

Chapter 2

Steps Towards the De Sitter Relativity

It is here introduced the *Erlangen program for Physics* by G. Arcidiacono, where the three relativities (Galilei, Poincaré and de Sitter) are a groupal “matryoshka”, i.e., three description levels of physical phenomena. The Arcidiacono transformations for the de Sitter group and the cosmological consequences are also introduced.

2.1 Some Recalls About Lie Groups¹

Symmetry is one of the most fundamental properties of nature. The branch of mathematics dealing with symmetry is the group theory. Let us recall that a group is a set G endowed with an associative operation denoted ab for group elements $a, b \in G$. The group also contains a unique identity element, denoted u , and each group element a has an inverse a^{-1} satisfying $aa^{-1} = a^{-1}a = u$. Lie groups lie at the intersection of two fundamental fields of mathematics: algebra and geometry. A Lie group is first of all a group, and secondly it is a differentiable manifold. Differentiable manifolds are the basic objects in differential geometry and they generalize to higher dimensions the curves and surfaces. A manifold is a topological space which resembles Euclidean space locally. A Lie group is a group which is also a differentiable manifold, with the property that the group operations are compatible with the differential structure. That is, the applications

$$(a) \quad G \times G \rightarrow G : (a, b) \rightarrow ab \quad (2.1)$$

$$(b) \quad G \rightarrow G : a \rightarrow a^{-1} \quad (2.2)$$

¹For a more extended treatment of Lie algebras and groups, the reader is referred to classical literature [1–7].

are differentiable. Informally, a Lie group is a group of continuous symmetries. A Lie algebra is a vector space V over some field K together with a binary operation $[\cdot, \cdot]: V \times V \rightarrow V$ called the Lie bracket, which satisfies the following axioms

(a) Bilinearity:

$$[a + b, c] = [a, c] + [b, c], \quad (2.3)$$

$$[a, b + c] = [a, b] + [a, c], \quad (2.4)$$

$$[\lambda a, b] = [a, \lambda b] = \lambda[a, b], \quad (2.5)$$

for all $a, b \in V$ and for all $\lambda \in K$.

(b) Anticommutativity:

$$[a, b] = -[b, a] \text{ for all } a, b \in V. \quad (2.6)$$

(c) The Jacobi identity:

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0 \text{ for all } a, b, c \in V. \quad (2.7)$$

From the second axiom we deduce

$$[a, b] + [b, a] = 0 \Rightarrow [a, a] + [a, a] = 2[a, a] = 0 \Rightarrow [a, a] = 0 \quad (2.8)$$

for all $a \in V$. For any associative algebra $(A, *)$ one can construct a Lie algebra where the Lie bracket is defined by commutator $[a, b] = a * b - b * a$.

If $\{e_1, \dots, e_n\}$ is a basis of V we can introduce the so-called structure constants defined by $[e_i, e_j] = c_{kij}e_k$, and these constants are useful to calculate the commutator of any pair $a, b \in V$. In fact $[a, b] = a_i b_j [e_i, e_j] = a_i b_j c_{kij} e_k$.

Any vector space V endowed with the identically zero Lie bracket becomes a Lie algebra. Such Lie algebras are called Abelian. The three-dimensional Euclidean space R^3 with the Lie bracket given by the cross product of vectors becomes a three-dimensional Lie algebra. After these premises, we analyze some groups of transformation which are particularly important in the study of physics. In an Euclidean n -dimensional space, the linear and homogeneous transformations

$$x'_i = \sum_k a_{ik} x_k \quad (2.9)$$

that leave invariable the quadratic form $x_1^2 + \dots + x_n^2$ are said to be orthogonal transformations; they form the orthogonal group O_n . The matrix $[a_{ik}]$ is orthogonal, that is its inverse is equal to its transpose. Moreover, the determinant of $[a_{ik}]$ is ± 1 , and the orthogonal transformations whose matrices have determinant equal to 1 form a subgroup called special orthogonal group $SO(n)$ which is the group of all

rotations about the origin of n -dimensional Euclidean space. The numbers of parameters of the orthogonal and special orthogonal group is $n(n-1)/2$. The orthogonal group is a subgroup of the general linear group GL_n with determinant of $[a_{ik}] \neq 0$ with n^2 parameters. We also quote the special linear group SL_n , the subgroup of GL_n consisting of matrices with determinant equal to 1. Even relevant is the unitary group $U(n)$ of dimension n^2 , consisting of $n \times n$ unitary complex matrices, which is a subgroup of the general linear group. Finally we quote the special unitary group denoted $SU(n)$ consisting of all $n \times n$ unitary complex matrices with determinant 1, whose dimension is $n^2 - 1$.

The mathematician Sophus Lie had the idea to consider, in the study of continuous groups of transformations, the infinitesimal elements of group, that is the elements that are in a neighborhood of the identity element. The properties of these infinitesimal elements characterize the properties of the group. If we consider a group of transformations with one parameter t

$$x'_i = x_i(x_1, \dots, x_n, t) \quad (2.10)$$

and we introduce the infinitesimal operator X of the group, it is possible to show that

$$x'_i = x_i + tXx_i + \frac{1}{2}t^2X^2x_i + \dots = e^{tX}x_i. \quad (2.11)$$

When the parameter t is considered infinitely small, we obtain the infinitesimal transformations

$$x'_i = x_i + tXx_i. \quad (2.12)$$

For example, the infinitesimal operator of rotations group on a plan is $X = x_1\partial_2 - x_2\partial_1$, and we have the infinitesimal transformations

$$x'_1 = x_1 - tx_2 \quad (2.13)$$

$$x'_2 = x_2 + tx_1 \quad (2.14)$$

Assigned a continuous group with r parameters, it is possible to calculate its infinitesimal operators and its infinitesimal transformations. Viceversa, assigning r infinitesimal operators $X_1, \dots, X_r$ and introducing the Lie bracket $[X_i, X_k] = X_iX_k - X_kX_i$ we have r infinitesimal operators, among them independent, that can produce a group with r parameters if and only if $[X_i, X_k] = \sum_s c_{iks}X_s$.

Two groups with the same structure constants are isomorphic in the neighborhood of the identity element. Cartan has given a complete classification of the simple groups, and he has found that there are four infinite families.

- (1) A_n groups; a model of these groups is $SU(n+1)$
- (2) B_n groups; a model of these groups is $SO(2n+1)$

- (3) C_n groups; a model of these groups is the symplectic group $S_p(2n)$
- (4) D_n groups; a model of these groups is $SO(2n)$.

Then there are other five possible simple groups and these are the so-called exceptional cases G_2, F_4, E_6, E_7 and E_8 . These cases are said exceptional because they do not fall into infinite series of groups of increasing dimension. G_2 has 14 dimensions, F_4 has 52 dimensions, E_6 has 78 dimensions, E_7 has 133 dimensions and E_8 248 dimensions. The E_8 algebra is the largest and most complicated of these exceptional cases and is often the last case of various theorems to be proved. E_8 is a very beautiful group; in fact it is the symmetry group of a particular 57-dimensional object.

2.2 Lie Groups of Spacetime

The spacetime of Newtonian physics is a four-dimensional manifold endowed with the following geometrical structures:

- (1) Absolute time
- (2) Absolute space
- (3) Absolute spatial distances
- (4) Absolute temporal distances.

The spacetime of special relativity, instead, is a four-dimensional manifold endowed with the following geometrical structures:

- (1) Relative time
- (2) Relative space
- (3) Absolute Space-Time distances.

Let us remember that Galileo group is the main group of classical physics and is formed by the composition of the following transformations.

- (a) Spatial rotations characterized by three parameters

$$x'_\mu = a_{\mu\nu}x_\nu \quad (2.15)$$

where $[a_{\mu\nu}]$ is a 3×3 orthogonal matrix whose determinant is $+1$.

- (b) Inertial motions (boosts) characterized by the three components of velocity

$$x'_\mu = x_\mu + v_\mu t \quad (2.16)$$

- (c) Spatial translations characterized by three parameters

$$x'_\mu = x_\mu + a_\mu \quad (2.17)$$

(d) Temporal translations characterized by only one parameter

$$x'_\mu = x_\mu, t' = t + t_0 \quad (2.18)$$

The global Galileo group dimension is 10. This group appears in the formulation of Galileo well know relativity principle. Moving on to relativistic physics, spatial rotations and boosts are blended in a unique operation: the rotations of a pseudo-Euclidean space (Minkowski spacetime) M_4 , characterized by 6 parameters,

$$x'_i = a_{ik}x_k \quad (2.19)$$

where $x_4 = ict$. These transformations, called Lorentz special transformations, form Lorentz proper group and, after joining reflections, the Lorentz extended group. Then we need to add the translations of M_4

$$x'_i = x_i + a_i \quad (2.20)$$

characterized by 4 parameters, which include spatial and temporal translations. By composing the transformations of these two groups, we obtain the Poincaré group with 10 parameters

$$x'_i = a_{ik}x_k + a_i \quad (2.21)$$

Poincaré group is relevant in the formulation of Einstein relativity principle. When $c \rightarrow \infty$ so that $v/c \ll 1$, Minkowski spacetime reduces to that of Newton and Poincaré group reduces to Galileo group. Let us synthesize the geometric structure of spacetime groups in the following scheme:

$$\begin{aligned} \text{GALILEO} &\Rightarrow \left\{ \begin{array}{l} \text{Rotations of 3 - dimensional space} \\ \text{Boosts} \\ 3 - \text{dimensional space translations} \\ \text{Time translations} \end{array} \right. \\ \text{POINCARÉ}' &\Rightarrow \left\{ \begin{array}{l} \text{Rotations of 4 - dimensional space} \\ 4 - \text{dimensional translations.} \end{array} \right. \end{aligned}$$

2.3 The Fantappié “Erlangen Program”

In 1872 Felix Klein (1849–1925) presented the so-called Erlangen program for geometry, centred upon the symmetry transformations groups. From 1952, Fantappié, basing on a similar idea and in perfect consonance with relativity spirit, proposed an Erlangen program for physics, whose essential idea was to individuate

physical laws starting from the transformations group which let them invariant. We observe here that there are infinite possible transformations groups which individuate an isotropic and homogeneous space-time. In order to build the next improvements in physics using the group extension method, we can follow the path indicated by the two groups we know to be valid description levels of the physical world: the Galileo group and the Poincaré group. It is useful to remember that the Galileo group is a particular case of the Poincaré one when $c \rightarrow \infty$, i.e. when it is not made use of the field notion and the interactions velocity is considered to be infinite. Staying within a quadri-dimensional spacetime and consequently considering only groups at 10 parameters and continuous transformations, Fantappiè showed [8] that the Poincaré group can be considered a limiting case of a broader group F depending with continuity on c and another parameter r . Moreover, this group cannot be further extended provided that only ten parameters are allowed. So we have the sequence:

$$G_{1+3}^{10} \rightarrow P_{1+3}^{10} \rightarrow F_{1+3}^{10}$$

where G , P and F are respectively the Galileo, Poincaré and Fantappiè (that is, de Sitter) group; when $r \rightarrow \infty$, the latter becomes the P group. It is shown that such sequence is univocal. The F group is the one of the five-dimensional rotations of a new spacetime: the maximally symmetric de Sitter universe at constant curvature $1/r^2$. We point out the de Sitter model is obtained without any reference to the gravitational interaction, differently from the General Relativity where the de Sitter universe is one of the possible solutions of the Einstein equations with cosmological constant. From a formal viewpoint we make recourse to five-dimensional rotations because in the de Sitter universe there appears a new constant r , which can be interpreted as the Universe radius. The group extension mechanism individuates an univocal sequence of symmetry groups; for each symmetry group we have a corresponding level of physical world description and a new universal constant, so providing the most general boundary conditions and constraining the form of the possible physical laws. The de Sitter group fixes the c and r constants and defines a new relativity for the inertial observers in de Sitter universe. In this sense we actually have a version of what is sought for in the Holographic Principle: the possibility to describe laws and boundaries in a compact and unitary way. In 1956 Giuseppe Arcidiacono proposed to study the de Sitter absolute universe by means of the tangent relative spaces where observers localize and describe the physical events by using the Beltrami-Castelnuovo P_4 projective representation. The Projective Special Relativity (PSR) is thus obtained, which collapses in the usual Special Relativity (SR) when $r \rightarrow \infty$ [9].

2.4 The Arcidiacono Transformations²

The de Sitter universe is represented by the surface of a five-dimensional hyperboloid H_4 in real time and a five-dimensional sphere S_4 in imaginary time. Arcidiacono considered the flat projective representation of H_4 , giving the spacetime P_4 which generalizes Minkowski spacetime. P_4 is defined as the interior of an hyperboloid with two sheets, with center O and semi-axis ir , called the Cayley-Klein *absolute*. The equation of the sheets is, in usual physical coordinates x, y, z, ict :

$$x^2 + y^2 + z^2 - c^2 t^2 + r^2 = 0 \quad (2.22)$$

The manifold (2.22) meets the time axis in correspondence of “singularities” $t = \pm t_0$ where $t_0 = r/c$ is the time the light takes to travel the Universe radius r . These “singularities” are purely geometrical, not physical; they are placed on the two sheets of the de Sitter horizon of the observer O.

The de Sitter transformations are represented by the projections that transform the absolute in itself. Let us rewrite Eq. (2.22) as $(x_1 = x, x_2 = y, x_3 = z, x_0 = ict)$:

$$x_1^2 + x_2^2 + x_3^2 + x_0^2 + r^2 = 0 \quad (2.23)$$

Introducing the homogeneous coordinates $\bar{x}_a (a = 1, \dots, 5)$ so defined ($i = 0, 1, 2, 3$):

$$x_i = r \bar{x}_i / \bar{x}_5 \quad (2.24)$$

it becomes

$$\bar{x}_1^2 + \bar{x}_2^2 + \bar{x}_3^2 + \bar{x}_4^2 + \bar{x}_5^2 = 0 \quad (2.25)$$

The coordinate $\bar{x}_5$ is determinated according to the Weierstrass condition:

$$\bar{x}^a \bar{x}_a = r^2 \quad a = 0, 1, 2, 3, 5 \quad (2.26)$$

The inverse relations of (2.24) are:

$$\bar{x}_i = x_i / A; \quad \bar{x}_5 = r / A \quad (2.27)$$

where $A^2 = 1 + x^i x_i / r^2 = 1 + \alpha^2 - \gamma^2$, with $\alpha = (x^k x_k)^{1/2} / r, k = 1, 2, 3$ and $\gamma = ct/r = t/t_0$. The requested transformation between the two O' and O observers consequently has the form:

²For a more extended treatment of arguments exposed in Sects. 1.4–1.5, the reader is referred to [10–14] and there quoted references.

$$\underline{x}'_a = A_{ab}\underline{x}_b \quad (2.28)$$

where A is an orthogonal matrix. Following the same standard method also used in SR, we get three families of transformations:

- (a) The spatial translations of parameter T along (we say) the x axis, given by the $(\underline{x}_5, \underline{x}_1)$ rotation:

$$\begin{aligned} \underline{x}'_0 &= \underline{x}_0 \\ \underline{x}'_5 &= -\underline{x}_1 \sin \theta + \underline{x}_5 \cos \theta \\ \underline{x}'_1 &= \underline{x}_1 \cos \theta + \underline{x}_5 \sin \theta \end{aligned} \quad (2.29)$$

Using (2.29) and putting $tg\theta = T/r = \alpha$, we derive the spacetime transformations with T parameter

$$\begin{cases} x' = \frac{x-T}{1+\alpha x/r}, & y' = \frac{y\sqrt{1+\alpha^2}}{1+\alpha x/r} \\ z' = \frac{z\sqrt{1+\alpha^2}}{1+\alpha x/r}, & t' = \frac{t\sqrt{1+\alpha^2}}{1+\alpha x/r} \end{cases} \quad (2.30)$$

Equations (2.30) reduce, for $r \rightarrow \infty$, to the well known spatial translations of Galileo and Einstein relativities.

- (b) The time translations of parameter T_0 given by the $(\underline{x}_5, \underline{x}_0)$ rotation:

$$\begin{aligned} \underline{x}'_1 &= \underline{x}_1 \\ \underline{x}'_5 &= -\underline{x}_0 \sin \theta_0 + \underline{x}_5 \cos \theta_0 \\ \underline{x}'_0 &= \underline{x}_0 \cos \theta_0 + \underline{x}_5 \sin \theta_0 \end{aligned} \quad (2.31)$$

Putting $tg\theta_0 = iT_0/t_0 = i\gamma$, we obtain ($k = 1,2,3$):

$$\begin{cases} x'_k = \frac{x_k\sqrt{1-\gamma^2}}{1-\gamma t/t_0} \\ t' = \frac{t-T_0}{1-\gamma t/t_0} \end{cases} \quad (2.32)$$

Also (2.32), when $r \rightarrow \infty$, is reduced to the known cases of Galileo and Einstein relativities.

- (c) The boosts of parameter V along (we say) the x axis, given by the $(\underline{x}_1, \underline{x}_0)$ rotation:

$$\begin{aligned} \underline{x}'_5 &= \underline{x}_5 \\ \underline{x}'_0 &= -\underline{x}_1 \sin \theta_1 + \underline{x}_0 \cos \theta_1 \\ \underline{x}'_1 &= \underline{x}_1 \cos \theta_1 + \underline{x}_0 \sin \theta_1 \end{aligned} \quad (2.33)$$

Putting $tg\theta_1 = iV/c = i\beta$, we reobtain the Lorentz transformations:

$$\begin{cases} x' = \frac{x - Vt}{\sqrt{1 - \beta^2}} & y' = y \\ t' = \frac{t - Vx/c^2}{\sqrt{1 - \beta^2}} & z' = z \end{cases} \quad (2.34)$$

The general composition of (a), (b) and (c) transformations forms the generic element of de Sitter projective group which, for two variables (x, t) and three parameters (T, T_0, V) , can be written as:

$$x' = \frac{Ax + [\beta + \gamma(\alpha - \beta\gamma)]ct + BT}{A(\gamma\beta - \alpha)x/r + (\gamma - \alpha\beta)t/t_0 + B} \quad (2.35)$$

$$t' = \frac{A\beta x/c + [1 + \alpha(\alpha - \beta\gamma)]t + BT_0}{A(\gamma\beta - \alpha)x/r + (\gamma - \alpha\beta)t/t_0 + B} \quad (2.36)$$

where:

$$A^2 = 1 + \alpha^2 - \gamma^2, B^2 = 1 - \beta^2 + (\alpha - \beta\gamma)^2, \alpha = x/r \quad \gamma = t/t_0 \quad (2.37)$$

For $r \rightarrow \infty$, we have $A = 1$ and $B = (1 - \beta^2)^{1/2}$ and, from (2.35, 2.36), the Poincaré group with three parameters (T, T_0, V) . The De Sitter universe with $1/r^2$ constant curvature shows an elliptic geometry in its hyper-spatial global aspect (Gauss-Riemann) and an hyperbolic geometry in its spacetime sections (Lobacevskij). Making the “natural” r unit of this two geometries tend towards infinity we obtain the parabolic geometry of Minkowski flat space.

2.5 Some Consequences

Let us consider a spatial interval χ on H_4 and its projection x on P_4 . In addition, let us consider a time interval τ on H_4 and its projection t on P_4 ; in both cases, an extreme of the interval coincides with the point-event of observation O.

The point-event O is the vertex of a light-cone and the private spacetime P_4 of O is the portion of this light-cone having the absolute as its boundary.

Let us consider two point-events A and B inside P_4 . The usual Pythagorean expression in the difference of A and B coordinates not represents a suitable notion of distance AB, because it is not invariant respect to projections. Given a AB straight line intersecting the absolute in R and S, the projective distance is given by the logarithm of the (ABRS) cross ratio:

$$AB = \left(\frac{t_0}{2}\right) \log(ABRS) = \left(\frac{t_0}{2}\right) \log(\overline{AR} \cdot \overline{BS})/(\overline{BR} \cdot \overline{AS}) \quad (2.38)$$

From the (2.38) we obtain:

$$\chi = r \operatorname{arctg}\left(\frac{x}{r}\right); \quad \tau = \frac{t_0}{2} \log\left(\frac{t_0 + t}{t_0 - t}\right). \quad (2.39)$$

Thus χ is bounded while x is not and, viceversa, τ is unbounded while t is. As a consequence of this latter result a new law of addition of durations holds:

$$T = \frac{T_1 + T_2}{1 + T_1 T_2 / t_0^2} \quad (2.40)$$

It is obtained by (2.32) and finds its physical meaning in the identity of the cronological distance of all the observers from the de Sitter horizon; in other words the “de Sitter time” $t_0 = r/c$ is the same for any P_4 observer, exactly as c .

Even the speed composition law is modified according to de Sitter transformations. Let us suppose a certain object moves respect to O with rectilinear uniform motion of speed V . If O in turn moves respect to the inertial observer O' with rectilinear uniform motion of speed U , O' sees the same object in rectilinear uniform motion at the speed W given by:

$$W = \frac{(1 + \alpha^2)U + (1 - \gamma^2)V + \alpha\gamma c(1 - UV/c^2)}{A(1 + UV/c^2)} \quad (2.41)$$

For the visible universe of the observer O' the condition $\alpha = \pm \gamma$ (that is $A = 1$) holds, and the previous equation can be simplified as:

$$W = \frac{U + V \pm \alpha^2 c(1 + U/c)(1 - V/c)}{1 + UV/c^2} \quad (2.42)$$

For $V = c$ then $W = c$, according to SR, while for $U = c$ we have:

$$W = c \pm 2\alpha^2 c \left(1 - \frac{V}{c}\right) \left(1 + \frac{V}{c}\right) \neq c \quad (2.43)$$

Equation (2.43) expresses the possibility of observing hyper- c velocity in PSR. The outcome is less strange than it can seem at first sight, because now the space-time of an observer is defined not only by the c constant but also by r , and the light-cone is at variable aperture. In fact from relations $A^2 = 1 + \alpha^2 - \gamma^2$, $B^2 = 1 - \beta^2 + (\alpha - \beta\gamma)^2$ the following reality condition for B immediately follows:

$$1 - \beta^2 + (\alpha - \beta\gamma)^2 \geq 0 \quad (2.44)$$

which is a quadratic inequality in β :

$$(1 - \gamma^2)\beta^2 + 2\alpha\gamma\beta - (1 + \alpha^2) \leq 0 \quad (2.45)$$

or

$$\frac{-\alpha\gamma - A}{1 - \gamma^2} \leq \beta \leq \frac{-\alpha\gamma + A}{1 - \gamma^2} \quad (2.46)$$

In straighter physical terms it means that when we observe a far region of Universe placed at a chronological distance comparable to $t_0 = r/c$, the relative speed of cosmic objects within that region can exceed c value, even if the region belongs to our past light-cone. For $B = 0$ we obtain the angular coefficients of the tangents to the Cayley-Klein absolute exiting from a point P of P_4 , representing the two extreme straight lines of the light-cone with vertex P. Differently from Special Relativity, the light-cone opening angle is not constant and depends on the point P according to the formula:

$$tg\theta = 2A/(\alpha^2 + \gamma^2) \quad (2.47)$$

From the (2.46) derives the variation of the light velocity C with time (c = local value):

$$C = \frac{c}{\sqrt{1 - \gamma^2}} \quad (2.48)$$

from which follows that $C \rightarrow \infty$ in the two singularities $t = \pm t_0$.

Exactly as in Special Relativity, the enlargement of the symmetry group involves a deep redefinition of mechanics. In PSR, the m mass of a body varies with velocity and distance according to

$$m = m_0 \frac{A^2}{B} \quad (2.49)$$

From (2.49) it follows that on the lightcone ($A = 0$) the mass is null, while on the absolute ($B = 0$) $m \rightarrow \infty$. The local mass of a body at rest varies with t according to:

$$m = m_0(1 - \gamma^2) \quad (2.50)$$

and it vanishes for $\gamma = \pm 1$, at the initial and respectively final instant. So the overall picture for an inertial observer in a De Sitter spacetime is that of a Universe coming into existence in a singularity at $-t_0$ time, expanding and collapsing at t_0 time and

where the light speed c is only locally constant. In the initial and final instants this speed is infinite and the mass of a given object is zero. In the projective scenario the space flatness is linked to the observer geometry in a universe at constant curvature. All this is linked to the fact that in PSR the translations and rotations are indivisible. In the singularities there is no “breakdown” of the physical laws because the global spacetime structure is univocally individuated by the group which is independent of the matter-energy distribution. In this case, the singularities in P_4 are—more properly—a horizon of events for the observer (de Sitter horizon).

In order to avoid misunderstandings is necessary take into account that x and t are distances from the observer O, and *the dependence of physical quantities on these distances cannot be verified by that observer by means of local measurements*. Locally, the SR theory is fully valid. Therefore, the de Sitter generalization of well known SR relations is purely formal and physically detectable consequences only exist for the propagation of signals over cosmological distances. This argument is detailed in successive chapters.

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