Translator's Preface

I first heard of this monograph—*Sur le problème des trois corps et les équations de la dynamique* by Henri Poincaré—around 1978 while I was an undergraduate at Cornell University or around 1982 while I was a graduate student in the astronomy department at Harvard University. I no longer remember the particular time or context, although there are a few conceivable possibilities.

What is clear, many years later, is that I was in an environment that recognized, respected, and understood (on some level) the importance of this monograph. And it did so despite two obstacles. The first was of course its age; it was published in 1890. The other obstacle, from the perspective of a US academic environment, was the language; it was written in formal French with a specialized vocabulary demanded by the subject matter. In 1982, I spoke French that was fully adequate for many purposes; yet it then seemed to me unlikely that I would be able to read and understand Poincaré's work, so I made no effort to try.

Together, this means that the monograph was a classic and inaccessible to a large readership even though its existence was well known.

On the way from 1985 (when I was awarded a Ph.D.) to 2014, my life and career experienced some strange twists and turns and sharp bumps and jolts. By then I had become an established, independent translator from French into English working mostly with complex technical subjects.

My command of French had improved and I had succeeded in adding to that base the complex skills of mental magic needed to translate. I had also developed the skills for researching and learning the particular terminology of specialized subject matter. In brief, I had acquired the language skills to make what had been inaccessible accessible.

One of the distressing realities of freelance work is the unpredictable switch between frenetic feeding frenzy and frustrating famine. In the spring of 2014, during one such famine, I started to look for stimulating intellectual activity to fill the time until the next feeding frenzy hit. I immediately focused my search on potential projects that could make a connection back to what I had once been: an astronomer and mathematical physicist.

In fairly short order I had a few ideas for projects involving dynamics or stability of rotating astrophysical fluids. I talked to some people. I tried to assess the effort and resources that might be needed. While this route seemed plausible, it did not grab hold of my interest and hang on.

At the same time, my interest in Henri Poincaré's work resurfaced; I was very interested in carefully understanding what Poincaré had written. What better way to do that than to translate his monograph? I quickly found that it was easy to find scanned images of his works online. (As an interested reader, it is worth your while to visit the web site hosted by the Université de Lorraine for the Henri Poincaré Papers and the bibliography in particular.) In addition to this monograph, I also looked at *Les méthodes nouvelles de la mécanique céleste* and his three books popularizing science (*La Science et l'hypothèse*, *La Valeur de la science* and *Science et méthode*). These last four books all had existing translations of unknown quality. (Not all translators are equal, far from it.) It was also clear that setting the equations in *Sur le problème* and *Les méthodes nouvelles* would require a significant effort. On the other hand, I recognized that I would likely find that effort satisfying.

Even more than that, I found Poincaré to be a compelling author.

I prepared a sample translation of a chapter from *La Science et l'hypothèse* and after discussions with Maria Ascher and Michael Fischer then at Harvard University Press, I decided to dive in and start translating *Sur le problème* motivated by my interest in the author and subject.

As paying translation work flowed in, I translated patents and documents for clinical trials and as that work ebbed, I went back to translating mathematical physics. In that way, I got two things that really interested me: stimulating intellectual activity, and close, detailed study of a monograph and author that had long interested me.

What you have before your eyes is the product of a labor of love.

The Monograph

This monograph is the foundational work in the theory of dynamical systems.

Before this work, it was generally thought that the motion of the planets in the solar system proceeded in orderly fashion, like a clockwork orrery, from a known past to a predictable future since their motion was governed by deterministic laws: Newton's laws of gravitation and motion. Work on the computational machinery for predicting planetary positions into the future had progressed through the nineteenth century with notable contributions from Joseph-Louis Lagrange, Pierre-Simon Laplace and Urbain Le Verrier. The discovery of Neptune by Johann Galle within 1° of the position predicted by Le Verrier was a major triumph.

The computational machinery for predicting future positions of planets under these deterministic laws involves the use of time series expansions. The practical value of these time series, at least over some time domains, was well established.

The mathematical convergence of the series predicting planetary positions for all time was unproven. Phrased another way, it was unknown whether the orbits of the planets in the solar system were stable for all time.

Providing this proof was the subject of a prize competition celebrating the 60th birthday of King Oscar II of Sweden in 1889. This monograph is the published result of Poincaré's entry in that prize competition. Readers wishing more information about the prize competition and the history surrounding this monograph would do well to consult the book by June Barrow-Green (Barrow-Green 1997).

To approach the prize problem, Poincaré did not start with the solutions for the planetary positions. Instead he approached from the other direction by studying the equations of dynamics in general, Hamiltonian form. In that way, he developed the tools and theorems for studying the equations of dynamics in order to understand the behavior of their solutions. The immediate result was many tools and theorems that are now central to dynamical systems theory: phase space, trajectories, recurrence, Poincaré maps and more. (See, at the end of this Preface, the *Notable Concepts* section).

Hilborn writes (Hilborn 2000) on page 62, "Even a cursory reading of the history of chaos, the definitive version of which is not yet to be written, shows that Poincaré knew about, at least in rough way, most of the crucial ideas of nonlinear dynamics and chaos." And on the same page, he continues, "It is safe to say, however, that if Poincaré had a Macintosh or IBM personal computer, then the field of nonlinear dynamics would be much further along in its development than it is today." This is a very interesting anachronism.

With this theory of dynamical systems, Poincaré proved that the convergence of the solutions of the equations of dynamics cannot be established in general for all time. The series expansions may converge and be useful in some cases and for some time domains, and diverge over longer time. We do not know, and we cannot know, whether the solar system is stable.

Anyone interested in the history of chaos or in understanding how well and completely Poincaré understood the crucial ideas of dynamical systems theory will find it rewarding to explore this monograph. Beyond my personal interest, I prepared this translation so you could explore this monograph for yourself. In (Lorenz 1993), Lorenz asks on p. 118, "Did [Poincaré] recognize the phenomenon of full chaos, where most solutions—not just special ones—are sensitively dependent and lack periodicity?" Reading Chap. 4, *Encounters with Chaos*, of (Lorenz 1993) I wonder how much (well, actually, how little) Lorenz knew of the content of Poincaré's work in this monograph and in *Les méthodes nouvelles de la mécanique céleste*. Only *Les méthodes nouvelles* and the last of the three popularizations are cited in his Bibliography. With this translation, you can read through this work by Poincaré for yourself to form your own informed answer.

Errors and Typos

In the Preface, H. Poincaré writes, "By drawing my attention to a delicate point, he enabled me to discover and rectify a significant error." Not only was the error significant, the timing was bad. As the sentence suggests, the error was found and corrected without knowledge of it becoming public and the correction and reprinting resulted in a substantially different monograph becoming available to the public (Barrow-Green 1997).

Despite this challenging publication history, the published version of the monograph that reached the public has a limited amount of lint, distracting errors of a typographic nature not affecting the fabric of the work. I have found 26. For example, on pages 32–33 the equation numbers advance from 3 to 5 and equation 4 does not appear anywhere else in this section. There is nothing to be done about an error like this during translation and so the error is repeated. On the other hand, on page 34 in equation 6, the first subscript x_n is incorrect and is easily corrected to x_1. In circumstances like this I have corrected the errors unobtrusively.

I firmly hope that I have not contributed to the lint (or worse) and I have been diligent in my efforts to avoid (or failing that to find and correct) errors large and small of my own creation.

The Translation

In preparing this translation, I have tried to keep before me several objectives.

The first is accessibility. At one level, this objective is true of any translation. The purpose of translation is to take a document which was written (and therefore accessible) in one language and fit for a particular purpose and render it in another language (and therefore accessible in that language too) where it is fit for the same purpose or some analog of that purpose. In this instance, I understand that purpose to be a scholarly presentation of Poincaré's ideas and approach to studying and understanding dynamical systems and particularly the general three-body problem. Implicit here are the ideas of time and audience: 125 years later the expected audience for this translation is English-speaking people knowledgeable in dynamical systems wishing to understand how a foundational classic of the field established and set its direction.

Looking deeper, there is also the issue of voice. In the translation, in contrast to this preface, I have tried to avoid speaking for myself, meaning retelling in my words what Poincaré wrote, and to follow closely what and how Poincaré wrote, meaning to let his voice come through, while respecting standards of grammar, syntax, and phrasing expected in contemporary professional US English.

Essential to both of these is the matter of accuracy. In preparing this translation, I have worked through and sought to understand what Poincaré was writing about so that I would be able to accurately present it in my translation. I have then checked and rechecked this translation to eliminate any misunderstanding, inconsistency, or infelicity that might have gotten through anyway. I am human so I can be certain that I have not been fully successful despite my best effort.

Fundamental to my effort and motivation is my opinion that this is a classic of our literature in the field that deserves to be understood and that Poincaré merits the recognition and credit that follows from that understanding.

References and Index

Poincaré provided in line references. In some cases the reference amounted to little more than a name and a subject area. The least detailed citation was, "it is useful to cite the work of Mr. Puiseux on the roots of algebraic equations." He did not provide an index.

A list of references Poincaré cited (including the above example) and an index have been prepared to accompany this translation. They can be found after the translation.

Notation

As a consequence of the objective of helping Poincaré's voice to come through clearly, in the sentences and words and in the equations and symbols, there are several considerations concerning his choice of notation and symbols which should be indicated in order to help the reader appreciate the content.

Vectors

Here Poincaré does not use vector notation. He consequently writes things like "x_1, x_2 and x_3" and "the x" where we would expect to see $\vec{x}$. Reading his work with this in mind, you will easily recognize places where equations could be written more compactly with vectors and connected notational machinery such as dot products. After some experience reading his equations and following his reasoning, I became comfortable with the use of subscripts and summations and could see that it presented some advantages.

Superscripts as Indices

Related, and perhaps a consequence of using subscripts to indicate components of a vector, is the use of superscripts to indicate the order of the term in a series expansion. Therefore, x_1^2 is the second term in a series expansion of the first component of the coordinate x. There is certainly a possibility for confusion with the square of the first component. In a footnote on page 96 Poincaré warns that these "are indices and not exponents." Now I have warned you too.

Full and Partial Derivatives

In the text Poincaré refers to partial derivatives and partial differential equations 11 times. In his equations he makes no distinction between full and partial derivatives and uses d consistently for both.

As an example consider equation 1 on page 5 which Poincaré writes:

$$\frac{dx_i}{dt} = \frac{dF}{dy_i}, \quad \frac{dy_i}{dt} = -\frac{dF}{dx_i}. \tag{1}$$

In this case only the derivatives with respect to time are full derivatives; in contrast the derivatives on the right-hand sides are partial derivatives. Using ∂ to indicate partial derivatives, these equations can be rewritten

$$\frac{dx_i}{dt} = \frac{\partial F}{\partial y_i}, \quad \frac{dy_i}{dt} = -\frac{\partial F}{\partial x_i}.$$

As a plausible rule of thumb, you can assume the derivatives with respect to time are full derivatives and derivatives with respect to other variables are partial derivatives. Poincaré does make frequent use of transformations and substitutions of variables and this can cause some uncertainty as to how a new variable is related to a variable from the equations in canonical form. In case of doubt about whether a specific derivative should be a full or partial derivative, the interested reader will need to work through the substitutions and changes made to the equations in canonical form as given above.

Hamiltonian

Poincaré does not mention the name Hamilton. Poincaré refers to the canonical form of the equations of dynamics or simply the canonical equations. What Poincaré calls the canonical equations are given by equation 1 from page 5 (among other places), repeated above.

With a simple comparison, the reader can verify that these are in fact Hamilton's equations. This involves identifying F with the Hamiltonian H, the x_i (linear variables in Poincaré's terminology) with the generalized coordinates q_i and the y_i (angular variables in Poincaré's terminology) with the generalized momenta p_i. This allows writing the canonical equations in a more familiar form:

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}.$$

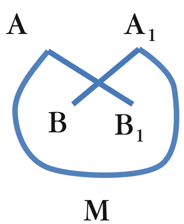
The Hamiltonian is the total energy and Poincaré notes on page 4 that it is conserved.

An important source of generality in Poincaré's results is his reliance on a generic Hamiltonian. In fact the most important constraint that Poincaré places on the Hamiltonian (see for example page 90) is that it can be expanded in a power series of a small parameter μ. In the case of the three-body problem, the small parameter is identified with the reduced mass of the third body. If the third body is massless, then $\mu = 0$. He is sparing in his reference to a specific form of the Hamiltonian; his discussion of the work of G.W. Hill starting on page 63 is a rare example of explicitly stating the form of the Hamiltonian.

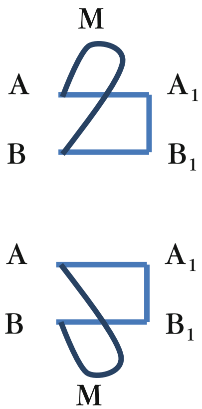
Section 4 of Chapter 2, Theorem III

In his proof of Theorem III on pages 74–76, Poincaré references four scenarios and provides two figures, Figs. 1 and 2. A reader who follows the proof will recognize that Scenario 2 corresponds to Fig. 1 and Scenario 3 corresponds to Fig. 2. For the reader's convenience, the following two sketches show Scenario 1 and the two variants of Scenario 4.

Scenario 1



Scenario 4



Notable Concepts

This section is provided as an aid to readers who, after consulting the table of contents, want a further indication where certain concepts are applied or discussed by Poincaré. It is therefore my effort to help readers find what they might be looking for or to make them aware of interesting things they might not have expected to find.

While this may seem similar to an index, it should be noted that unlike an index there is no effort to be comprehensive either in selecting concepts or in identifying all the places were the selected concepts occur. The discussion here, when there is any, is deliberately kept brief since the reader has access directly to Poincaré's own words for expansion and discussion.

This selection inherently reflects my background and therefore my ability to recognize certain concepts; other readers of Poincaré's work will make discoveries matching their interests.

Asymptotic solutions

p. 124: Definition

Asymptotic stability; Stability in the meaning of Poisson

p. 58: Definition

Asymptotic surfaces

p. 125, 161: Definition

Characteristic exponents

Section 2 of Chap. 3
p. 84: Definition
p. 86–87: For dynamical systems, they are equal pairwise and of opposite sign.
p. 108: Proof they can be expanded in $\sqrt{\mu}$

Collisionless

p. 11–12: Collisions result in singular points in Newton's law of gravitation preventing convergence of series expansions. The problems considered must therefore be collisionless.

Commensurable mean motions

p. 90–91: The existence of periodic solutions of the three-body problem requires that the mean motions of the three bodies be commensurable. See also *Small Divisors*.

Contactless surface

p. 56: Definition
p. 73: Lemma II

Delaunay variables (as used by Tisserand)

p. 151: Introduced. Relationships to semi-major axis, eccentricity and mean motion are given. With these variables, the equations for the orbital elements can be written as canonical equations of dynamics.

Density of trajectories

p. 58: Theorem I
p. 60: Corollary
p. 212: In a region of libration

Doubly asymptotic (now known as homoclinic) trajectories

p. 200: Definition

Duffing's equation

p. 103 (equation 18) and 157 (unnumbered): As written, this form of the equation is undamped and weakly forced. Poincaré provides a change of variables that allows the equation to be rewritten in the canonical form of the equations of dynamics.

Existence of periodic solutions for small μ

p. 91–94: Proof

G.W. Hill

p. 63–64: Discussed in terms of integral invariants, bounded solutions.
p. 65–68: Results extended by Poincaré and Bohlin.
p. 68–69: Hill's bounded solutions for the restricted three-body problem do not extend to the general three-body problem.

Hamiltonian, explicit forms

p. 41: Provides definition in terms of kinetic and potential energy
p. 51: For inverse square law
p. 63: G. W. Hill
p. 103 and 157: For Duffing's equation
p. 151: In terms of *Delaunay variables*
p. 196: Weakly coupled pendulum and spring, used as a counterexample. (For further discussion see (Holmes 1990) section 5 where this is equation 5.1 with $\varphi(y)$ (Poincaré) identified with $\sin q_1$ (Holmes). Poincaré assumes $\varphi(y) = \sin y$ on page 198.)

Homoclinic trajectories

See *Doubly asymptotic trajectories*.

Hyperboloid

p. 163: An asymptotic surface with two sheets

Infinitesimal contact transformation; Perturbation of Hamilton's equations

p. 37–38: A first-order Taylor series expansion of Hamilton's equations near a known solution is used to study a second infinitesimally close solution. This leads to relationships between the two solutions.

Integral Invariants

Section 2 of Chap. 2
p. 43: Definition.
p. 44–45: Proof of Poincaré's theorem.

Invariant Curve

p. 69–70: Definition

Libration

p. 206: Orbital libration near Lagrange points.
p. 209–212: In a region of libration, there are infinitely many closed trajectories corresponding to periodic solutions.

Linear and angular variables

p. 6: Definition. These take the place of coordinates and conjugate momentums.

Lunar problem

p. 150: Stated

Lindstedt

Last paragraph of Poincaré's Preface, p. 69 and Sect. 2 of Chap. 6

Liouville's Theorem

p. 45–46: Discussion; Volume in phase space of an incompressible fluid is an integral invariant.
p. 47–49: Proof

Newton's Law of Gravitation

p. 11

Orbital Elements

See *Delaunay variables*.
p. 90: Defines mean motion (n_i), longitude of pericenter (ϖ_i)

Periodic Forcing

p. 86: The assumptions allow either a time independent Hamiltonian or periodic forcing with period 2π.

Periodic Solutions of the First Kind

p. 89: Proof of the existence of solutions of the equations of dynamics for $\mu = 0$ which continue to exist as periodic solutions for small values of μ.

Periodic Solutions of the Second Kind

Section 1 of Chap. 6: These solutions do not exist for $\mu = 0$, but do exist for $\mu > 0$.
p. 212: Periodic solutions of the second kind disappear as μ decrease continuously to 0.

Poincaré map

p. 56: Discusses a point P_0 where a trajectory intersects a portion of surface S and the recurrences $(P_1, P_2, \ldots P_n)$ of that point.

Poincaré's theorem

p. 45–46: Proof

Poisson Bracket

p. 40: Defined (but not explicitly named).
p. 51: Used in proof that a quantity is an integral invariant.

Poisson Stability

p. 58: Definition

Poisson's Theorem

p. 40: The Poisson bracket of two, constant integrals of the canonical equations is a constant.

Recurrence

p. 56: Definition

Recurrence and Integral Invariants

p. 57: Extends integral invariants to areas related by recurrence.

Recurrence Theorem

p. 58: Theorem I

Small divisors

p. 239: Avoided by commensurability relation between mean motions. See also *Commensurable Mean Motions*

State Space (now commonly called phase space)

p. 5: No explicit definition or discussion, but used throughout. See p. 6, "The state of the system could then be represented by a point." For higher dimensions, visualization is a problem (p. 6) and one is dependent on "analytical language" (p. 69).

Temporary (linear) Stability

p. 88: Definition

Testing convergence of series (Cauchy, Weierstass, Kowalevski, Poincaré)

Section 2 of Chap. 1: Testing convergence by term-by-term comparison of series in question to a converging, reference series.
Section 3 of Chap. 1: Extension by Kowalevski (variously spelled, including Kovalevskaya) and Poincaré.

Tisserand relation

p. 152, unnumbered equation: The quantity $F_0 = 1/2a + G$ is the zeroth order term in the expansion in μ of the Hamiltonian for the planar (zero inclination) three-body problem. The Tisserand relation states that F_0 is unchanged after a close encounter of a comet with Jupiter. The full Hamiltonian is an integral invariant and for very small μ (for example a comet or spacecraft coming close to Jupiter) F_0 is very nearly an integral invariant.

Total Energy is an integral invariant

p. 43–44: Proof

Total energy along an arc in phase space is an integral invariant

p. 51–52: Proof

Trajectory

p. 5: Definition

Trajectory surface

p. 56–57: Definition

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Le Laboratoire d'Histoire des Sciences et de Philosophie - Archives Henri-Poincaré. CNRS/Université de Lorraine. http://henripoincarepapers.univ-lorraine.fr allowed the use of the images of the original figures from their scanned copy.

Norwood, MA, USA Bruce D. Popp
December 2016

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Author's Preface

This work studying the three-body problem is a reworking of the monograph that I had presented to the Competition for the prize established by His Majesty the King of Sweden. This reworking had become necessary for several reasons. Short on time, I had needed to present several results without proof; with the help of the indications which I had provided, the reader would have been able to reconstruct the proofs only with great difficulty. I had first considered publishing the initial text and accompanying it with explanatory notes; the result was a multiplication of these notes and reading the monograph would have been demanding and unpleasant.

I therefore preferred to merge these notes into the body of the work and this merging had the advantage of avoiding some restatements and better positioning the ideas in logical order.

I must heartily acknowledge the contribution of Mr. Phragmén who not only reviewed the proofs with great care, but who—having read the monograph with attention and having penetrated the meaning with utmost finesse—showed me places where he thought additional explanations were necessary to bring out the full insight of my thoughts. The elegant form that I gave to the calculation of S_i^m and T_i^m at the end of §12 is due to him. By drawing my attention to a delicate point, he enabled me to discover and rectify a significant error.

In some of the additions that I made to the initial monograph, I limited myself to reviewing some previously known results. Since these results are scattered in many works and I had made frequent use of them, I thought the reader would be well served by sparing them tedious searches; furthermore, I was often led to apply these theorems in a different form than first given to them by their author and it was indispensable to present them in this new form. In Chap. 1 (Part I), these existing theorems—some of which are even classics—are developed side-by-side with some new propositions.

I am still a long ways from having fully resolved the problem that I have taken on. I have limited myself to demonstrating the existence of some remarkable specific solutions which I call periodic solutions, asymptotic solutions and doubly asymptotic solutions. I have more closely studied a particular case of the three-body problem in which one of the masses is zero and the motion of the two others is circular. I have recognized that in this case the three bodies will return arbitrarily close to their initial position infinitely many times, unless the initial conditions of the motion are exceptional.

As can be seen, these results show us very little about the problem's general case, but they could have some worth, because they are rigorously established, whereas the three-body problem had until now seemed accessible only by methods of successive approximation where the absolute rigor, which is required in other areas of mathematics, is given away cheaply.

But I would especially like to draw the reader's attention to the negative results which are developed at the end of the monograph. For example, I established that apart from known integrals the three-body problem does not comprise any analytic and one-to-one integral. Many other circumstances lead us to expect that the full solution, if it can ever be discovered, will demand analytical tools which are absolutely different from those which we have and which are infinitely more complicated. The more thought given to the propositions that I demonstrate later on, the better it will be understood that this problem was incredible difficulties that has certainly suggested by the lack of success in prior efforts, but I think I have brought out the nature and the immensity even better.

I also show that most of the series used in celestial mechanics and in particular those of Mr. Lindstedt, which are the simplest, are not convergent. I am sorry in that way to have thrown some discredit on the work of Mr. Lindstedt or on the more detailed work of Mr. Gyldén; nothing could be farther from my thoughts. The methods that they are proposing retain all their practical value. In fact the value that can be drawn from a numeric calculation using divergent series is known and the famous Stirling series is a striking example. Because of an analogous circumstance, the tried-and-true developments in celestial mechanics have already rendered such great service and are called on to render even greater service.

One of the series, which I will make use of later and whose divergence I will furthermore prove, has a significant analogy with the development proposed to the Stockholm Academy May 9, 1888 by Mr. Bohlin. Since his monograph was printed a few months later, I was unaware of it at the time the competition closed, meaning June 1, 1888. That means that I did not cite Mr. Bohlin's name and I am making an effort here to give him the justice he deserves. (See Supplement to the minutes of the Stockholm Academy, Volume 14 and Astronomische Nachrichten, Number 2883.)[1]

Paris, France Henri Poincaré

[1]Translator: It appears that the reference should be to number 2882, *Zur Frage der Convergenz der Reihenentwickelungen in der Störungstheorie*.

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