

---

# Contents

<b>1</b>	<b>Probability</b>	<b>1</b>
1.1	Sample Spaces and Events	1
1.1.1	The Sample Space of an Experiment	1
1.1.2	Events	3
1.1.3	Some Relations from Set Theory	4
1.1.4	Exercises: Section 1.1 (1–12)	5
1.2	Axioms, Interpretations, and Properties of Probability	7
1.2.1	Interpreting Probability	9
1.2.2	More Probability Properties	11
1.2.3	Determining Probabilities Systematically	13
1.2.4	Equally Likely Outcomes	14
1.2.5	Exercises: Section 1.2 (13–30)	14
1.3	Counting Methods	18
1.3.1	The Fundamental Counting Principle	18
1.3.2	Tree Diagrams	19
1.3.3	Permutations	20
1.3.4	Combinations	22
1.3.5	Exercises: Section 1.3 (31–49)	25
1.4	Conditional Probability	29
1.4.1	The Definition of Conditional Probability	30
1.4.2	The Multiplication Rule for $P(A \cap B)$	32
1.4.3	The Law of Total Probability and Bayes' Theorem	34
1.4.4	Exercises: Section 1.4 (50–78)	37
1.5	Independence	43
1.5.1	$P(A \cap B)$ When Events Are Independent	44
1.5.2	Independence of More than Two Events	45
1.5.3	Exercises: Section 1.5 (79–100)	47
1.6	Simulation of Random Events	51
1.6.1	The Backbone of Simulation: Random Number Generators	51
1.6.2	Precision of Simulation	55
1.6.3	Exercises: Section 1.6 (101–120)	56
1.7	Supplementary Exercises (121–150)	60
<b>2</b>	<b>Discrete Random Variables and Probability Distributions</b>	<b>67</b>
2.1	Random Variables	67
2.1.1	Two Types of Random Variables	69
2.1.2	Exercises: Section 2.1 (1–10)	70

2.2	Probability Distributions for Discrete Random Variables . . . . .	71
2.2.1	A Parameter of a Probability Distribution . . . . .	74
2.2.2	The Cumulative Distribution Function . . . . .	75
2.2.3	Another View of Probability Mass Functions . . . . .	78
2.2.4	Exercises: Section 2.2 (11–28) . . . . .	79
2.3	Expected Value and Standard Deviation . . . . .	83
2.3.1	The Expected Value of $X$ . . . . .	83
2.3.2	The Expected Value of a Function . . . . .	86
2.3.3	The Variance and Standard Deviation of $X$ . . . . .	88
2.3.4	Properties of Variance . . . . .	90
2.3.5	Exercises: Section 2.3 (29–48) . . . . .	91
2.4	The Binomial Distribution . . . . .	95
2.4.1	The Binomial Random Variable and Distribution . . . . .	97
2.4.2	Computing Binomial Probabilities . . . . .	99
2.4.3	The Mean and Variance of a Binomial Random Variable . . . . .	101
2.4.4	Binomial Calculations with Software . . . . .	102
2.4.5	Exercises: Section 2.4 (49–74) . . . . .	102
2.5	The Poisson Distribution . . . . .	107
2.5.1	The Poisson Distribution as a Limit . . . . .	107
2.5.2	The Mean and Variance of a Poisson Random Variable . . . . .	110
2.5.3	The Poisson Process . . . . .	110
2.5.4	Poisson Calculations with Software . . . . .	111
2.5.5	Exercises: Section 2.5 (75–89) . . . . .	111
2.6	Other Discrete Distributions . . . . .	114
2.6.1	The Hypergeometric Distribution . . . . .	114
2.6.2	The Negative Binomial and Geometric Distributions . . . . .	117
2.6.3	Alternative Definition of the Negative Binomial Distribution . . . . .	120
2.6.4	Exercises: Section 2.6 (90–106) . . . . .	120
2.7	Moments and Moment Generating Functions . . . . .	123
2.7.1	The Moment Generating Function . . . . .	125
2.7.2	Obtaining Moments from the MGF . . . . .	127
2.7.3	MGFs of Common Distributions . . . . .	128
2.7.4	Exercises: Section 2.7 (107–128) . . . . .	129
2.8	Simulation of Discrete Random Variables . . . . .	131
2.8.1	Simulations Implemented in R and Matlab . . . . .	134
2.8.2	Simulation Mean, Standard Deviation, and Precision . . . . .	135
2.8.3	Exercises: Section 2.8 (129–141) . . . . .	138
2.9	Supplementary Exercises (142–170) . . . . .	140
<b>3</b>	<b>Continuous Random Variables and Probability Distributions . . . . .</b>	<b>147</b>
3.1	Probability Density Functions and Cumulative Distribution Functions . . . . .	147
3.1.1	Probability Distributions for Continuous Variables . . . . .	148
3.1.2	The Cumulative Distribution Function . . . . .	152
3.1.3	Using $F(x)$ to Compute Probabilities . . . . .	154
3.1.4	Obtaining $f(x)$ from $F(x)$ . . . . .	155
3.1.5	Percentiles of a Continuous Distribution . . . . .	156
3.1.6	Exercises: Section 3.1 (1–18) . . . . .	158
3.2	Expected Values and Moment Generating Functions . . . . .	162
3.2.1	Expected Values . . . . .	162
3.2.2	Moment Generating Functions . . . . .	166
3.2.3	Exercises: Section 3.2(19–38) . . . . .	168

3.3	The Normal (Gaussian) Distribution . . . . .	172
3.3.1	The Standard Normal Distribution . . . . .	173
3.3.2	Non-standardized Normal Distributions . . . . .	175
3.3.3	The Normal MGF . . . . .	178
3.3.4	The Normal Distribution and Discrete Populations . . . . .	179
3.3.5	Approximating the Binomial Distribution . . . . .	180
3.3.6	Normal Distribution Calculations with Software . . . . .	182
3.3.7	Exercises: Section 3.3 (39–70) . . . . .	182
3.4	The Exponential and Gamma Distributions . . . . .	187
3.4.1	The Exponential Distribution . . . . .	188
3.4.2	The Gamma Distribution . . . . .	190
3.4.3	The Gamma MGF . . . . .	193
3.4.4	Gamma and Exponential Calculations with Software . . . . .	193
3.4.5	Exercises: Section 3.4 (71–83) . . . . .	194
3.5	Other Continuous Distributions . . . . .	196
3.5.1	The Weibull Distribution . . . . .	196
3.5.2	The Lognormal Distribution . . . . .	199
3.5.3	The Beta Distribution . . . . .	201
3.5.4	Exercises: Section 3.5 (84–100) . . . . .	202
3.6	Probability Plots . . . . .	205
3.6.1	Sample Percentiles . . . . .	205
3.6.2	A Probability Plot . . . . .	206
3.6.3	Departures from Normality . . . . .	209
3.6.4	Beyond Normality . . . . .	211
3.6.5	Probability Plots in Matlab and R . . . . .	213
3.6.6	Exercises: Section 3.6 (101–111) . . . . .	213
3.7	Transformations of a Random Variable . . . . .	216
3.7.1	Exercises: Section 3.7 (112–128) . . . . .	220
3.8	Simulation of Continuous Random Variables . . . . .	221
3.8.1	The Inverse CDF Method . . . . .	221
3.8.2	The Accept–Reject Method . . . . .	224
3.8.3	Built-In Simulation Packages for Matlab and R . . . . .	227
3.8.4	Precision of Simulation Results . . . . .	227
3.8.5	Exercises: Section 3.8 (129–139) . . . . .	228
3.9	Supplementary Exercises (140–172) . . . . .	230
<b>4</b>	<b>Joint Probability Distributions and Their Applications . . . . .</b>	<b>239</b>
4.1	Jointly Distributed Random Variables . . . . .	239
4.1.1	The Joint Probability Mass Function for Two Discrete Random Variables . . . . .	240
4.1.2	The Joint Probability Density Function for Two Continuous Random Variables . . . . .	241
4.1.3	Independent Random Variables . . . . .	245
4.1.4	More Than Two Random Variables . . . . .	246
4.1.5	Exercises: Section 4.1 (1–22) . . . . .	249
4.2	Expected Values, Covariance, and Correlation . . . . .	255
4.2.1	Properties of Expected Value . . . . .	256
4.2.2	Covariance . . . . .	257
4.2.3	Correlation . . . . .	259

4.2.4	Correlation Versus Causation . . . . .	262
4.2.5	Exercises: Section 4.2 (23–42) . . . . .	262
4.3	Properties of Linear Combinations . . . . .	264
4.3.1	The PDF of a Sum . . . . .	268
4.3.2	Moment Generating Functions for Linear Combinations . . . . .	270
4.3.3	Exercises: Section 4.3 (43–65) . . . . .	272
4.4	Conditional Distributions and Conditional Expectation . . . . .	277
4.4.1	Conditional Distributions and Independence . . . . .	279
4.4.2	Conditional Expectation and Variance . . . . .	280
4.4.3	The Laws of Total Expectation and Variance . . . . .	281
4.4.4	Exercises: Section 4.4 (66–84) . . . . .	286
4.5	Limit Theorems (What Happens as $n$ Gets Large) . . . . .	290
4.5.1	Random Samples . . . . .	290
4.5.2	The Central Limit Theorem . . . . .	293
4.5.3	Other Applications of the Central Limit Theorem . . . . .	297
4.5.4	The Law of Large Numbers . . . . .	299
4.5.5	Exercises: Section 4.5 (85–102) . . . . .	300
4.6	Transformations of Jointly Distributed Random Variables . . . . .	302
4.6.1	The Joint Distribution of Two New Random Variables . . . . .	303
4.6.2	The Joint Distribution of More Than Two New Variables . . . . .	306
4.6.3	Exercises: Section 4.6 (103–110) . . . . .	307
4.7	The Bivariate Normal Distribution . . . . .	309
4.7.1	Conditional Distributions of $X$ and $Y$ . . . . .	311
4.7.2	Regression to the Mean . . . . .	312
4.7.3	The Multivariate Normal Distribution . . . . .	312
4.7.4	Bivariate Normal Calculations with Software . . . . .	313
4.7.5	Exercises: Section 4.7 (111–120) . . . . .	313
4.8	Reliability . . . . .	315
4.8.1	The Reliability Function . . . . .	315
4.8.2	Series and Parallel Designs . . . . .	317
4.8.3	Mean Time to Failure . . . . .	320
4.8.4	Hazard Functions . . . . .	321
4.8.5	Exercises: Section 4.8 (121–132) . . . . .	323
4.9	Order Statistics . . . . .	326
4.9.1	The Distributions of $Y_n$ and $Y_1$ . . . . .	326
4.9.2	The Distribution of the $i$ th Order Statistic . . . . .	328
4.9.3	The Joint Distribution of the $n$ Order Statistics . . . . .	329
4.9.4	Exercises: Section 4.9 (133–142) . . . . .	331
4.10	Simulation of Joint Probability Distributions and System Reliability . . . . .	332
4.10.1	Simulating Values from a Joint PMF . . . . .	332
4.10.2	Simulating Values from a Joint PDF . . . . .	334
4.10.3	Simulating a Bivariate Normal Distribution . . . . .	336
4.10.4	Simulation Methods for Reliability . . . . .	338
4.10.5	Exercises: Section 4.10 (143–153) . . . . .	340
4.11	Supplementary Exercises (154–192) . . . . .	342
<b>5</b>	<b>The Basics of Statistical Inference . . . . .</b>	<b>351</b>
5.1	Point Estimation . . . . .	352
5.1.1	Estimates and Estimators . . . . .	354
5.1.2	Assessing Estimators: Accuracy and Precision . . . . .	357
5.1.3	Exercises: Section 5.1 (1–23) . . . . .	360

5.2	Maximum Likelihood Estimation . . . . .	366
5.2.1	Some Properties of MLEs . . . . .	372
5.2.2	Exercises: Section 5.2 (24–36) . . . . .	373
5.3	Confidence Intervals for a Population Mean . . . . .	375
5.3.1	A Confidence Interval for a Normal Population Mean . . . . .	376
5.3.2	A Large-Sample Confidence Interval for $\mu$ . . . . .	380
5.3.3	Software for Confidence Interval Calculation . . . . .	381
5.3.4	Exercises: Section 5.3 (37–50) . . . . .	382
5.4	Testing Hypotheses About a Population Mean . . . . .	386
5.4.1	Hypotheses and Test Procedures . . . . .	386
5.4.2	Test Procedures for Hypotheses About a Population Mean $\mu$ . . . . .	388
5.4.3	$P$ -Values and the One-Sample $t$ Test . . . . .	389
5.4.4	Errors in Hypothesis Testing and the Power of a Test . . . . .	392
5.4.5	Software for Hypothesis Test Calculation . . . . .	395
5.4.6	Exercises: Section 5.4 (51–76) . . . . .	396
5.5	Inferences for a Population Proportion . . . . .	401
5.5.1	Confidence Intervals for $p$ . . . . .	401
5.5.2	Hypothesis Testing for $p$ . . . . .	403
5.5.3	Software for Inferences about $p$ . . . . .	405
5.5.4	Exercises: Section 5.5 (77–97) . . . . .	405
5.6	Bayesian Inference . . . . .	409
5.6.1	The Posterior Distribution of a Parameter . . . . .	410
5.6.2	Inferences from the Posterior Distribution . . . . .	413
5.6.3	Further Comments on Bayesian Inference . . . . .	413
5.6.4	Exercises: Section 5.6 (98–106) . . . . .	414
5.7	Supplementary Exercises (107–138) . . . . .	416
<b>6</b>	<b>Markov Chains</b> . . . . .	<b>423</b>
6.1	Terminology and Basic Properties . . . . .	423
6.1.1	The Markov Property . . . . .	426
6.1.2	Exercises: Section 6.1 (1–10) . . . . .	428
6.2	The Transition Matrix and the Chapman–Kolmogorov Equations . . . . .	431
6.2.1	The Transition Matrix . . . . .	431
6.2.2	Computation of Multistep Transition Probabilities . . . . .	432
6.2.3	Exercises: Section 6.2 (11–22) . . . . .	436
6.3	Specifying an Initial Distribution . . . . .	440
6.3.1	A Fixed Initial State . . . . .	443
6.3.2	Exercises: Section 6.3 (23–30) . . . . .	444
6.4	Regular Markov Chains and the Steady-State Theorem . . . . .	446
6.4.1	Regular Chains . . . . .	446
6.4.2	The Steady-State Theorem . . . . .	448
6.4.3	Interpreting the Steady-State Distribution . . . . .	450
6.4.4	Efficient Computation of Steady-State Probabilities . . . . .	451
6.4.5	Irreducible and Periodic Chains . . . . .	453
6.4.6	Exercises: Section 6.4 (31–43) . . . . .	454
6.5	Markov Chains with Absorbing States . . . . .	457
6.5.1	Time to Absorption . . . . .	458
6.5.2	Mean Time to Absorption . . . . .	461
6.5.3	Mean First Passage Times . . . . .	465
6.5.4	Probabilities of Eventual Absorption . . . . .	466
6.5.5	Exercises: Section 6.5 (44–58) . . . . .	469

6.6	Simulation of Markov chains . . . . .	472
6.6.1	Exercises: Section 6.6 (59–66) . . . . .	479
6.7	Supplementary Exercises (67–82) . . . . .	481
<b>7</b>	<b>Random Processes</b> . . . . .	<b>489</b>
7.1	Types of Random Processes . . . . .	489
7.1.1	Classification of Processes . . . . .	493
7.1.2	Random Processes Regarded as Random Variables . . . . .	493
7.1.3	Exercises: Section 7.1 (1–10) . . . . .	494
7.2	Properties of the Ensemble: Mean and Autocorrelation Functions . . . . .	496
7.2.1	Mean and Variance Functions . . . . .	496
7.2.2	Autocovariance and Autocorrelation Functions . . . . .	499
7.2.3	The Joint Distribution of Two Random Processes . . . . .	502
7.2.4	Exercises: Section 7.2 (11–24) . . . . .	503
7.3	Stationary and Wide-Sense Stationary Processes . . . . .	504
7.3.1	Properties of Wide-Sense Stationary Processes . . . . .	508
7.3.2	Ergodic Processes . . . . .	511
7.3.3	Exercises: Section 7.3 (25–40) . . . . .	514
7.4	Discrete-Time Random Processes . . . . .	516
7.4.1	Special Discrete Sequences . . . . .	518
7.4.2	Exercises: Section 7.4 (41–52) . . . . .	520
7.5	Poisson Processes . . . . .	522
7.5.1	Relation to Exponential and Gamma Distributions . . . . .	524
7.5.2	Combining and Decomposing Poisson Processes . . . . .	526
7.5.3	Alternative Definition of a Poisson Process . . . . .	528
7.5.4	Nonhomogeneous Poisson Processes . . . . .	530
7.5.5	The Poisson Telegraphic Process . . . . .	531
7.5.6	Exercises: Section 7.5 (53–72) . . . . .	532
7.6	Gaussian Processes . . . . .	535
7.6.1	Brownian Motion . . . . .	536
7.6.2	Brownian Motion as a Limit . . . . .	538
7.6.3	Further Properties of Brownian Motion . . . . .	538
7.6.4	Variations on Brownian Motion . . . . .	541
7.6.5	Exercises: Section 7.6 (73–85) . . . . .	541
7.7	Continuous-Time Markov Chains . . . . .	544
7.7.1	Infinitesimal Parameters and Instantaneous Transition Rates . . . . .	546
7.7.2	Sojourn Times and Transitions . . . . .	548
7.7.3	Long-Run Behavior of Continuous-Time Markov Chains . . . . .	552
7.7.4	Explicit Form of the Transition Matrix . . . . .	554
7.7.5	Exercises: Section 7.7 (86–97) . . . . .	556
7.8	Supplementary Exercises (98–114) . . . . .	559
<b>8</b>	<b>Introduction to Signal Processing</b> . . . . .	<b>563</b>
8.1	Power Spectral Density . . . . .	563
8.1.1	Properties of the Power Spectral Density . . . . .	566
8.1.2	Power in a Frequency Band . . . . .	569
8.1.3	White Noise Processes . . . . .	570
8.1.4	Power Spectral Density for Two Processes . . . . .	572
8.1.5	Exercises: Section 8.1 (1–21) . . . . .	573
8.2	Random Processes and LTI Systems . . . . .	576
8.2.1	Statistical Properties of the LTI System Output . . . . .	577

---

8.2.2	Ideal Filters . . . . .	580
8.2.3	Signal Plus Noise . . . . .	583
8.2.4	Exercises: Section 8.2 (22–38) . . . . .	586
8.3	Discrete-Time Signal Processing . . . . .	589
8.3.1	Random Sequences and LTI Systems . . . . .	591
8.3.2	Random Sequences and Sampling . . . . .	593
8.3.3	Exercises: Section 8.3 (39–50) . . . . .	595
<b>Appendix A: Statistical Tables . . . . .</b>		<b>597</b>
<b>Appendix B: Background Mathematics . . . . .</b>		<b>609</b>
<b>Appendix C: Important Probability Distributions . . . . .</b>		<b>615</b>
<b>Answers to Odd-Numbered Exercises . . . . .</b>		<b>621</b>
<b>References . . . . .</b>		<b>637</b>
<b>Index . . . . .</b>		<b>639</b>

<http://www.springer.com/978-3-319-52400-9>

Probability with Applications in Engineering, Science,  
and Technology

Carlton, M.A.; Devore, J.L.

2017, XXVI, 610 p. 209 illus., 178 illus. in color.,

Softcover

ISBN: 978-3-319-52400-9