Algorithmic Developments of Information Granules of Higher Type and Higher Order and Their Applications

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Abstract. Information granules are conceptual entities using which experimental data are conveniently described and in the sequel their processing is realized at the higher level of abstraction. The central problem is concerned with the design of information granules. We advocate that a principle of justifiable granularity can be used as a sound vehicle to construct information granules so that they are (i) experimentally justifiable and (ii) semantically sound. We elaborate on the algorithmic details when forming information granules of type-1 and type-2. It is also stressed that the construction of information granule realized in this way follows a general paradigm of elevation of type of information granule, say numeric data (regarded as information granules of type-0) give rise to information granule of type-1 while experimental evidence coming as information granules of type-1 leads to the emergence of a single information granule of type-2. We discuss their direct applications to the area of system modeling, in particular showing how type-*n* information granules are used in the augmentation of numeric models.

Keywords: Granular computing · Information granules · Type and order of information granules · Principle of justifiable granularity · Coverage · Specificity

1 Introduction

Information granules are omnipresent. They are regarded as a synonym of abstraction. They support ways of problem solving through problem decomposition. Information granularity is central to perception and reasoning about complex systems. It also become essential to numerous pursuits in the realm of analysis and design of intelligent systems. Granular Computing forms a general conceptual umbrella, which embraces the well-known constructs including fuzzy sets, rough, sets, intervals and probabilities.

The ultimate objective of the study is to focus on the concepts, roles, and design of information granules of higher type and higher order. We offer a motivation behind the emergence of information granules of higher type. As of now, they become quite visible in the form of type-2 fuzzy sets – these constructs form the current direction of

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intensive research in fuzzy sets, especially at its applied side. Several ways of forming (designing) information granules are outlined; it is demonstrated that clustering arises as a general way of transforming data into clusters (information granules). Another alternative comes in the form of the principle of justifiable granularity, which emphasizes a formation of information granules as a result of an aggregation of available experimental evidence and quantification of its diversity.

The structure of the paper reflects a top-down organization of the overall material. To make the study self-contained, we start with an exposure of the essential prerequisites (Sect. 2). In Sect. 3, we discuss main ways of building information granules; here the proposed taxonomy embraces a suite of key methods. Section 4 elaborates on the essence of information granules of higher type and higher order. We present them both in terms of their conceptual underpinnings and compelling motivating arguments as well as discuss ways of their construction. In Sect. 5, we focus on the direct usage of such information granules; it is advocated that the higher type of information granularity is associated with the realization of models, in particular fuzzy models, of increased experimental relevance.

2 Information Granules and Granular Computing: Essential Prerequisites

Information granules forming the Granular Computing are conceptual entities that support all processing realized in this environment. For the completeness of the study, we briefly recall some principles behind this paradigm.

2.1 Agenda of Granular Computing

Information granules are intuitively appealing and convincing constructs, which play a pivotal role in human cognitive and decision-making activities. We perceive complex phenomena by organizing existing knowledge along with available experimental evidence and structuring them in a form of some meaningful, semantically sound entities. In the sequel, such entities become central to all ensuing processes of describing the world, reasoning about the surrounding environment and supporting various decision-making activities. The term information granularity itself has emerged in different contexts and numerous areas of application. It carries various meanings. One can refer to Artificial Intelligence in which case information granularity is central to a way of problem solving through problem decomposition where various subtasks could be formed and solved individually. In general, as stressed by Zadeh [20], by information granule one regards a collection of elements drawn together by their closeness (resemblance, proximity, functionality, etc.) articulated in terms of some useful spatial, temporal, or functional relationships. In a nutshell as advocated in [9–13, 18], Granular Computing is about representing, constructing, processing, and communicating information granules.

We can refer here to some areas, which deliver compelling evidence as to the nature of underlying processing and interpretation in which information granules play a pivotal role. The applications include image processing, processing and interpretation of time series, granulation of time, design of software systems. Information granules are examples of abstractions. As such, they naturally give rise to hierarchical structures: the same problem or system can be perceived from different viewpoints and at different levels of specificity (detail) depending on the complexity of the problem, available computing resources, and particular needs and tasks to be addressed. A hierarchy of information granules is inherently visible in processing of information granules. The level of detail (which is represented in terms of the size of information granules) becomes an essential facet facilitating a way a hierarchical processing of information positioned at different levels of hierarchy and indexed by the size of information granules.

Such commonly encountered and simple examples presented above are convincing enough to highlight several essential features:

(a) information granules are the key components of knowledge representation and processing,
(b) the level of granularity of information granules (their size, to be more descriptive) becomes crucial to the problem description and an overall strategy of problem solving,
(c) hierarchies of information granules support an important aspect of perception of phenomena and deliver a tangible way of dealing with complexity by focusing on the most essential facets of the problem and,
(d) there is no universal level of granularity of information; the size of granules becomes problem-oriented and user dependent.

2.2 The Landscape of Information Granules

There are numerous well-known formal settings in which information granules can be expressed and processed. Here we identify several commonly encountered conceptual and algorithmic platform:

Sets (intervals) realize a concept of abstraction by introducing a notion of dichotomy: we admit element to belong to a given information granule or to be excluded from it. Along with set theory comes a well-developed discipline of interval analysis. Alternatively to an enumeration of elements belonging to a given set, sets are described by characteristic functions taking on values in {0,1}.

Fuzzy sets provide an important conceptual and algorithmic generalization of sets. By admitting partial membership of an element to a given information granule we bring an important feature which makes the concept to be in rapport with reality. It helps working with the notions where the principle of dichotomy is neither justified nor advantageous. The description of fuzzy sets is realized in terms of membership functions taking on values in the unit interval. Formally, a fuzzy set A is described by a membership function mapping the elements of a universe $\mathbf{X}$ to the unit interval [0,1].

Shadowed sets [15] offer an interesting description of information granules by distinguishing among elements, which fully belong to the concept, are excluded from it and whose belongingness is completely *unknown*. Formally, these information granules are described as a mapping X: $\boldsymbol{X} \rightarrow \{1, 0, [0,1]\}$ where the elements with the membership quantified as the entire [0,ccc1] interval are used to describe a shadow of the construct. Given the nature of the mapping here, shadowed sets can be sought as a granular description of fuzzy sets where the shadow is used to localize unknown membership values, which in fuzzy sets are distributed over the entire universe of discourse. Note that the shadow produces non-numeric descriptors of membership grades.

Probability-oriented information granules are expressed in the form of some probability density functions or probability functions. They capture a collection of elements resulting from some experiment. In virtue of the concept of probability, the granularity of information becomes a manifestation of occurrence of some elements. For instance, each element of a set comes with a probability density function truncated to [0,1], which quantifies a degree of membership to the information granule.

Rough sets [7, 8] emphasize a roughness of description of a given concept X when being realized in terms of the indiscernibility relation provided in advance. The roughness of the description of X is manifested in terms of its lower and upper approximations of the resulting rough set.

2.3 Key Characterization of Information Granules

Information granules as being more general constructs as numeric entities, require a prudent characterization so that their nature can be fully captured. There are two main characteristics that are considered here.

Coverage

The concept of coverage of information granule, cov(.) is discussed with regard to some experimental data existing in $\boldsymbol{R}^n$, that is$\{\boldsymbol{x}_1, \boldsymbol{x}_2,\ldots, \boldsymbol{x}_N\}$. As the name itself stipulates, coverage is concerned with an ability of information granule to represent (cover) these data. In general, the larger number of data is being "covered", the higher the coverage of the information granule. Formally, the coverage can be sought as a non-decreasing function of the number of data that are represented by the given information granule A. Depending upon the nature of information granule, the definition of cov(A) can be properly refined. For instance, when dealing with a multidimensional interval (hypercube) A, cov(A) in its normalized form is related with the normalized cardinality of the data belonging to A, $\mathrm{cov}(A) = \frac{1}{N} card\{\boldsymbol{x}_k | x_k \in A\}$. For fuzzy sets, the coverage is realized as a σ-count of A, where we combine the degrees of membership of $\boldsymbol{x}_k$ to A, $\mathrm{cov}(A) = \frac{1}{N}\sum_{k=1}^{N} A(\boldsymbol{x}_k)$.

Specificity

Intuitively, the specificity relates to a level of abstraction conveyed by the information granules. The higher the specificity, the lower the level of abstraction. The monotonicity property holds: if for the two information granules A and B one has $A \subset B$ (when the inclusion relationship itself is articulated according to the formalism in which A and B have been formalized) then specificity, sp(.), [16] satisfies the following inequality: sp(A) $\geq$ sp(B). Furthermore for a degenerated information granule comprising a single element $\boldsymbol{x}_0$ we have a boundary condition sp($\{\boldsymbol{x}_0\}$) = 1. In case of a one-dimensional interval information granules, one can contemplate expressing specificity on a basis of the length of the interval, say sp(A) = exp(−length(A)); obviously the boundary condition specified above holds here. If the range *range* of the data is available (it could be easily determined), say, then sp(A) = 1- $|b\text{-}a|$/length (*range)* where A = [a, b], *range* = [$\min_k x_k$, $\max_k x_k$].

The realizations of the above definitions can be augmented by some parameters to offer some additional flexibility. It is intuitively apparent that these two characteristics are associated: the increase in one of then implies a decrease in another: an information granule that "covers" a lot of data cannot be overly specific and vice versa. This is not surprising at all: higher coverage relates to the increasing level of abstraction whereas higher specificity is about more details being captured by the corresponding information granule.

3 Design of Information Granules

Before information granules can be used, they need to be constructed. There is an urgent need to build to come up with an efficient way of forming them to reflect the existing experimental evidence and some predefined requirement. Here we recall two categories of methods. Clustering is the one of them. Clustering techniques transform data into a finite number of information granules. The second class of methods involves the principle of justifiable granularity, which directly dwells on the characteristics of information granules (coverage and specificity) and builds an information granule, which offers an optimization of these characteristics.

3.1 Fuzzy C-Means – Some Brief Focused Insights

Objective function-based clustering is sought as one of the vehicles to develop information granules [1, 17]. In what follows, we briefly recall the essence of the method and elaborate on the format of the results. We consider a collection of n-dimensional numeric data z_1, z_2, …, z_N. A formation of information granules is realized by minimizing an objective function expressing a spread of data around prototypes (centroids)

$$Q = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{m} \|\boldsymbol{x}_k - \boldsymbol{v}_i\|^2 \tag{1}$$

where c stands for the number of clusters (information granules). The description of the clusters is provided in the form of a family of prototypes $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, .., $\boldsymbol{v}_c$ defined in the data space and a partition matrix $U = [u_{ik}]$, i = 1, 2…, c; k = 1,2,…, N, $m > 1$. It is worth noting that the above-stated objective function is the same as being used in the Fuzzy C-Means (FCM) [1] however in the context of our discussion one could consider other forms of information granules. Note that in the FCM algorithm, the individual rows of the partition matrix are discrete membership functions of the information granules expressed by means of fuzzy sets. In this case, the parameter m standing in the above expression is referred to as a fuzzification coefficient. If one considers sets rather than fuzzy sets, one arrives at the Boolean partition matrix and the method comes as the K-Means algorithm. There are generalizations of the method engaging fuzzy sets of type-2 [2] or rough sets [5, 6].

There are two fundamental design issues that are inherently associated with fuzzy clustering (and clustering, in general) that is (a) a choice of the number of clusters and a selection of the value of the fuzzification coefficient (m), and (b) evaluation of the quality of the constructed clusters and interpretation of results. This task, which is highly relevant when dealing with the optimization of the parameters of the clustering algorithm, implies the usefulness of the clustering results used afterwards in fuzzy modeling and fuzzy classification. Various cluster validity indexes [17, 19] are used to assess the suitability of fuzzy clusters. Different cluster validity indexes can lead to quite distinct results. This is not surprising as each cluster validity index comes with some underlying rationale and in this way prefers a certain structure of clusters (and their ensuing number). On the other hand, a reconstruction criterion [14], emphasizes the quality of clusters being sought as information granules. The criterion is concerned with the evaluation of the quality of information granules (clusters) to describe the data. In essence, one described the available data in terms of information granules (clusters) and then using this characterization decodes (de-granulates) the original data. This transformation, referred to as a granulation-degranulation process leads to inevitable loses which are quantified in terms of a reconstruction error. The value of the error becomes minimized by optimizing the values of the key parameters of the clustering method (such as the fuzzification coefficient m and the number of clusters c). Crucial to the discovery of the structure is the data is a data space in which the clustering takes place.

3.2 The Principle of Justifiable Granularity

The principle of justifiable granularity [10, 12] delivers a comprehensive conceptual and algorithmic setting to develop an information granule. The principle is general as it shows a way of forming information granule without being restricted to certain formalism in which information granularity is expressed and a way experimental evidence using which this information granule comes from. For illustrative purposes, we consider a simple scenario. Let us denote one-dimensional numeric data of interest (for which an information granule is to be formed) by $\mathbf{Z} = \{z_1, z_2, \ldots, z_N\}$. Denote the largest and the smallest element in $\mathbf{Z}$ by $z_{\min}$ and z_{max}, respectively. On a basis of $\mathbf{Z}$ we are form an information granule A so that it attempts to satisfy two intuitively requirements of coverage and specificity. The first one implies that the information granule is justifiable, viz. it embraces (covers) as many elements of $\mathbf{Z}$ as possible. The second one is to assure that the constructed information granule exhibits a well-defined semantics by being specific enough. For instance, when constructing a fuzzy set, say a one with a triangular membership function, we start with a numeric representative of $\mathbf{Z}$, say a mean or a modal value (denoted here by m) and then separately determine the lower bound (a) and the upper bound (b). In case of an interval A, we start with a modal value and then determine the lower and upper bound, Fig. 1.

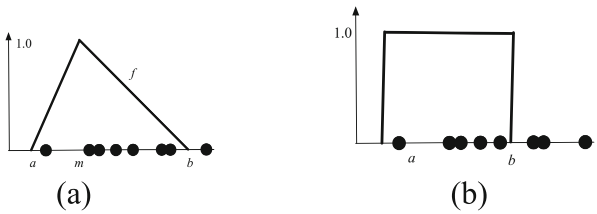

Fig. 1. Formation of information granules with the use of the principle of justifiable granularity: (a) triangular membership function, (b) interval (characteristic function). The design is realized by moving around the bounds a and b so that a certain optimization criterion is maximized

The construction of the bounds is realized in the same manner for the lower and upper bound so in what follows we describe only a way of optimizing the upper bound (b). The coverage criterion is expressed as follows

$$cov(A) = \sum_{z_k : z_k \in [m,b]} f(z_k) \tag{2}$$

where f is a decreasing linear portion of the membership function. For an interval (set) form of A, the coverage is expressed as a normalized count of the number of data included in the interval $[m, b]$,

$$cov(A) = card\{z_k | z_k \in [m, b]\} \tag{3}$$

The above coverage requirement states that we reward the inclusion of z_i in A. The specificity $sp(A)$ is realized as one of those specified in the previous section. As we intend to maximize coverage and specificity and these two criteria are in conflict, an optimal value of b is the one, which maximizes the product of the two requirements

$$Q(b) = cov(A) * sp(A)^{\gamma} \tag{4}$$

Furthermore the optimization performance index is augmented by an additional parameter γ used in the determination of the specificity criterion, $sp(A)^{\gamma}$ and assuming non-negative values. It helps control an impact of the specificity in the formation of the information granule. The higher the value of γ, the more essential the impact of specificity on A becomes. If γ is set to zero, the only criterion of interest is the coverage. Higher values of γ underline the importance of specificity as a resulting A gets more specific. The result of optimization comes in the form $b_{\text{opt}} = \arg\max_b Q(b)$. The optimization of the lower bound of the fuzzy set (a) is carried out in an analogous way as above yielding $a_{\text{opt}} = \arg \text{Max}\ Q(a)$.

Several observations are worth making here. First, the approach exhibits a general character and the principle is applicable to any formalism of information granules; here we just highlighted the case of sets and fuzzy sets. Second, it is visible that a single information granule represents a collection of many experimental data in a compact form.

4 Higher Type and Higher Order Information Granules

Information granules we discussed so far come with an inherent numeric description: intervals are described by two numeric bounds (*a* and *b*), fuzzy sets are described by *numeric* membership functions, probability functions (probability density functions) are *numeric* mappings. One may argue whether such a request is meaningful and does not create any restriction. In particular, with regard to fuzzy sets, this was a point of a visible criticism in the past: what is fuzzy about fuzzy sets? Obviously, the same issue could be formulated with respect to sets or probabilities. There are some interesting generalizations of information granules in which this type of requirement can be relaxed. This gives rise to the concept of information granules of type-2, type-3, and type-*n*, in general etc. The other direction of generalization deals with the nature of the space over which information granules are formed, which leads to information granules of higher order.

4.1 Higher Type Information Granules

By information granules of *higher type* (2^{nd} type and n^{th} type, in general) we mean granules in the description of whose we use information granules rather than numeric entities. For instance, in case of type-2 fuzzy sets we are concerned with information granules- fuzzy sets whose membership functions are granular. As a result, we can talk about interval-valued fuzzy sets, fuzzy fuzzy sets (or fuzzy^2 sets, for brief), probabilistic sets, uncertain probability, and alike. The grades of belongingness are then intervals in [0,1], fuzzy sets with support in [0,1], probability functions truncated to [0,1], etc. In case of type-2 intervals we have intervals whose bounds are not numbers but information granules and as such can be expressed in the form of intervals themselves, fuzzy sets, rough sets or probability density functions. Information granules have been encountered in numerous studies reported in the literature; in particular stemming from the area of fuzzy clustering in which fuzzy clusters of type-2 have been investigated [2] or they are used to better characterize a structure in the data and could be based upon the existing clusters. Fuzzy sets and interval-valued fuzzy sets form an intensive direction of research producing a number of approaches, algorithms, and application studies.

The development of information granules of higher type can be formed on a basis of information granules of lower type. The principle of justifiable granularity plays here a pivotal role as it realizes an elevation of type of information granularity. Refer to the discussion in the previous section. We started with experimental evidence formed by a *collection* of numeric data (viz. information granules of type-0) and form a *single* information granule of type-1. There is an apparent effect of elevation of the type of information granularity. If the available experimental evidence comes as information granules of type-1 then the result becomes an information granule of type-2. Likewise, if we start with a collection of type-2 information granules forming experimental evidence, the result becomes an information granule of type-3, etc. In particular, the principle of justifiable granularity can be regarded as a vehicle to construct type-2 fuzzy sets.

4.2 Higher Order Information Granules

Information granules, which are defined in the space (universe of discourse) whose elements are individual items, are called information granules of order-1. If the space itself is formed as a collection of information granules then any information granule defined over a space of information granules is referred to as information granules of order-2. The constructs could formed recursively thus forming information granules of order-3, 4, etc. It is worth noting that one can envision information granules of higher order and higher type.

The four alternatives that might arise here are displayed below, see Fig. 2. They capture the semantics of the resulting constructs.

	type-1	type-2
order-1	$A: \boldsymbol{X} \to [0,1]$ $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$	$A: \boldsymbol{X} \to P([0,1])$ $A: X \to F([0,1])$ $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$
order-2	$A: \boldsymbol{A} \to [0,1]$ $\boldsymbol{A} = \{R_1, R_2, \ldots, R_c\}$	$A: \boldsymbol{A} \to P([0,1])$ $A: \boldsymbol{A} \to F([0,1])$ $\boldsymbol{A} = \{R_1, R_2, \ldots, R_c\}$

Fig. 2. Examples of four categories of information granules of type-2 and order-2; *P, F*- families of intervals and fuzzy sets, respectively; $\boldsymbol{A} = \{R_1, R_2, \ldots, R_c\}$- a collection of reference information granules

5 Selected Application Areas

In this section, we elaborate on several applications of information granules of higher type and higher order.

5.1 Fuzzy Modeling

The involvement of fuzzy sets of higher type, in particular type-2 fuzzy sets and interval–valued fuzzy sets have triggered a new direction in fuzzy modeling. A general motivation behind these models relates with the elevated generality of the concepts of fuzzy sets of higher type, which translates into a higher flexibility of type-2 fuzzy models. While this argument is valid, there are a number of ongoing challenges. This concerns an increase of complexity of the development schemes of such fuzzy models.

A significantly larger number of their parameters (in comparison with the previously considered fuzzy models) require more elaborate estimation mechanisms. This has immediately resulted in essential optimization challenges (which owing to the engagement of more advance population-based optimization tools have been overcome to some extent but at expense of intensive computing). At the end, type-2 fuzzy models are assessed as numeric constructs with the chain of transformations: type reduction (from type-2 to type-1) followed by defuzzification (reduction from type-1 to type-0 information granules, viz. numbers) thus resulting in a numeric construct.

Ironically, in spite of all significant progress being observed, fuzzy models seem to start losing identity, which was more articulated and visible at the very early days of fuzzy sets. While one may argue otherwise, there is a visible identity crisis: at the end of the day fuzzy models have been predominantly perceived and evaluated as numeric constructs with the quality expressed at numeric level (through accuracy measures).

5.2 Embedding Fuzzy Models: A Granular Parameter Space Approach

The concept of the granular models form a generalization of numeric models no matter what their architecture and a way of their construction are. In this sense, the conceptualization offered here are of general nature. They also hold for any formalism of information granules. A numeric model M_0 constructed on a basis of a collection of training data ($\boldsymbol{x}_k$, target$_k$), $\boldsymbol{x}_k \in \boldsymbol{R}^n$ and $target_k \in \boldsymbol{R}$ comes with a collection of its parameters $\boldsymbol{a}_{\text{opt}}$ where $\boldsymbol{a} \in \boldsymbol{R}^p$. Quite commonly, the estimation of the parameters is realized by minimizing a certain performance index Q (say, a sum of squared error between $target_k$ and $M_0(\boldsymbol{x}_k)$), namely $\boldsymbol{a}_{\text{opt}} = \arg \text{Min}_{\boldsymbol{a}}\ Q(\boldsymbol{a})$. To compensate for inevitable errors of the model (as the values of the index Q are never equal identically to zero), we make the parameters of the model information granules, resulting in a vector of information granules $\boldsymbol{A} = [A_1\ A_2 \ldots\ A_p]$ built around original numeric values of the parameters $\boldsymbol{a}$. In other words, the fuzzy model is embedded in the *granular* parameter space. The elements of the vector $\boldsymbol{a}$ are generalized, the model becomes granular and subsequently the results produced by them are information granules. Formally speaking, we have

- granulation of parameters of the model $\boldsymbol{A} = G(\boldsymbol{a})$ where G stands for the mechanisms of forming information granules, viz. building an information granule around the numeric parameter
- result of the granular model for any $\boldsymbol{x}$ producing the corresponding information granule Y, $Y = M_1(\boldsymbol{x}, \boldsymbol{A}) = G(M_0(\boldsymbol{x})) = M_0(\boldsymbol{x}, G(\boldsymbol{a}))$.

Information granulation is regarded as an essential design asset [10]. By making the results of the model granular (and more abstract in this manner), we realize a better alignment of $G(M_0)$ with the data. Intuitively, we envision that the output of the granular model "covers" the corresponding target. Formally, let cov(*target*, *Y*) denote a certain coverage predicate (either Boolean or multivalued) quantifying an extent to which target is included (covered) in *Y*.

The design asset is supplied in the form of a certain allowable level of information granularity ε which is a certain non-negative parameter being provided in advance.

We allocate (distribute) the design asset across the parameters of the model so that the coverage measure is maximized while the overall level of information granularity serves as a constraint to be satisfied when allocating information granularity across the model, namely $\sum_{i-1}^{p} \varepsilon_i = e$ The constraint-based optimization problem reads as follows

$$\max_{\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_p} \sum_{k=1}^{N} \mathrm{cov}(target_k \in Y_K)$$

subject to

$$\sum_{i=1}^{P} \varepsilon_i = e \text{ and } \varepsilon_i \geq 0 \tag{5}$$

The monotonicity property of the coverage measure is obvious: the higher the values of e, the higher the resulting coverage. Hence the coverage is a non-decreasing function of ε.

Along with the coverage criterion, one can also consider the specificity of the produced information granules. It is a non-increasing function of e. The more general form of the optimization problem can be established by engaging the two criteria leading to the two-objective optimization problem. The problem can be re-structured in the following form in which the objective function is a product of the coverage and specificity-determine optimal allocation of information granularity [ε_1 ε_2,…, ε_p] so that the coverage and specificity criteria become maximized.

Plotting these two characteristics in the coverage–specificity coordinates offers a useful visual display of the nature of the granular model and possible behavior of the behavior of the granular model as well as the original model. There are different patterns of the changes between coverage and specificity. The curve may exhibit a monotonic change with regard to the changes in e and could be approximated by some linear function. There might be some regions of some slow changes of the specificity with the increase of coverage with some points at which there is a substantial drop of the specificity values. A careful inspection of these characteristics helps determine a suitable value of ε – any further increase beyond this limit might not be beneficial as no significant gain of coverage is observed however the drop in the specificity compromises the quality of the granular model.

The global behavior of the granular model can be assessed in a global fashion by computing an area under curve (*AUC*) of the coverage-specificity curve. Obviously, the higher the *AUC* value, the better the granular model. The *AUC* value can be treated as an indicator of the global performance of the original numeric model produced when assessing granular constructs built on their basis. For instance, the quality of the original numeric models M_0 and M_0' could differ quite marginally but the corresponding values of their *AUC* could vary quite substantially by telling apart these two models. For instance, two neural networks of quite similar topology may exhibit similar performance however when forming their granular generalizations, those could differ quite substantially in terms of the resulting values of the *AUC*.

As to the allocation of information granularity, the maximized coverage can be realized with regard to various alternatives as far as the data are concerned: (a) the use of the same training data as originally used in the construction of the model, (b) use the testing data, and (c) usage of some auxiliary data.

5.3 Granular Input Spaces in Fuzzy Modeling

The underlying rationale behind emergence of granular input spaces deals with an ability to capture and formalize the problem at the higher level of abstraction by adopting a granular view of the input space in which supporting system modeling and model construction are located. Granulation of input spaces is well motivated and often implied by the computing economy or a flexibility and convenience they offer to they offer when capturing the. Here we would like to highlight some illustrative examples, especially those commonly visible in some temporal or spatial domains.

Granular input spaces deliver an important, unique, and efficient design setting for the construction and usage of fuzzy models: (i) information granulation of a large number of data (in case of streams of data) leads to a far smaller and semantically sound entities facilitating and accelerating the design of fuzzy models, and (ii) the results of fuzzy modeling are conveyed at a suitable level of specificity suitable for solving a given problem. In the sequel, information granules used to construct a model, viz. a mapping between input and output information granules.

5.4 Rule-Based Models and Their Augmentation with Schemes of Allocation of Information Granularity

Functional rules (Takagi-Sugeno format of the conditional statements) link any input space with the corresponding local model whose relevance is confided to the region of the input space determined by the fuzzy set standing in the input space (A_i). The local character of the conclusion makes an overall development of the fuzzy model well justified: we fully adhere to the modular modeling of complex relationships. The local models (conclusions) could vary in their diversity; in particular local models in the form of constant functions (m_i) are of interest

$$\text{- if } x \text{ is } A_i \text{ then } y \text{ is } m_i \tag{6}$$

These models are equivalent to those produced by the Mamdani-like rules with a weighted scheme of decoding (defuzzification). There has been a plethora of design approaches to the construction of rule-based models, cf. [3, 4].

Information granularity emerges in fuzzy models in several ways by being present in the condition parts of the rules, their conclusion parts and both. In a concise way, we can describe this in the following way (below the symbol $G(.)$ underlines the granular expansion of the fuzzy set construct abstracted from their detailed numeric realization or a granular expansion of the numeric mapping).

(i) *Information granularity associated with the conditions of the rules*. We consider the rules coming in the format

$$\text{- if } G(A_i) \text{ then} f_i \tag{7}$$

where $G(A_i)$ is the information granule forming the condition part of the i-th rule. An example of the rule coming in this format is the one where the condition is described in terms of a certain interval-valued fuzzy set or type-2 fuzzy set, $G(A_i)$.

(ii) *Information granularity associated with the conclusion part of the rules*. Here the rules take on the following form

$$\text{- if } \boldsymbol{x} \text{ is } A_i \text{ then } G(f_i) \tag{8}$$

with $G(f_i)$ being the granular local function. The numeric mapping f_i is made more abstract by admitting their parameters being information granules. For instance, instead of the numeric linear function f_i, we consider $G(f_i)$ where $G(f_i)$ is endowed with parameters regarded as intervals or fuzzy numbers. In this way, we have $f_i(A_0, A_1, \ldots, A_n) = A_{i0} + A_{i1}x_1 + \ldots A_{in}x_n$ with the algebraic operations carried out on information granules (in particular adhering to the algebra of fuzzy numbers).

(iii) *Information granularity associated with the condition and conclusion parts of the rules*. This forms a general version of the granular model and subsumes the two situations listed above. The rules read now as follows

$$\text{- if } \ G(A_i) \text{ then } G(f_i) \tag{9}$$

The augmented expression for the computations of the output of the model generalizes the expression used in the description of the fuzzy models (8). We have

$$Y = \sum_{\substack{\mathrm{i}=1 \\ \oplus}}^{c} (G(A_i(\boldsymbol{x}) \otimes G(f_i)) \tag{10}$$

where the algebraic operations shown in circles $\otimes$ and $\oplus$ reflect that the arguments are information granules instead of numbers (say, fuzzy numbers). The detailed calculations depend upon the formalism of information granules being considered. Let us stress that Y is an information granule. Obviously, the aggregation presented by (10) applies to (i) and (ii) as well; here we have some simplifications of the above stated formula.

There are no perfect models. Information granularity augmenting existing (numeric) models results in a granular model and makes it more in rapport with reality. In a general way, we can think of a certain general way of forming a granular model at successively higher levels of abstraction. Subsequently the representation (model) of a real system S can be symbolically described through the following relationship

$$S \approx M \oplus G(M) \oplus G^2(M) \oplus \ldots \oplus G^t(M) \tag{11}$$

where the symbols M, $G(M)$, $G^2(M)$, … $G^t(M)$ stand for an original (numeric) model, granular model built with the use of information granules of type-1, $G(M)$, granular model realized with the use of information granules of type-2, $G^2(M)$,.., and information granules of type-t, etc. The symbol is used to denote the enhancements of the modeling construct aimed to model S. The models formed in this way are displayed in Fig. 3.

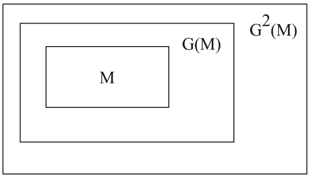

Fig. 3. A hierarchy of granular models: from numeric constructs to granular models with information granules of higher type

Noticeable is the fact that successive enhancements of the model emerge at the higher level of abstraction engaging information granules of the increasing type.

6 Conclusions

In the study, we have presented a general framework of Granular Computing and elaborated on their generalizations coming in the form of information granules of higher type and higher order. We offered a brief overview of fuzzy rule-based models and demonstrated that in light of new challenging modeling environments, there is a strongly motivated emergence of granular fuzzy models where the concept of information granularity and information granules of higher type/order play a pivotal role. The fundamentals of Granular Computing such as the principle of justifiable granularity and an optimal allocation of information granularity are instrumental in the construction of the granular models.

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