of the propagation of radio waves through this plasma. Then, to mitigate the ionospheric distortions of SAR images, we propose to probe the terrain, and hence the ionosphere, on two distinct carrier frequencies. The resulting two images appear shifted with respect to one another, and the magnitude of the shift (which undergoes a weak linear variation across the image in its own right) allows one to evaluate the total electron content (TEC) in the ionosphere. Knowing the TEC, one can correct the matched filter and hence improve the quality of the image. Robustness of the proposed approach can subsequently be increased by applying an image registration technique (feature-based or area-based) to the two images obtained on two frequencies. The latter leads to a precise estimate of the TEC and hence to a very accurate correction of the matched filter. In Chapter 3, we also analyze some additional factors that may affect the spaceborne SAR performance. Those include the Ohm conductivity of the ionospheric plasma, which is due to the collisions of electrons with other particles, as well as the possible horizontal variation of the ionospheric parameters.

The key phenomenon of ionospheric turbulence and its implications for spaceborne SAR are examined in Chapter 4. The ionospheric turbulence manifests itself as random fluctuations of the electron number density. The effect of the latter on SAR is evaluated in the statistical sense and shown to be stronger for the azimuthal resolution than the range resolution. The development of an efficient strategy for mitigating the adverse impact of the ionospheric turbulence on the performance of spaceborne SAR is left for future study.

In Chapter 5, we discuss the anisotropic phenomena relevant for SAR imaging and for the first time in this book take into account the vector nature of the interrogating electromagnetic field. The ionospheric plasma becomes anisotropic (gyrotropic) due to the magnetic field of the Earth. The propagation of radio waves in a gyrotropic medium is accompanied by the Faraday rotation, which is a slow rotation of the polarization plane with distance. For single-polarization spaceborne SAR imaging, the Faraday rotation presents an additional source of mismatch between the received signal and the matched filter and hence causes additional image distortions. In particular, the image of a point target may have its intensity peak split in the range direction. To tell between the cases of low reflectivity and those where the low antenna signal is due to the Faraday rotation, we employ image autocorrelation analysis. It helps us obtain the parameters of the Faraday rotation and quantify its impact on the image, which then allows us to correct the matched filter accordingly.

Chapter 6 presents a mathematical analysis of the start-stop approximation for SAR imaging. This analysis is needed because the image may be affected by the factors that the start-stop approximation inherently neglects. First and foremost, those factors are the displacement of the antenna during the pulse round-trip time between the platform trajectory and the Earth's surface and the Doppler frequency shift. In Chapter 6, we show that both phenomena can be accounted for by appropriately correcting the matched filter. This, in turn, requires computing the emitted and scattered field with the help of the Lorentz transform. If the filter is corrected, then the effect of the antenna motion on the SAR image becomes negligibly small. Otherwise, the image gets shifted and also distorted. For the typical parameters that pertain to spaceborne SAR imaging and that we use throughout the book, neither the magnitude of the shift nor that of the distortions is substantial. However, for other imaging settings, the distortions due to the start-stop approximation may become significant, which is a phenomenon not commonly discussed in the SAR literature.

In Chapter 7, we return to the assumption of weak scattering. It is standard for the theoretical analysis of SAR imaging, as it helps linearize the inverse scattering problem via the first Born approximation. Yet it is inconsistent with another common assumption that the interrogating waves do not penetrate into the target material and get scattered off its surface only, which essentially means that the scattering is strong. We address this and other existing inconsistencies, such as the absence of the resonant Bragg scale in scattering, by introducing a new model for radar targets that allows us to compute the scattered field from first principles. Specifically, the scattering medium is considered in the form of a horizontally inhomogeneous yet isotropic dielectric half-space. The new model renders the assumption of weak scattering unnecessary, but still keeps the overall inverse scattering problem linear. It also allows one to interpret the observable quantity in SAR imaging as a slowly varying amplitude of the Bragg harmonic in the spectrum of ground reflectivity. Our analysis in Chapter 7 takes into account the polarization of the incident and scattered field. We also show the relation between our new model for radar targets and two well-known models of surface scattering: the one based on the Leontovich approximation and the one that employs the notion of rough surfaces.

In Chapter 8, we consider anisotropic targets. Namely, we interpret the target as a weakly conductive birefringent dielectric and derive a necessary and sufficient condition under which this model allows one to reconstruct all the existing degrees of freedom in the scattered signal. Prior studies in the literature have introduced those degrees of freedom phenomenologically.

Finally, in Chapter 9, we summarize the results presented in the previous chapters of the book, outline the conclusions, and identify the key outstanding issues that, in our opinion, are most interesting and challenging for future investigation. In that regard, we note that as of yet each phenomenon that affects the spaceborne SAR imaging and is covered in the book has been covered in isolation from the others. Therefore, in the future, it will be very important to study the interactions between the various phenomena (e.g., the stochasticity of the ionosphere and its anisotropy). Among other important issues are the mitigation of the random component of ionospheric distortions, i.e., the distortions due to the turbulence, and the treatment of dispersive targets and the distinction between the dispersion at the target and that in the ionosphere.

Each chapter of the book (except Chapter 1) is concluded with a chapter summary that presents a succinct account of what has been done in this chapter. The summaries for Chapters 2 through 8 also contain bulleted lists of the most important concepts and equations.

The dependence between the chapters is schematically shown in Figure 1.

Earlier versions of the analyses and results presented in this monograph have been reported in a series of journal papers that appeared between 2009 and 2015 (see [1–7]) as well as in the PhD dissertation of the second author (see [8]). When preparing the manuscript of this book, we have very substantially improved and refined most of those results and included a number of new results as well. As far as the developments that have stayed relatively unchanged, we are grateful to SIAM and IOP for their permission to republish certain materials from [1–7].

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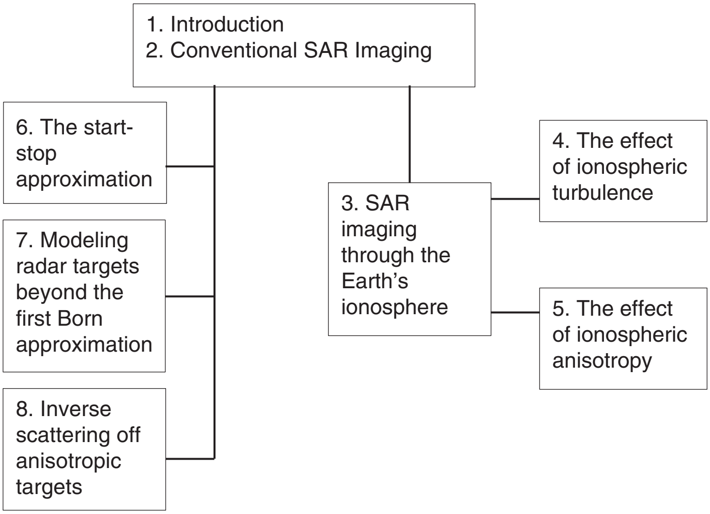

Fig. 1 Schematic dependence between the chapters of the book

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The book is intended for applied mathematicians (from academia, government laboratories, and the industry) interested in the area of radar imaging or, more generally, remote sensing, as well as physicists and electrical/electronic engineers who develop/operate spaceborne SAR sensors and perform the data processing. The book may also be useful for researchers and practitioners working on other types of imaging. Moreover, it is accessible by graduate students in applied mathematics, physics, engineering, and related disciplines.

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