

Preface

The recent studies on fractional differential equations indicate that a variety of interesting and important results concerning existence and uniqueness of solutions, stability properties of solutions, and analytic and numerical methods of solutions for these equations have been obtained, and the surge for investigating more and more results is underway [36, 37]. The tools of fractional calculus have played a significant role in improving the modeling techniques for several real-world problems. Nowadays, fractional-order differential equations appear extensively in a variety of applications such as diffusion processes, chaos, thermo-elasticity, biomathematics, fractional dynamics, etc. [87, 118, 142, 183, 187]. One of the characteristics of operators of fractional order is their nonlocal nature accounting for the hereditary properties of many phenomena and processes involved. For the recent development of the topic, we refer the reader to a series of books and papers [1, 6, 8, 9, 40, 41, 91, 96, 114, 121, 133, 141, 180]. However, it has been noticed that most of the work on the topic is based on Riemann-Liouville, and Caputo-type fractional differential equations. Another kind of fractional derivatives that appears side by side to Riemann-Liouville and Caputo derivatives in the literature is the fractional derivative due to Hadamard, introduced in 1892 [89], which contains logarithmic function of arbitrary exponent in the kernel of the integral appearing in its definition. Hadamard-type integrals arise in the formulation of many problems in mechanics such as in fracture analysis. For details and applications of Hadamard fractional derivative and integral, we refer the reader to the works in [51–53, 94, 96–98].

The main idea for writing this book is to focus on the recent development of fractional differential equations, integrodifferential equations, and inclusions and inequalities involving Hadamard derivative and integral. In precise terms, we address the issues related to initial and boundary value problems involving Hadamard-type differential equations and inclusions as well as their functional counterparts. Much of the material presented in this book is based on the recent research of the authors on the topic.

The book is organized as follows. Chapter 1 contains fundamental concepts of multivalued analysis, differential inclusions, and Hadamard fractional calculus. We also describe a number of fixed-point theorems used to establish the existence results for the proposed problems. Included among the fixed-point theorems recognized by their names are Amini-Harandi, Boyd and Wong, Covitz and Nadler, Dhage, Guo-Krasnosel'skii, Krasnosel'skii, Krasnosel'skii-Zebreiko, Leggett-Williams, Leray-Schauder nonlinear alternative for single and multivalued maps, O'Regan, Petryshyn, and Sadovski.

Chapter 2 is devoted to the study of existence of solutions for initial and boundary value problems of fractional-order Hadamard-type functional and neutral functional differential equations and inclusions with both retarded and advanced arguments.

The objective of Chapter 3 is to investigate fractional integral boundary value problems involving Hadamard fractional derivative and integral for nonlocal fractional differential equations and inclusions. We establish some existence and uniqueness results for the given problems by means of classical fixed-point theorems.

In Chapter 4, we introduce a new class of mixed initial value problems involving Hadamard derivative and Riemann-Liouville fractional integrals. Existence and uniqueness results for the given problems are obtained with the help of standard fixed-point theorems. The purpose of Chapter 5 is to study nonlocal boundary value problems of Riemann-Liouville fractional differential equations and inclusions equipped with Hadamard fractional integral boundary conditions. In Chapter 6, we switch onto the study of coupled systems of Hadamard- and Riemann-Liouville-type fractional differential equations with coupled and uncoupled nonlocal Hadamard fractional boundary conditions.

Chapter 7 studies nonlinear Langevin equations and inclusions involving Hadamard-Caputo-type fractional derivatives with nonlocal fractional integral conditions. Then we extend our study to coupled systems of Langevin equation with fractional integral conditions. In Chapter 8, we investigate a nonlinear boundary value problem of impulsive hybrid multi-orders Caputo-Hadamard fractional differential equations with nonlinear integral boundary conditions. In Chapter 9, we study the existence of solutions for initial and boundary value problems of hybrid fractional differential equations and inclusions of Hadamard type. In Chapter 10, we develop some fractional integral inequalities using the Hadamard fractional integral. Several new integral inequalities are obtained by using Young and weighted AM-GM inequalities. Many special cases are also discussed. Moreover, a Grüss-type Hadamard fractional integral inequality is obtained. Chapter 11 is concerned with the existence criteria of positive solutions for fractional differential equations of Hadamard type with integral boundary condition on infinite intervals.

We are grateful to the senior editor, Marc Strauss, for his continued support and encouragement during the preparation of this text.

Jeddah, Saudi Arabia

Jeddah, Saudi Arabia

Ioannina, Greece

Bangkok, Thailand

November 2016

Bashir Ahmad

Ahmed Alsaedi

Sotiris K. Ntouyas

Jessada Tariboon

Hadamard-Type Fractional Differential Equations,
Inclusions and Inequalities

Ahmad, B.; Alsaedi, A.; Ntouyas, S.K.; Tariboon, J.

2017, XIII, 414 p. 1 illus., Hardcover

ISBN: 978-3-319-52140-4