Chapter 2
Acceleration of Gravity

Introduction

The Greek philosopher Aristotle (384–322 B.C.) considered the Universe spherical, finite, and completely filled with *aether*. According to his model the celestial bodies (e.g., Moon and planets) are localized each one on a different "sphere," and all the Universe is entirely immerse in a bigger sphere, where are the stars. The Earth is spherical and in rest, and its center coincides with the center of the Universe. Aristotle did not accept the idea of vacuum—in his vision, the elements that compose all things and terrestrial beings are different from the aether, substance of divine origin, eternal and imperishable that forms and fills "the Skies," which include the spheres of the Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, and, finally, the sphere of the stars. In the sublunary world, below the sphere of the Moon, there is no place for aether, only for the four elements: earth, water, air, and fire. The natural movement of these sublunary objects is not a circular and uniform motion, only reserved to what is perpetual and perfect, but rectilinear and directed to the center of the Universe, or to the Earth, or its inverse, driven away from the center (which would be the case of fire and air). As an example, the water is lighter then earth (less dense, according to modern science), but both follow their natural movement towards the center, being water above earth, and thus Aristotle explains the reason why seas are covering the surface of the Earth. Water and earth are heavy (*gravitas*) elements. These bodies tend to fall towards the center of the Universe due to their heaviness (gravity), which is for Aristotle an intrinsically quality of this matter. So, the gravity impels the heavy bodies to find its natural place.

Unlike Aristotle, who affirmed that the speed of a body in free fall is proportional to its weight, Galileo Galilei (1564–1642) refuses any statement a priori (in advance) and starts to observe falling bodies, proposing theoretical models using mathematical language and conducting experiments in an organized and systematic way. Thereby, we have the beginning of the scientific method, which is based on the structuring of theoretical abstract models, usually suggested by a

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question that the researcher makes about a physical phenomenon. The model allows the discovery of the consequences following the hypotheses that have been imposed. The researcher should always find in the experiment the verdict of nature to validate the proposed model. It can be said that Galileo freed science from the Aristotelian essences and medieval magic, which for more than 2000 years had impaired its development [1].

Experimental Development

The experimental apparatus[1] dedicated to the study of free fall is shown in Fig. 2.1. The setup has five photosensors, a metallic sphere, an electromagnet, and a stopwatch available to measure up to four intervals of time. Similar setups can be made in didactic laboratories, as exemplified in reference [2], that uses blue LEDs combined with photodiodes, an electromagnet and an *Arduino* acquisition board.[2]

A metallic sphere is abandoned from the initial position, which can be considered zero, as indicated in the ruler of the experimental apparatus. The problem can be approached considering that we know nothing about the physical lows governing

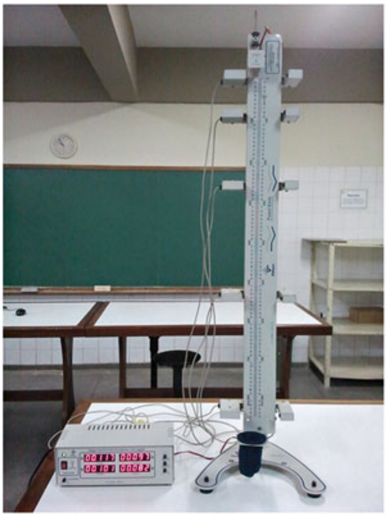

Fig. 2.1 Didactical apparatus dedicated to study free fall. The setup has five photosensors, a metallic sphere, an electromagnet, and a stopwatch available to measure up to four time intervals

[1] The didactical equipment used is manufactured by the company CIDEPE (Centro Industrial de Equipamentos de Ensino e Pesquisa—www.cidepe.com.br). The experimental development as well as the data analysis presented here is useful for any similar commercial or homemade equipment.

[2] Available in: <http://www.arduino.cc/>. Accessed in: 11/11/2016.

the movement, only the fact that its speed varies in the vertical direction (after all, the sphere is initially at rest and, after being abandoned, reaches the table with non-zero speed). Thereby, we can affirm that the movement of the sphere is accelerated. The simplest hypothesis is to assume that the acceleration, a, is constant. In other words, the rate of change of its speed is independent of time:

$$a(t) = \frac{dv}{dt} = \text{const.} \tag{2.1}$$

In this case, what are the expected results for the experiment? Integrating Eq. (2.1) we expect that the velocity $v(t)$ varies linearly with time:

$$v(t) = v_0 + at \tag{2.2}$$

being v_0 the initial velocity at $t = 0$.

The instantaneous velocity is defined as the position's rate of change of the observing object as a function of time, $v(t) = dx/dt$. The Eq. (2.2) can then be integrated and the position of the falling object can be obtained, having a quadratic of time:

$$x(t) = x_0 + v_0 t + \frac{a}{2}t^2, \tag{2.3}$$

where x_0 is the initial position, in other words, the position of the object observed in $t = 0$.

Aiming to verify the proposed model easily, we can prepare the experiment in a way to simplify the data analysis by putting the metal sphere at initial position equal to zero ($x_0 = 0$), and by starting the time measurement at the beginning of the fall, that is $v_0 = 0$. If we do so, Eqs. (2.2) and (2.3) become, respectively:

$$v(t) = at \tag{2.4}$$

$$x(t) = \frac{a}{2}t^2 \tag{2.5}$$

Observing Eq. (2.5), it is possible to realize if the proposal of a constant acceleration is compatible with the obtained data of position and time. To visualize it, we will plot the graph of the position of the sphere as a function of the square of time, $x \times t^2$. This procedure is called linearization. If the graph shows a linear tendency, it is an excellent indication that the model proposed of constant acceleration can describe the free fall of bodies at least near the Earth's surface, where the experiment is performed. We can then obtain the value of this constant acceleration from the slope of the straight line fitted to the experimental data. According to the proposed model, the slope is equal to the coefficient that multiplies t^2 in Eq. (2.5), $a/2$. If this is so, doubling the slope should give us the value of the acceleration.

Now we are able to test the hypothesis if the acceleration imposed to the bodies while falling is constant.

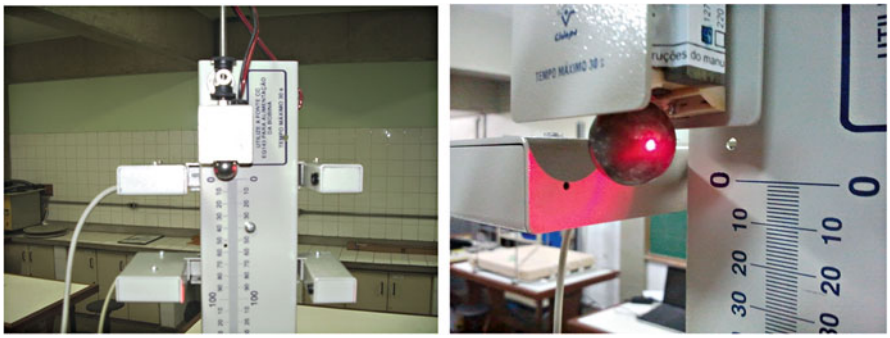

Fig. 2.2 Initial position of the metallic sphere. It is important to align the lower part of the sphere as close as possible to the zero position. This means that the lower part of the sphere's shadow should be as close as possible to the photosensor's aperture at the imminence of the start of the experiment (the photosensor emits an electronic signal when the laser is blocked by the sphere)

Let's abandon the metallic sphere from zero position, as it is shown in Fig. 2.2. The first photosensor is at position zero. The photosensor used in this experiment consists of a small source of light (diode) whose emission is detected on the opposite side by a photodiode, both mounted in a U-shaped metallic structure. The sphere is detected at the instant it blocks the light on the photodiode (see Fig. 2.2). At this moment, an electronic signal is sent to the stopwatch to start the measurement. The lower part of the sphere should be aligned with the zero position and also very close to the laser beam that goes to the photosensor (1 mm or less), but not enough to start it. This can be achieved adjusting the electromagnet position until the limitrophe position to start the stopwatch is found by just looking at the lower part of the sphere's shadow produced by the laser, and set it as close as possible to the photosensor's aperture at the imminence of starting it. It is important to avoid keeping the electromagnet on for too long, otherwise the circuit can be damaged.

The photosensors should be separated in such a way that the measured time intervals of the movement should not be lower than tens of microseconds (just try few different configurations of the photosensor positions before starting the final measurement). This is due to the stopwatch resolution,[3] which is 10^{-3} s (or 1 ms). Assuming that the uncertainty[4] is its own resolution, any measurement would have a relative uncertainty given by[5]:

[3]Resolution is the smallest difference between indications of a display device that can be significantly perceived [3].

[4]The uncertainty of a measurement is a parameter that characterizes the dispersion of the values that can be attributed to this measurement. This parameter can be a standard deviation, or multiple of it, or half of an interval that corresponds to a stated level of confidence. Preferably, the uncertainty should be declared with one significant digit. In cases of higher precision it is possible to express the uncertainty with two significant digits [3].

[5]In case of an apparatus with digital displays, the resolution corresponds to the digital increment. In the case of an apparatus that uses analogical displays, the resolution should be estimated by the experimentalist [3].

Table 2.1 Position and time data set obtained for the free fall of a metallic sphere. Each interval of time has an associated uncertainty estimated as ± 1 ms. When the intervals are added we need take into account that the final uncertainty increases

x (mm)	t_{abs} (ms)		t^2_{abs} (10^4 ms^2)
0 ± 1	0	0	0
88 ± 1	117	117 ± 1	1.37 ± 0.02
258 ± 1	117+97	214 ± 2	4.58 ± 0.09
535 ± 1	117+97+101	315 ± 3	9.9 ± 0.2
834 ± 1	117+97+101+82	397 ± 4	15.8 ± 0.3

$$\frac{\delta t}{\Delta t}(\%) = \frac{1}{\Delta t} \times 100\%$$

For example, time intervals measured between 20 ms and 10 ms give relative uncertainties of 5–10%. In this experiment, relative uncertainties on time are around 1%.

To start the experiment, the metallic sphere should be positioned at the electromagnet while it is turned on. The stopwatch should then be reset and the electromagnet turned off, releasing the sphere. If all the steps described earlier were accurately done, the initial velocity can be considered zero at first approximation. This is the crucial point of this experiment, since we are assuming that the initial velocity is zero, allowing us to investigate $x \times t^2$ as being linear.

Table 2.1 shows a data set from the photosensor's position, x, as a function of time represented by t_{abs}. The photosensor's position measurements (in other words, positions of the small apertures in the metallic structure that correspond to the photosensors as shown in Fig. 2.3) can be made by the following procedure: we know that the aperture is localized at the middle of the structure and its width is 22 mm. Adding half of this value (11 mm) to the position of the superior base we can find the desired measurement. In the example shown in Fig. 2.3, the position of the second photosensor is given by 11 mm + 247 mm = 258 mm.

It is necessary now to evaluate the uncertainty associated with the measurement of the photosensor's position. We avoid to estimate the uncertainty by statistical methods (**Type A uncertainty**[6]) and use the evaluation known in metrology **Type B uncertainty**.[7] This uncertainty can be estimated, in a very conservative way, as

[6]The method of evaluation of **Type A uncertainty** is based in statistical analysis on a series of observations that can be characterized by experimental standard deviations. In metrology, the best estimate of a physical quantity x that varies randomly is the arithmetic mean value x_m of an n number of measurements. The standard deviation σ characterizes the variability of the measured values, in other words, the dispersion around the mean value. In general, the standard deviation of the mean value $\sigma_m = \sigma/(n)^{1/2}$ is useful to qualify how the mean value represents the physical quantity to be measured [3].

[7]The evaluation of **Type B uncertainty** is based in a different method than those of statistical analysis on a series of observations. It can also be characterized by standard deviations estimated by assuming probability distributions based on the researcher's experience or other kind of

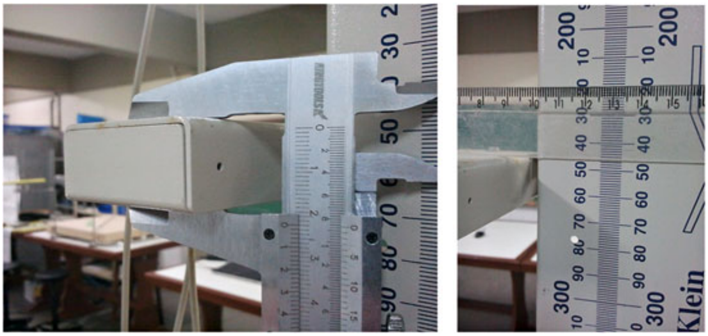

Fig. 2.3 The position of the second photosensor can be obtained by adding half of its width (0.5×22 mm = 11 mm) to the position of the its superior part. In this example its value is 11 mm + 247 mm = 258 mm

± 1 mm.[8] This kind of uncertainty measurement evaluation arises from an estimate based on the past experience and judgment of the experimentalist, and not from the statistic of several measurements.

The measurements related to the four time intervals can be seen at the stopwatch display shown in Fig. 2.1. The uncertainty of each measurement can be estimated as ± 1 ms. It is possible to estimate this value by making some launches and observing that the intervals of time can change approximately 1 ms. It is important to note that when you add two intervals of time, the associated uncertainty of the result increases. Conservatively, we can adopt that the result of the sum of two intervals of time (this is also valid for the difference) is the sum of each associated uncertainty. This way of estimating the uncertainty is called ***maximum possible uncertainty*** [3]. Using as an example the calculation of the absolute time associated with the position of the third photosensor (258 mm) it is easy to see that:

$$t_{\text{abs}} = 117 + 97 = 214\,\text{ms}$$

But it also can fluctuate between:

$$t_{\text{abs}+} = (117 + 1) + (97 + 1) = 216\,\text{ms}$$

observations. The correct use of the set of available information for the evaluation of the Type B uncertainty claims for the researcher's experience and wide knowledge, and this ability can be learned in practice. An evaluation Type B can be as trustable as an evaluation type A, especially in a measurement situation where an evaluation type A is based on a comparatively small number of statistical independent observations [3].

[8]For example, the experimentalist can ask one or two colleagues to make the same measurement and evaluate the result of the fluctuations. This is a simple method that can help in the evaluation of a type B uncertainty.

$$t_{\text{abs}-} = (117 - 1) + (97 - 1) = 212\,\text{ms}$$

It is clear in the example above that the maximum possible variation of the value of t_{abs} is ± 2 ms, which corresponds the sum in absolute values of each uncertainty. It is not difficult to realize that for an N number of terms to be added, the maximum possible uncertainty will be the sum of the absolute values of the uncertainties of each associated term. Table 2.1 shows the data set obtained for the free fall of a metallic sphere.

The last column of Table 2.1 shows the associated values of the square of absolute time. Below we show how it is possible to obtain an estimate of its uncertainty. In this case, a possible way would be to derive the square of absolute time and obtain the uncertainty for each measurement:

$$\delta\left(t^2\right) = \frac{d(t^2)}{dt}\delta t = 2t\delta t, \tag{2.6}$$

being $\delta(t^2)$ the associate uncertainty of the quantity t^2. As an example, we can calculate the associated uncertainty of the measurement $(214 \pm 2)^2$.

$$t^2 = 45796$$
$$\delta\left(t^2\right) = 2 \times 214 \times 2 = 856$$

As the uncertainty should present with one or two significant digits [3], we have:

$$\delta\left(t^2\right) = 9 \times 10^2$$

And the measurement can be written as:

$$t^2 = (458 \pm 9) \times 10^2\text{ms}^2 = (4.58 \pm 0.09) \times 10^4\text{ms}^2$$

Analysis of the Experimental Data

The graph of $x \times t^2$ is presented in Fig. 2.4. The associated uncertainties to the positions are smaller than the size of the representation of the experimental data points. But the uncertainties associated with the square of time increase and are visible on the graph at the last three data points. The dashed lines were traced aiming to delimit the area where possible straight lines would be "acceptable" as a fit. The central line shows what would be considered the best fit. Note that the fit is made in an empirical way without any formal mathematical justification, based only in reasonability. The uncertainties of the experimental points can help us to choose the area between the dashed lines. If the uncertainties were higher the area

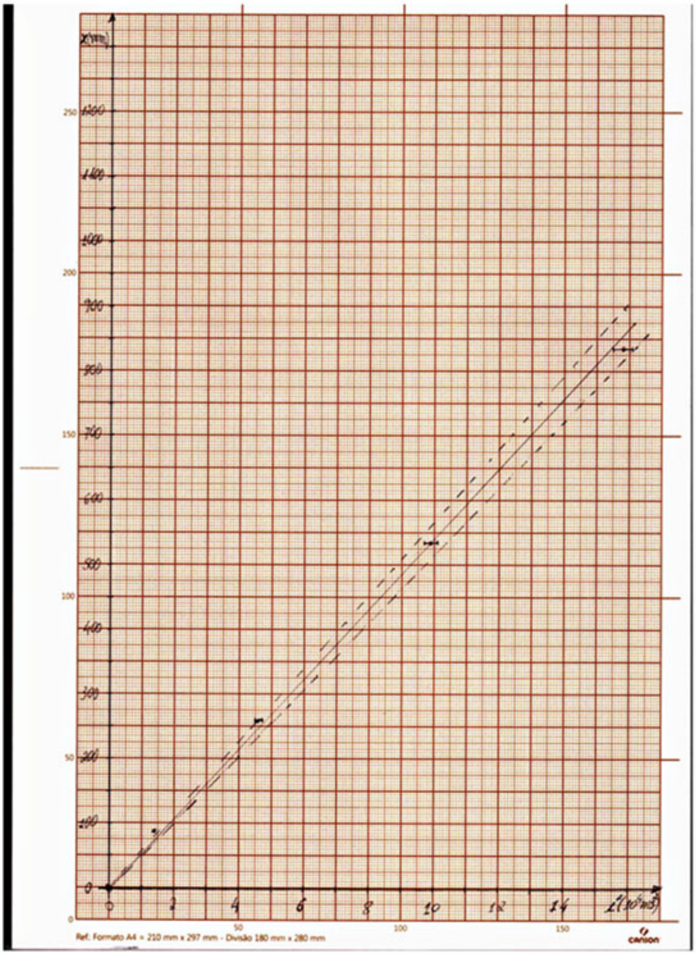

Fig. 2.4 Graph of $x \times t^2$. Note the two dashed lines delimiting the possible linear fits that would be "acceptable." The central line is the best fit that visually adjusts to the experimental data

between the dashed lines would also be larger. This procedure allows the obtaining of an estimation of the slope's uncertainty and, thus, the uncertainty of the free fall acceleration of the bodies.

Using any two points included in the central linear fit and reasonably far from each other, for example, (0;0) and (10;540), respecting the units, we obtain the slope value that corresponds to half of the acceleration:

$$\frac{a}{2} = \frac{(540 - 0)\text{mm}}{(10 - 0) \times 10^4 \text{ms}^2} = \frac{0.540\text{m}}{10 \times 10^4 \times 10^{-6}\text{s}^2} = 5.40\,\text{m/s}^2$$

Proceeding in a similar way, we can obtain the slopes related to the dashed straight lines 5.15 m/s^2 and 5.65 m/s^2. Now it is possible to evaluate the acceleration and its uncertainty (it is only needed to subtract 5.15 from 5.64 and divide the result by 2):

$$\frac{a}{2} = 5.40 \pm 0.25 \text{ m/s}^2$$

Remember that the uncertainty can be written with one or two significant digits. Therefore, by this method, the value of the acceleration is:

$$a = 10.8 \pm 0.5 \text{ m/s}^2$$

At first glance the linear fit could be considered reasonable, but we can compare the obtained result with the official value of the local acceleration of gravity[9] measured by the Brazilian National Observatory (O.N.), $g = 9.7877394 \pm 0.0000002$ m/s^2. This can be made by calculating the relative error, but first it is important to define the absolute error. The absolute error is the result of a measurement subtracted by its true value, and here the true value is the standard value, in other words, the value of the local acceleration of gravity measured by the O.N. The relative error is given by the ratio of the absolute error by the standard value, normally expressed in percentage:

$$\text{Error}_{\text{relative}} = \left(\frac{10.8 - 9.7877394}{9.7877394}\right) \times 100\% \approx +10\%$$

Figure 2.5 shows that the standard value of the acceleration of gravity does not fall within the range of the estimated uncertainty. In order to understand that, it

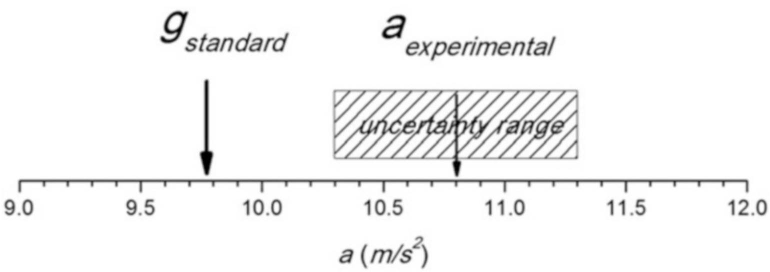

Fig. 2.5 Comparison between experimental and standard values of the acceleration of gravity

[9]Didactic laboratory of physics—Instituto Federal de Educação, Ciência e Tecnologia do Rio de Janeiro (IFRJ), *campus* Nilópolis, State of Rio de Janeiro, Brazil.

is important to review some of the assumptions made during the experiment. The adjustment of the initial position at zero seems to be reasonable, but what would be the consequences if the initial velocity was a bit higher than zero? The first consequence would be that: it would not be possible anymore to expect that the experimental data would be fitted by a straight line as proposed by our theoretical model shown at the graph in Fig. 2.4.

Assuming the sphere starts the stopwatch 1 mm after it begins to fall (this is a reasonable estimation, as it is quite difficult to adjust the separation between the low part of the sphere and the laser to be less than 1 mm apart without the sphere starting the stopwatch before it begins to fall—observe the photo on the right shown in Fig. 2.2), we can use the experimental value of the acceleration itself to obtain an estimation of the initial velocity in $t = 0$. Using the equation that relates the final velocity and the distance that an object travels, which is valid for movements with constant acceleration (known as Torricelli equation), we have:

$$v_0 = \sqrt{2a\Delta s} = \sqrt{2 \times 10.8\,\mathrm{m/s^2} \times 10^{-3}\,\mathrm{m}} \approx 0.15\,\mathrm{m/s}$$

If the initial velocity is no longer zero, how can we correct the graph shown in Fig. 2.4? Equation (2.3) would be written as:

$$x(t) = v_0 t + \frac{a}{2}t^2 \qquad (2.7)$$

The correction can be made using the displacement of the position x in each data point by a factor $(-v_0 t)$. We have now a corrected value for each position of the sphere in function of time, and it can be linearized anew.

$$\begin{aligned} x(t) - v_0 t &= \frac{a}{2}t^2 \\ x_{\mathrm{corr}} &= \frac{a}{2}t^2 \end{aligned} \qquad (2.8)$$

Table 2.2 presents the correct values for the position as a function of time and the graph shown in Fig. 2.6 includes now the corrected data points (labeled as triangles). It is possible to make a linear fit and obtain an estimation of the slope equal

Table 2.2 Experimental data of position, corrected position (x_{corr}), and squared absolute time of the metallic sphere in free fall

x (mm)	$x_{corr} = x - v_0 t$	t^2_{abs} (10^4 ms^2)
0 ± 1	0 ± 1	0
88 ± 1	70 ± 1	1.37 ± 0.02
258 ± 1	226 ± 1	4.58 ± 0.09
535 ± 1	488 ± 1	9.9 ± 0.2
834 ± 1	775 ± 1	15.8 ± 0.3

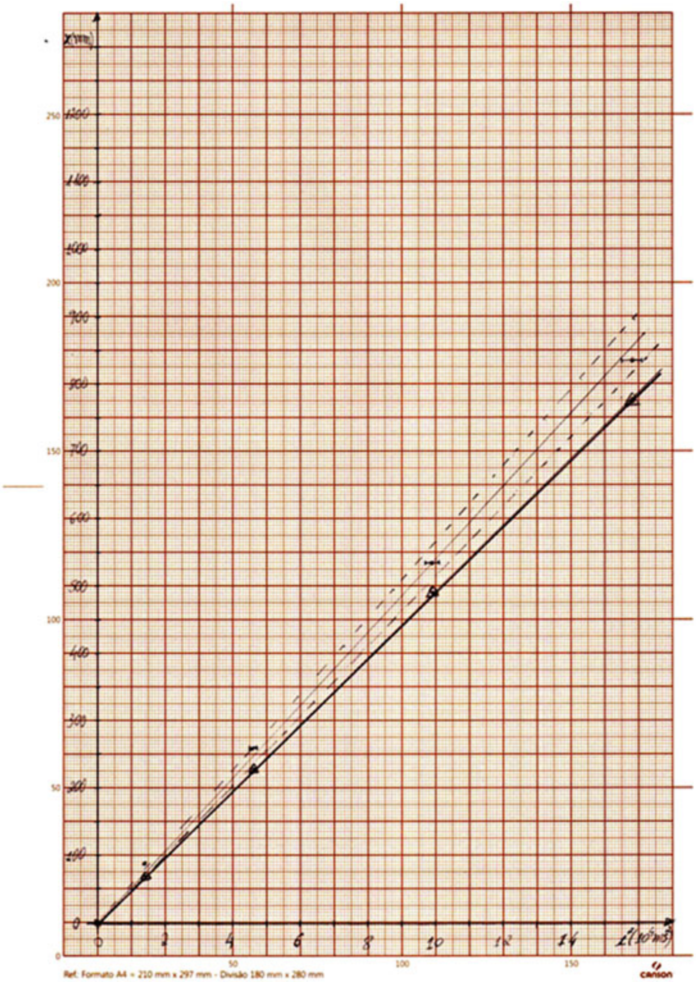

Fig. 2.6 Graph of $x \times t^2$. The data points labeled as triangles correspond to the corrected experimental values (x_{corr}). The slope of the new linear fit corresponds to 4.9 m/s^2, which corresponds to an acceleration of 9.8 m/s^2

to 4.9 m/s^2, which corresponds to a new value of 9.8 m/s^2 to the local acceleration of gravity. This number is much closer to the standard value.

Note that it was necessary to make a first measurement of the acceleration, and afterwards, improve it using a correction and taking the first acceleration measurement as a reference value. This is a very common procedure in experimental physics.

There is another mathematically more elegant procedure to directly obtain the values of acceleration and initial velocity, which is called "least-squares method." Applying this method to the experimental data in the graph $x \times t$, we should expect to fit a parabola, as shown in Eq. (2.3).

Least-Squares Method

The least-squares method is discussed here in a simplified enough way to meet the needs of this experiment. However, the discussion presented here is useful to the understanding of the fitting mechanism made by computer programs, differing only in computing capability. Eq. (2.3) presents the position of the sphere as a function of time and will be used considering the initial position $x_0 = 0$. It is expected that the graph of $x \times t$ shows experimental data distributed along an arch of a parabola, whose equation can be written as:

$$x(t) = At + Bt^2 \tag{2.9}$$

In the present case, A corresponds to the initial velocity and B to half of the acceleration (supposed constant) near to the surface of the Earth.

The least-squares method consists in calculating the coefficients A and B in a way that the distance between each of the N experimental points and the fitted curve given by Eq. (2.9) is minimized. It is possible to define the function χ^2 that quantifies these differences:

$$\chi^2 = \sum_{i=1}^{N} [x_i - x(t_i)]^2 = \sum_{i=1}^{N} \left[x_i - At_i - Bt_i^2\right]^2 \tag{2.10}$$

The sum extends for all N experimental points. The partial derivatives of function χ^2 related to the coefficients A and B result in a system of equations that can be easily solved:

$$A = \frac{s_{1x}s_4 - s_{2x}s_3}{s_2 s_4 - s_3^2} \quad \text{and} \quad B = \frac{s_{2x}s_2 - s_{1x}s_3}{s_2 s_4 - s_3^2} \tag{2.11}$$

and

$$s_2 = \sum_{i=1}^{N} t_i^2;\ \ s_3 = \sum_{i=1}^{N} t_i^3;\ \ s_4 = \sum_{i=1}^{N} t_i^4;\ \ s_{1x} = \sum_{i=1}^{N} x_i t_i \ \text{ and } \ s_{2x} = \sum_{i=1}^{N} x_i t_i^2 \tag{2.12}$$

Table 2.3 presents the constants shown in Eq. (2.12). Using the relations of Eq. (2.11) we find:

Table 2.3 Data used to calculate the coefficients A and B using the least-squares method

x (m)	t (s)	t^2 (s^2)	t^3 (s^3)	t^4 (s^4)	xt (ms)	xt^2 (ms^2)
0.000	0.000	0.000000	0.000000	0.000000	0.000000	0.000000
0.088	0.117	0.013689	0.001602	0.000187	0.010296	0.001205
0.258	0.214	0.045796	0.009800	0.002097	0.055212	0.011815
0.535	0.315	0.099225	0.031256	0.009846	0.168525	0.053085
0.834	0.397	0.157609	0.062571	0.024841	0.331098	0.131446
		$s_2 = \sum_{i=1}^{N} t_i^2 = 0.31632$	$s_3 = \sum_{i=1}^{N} t_i^3 = 0.10523$	$s_4 = \sum_{i=1}^{N} t_i^4 = 0.03697$	$s_{1x} = \sum_{i=1}^{N} x_i t_i = 0.56513$	$s_{2x} = \sum_{i=1}^{N} x_i t_i^2 = 0.19755$

$$A = \frac{s_{1x}s_4 - s_{2x}s_3}{s_2 s_4 - s_3^2} = 0.16948\,\text{m/s} \quad \text{and} \quad B = \frac{s_{2x}s_2 - s_{1x}s_3}{s_2 s_4 - s_3^2} = 4.86106\,\text{m/s}^2$$

From the values of A and B we obtain both the initial velocity and the acceleration:

$$v_0 = A = 0.17\,\text{m/s} \quad \text{and} \quad a = 2B = 9.72\,\text{m/s}^2$$

Note that the initial velocity is not zero, but very close to the estimation obtained assuming that the movement begins to be tracked after the sphere travels approximately 1 mm, resulting in an initial velocity of 0.15 m/s. The obtained value of the acceleration is a bit lower than the one obtained via linear fit, reducing the relative error to approximately 0.7%.

The graph shown in Fig. 2.7 exhibits the parabolic curve plotted with the aid of 32 points (see Table 2.4) obtained by the fitting function $x(t) = 0.16948\,t + 0.00486106\,t^2$, whose parameters calculated using the least-squares method are A = 0.16948 mm/ms and B = 0.00486106 mm/ms^2. The idea is to generate a number of points large enough to "connect them," generating the fitting curve. The higher the number of points the easier it is to trace the curve.

The uncertainty calculations of the obtained parameters by the least-squares method can also be performed; however, such calculations are beyond the scope of this book. For further details, reference [3] is indicated.

To close this chapter, it would be interesting to indicate an experiment to show that objects with different masses, when the air resistance can be neglected, fall with the same acceleration. If two spheres with different masses are abandoned simultaneously, they should reach the ground at the same time. This is a simple experiment, though there is an important detail: the two bodies should be abandoned simultaneously. Professor João Canalle published a very interesting article

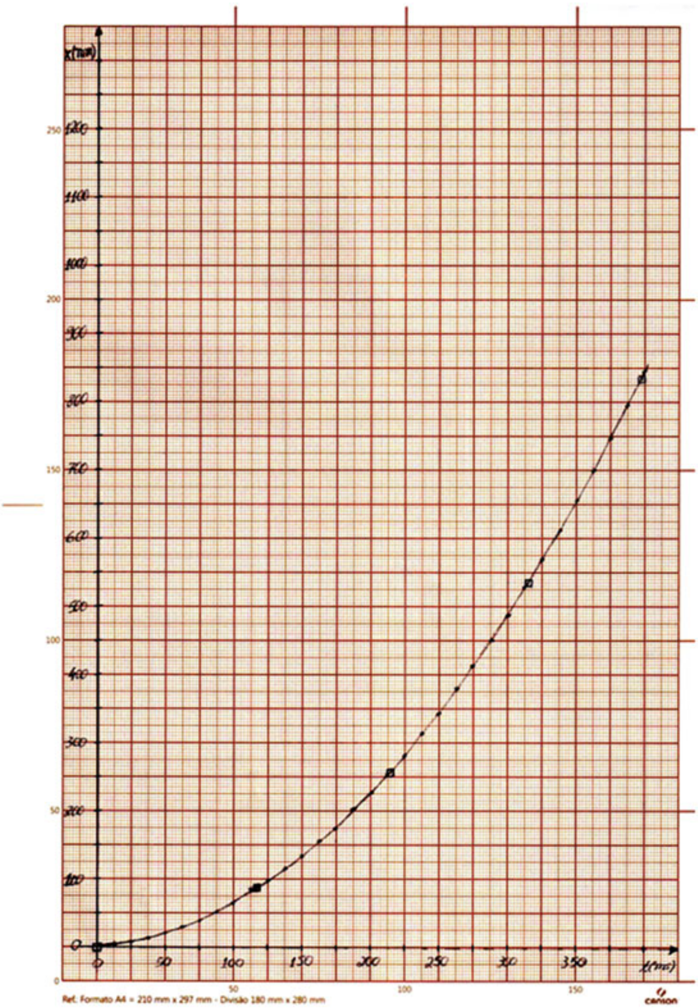

Fig. 2.7 Graph $x \times t$. The five experimental data points are labeled as squares. The parabolic curve was plotted with the aid of 32 points (see Table 2.4) obtained by the fitting function $x(t) = 0.16948\,t + 0.00486106\,t^2$, whose parameters obtained using the least-squares method are $A = 0.16948$ mm/ms and $B = 0.00486106$ mm/ms^2

that explains in detail how to build an apparatus that guarantee simultaneous free fall, called "Free fall mouse trap" [4]. Professor Canalle suggests the use of a small metallic mousetrap, screws, wood blocks, and, of course, two bodies with different masses, e.g., a sphere of glass and one of steel.

Table 2.4 Position and time data obtained by the fitting function $x(t) = 0.16948\,t + 0.00486106\,t^2$, whose parameters obtained by the least-squares method are $A = 0.16948$ mm/ms and $B = 0.00486106$ mm/ms^2

t (ms)	x (mm)	t (ms)	x (mm)	t (ms)	x (mm)	t (ms)	x (mm)
0.0	0	112.5	81	225.0	284	337.5	611
12.5	3	125.0	97	237.5	314	350.0	655
25.0	7	137.5	115	250.0	346	362.5	700
37.5	13	150.0	135	262.5	379	375.0	747
50.0	21	162.5	156	275.0	414	387.5	796
62.5	30	175.0	179	287.5	451	400.0	846
75.0	40	187.5	203	300.0	488		
87.5	52	200.0	228	312.5	528		
100.0	66	212.5	256	325.0	569		

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