Preface

This is a heavily reworked and considerably shortened version of the first edition of this text. Especially, considerable "extra" material and background material have now been either removed or moved to the appendices. Moreover, some important rearrangement of chapters has taken place to facilitate its intended use as a text.

Chapters 1–5 provide the measure-theoretic mathematical foundation for the rest of the text. Then Chapter 6 (Distribution and Quantile Functions) and Chapter 7 (Independence and Conditional Expectation) hone some tools geared to probability theory. Appendix A (see page 417) provides a brief introduction to elementary probability theory that could be useful for some mathematics students (This Appendix A begins on page 417). A very useful version of this text could end at this page; omit Sections 10.5–10.9, and greatly slim down or eliminate the later parts of Chapters 7 and 13 (intentionally paired), Chapters 12 and 15 (again, intentionally paired), Chapter 6, Chapter 11, and Chapter 14.

The classical weak law of large numbers (WLLN) and strong law of large numbers (SLLN) as presented in Sections 8.2–8.4 are particularly complete, and they also emphasize the important role played by the behavior of the maximal summand. Presentation of good inequalities is emphasized in the entire text, and Chapter 8 is a good example. (Also, there is a very general collection of characterizations of the WLLN in Section C.1, that is then specialized carefully to the context of the behavior of the sample variance as an estimator. This Appendix C will also be appealed to in the optional Sections 10.5 (for a very general CLT for sums of row independent rvs) and 10.6 (where the domain of attraction of the Normal distribution is characterized via many very different looking equivalent conditions)).

The classical central limit theorem (CLT) and its Lindeberg, Liapunov, and Berry–Esseen generalizations are presented in Chapter 10 using the characteristic function (chf) methods introduced in Chapter 9. Many statistical applications also appear in Chapter 10.

A form of the most general CLT for "negligible pieces" is found in the optional Section 10.5, along with a more statistical variant. This is specialized to the iid case in Section 10.6—where many versions of necessary and sufficient conditions are presented. These Section 10.6 variants are justified primarily in Sections C.1–C.3. Conditions for both the weak bootstrap and the strong bootstrap are also developed in Section 10.8, and a universal bootstrap CLT based on light trimming of the sample is presented in Section 10.9. This approach emphasizes a statistical perspective. Gamma and Edgeworth approximations appear at the end of Chapter 11. The early parts of Chapter 11 deal with infinitely divisible and stable rvs. One of the main objectives in this second edition is to make it easier for many instructors to pick and choose from such topics (All references to the Stein method in the first edition have been removed because of a basic problem with that presentation.)

Both the distribution function (df $F(.)$) and the quantile function (qf $K(\cdot) \equiv F^{-1}(\cdot)$) are emphasized throughout (quantile functions are important to statisticians). In Chapter 6, much general information about both dfs, qfs, and the Winsorized variance is developed. The text includes presentations showing how to exploit the inverse transformation $X \equiv K(\xi)$ with $\xi \cong$ Uniform(0, 1). In particular, Section C.6 inequalities relating the qf and the Winsorized variance to some empirical process results of Chapter 12 are used in Chapter 15 of this text to treat trimmed means and L-statistics, rank and permutation tests, and sampling from finite populations (Even more of this appears in the first addition of this text).

Chapter 13 provides quite a strong set of results for martingales. (The first edition includes even more topics and examples. Especially, there is a nice treatment of predictable variation.)

Chapter 14 considers convergence in law on more general metric spaces.

I have learned much through my association with David Mason, and I would like to acknowledge that here. Especially (in the context of this text), Theorem 12.10.3 is a beautiful improvement on Theorem 12.10.2, in that it still has the potential for necessary and sufficient results. I really admire the work of Mason and his colleagues. It was while working with David that some of my present interests developed. In particular, a useful companion to Theorem 12.10.3 is knowledge of quantile functions. Section 6.6 and Appendix C owe a debt to what I have compiled and produced on that topic while working on various applications, partially with David.

Jon Wellner has taught from several versions of this text. In particular, he typed an earlier version and thus gave me a major critical boost. That head start is what turned my thoughts to writing a text for publication. The Hoffman–Jorgensen inequalities in Section 8.10 came from him. He has also formulated a number of exercises, suggested various improvements, and offered good suggestions and references regarding predictable processes. My thanks to Jon for all of these contributions (Obviously, whatever problems may remain lie solely with me.)

My thanks go to John Kimmel for his interest in the first version of this text, and for his help and guidance through the various steps and decisions.

This is intended as a textbook, not as a research manuscript. Accordingly, it is somewhat lightly referenced. There is a section at the end that contains some discussion of the literature.

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