Chapter 2
Basic Notions

2.1 Network Graph Notions

Classical modeling of network systems uses the graph theory notions to define the graph of network. The following fundamental notions are used:

The branch (*edge*) is a basic axiomatic notion and is defined as any element, represented by a line segment, with specified two different ends: *terminals plus* (+) and *terminal minus* (−) designating positive branch direction from terminal minus to terminal plus.

The node (*vertex*) is a basic axiomatic notion and is defined as a point designating one or more branch terminals.

The connection or *interconnection* of two or more branches means that they have one common terminal.

The planar graph is a graph, which if drawn in planar plane has not crossing branches.

The tree is defined as any minimal set of interconnected branches, called *tree branches*, which contain all nodes in network system. From the definition of a tree, it follows that it is an open subgraph of a network system graph.

The cotree is defined as a set of branches, which does not belong to any tree. The sets of tree branches and cotree branches are *complementary* if they are disjoint. Cotree is a dual concept, in any sense, to the tree.

2.2 Electrical Network Notions

Graph theory provides an excellent tool for visualizing and developing the model of network graph. However, graph theory is of little use in modeling the physical states of the network system and interrelations of branch currents and branch voltages. Furthermore, even representation of network by a graphical model has

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serious drawbacks from the point of view of development of the mathematical model of network. Hence, graph theory is used as an aid in developing the model. In what follows, the final model presented is algebraic in character and is expressed in terms of linear algebra. Taking into account the above definitions, the basic notions used in mathematical modeling of network system are defined as follows:

The directed electrical branch (called branch in what follows) is a basic notion and is defined as any electrical element (object or earth or air), represented by a line segment, which has two specified end terminals: plus terminal and minus terminal, designating branch direction.

The electrical network system (called network system) is a set of connected branches, which are electrically interconnected and electrically active. Electrical interconnection means that the electrophysical quantities of all branches (e.g., branch currents) are dependent on each other. In other words, in network system, the electrophysical quantity of every branch (e.g., branch current) depends on electrophysical quantities of all other branches, and the system has the ability to be active (possesses electricity source).

Electrical network system is *closed* if each electrical branch is connected at least with two or more other branches and is *open* if there are branches connected with one terminal only to the other branches.

The branch current is a quantity, which characterizes the amount of medium (electric charge) transmitted along the branch in a unit time interval. It can be measured at any one point of branch, assuming that there are no losses or dissipation of the medium in the branch.

The branch voltage is a quantity, which characterizes the difference of electric "pressure" or "stress" being exerted and measured on the branch between plus terminal and minus terminal.

The above quantities are fundamental notions used in the mathematical modeling of electrical systems. In order to derive the algebraic relations precisely, it is necessary to define the algebraic sign of the branch current and branch voltage values in relation to the branch direction. The choice of sign is optional. In what follows, we assume:

Branch current has *plus sign* if current is flowing from branch terminal plus to branch terminal minus. Branch voltage has *plus sign* if the voltage value measured along the branch is increasing in the direction from terminal minus to terminal plus.

The branch parameters are the constant coefficients of branch current as a function of branch voltage, called *branch admittances* or the constant coefficients of branch voltage as a function of branch current, called *branch impedances*, and are the constant coefficients of electricity sources: *the ideal current sources* and *the ideal voltage sources*.

Real electrical branches are often combinations of the above main kinds, e.g., in power system analysis, there are sending (generating) branches, which are combination of ideal voltage sources and impedances and receiving (load-consuming) branches, which are treated as current sources and admittances.

The node-set is defined as a set of branches connected together to one common terminal (node of a graph). *The directed node-set* can be defined as a set of network branches, which connect one node with the rest of network. The branches defining a directed node-set are connected to the node (branch terminal) either by the terminal plus or by the terminal minus. So there are two options and consequently two possible definitions of the *direction* of node-set. In the first option, the direction of node-set is the same as the direction of branches directed toward common terminal, and in the second option, it is the same as the direction of branches directed away from the common terminal. From the definition of the node-set, it follows that the sum of all node-sets is equal to zero. This follows because each branch is represented in one node-set by plus terminal and in another node-set by minus terminal. Removing any one node-set in the network must leave a sum of the remaining node-sets not equal to zero. The number of all node-sets minus any one is denoted by n and is a number of linearly independent node-sets.

The cut-set is defined as a set of network branches connecting two parts of network. It is the generalization of node-set. Note that a node-set is also connecting two parts of network, because it connects one node with the rest of network, so is a special kind of cut-set. Graphically, cut-set is a set of branches cut by a line disconnecting two parts of network. From the definition of a cut-set, it follows that the number of cut-sets is equal to the number of all subsets of the $n + 1$ node-sets, excluding the empty subset and the whole subset, so the number of cut-sets is equal to $2^{n+1} - 2$.

The path of branches is intuitively defined as any set of two or more branches and nodes (branch terminals) connected in series (one branch after another). The number of nodes is equal to the number of branches plus one. The path of branches is closed if two ends of path terminals are one terminal. The direction of path is optional and may be defined equal to the direction of any branch in path.

The loop is generally defined as a closed path which is not crossing itself. Intuitively, it is a loop or a mesh in planar networks.

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