Chapter 2
The Nature of Modal Individuals

2.1 Introduction

In this chapter, I discuss, first, the nature of the proposal, according to which it is a 'transcendental precondition' of the way in which we speak and think about individuals in modal settings that they are categorized as world lines (Sect. 2.2). I then proceed to formulate a formal semantics of a quantified modal language in which quantifiers range over world lines (Sects. 2.3 and 2.4). I discern a general notion of content and show that both worlds and worlds lines can be seen as *modal unities* (Sects. 2.5 and 2.6). Contents are structures of interrelated modal unities. I close the chapter by clarifying how my world line framework is related to competing semantic and metaphysical views, notably those developed by Saul Kripke, David Lewis, and Kit Fine (Sect. 2.7).

2.2 Transcendental Preconditions

In Chap. 1, I spoke of a transcendental interpretation of world lines and suggested that construing individuals as world lines is a 'transcendental precondition' of our possibility to think and talk about them in modal settings. Saying so is not informative unless the relevant notion of precondition is clarified.

When discussing transcendental arguments, we are typically confronted with an inference that has two premises: a factual one, X, and a conditional one, 'necessarily, if X, then Y'. Here, X is typically a statement according to which we have certain experiences or thoughts or are capable of using language meaningfully in certain ways, and Y is often a proposition the skeptic doubts; it states either how things as a matter of fact are (strong version) or how we think they are (modest version). The reasoning is meant to establish that Y is a *transcendental precondition* of X. Here, Y is obtained from the two premises by modus ponens. I will refer to the conditional premise as a *transcendental claim*. In a transcendental claim 'necessarily,

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if X, then Y', the statement X is its *antecedent* and Y its *consequent*. As Barry Stroud [117, p. 156] stresses, the reasoning just described is in no way special. We should not take the *inference* to constitute a transcendental argument. Rather, a transcendental argument is an argument by means of which we could try to establish the transcendental claim (the conditional premise).

People often use phrases like 'Y is a necessary condition for the possibility of X' and 'In order for X to be possible, the condition Y must hold' when wishing to explain what a transcendental argument is supposed to establish. Superficially, it looks as though *possibility* qualifies the antecedent X and *necessity* qualifies the consequent Y in such formulations. In reality, it is the conditional 'if X, then Y' that is qualified by *necessity*. This is a case where grammar easily leads us astray. For example, when discussing Kant's philosophy, Sebastian Gardner [32, p. 30] writes, 'A transcendental proof has the peculiarity that it converts a possibility into a necessity: by saying under what conditions experience of objects is possible, transcendental proofs show those conditions to be necessary for us to the extent that we are to have experience of objects at all'. Here, the crucial expression is the hidden conditional of 'to the extent that'. No possibility is turned into anything, and nothing is turned into a necessity. It is just said that in any possible occasion in which we have experience of objects, a certain condition holds. This is what saying 'necessarily, if we have experience of objects, then such-and-such a condition holds' means.[1]

Following Quassim Cassam [13, pp. 83, 85], a distinction between *world-directed* and *self-directed* transcendental claims can be made. The antecedent of a transcendental claim of the former type states that we have certain experiences or thoughts. Its consequent states that the world in which these thoughts or experiences occur is a certain way. A world-directed transcendental claim itself states, then, that the world's being a certain way is a necessary condition for our having the thoughts or experiences mentioned in the antecedent. The antecedent of a self-directed transcendental claim, again, states that we have certain cognitive achievements, and its consequent states that our cognitive faculties are thus and so. Consequently, self-directed transcendental claims state that unless the thinking self has certain cognitive faculties, it lacks the cognitive achievements referred to in the antecedent. Self-directed transcendental claims can be seen as statements of *conceptual necessity*: they affirm that the employment of such-and-such concepts (those mentioned in the consequent) is necessary for our knowledge or experience or meaningful discourse.

Stroud argued in his famous 1968 paper that attempts to establish world-directed transcendental claims are deeply problematic: instead of proving that the *factual truth* of a proposition Y is necessary for our having such-and-such experiences, they merely appear to show that our *believing Y to be true* is necessary for those experiences [116]. This leaves still open the possibility of arguing for modest claims about connections between different ways of thinking [117, pp. 165–6], thereby in effect shifting attention toward self-directed transcendental arguments.

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[1] This said, certainly nothing prevents X and Y from being modal statements—being, for example, of the form *possibly* Z or *necessarily* Z. I merely wish to stress that this is not a part of what saying 'Y is a necessary condition for the possibility of X' means.

I am interested in self-directed transcendental claims whose antecedent is a statement according to which we can speak meaningfully of objects exhibiting modal and temporal behavior and whose consequent states that we employ certain concepts to think about those objects. Cassam [13] takes up the question of whether self-directed transcendental claims are independent of the *subjective origin thesis* (SOT), according to which the cognitive faculties that a transcendental argument portrays as preconditions of our cognitive achievements are wholly subjective in nature. According to SOT, self-directed transcendental claims are committed to transcendental idealism. Cassam describes a way of viewing transcendental claims that—without rendering them superfluous—allows them to avoid a commitment to SOT, thereby making them compatible with a form of realism. Seen in this way, transcendental preconditions are taken to reflect the nature of mind-independent objects.

The realist position Cassam develops consists of seeing transcendental preconditions as *objectively necessary conditions*. These are world-dependent conditions, grounded at least partly in the nature of objects as they are in themselves. Cassam defines objectively necessary conditions as *conditionally* conceptually necessary conditions [13, pp. 103–4]. In this sense, Y is an objectively necessary condition of X iff Y is a conceptually necessary condition of X *given* certain assumptions about the objects as they are in themselves. If we write '[objective](X, Y)' for 'Y is objectively necessary for X' and '[conceptual](X, Y)' for 'Y is conceptually necessary for X', then affirming [objective](X, Y) means affirming a statement of the following form:

1. If Z, then [conceptual](X, Y),

where Z is a statement about the nature of those external objects to which our knowledge pertains or about which we can meaningfully speak according to the statement X. For example, Z could be the statement that the objects of our empirical knowledge are in themselves spatiotemporal, X and Y being, respectively, the statements that we have empirical knowledge of spatiotemporal objects and that we employ the concept of persisting space-occupying substance when thinking about these objects [ibid. p. 106]. Since [conceptual](X, Y) itself is a statement according to which Y is a necessary condition of X, the claim (1) amounts to (2):

2. If Z, then necessarily, if X, then Y.

Cassam refers to the realist position he describes as *conceptualist realism* [ibid. p. 104].

In the context of conceptualist realism, transcendental arguments are primarily arguments for conditional statements of the form (1). As Cassam sees it, for the conceptualist realist, such transcendental arguments have an *explanatory* role, not an anti-skeptical role [13, p. 109]. In a transcendental argument intended as establishing (1), the *explanandum* is the statement [conceptual](X, Y) and the *explanans* the whole conditional claim (1). According to the explanation in question, the conceptualizations mentioned in the statement Y are important for our cognitive achievements mentioned in the statement X, since the reality to which those cognitive achievements pertain is as described by Z. The claim (1) may of course hold even if Z is false. If Z

happens to be true, we can infer the corresponding claim of conceptual necessity—i.e., the claim [conceptual](X, Y)—with the help of (1).

We may now distinguish two versions of the 'transcendental interpretation' of world lines as sketched in Sect. 1.5: the *realist* and the *idealist* version. Consider the following statements P_1, P_2, Q, and R:

P_1: Some external (mind-independent) objects are temporally extended and have modal properties.
P_2: Some appearances (objects of experience conforming to our mode of cognition) are temporally extended and have modal properties.
Q: It is meaningful to speak of temporally extended objects and to talk about their counterfactual behavior.
R: Objects we speak of in modal settings are conceptualized as world lines.

In Q and R, by speaking of 'objects', I mean *physical objects*—as opposed to intentional objects. The conceptualist realist must provide a transcendental argument for the statement (3), whereas the transcendental idealist must establish (4):

3. If P_1, then [conceptual](Q, R)
4. [conceptual](Q, R).

Here, (4) is the statement that in order for us to meaningfully speak of temporally extended objects with modal properties, these objects *must be thought of as* world lines. This is precisely the thesis put forward by the transcendental interpretation of world lines. As explained in Sect. 1.3, thinking of modal individuals as world lines means holding that *individuals* and *worlds* are mutually independent but interacting 'modal unities'. The cross-world behavior of individuals does not reduce to, nor is supervenient on, world-internal local features. World lines are not determined by worlds, and worlds are not determined by world lines. An individual is realized in some but not necessarily all worlds. A world realizes some but not necessarily all individuals. Each version of the transcendental interpretation of world lines can be characterized as a conjunction of three statements that jointly entail R:

5. *Realist variant*: P_1 & Q & If P_1, then [conceptual](Q, R)
6. *Idealist variant*: P_2 & Q & [conceptual](Q, R).

In each case, the first statement describes how the relevant philosophical position views the nature of the objects spoken about; the second statement affirms the antecedent of the transcendental claim (4); and the third statement is the claim whose proof would constitute the relevant transcendental argument. In the realist case, such an argument is potentially easier to produce: it suffices to argue for (4) on the condition that P_1 holds, instead of establishing (4) categorically, as required in the idealist case. All three conjuncts of (5) are needed to derive R. By contrast, R is entailed already by the latter two conjuncts of (6).

The truth of the realist claim (3) can be seen as *explaining* our cognitive faculty of thinking of objects of meaningful discourse in a certain way (namely, as world lines).

For the conceptual realist, the condition R is *objectively* necessary, grounded in the truth of P_1, and the claim Q is understood as concerning our capacity to talk about external objects. The idealist variant differs from its realist cousin in that the claim (4) must be established unconditionally, and Q is understood as a claim about our capacity to speak of the temporal and modal behavior of *appearances* (as opposed to objects as they are in themselves). The idealist takes the relevant necessary condition R to originate in 'the subjective constitution of our mind' [cf. A23/B38], not in how the world is.[2] In order to show that self-directed transcendental claims are not committed to SOT, it suffices to argue that the idea of a conditionally conceptually necessary condition is coherent; cf. [13, pp. 104–5]. To this end, the realist need not be able to categorically rule out the idealist position P_2.

To defend either variant of the transcendental interpretation, the specific task is to produce the relevant transcendental argument.[3] For the realist, this means arguing for (3). The idealist needs to argue for (4). In both cases, we need an argument that is semantic by nature and consists of two steps. First, the meaning of the statement R must be clarified. We need a sufficiently comprehensive semantic theory that explicates what it means for quantifiers to range over world lines. This will show that it is at least intrinsically coherent to claim that meaningful discourse about individuals in modal settings is based on construing individuals as world lines. Second, grounds must be given for preferring world line semantics over alternative semantic accounts. I must back up my analysis by explicit comparisons between my view and views according to which cross-world identity is a simple notion, unproblematically transferrable from extensional to modal settings. If I succeed in showing that one must adopt the semantics of world lines to account for our actual meaningful discourse about temporally extended objects with modal properties, I have ipso facto provided a transcendental argument for the transcendental claim (4).

The choice between the realist and the idealist version of the transcendental interpretation must be based on general philosophical considerations. Cassam's argumentation in his 1999 paper shows that the transcendental idealist faces considerable difficulties in maintaining that the realist cannot detach self-directed transcendental arguments from the subjective origin thesis. Within the confines of this book, I naturally cannot undertake an overarching defense of either transcendental idealism or (conceptualist) realism. What I say is compatible with either viewpoint. I am sympathetic to realism: I take it that we must postulate external objects. In addition to external physical objects, however, there are objects of thought. My goal is not to explain them away. Indeed, my overall semantic framework aims to defend a supplementary transcendental claim concerning intentional objects:

7. [conceptual](Q', R'),

where Q' and R' are the following statements:

Q': It is meaningful to speak of intentional objects.

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[2] All references of the form An/Bm or An or Bm are to Kant [60].

[3] The unspecific task would be to argue for P_1 or for P_2. However, it is beyond the scope of this book to undertake a global defense of realism or transcendental idealism.

R': Intentional objects are conceptualized as world lines.

Here, (7) affirms that in order for us to meaningfully speak of intentional objects, they *must be thought of as* world lines. I will argue that intentional objects must be viewed as being intrinsically modal—as world lines defined over the set of worlds compatible with an intentional state of an agent (e.g., the agent's perceptual experience or beliefs).[4] Chapters 3 and 4 provide a detailed discussion of intentional objects and their relation to physical objects. Until then, my analysis will remain schematic.

2.3 World Line Semantics

I proceed to describe schematically a semantics of a quantified modal language. Its quantifiers range over world lines. I refer to it as *world line semantics*. At this schematic level, I pay no attention to the nature of world lines. I merely wish to give a precise idea of how the semantics looks like. In Sect. 3.4, I enrich the framework to make it useful for discussing the contrast between intentional and physical objects.

Let *Var* be a set of variables and τ a *relational vocabulary*: a set of predicate symbols, each with an associated positive arity indicating how it syntactically combines with variables to form atomic formulas. For simplicity, sometimes I refer to predicate symbols as *predicates*. (In this sense, predicates are always linguistic entities.) The quantified modal language $L_0[\tau]$ of vocabulary τ is built according to the following syntax:

$$\phi ::= Q(x_1, \ldots, x_n) \mid x_1 = x_2 \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box\phi \mid \exists x\phi,$$

where n is a positive integer, the symbols $x, x_1, x_2, \ldots, x_n$ all belong to *Var*, and Q is an n-ary predicate belonging to τ.[5] Both *predications* $Q(x_1, \ldots, x_n)$ and *identities* $x_1 = x_2$ are *atomic* formulas. The operators $\vee$, $\rightarrow$, $\Diamond$, and $\forall$ are definable from the operators $\neg$, $\wedge$, $\Box$, and $\exists$ in the usual manner.

Models of vocabulary τ are structures $M = \langle W, R, \mathfrak{I}, Int\rangle$. Here, W is a non-empty set. Every member w of W has a specified non-empty domain $dom(w)$. Further, R is a binary relation on W, and *Int* is a function assigning to every n-ary

[4] I take it that neither physical nor intentional objects should be considered as local objects, but both types of objects must be viewed as world lines. In the case of intentional objects, this is so even if we limit attention to objects that are not thought of as being temporally extended or having modal properties. This is because there is normally a great variety of scenarios compatible with an agent's intentional state; intentional objects are construed as world lines considered in relation to the totality of all such scenarios.

[5] The syntax is conveniently specified in Backus–Naur form. It should be understood as follows. If Q is a predicate and the x_i are variables, both $Q(x_1, \ldots, x_n)$ and $x_1 = x_2$ are formulas. The result of prefixing a formula by $\neg$, $\Box$, or $\exists x$ is likewise a formula. Finally, the result of combining a formula with a formula using $\wedge$ is a further formula. The symbol ϕ represents schematically any expression generated by the grammar, and distinct occurrences of ϕ need not represent the same expression.

predicate Q of τ and element w of W a subset $Int(Q, w)$ of $dom(w)^n$. Finally, $\mathfrak{I}$ is a collection of sets $\mathfrak{I}_w$ with $w \in W$. Each element of $\mathfrak{I}_w$ is a non-empty partial function on W, assigning an element of $dom(w')$ to every w' on which this partial function is defined.

The set W is the *domain* of M, in symbols $dom(M)$. Elements of W are referred to as *worlds*. Elements of the sets $dom(w)$, again, are termed *local objects*. If $w \neq w'$, in accordance with my assumptions about cross-world identity, I take it that elements of $dom(w)$ cannot be compared in terms of identity or numerical distinctness with elements of $dom(w')$.[6] The component R of the model is an *accessibility relation*. If $w \in W$, I write $R(w)$ for the set $\{w' : R(w, w')\}$. The component Int of the model is called an *interpretation*. Since each local object belongs to a unique world, the world in question is in principle recoverable from the object, and we could define the interpretation of an n-ary predicate Q directly as a subset of the set $\bigcup_{w \in W} dom(w)^n$ without relativizing the interpretation to a world. I stay with the definition given, however, as it makes the discussion of interpretations more transparent and in fact allows a smoother generalization when constant symbols will be allowed in the syntax (Sect. 3.4). The elements of the sets $\mathfrak{I}_w$ are *world lines over* W. Note that when calling these non-empty partial functions ‘world lines’, I am following the liberalized terminology agreed upon in Sect. 1.3. World lines themselves, in the sense in which I have spoken of them in connection with the transcendental interpretation, *are* not partial functions, but there is a one-to-one correspondence between world lines and suitable partial functions, which is why this abuse of terminology is harmless, and we may well utilize partial functions as surrogates of world lines in the formulation of the semantics. In order not to render the terminology needlessly heavy, I use the term ‘world line’ in both cases. There should be no serious risk of confusion. If $\mathbf{I} \in \mathfrak{I}_w$ and w' is a world on which $\mathbf{I}$ is defined, its value $\mathbf{I}(w')$ is the *realization* of $\mathbf{I}$ in w'. The domain of the partial function $\mathbf{I}$ is its *modal margin*, denoted $marg(\mathbf{I})$. The *domain of world lines* of M is the set $\bigcup_{w \in W} \mathfrak{I}_w$, denoted $\mathrm{WL}(M)$.

An *assignment* in M is a function of type $Var \rightarrow \mathrm{WL}(M)$. Thus, the values of variables are world lines, not local objects. If g is an assignment defined on x, then $g(x)$ is a world line. If this world line is realized in world w, the result $g(x)(w)$ of applying the function $g(x)$ to the world w is a local object that belongs to the domain of w. If g is an assignment and $\mathbf{I}$ is a world line, $g[x := \mathbf{I}]$ stands for the assignment that differs from g at most in that it assigns $\mathbf{I}$ to x. That is, if v is a variable, then $g[x := \mathbf{I}](v) = g(v)$ if v is by syntactic criteria distinct from x, whereas $g[x := \mathbf{I}](v) = \mathbf{I}$ if, syntactically, v equals x. The semantics of L_0 is defined by recursively specifying what it means for a formula ϕ to be *satisfied* in a model M

[6]There is a sense in which the question ‘Is the set $dom(w) \cap dom(w')$ non-empty?’ is ill formed: in any single world w, we only encounter its local objects, and remaining within w, we can never effect the relevant comparisons allowing us to meaningfully affirm that every $a \in dom(w)$ is distinct from every $a' \in dom(w')$. From a metatheoretic perspective, we can answer this question. Worlds partition the class of local objects into cells so that local objects a and a' can be compared in terms of numerical identity iff they belong to the same cell. The set $dom(w) \cap dom(w')$ is empty if w and w' correspond to distinct cells of this partition; otherwise, w equals w'. Cf. footnote 15 in Sect. 4.5.

at a world w under an assignment g, denoted $M, w, g \models \phi$. Here are the semantic clauses:

- $M, w, g \models Q(x_1, \ldots, x_n)$ iff for all $1 \leq i \leq n$, the world line $g(x_i)$ is realized in the world w, and the tuple $\langle g(x_1)(w), \ldots, g(x_n)(w)\rangle$ belongs to $Int(Q, w)$.
- $M, w, g \models x_1 = x_2$ iff the world lines $g(x_1)$ and $g(x_2)$ are both realized in the world w, and the local object $g(x_1)(w)$ is the same as the local object $g(x_2)(w)$.
- $M, w, g \models \neg\phi$ iff $M, w, g \not\models \phi$.
- $M, w, g \models (\phi \wedge \psi)$ iff $M, w, g \models \phi$ and $M, w, g \models \psi$.
- $M, w, g \models \Box\phi$ iff for all worlds w' with $R(w, w')$, we have: $M, w', g \models \phi$.
- $M, w, g \models \exists x\phi$ iff there is $\mathbf{I} \in \mathfrak{I}_w$ such that $M, w, g[x := \mathbf{I}] \models \phi$.

The most distinctive feature of this semantics is the way it treats quantifiers. In a world w, the quantifier $\exists$ ranges over the set $\mathfrak{I}_w$. Elements of $\mathfrak{I}_w$ are said to be *available* in w. Being available in w is not the same as being realized in w. It is not required that every world line available in w be realized in w, nor that every world line realized in w be available in w. The possibility of considering these features independently of each other will be important for analyzing quantification into modal contexts and for accommodating a sense of ‘there is’ without ontological commitments when discussing intentional objects. If the quantifier $\exists x$ is evaluated in world w, this results in assigning as the value of x an element $\mathbf{I}$ of $\mathfrak{I}_w$, a certain world line. When the evaluation reaches an atomic formula, say $P(x)$, it is checked whether the value $\mathbf{I}$ of x is realized in the world w' that is current *then*. If so, it is further checked whether the local object $\mathbf{I}(w')$ belongs to the set $Int(P, w')$. Note that the world w' can very well be distinct from w. This may happen if syntactically between the quantifier and the atomic formula there are occurrences of modal operators—for example, if the formula evaluated at w is $\exists x\Box P(x)$. Because availability does not entail realization, we can quantify into a modal context in w without getting thereby committed to the existence of the value of the relevant quantified variable in w—recalling that by ‘existence’ of a world line in a world, we mean its being realized in that world.[7]

The accessibility relation R may be relative to an agent. It could, for example, determine which alternatives the agent’s perceptual experience leaves open in a given scenario (perceptual accessibility) or which scenarios are compatible with what an agent believes in a fixed scenario (doxastic accessbibility). The specific properties of the relation R depend on the modality it is taken to represent. In the case of a veridical perceptual experience, the relation must be reflexive, but not in connection with arbitrary perceptual experience and belief. I could introduce several types of modal operators into the syntax, each type having its associated accessibility relation as a component in the models. In fact, I will (Sect. 3.4). But, at this schematic phase, doing so would be a pointless complication.

[7] The distinction between availability and realization is considered in detail in Sect. 3.3. For a recent discussion of quantifying-in in general attitudinal contexts, see Jespersen [56].

As explained in Sect. 1.3, in my semantics, predicate symbols are suited for expressing properties that an individual has in a world depending on how it is in that world. According to the semantics above, predicate symbols are applied to local objects. If Q is an n-ary predicate symbol and a tuple $\langle a_1, \ldots, a_n\rangle$ belongs to the interpretation of Q in a world w, then the a_i are local objects (and belong to the domain of w). Predicate symbols are *not* applied to world lines. However, for every n-ary predicate Q of vocabulary τ, the semantics of $L_0[\tau]$ induces an $(n+1)$-ary relation R_Q as follows: $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w\rangle \in R_Q$ iff all world lines $\mathbf{I}_1, \ldots, \mathbf{I}_n$ are realized in w and $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w)\rangle \in Int(Q, w)$. Consequently, we may view formulas $Q(x_1, \ldots, x_n)$ as n-ary 'intensional predicates'. An n-tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle$ of world lines satisfies the intensional predicate $Q(x_1, \ldots, x_n)$ in a world w iff $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w\rangle \in R_Q$.

In the same way as the interpretation of an n-ary predicate symbol Q in a world w is the n-ary relation $Int(Q, w)$ on $dom(w)$, the fixed interpretation of the identity symbol in a world w is the relation of extensional identity on w—i.e., the binary identity relation $dom(w)$. Those philosophers who are perplexed by the very idea of a binary identity relation might prefer a formulation of the semantics of L_0 that does not expressly make use of such an identity relation.[8] It is, in fact, easy to reformulate the semantics of L_0 so as to meet this desideratum. Let K be the ternary relation of *co-realization* defined, relative to a given model M, as follows: whenever $\mathbf{I}, \mathbf{J} \in \bigcup_{v\in W} \mathfrak{I}_v$ and $w \in W$, let $\langle \mathbf{I}, \mathbf{J}, w\rangle \in K$ iff both world lines $\mathbf{I}$ and $\mathbf{J}$ are realized in w and the local object $\mathbf{I}(w)$ is the same as the local object $\mathbf{J}(w)$. We may take the identity symbol to stand for the relation K in M, because the above satisfaction condition of identity formulas can be equivalently formulated in terms of K: we have $M, w, g \models x_1 = x_2$ iff $\langle g(x_1), g(x_2), w\rangle \in K$. Now, for having $\langle \mathbf{I}, \mathbf{J}, w\rangle \in K$, it is neither sufficient nor necessary that the world lines $\mathbf{I}$ and $\mathbf{J}$ be identical: $\langle \mathbf{I}, \mathbf{I}, w\rangle \notin K$ if $\mathbf{I}$ is not realized in w, and we may well have $\mathbf{I}(w) = \mathbf{J}(w)$ and therefore $\langle \mathbf{I}, \mathbf{J}, w\rangle \in K$ even if the world lines $\mathbf{I}$ and $\mathbf{J}$ are distinct—i.e., even if there is a world v in which the two world lines $\mathbf{I}$ and $\mathbf{J}$ do not have identical realizations (either because their realizations in v are distinct or because one but not the other fails to be realized in v). Thus, the relation K may seem less problematic as an interpretation of the identity symbol than a binary identity relation that every object bears to itself and to itself only. However, we do not get rid of binary identity relations (as certain relations-in-extension) just by reformulating our semantics so that an identity relation does not *explicitly* appear as the interpretation of the identity symbol in a world. Namely, the fact remains that

[8] Cf. the discussion in footnote 5 in Sect. 1.1.

world lines $\mathbf{I}$ and $\mathbf{J}$ being co-realized in a world *means* that they are realized in that world and that their realizations are extensionally identical.[9]

I assume the standard notion of free variable of a formula: an occurrence of a variable x is free in ϕ if it does not lie in ϕ in the scope of a quantifier carrying the variable x.[10] A formula containing no free variables is a *sentence*. I write $\phi(x_1, \ldots, x_n)$ to indicate that $x_1, \ldots, x_n$ are n distinct variables and that the free variables of ϕ are exactly the variables $x_1, \ldots, x_n$. If $g(x_i) = \mathbf{I}_i$ for all $1 \leq i \leq n$, we may express the condition $M, w, g \models \phi$ by writing $M, w, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi(x_1, \ldots, x_n)$, without thereby suppressing any relevant information.[11] If ϕ is a sentence, its satisfaction is entirely independent of the assignment considered. We may write $M, w \models \phi$ when ϕ is a sentence and $M, w, g \models \phi$ holds for at least one assignment g. When this condition holds, we say that ϕ is *true* in M at w. A formula ϕ of vocabulary τ is *valid* if for all models M of vocabulary τ, worlds $w \in dom(M)$, and assignments g in M, we have $M, w, g \models \phi$. A formula ϕ is *refutable* (respectively, *satisfiable*) if ϕ (respectively, $\neg\phi$) is not valid. Let us take some examples of how the semantics works.

Example 2.1 The formula $P(x)$ may fail to be satisfied in M at w under g for two reasons: either because the world line $g(x)$ is not realized in w in the first place or because it is but its realization $g(x)(w)$ lies outside the set $Int(P, w)$. Consequently, the negated formula $\neg P(x)$ can be satisfied in M at w under g for two reasons. The formula $x = x$ is *not* valid. In fact, $M, w, g \models x = x$ iff $g(x)$ is realized in w. The formulas $\forall x(P(x) \vee \neg P(x))$ and $\forall x\, x = x$ are not equivalent. The former *is* valid. Namely, suppose $\mathbf{I} \in \mathcal{I}_w$. If $\mathbf{I}$ is realized in w, the realization $\mathbf{I}(w)$ either does or does not belong to $Int(P, w)$, satisfying correspondingly either the left or the right disjunct of $P(x) \vee \neg P(x)$ at w. If, again, $\mathbf{I}$ is not realized in w, it satisfies the right

[9] Hintikka [40] showed that in first-order logic, the semantics of quantifiers can be interpreted 'inclusively' or 'exclusively' and that in the latter case, the expressive power of first-order logic is not diminished by disallowing the use of the identity symbol in the syntax (supposing we restrict attention to vocabularies in which neither constant nor function symbols occur). Unlike in the standard 'inclusive' interpretation, according to the 'exclusive' interpretation, a formula $\exists x\, \phi(x, x_1, \ldots, x_n)$ is satisfied in a model $\mathcal{M}$ under an assignment Γ iff there is in the domain of $\mathcal{M}$ an individual b *other than* any of the individuals $\Gamma(x_1), \ldots, \Gamma(x_n)$ such that $\Gamma[x/b]$ satisfies the formula $\phi(x, x_1, \ldots, x_n)$ in $\mathcal{M}$: the range of x *excludes* the values of all variables $x_1, \ldots, x_n$ that are free in the scope of the quantifier $\exists x$. Wehmeier [125] attempts to argue that we can dispense with the binary relation of identity, and as a partial motivation, he refers to Hintikka's result. However, as Hintikka [40, p. 228] himself stressed and contrary to what Wehmeier [125, Sect. 2] suggests, Hintikka's result merely shows that we do not need the *identity symbol* in the syntax of first-order logic; the result by no means suggests that we can dispense with the *notion of identity*. The exclusive interpretation of quantifiers merely provides an alternative way of dealing with the notion of extensional identity in first-order logic.

[10] Given the syntax of L_0, the only possible quantifier meeting this criterion could be $\exists x$. The obvious syntactic notion of subformula gives rise to the notion of scope in the usual way.

[11] As in first-order logic, also in L_0, the satisfaction of a formula ϕ under an assignment g evidently depends only on the values of g on *those* variables that are free in ϕ.

disjunct at w. By contrast, if $\mathbf{I}$ is not realized in w, it fails to satisfy $x = x$ at w.[12] □

Let Q be a fixed unary predicate. The formulas BF and CBF are, respectively, the *Barcan formula* and the *converse Barcan formula*:

BF $\quad \Diamond \exists x\, Q(x) \to \exists x \Diamond Q(x)$
CBF $\quad \exists x \Diamond Q(x) \to \Diamond \exists x\, Q(x).$

For reasons to be explicated in Sect. 5.4, I will *not* say that all those formulas are Barcan formulas that are obtained from BF by replacing $Q(x)$ by a formula with x as its sole free variable. A similar remark applies to CBF. Accordingly, BF and CBF should not be seen as schemata in the usual sense.

Example 2.2 Neither BF nor CBF is valid. Let $w_1 \neq w_2$, and consider a model $M = \langle W, R, \mathfrak{I}, Int\rangle$ defined as follows:

- $W = \{w_1, w_2\}$ and $R = \{\langle w_1, w_2\rangle\}$
- $\mathfrak{I} = \{\mathfrak{I}_{w_1}, \mathfrak{I}_{w_2}\}$, where $\mathfrak{I}_{w_1} = \varnothing$ and $\mathfrak{I}_{w_2} = \{\mathbf{I}\}$ with $marg(\mathbf{I}) = \{w_2\}$
- $Int(Q, w_1) = \varnothing$ and $Int(Q, w_2) = \{a\}$, where $a = \mathbf{I}(w_2)$.

We have $M, w_1 \models \Diamond \exists x\, Q(x)$, since w_2 is R-accessible from w_1 and $M, w_2 \models \exists x\, Q(x)$. The latter condition holds because $\mathbf{I} \in \mathfrak{I}_{w_2}$ and $\mathbf{I}(w_2) = a \in Int(Q, w_2)$. Yet, $M, w_1 \not\models \exists x \Diamond Q(x)$. This follows from the fact that the set $\mathfrak{I}_{w_1}$ is empty. We may conclude that BF is refutable. In order to see that also CBF is refutable, consider the model $M' = \langle W', R', \mathfrak{I}', Int'\rangle$, where $W' = W$ and $R' = R$ and $Int' = Int$ and $\mathfrak{I}' = \{\mathfrak{I}'_{w_1}, \mathfrak{I}'_{w_2}\}$, where $\mathfrak{I}'_{w_1} = \{\mathbf{I}\}$ and $\mathfrak{I}'_{w_2} = \varnothing$. Note that the very same world line that is available in w_2 in M is available in w_1 in M'. Now, $M', w_1 \models \exists x \Diamond Q(x)$, since $\mathbf{I} \in \mathfrak{I}_{w_1}$ and $R(w_1, w_2)$ and $\mathbf{I}$ is realized in w_2, satisfying $\mathbf{I}(w_2) = a \in Int(Q, w_2)$. Still, $M', w_1 \not\models \Diamond \exists x\, Q(x)$, because the only world accessible from w_1 is w_2 and the set $\mathfrak{I}_{w_2}$ is empty. Even though $\mathbf{I}$ is realized in w_2, neither it nor any other world line is available in w_2. Consequently, there is no world line $\mathbf{J} \in \mathfrak{I}_{w_2}$ satisfying $M', w_2, x := \mathbf{J} \models Q(x)$. □

[12] No atomic formula $P(x)$ can be satisfied by a value $x := \mathbf{I}$ in a world w unless the world line $\mathbf{I}$ is realized in w. Thus, if ‘haired’ and ‘bald’ are construed as atomic predicates and $\mathbf{I}$ is not realized in w, the assignment $x := \mathbf{I}$ satisfies neither haired(x) nor bald(x) in w. As these predicates are used in English, qualifying anything as haired or bald in a context w would perhaps be taken to *presuppose* that the thing is present in w, rather than its presence in w being viewed as part of what is affirmed by such a qualification. In any event, natural-language semantics agrees that ‘haired’ and ‘bald’ cannot be satisfied in w by anything not realized in w. The same holds for such predicates as ‘unfair’. If the value $\mathbf{I}$ of x is realized in w, we can safely say that $x := \mathbf{I}$ satisfies unfair(x) in w iff it satisfies $\neg$fair(x) in w, but generally, we could have $w, x := \mathbf{I} \models \neg$fair$(x)$ because $\mathbf{I}$ is not realized in w. In that case, we would not have $w, x := \mathbf{I} \models$ unfair(x), since this would require that $\mathbf{I}$ be realized in w. That is, unfair(x) is not simply the negation of fair(x). This said, unfair(x) can be defined in terms of fair(x): the formula unfair(x) is equivalent to $x = x \wedge \neg$fair(x), since this latter is satisfied in w by $x := \mathbf{I}$ iff $\mathbf{I}$ is realized in w and $\mathbf{I}(w)$ fails to be fair. One could define a strong notion of negation ($\sim$) in L, by stipulating that $\sim P(x)$ means $x = x \wedge \neg P(x)$. Then, unfair(x) could indeed be defined as a negation of fair(x) in a certain sense, because unfair(x) is equivalent to $\sim$fair(x).

The above example shows that the semantics of L_0 validates neither BF nor CBF. It is important to note that this fact has nothing to do with local objects being world-bound. Values of variables are world lines, not local objects. The reason why BF is refutable is that world lines available to be quantified over in one world need not be available to be quantified over in another world. As for CBF, its counter-models must utilize a further property of the semantics of L_0, as well—namely, the fact that a world line may be realized in a world (and satisfy an atomic predicate therein) without being available in that world.

2.4 Types of Predicates

In world line semantics, an obvious distinction can be made between 'extensional' and 'intensional' predicates. The former apply to local objects (realizations of world lines), whereas the latter apply to world lines themselves. In models of L_0, elements of the non-logical vocabulary are treated as extensional predicates, which, in particular, satisfy the 'domain constraint': the interpretation of an n-ary predicate symbol in w is a subset of the n-th Cartesian power of $dom(w)$. However, formulas of L_0 give rise to intensional predicates. It has already been noted that an atomic formula $Q(x_1, \ldots, x_n)$ can be seen as an n-ary intensional predicate that is satisfied by a tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ of world lines in a world w iff we have $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n, w \rangle \in R_Q$, where R_Q is an $(n+1)$-ary relation induced by the semantics of L_0 in the way explained in Sect. 2.3. In fact, any formula $\phi(x_1, \ldots, x_n)$ with n free variables can be considered as an intensional n-ary predicate that is atomic or complex, depending on whether the formula $\phi(x_1, \ldots, x_n)$ is atomic or complex. The predicate $\phi(x_1, \ldots, x_n)$ applies in a model M at a world w to those n-tuples of world lines that satisfy it in M at w. More generally, the semantics specified in Sect. 2.3 determines for all models M and L_0-formulas ϕ of n free variables a certain set of $(n+1)$-tuples—namely, the sequence of those 'parameters of evaluation' that satisfy ϕ in M.

Definition 2.1 (*Semantic value*) Let M be a model, and let $\phi(x_1, \ldots, x_n)$ be a formula of the language L_0. The *semantic value* $|\phi(x_1, \ldots, x_n)|^M$ of ϕ in M is the set of all $(n+1)$-tuples $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in dom(M) \times \mathrm{WL}(M)^n$ such that

$$M, w, x_1 := \mathbf{I}_1, \ldots, x_n := \mathbf{I}_n \models \phi(x_1, \ldots, x_n).$$

If ϕ is a sentence, then $|\phi|^M$ is a (possibly empty) subset of $dom(M)$—namely, the set of worlds w at which ϕ is true in M. □

Since the semantics of atomic formulas of L_0 is formulated in terms of extensional predicates, in fact all *intensional* predicates induced by L_0-formulas are analyzable in terms of extensional predicates. This holds trivially for *atomic* intensional predicates built from n-ary predicate symbols. A tuple $\langle \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ of world lines satisfies such an intensional predicate $Q(x_1, \ldots, x_n)$ at a world w iff all these world lines are realized in w and the tuple $\langle \mathbf{I}_1(w), \ldots, \mathbf{I}_n(w)\rangle$ satisfies the *extensional* predicate Q in w. Those dispositional predicates that can be represented by using complex L_0-formulas are likewise analyzable in terms of extensional predicates. Consider the intensional predicate $\Box(P(x) \rightarrow Q(x))$ that a world line satisfies in w if in all accessible scenarios in which it satisfies P, it satisfies Q, as well. The world line $\mathbf{I}$ satisfies this dispositional predicate in w iff in all those worlds w' accessible from w in which $\mathbf{I}$ is realized and in which its realization $\mathbf{I}(w)$ satisfies the extensional predicate P, this realization $\mathbf{I}(w)$ also satisfies the extensional predicate Q. As I said in Sect. 1.3, I do not wish to suggest that all philosophically interesting atomic predicates for some reason must ascribe local properties, and a fortiori I do not wish to claim that all intensional predicates must be analyzable in terms of local properties. However, some intensional predicates are so analyzable—cases in point being all *those* intensional predicates that are induced by L_0-formulas. In this book, I confine attention to intensional predicates of this kind.

If $\phi(x_1, \ldots, x_n)$ is an L_0-formula and $y_1, \ldots, y_n$ are variables, all of which are free for every variable $x_1, \ldots, x_n$ in ϕ, then $\phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ stands for the result of uniformly replacing all free occurrences of x_i in ϕ by y_i for all $1 \leq i \leq n$.[13] The following notions are useful when discussing varieties of predicates.

Definition 2.2 Let $\phi(x_1, \ldots, x_n)$ be a predicate in L_0. It is *existence-entailing* if the formula $\phi(x_1, \ldots, x_n) \rightarrow \bigwedge_{1\leq i\leq n} x_i = x_i$ is valid. It is *pro mundo* if the formula $\big(\phi(x_1, \ldots, x_n) \wedge \bigwedge_{1\leq i\leq n} x_i = y_i\big) \rightarrow \phi[x_1 /\!\!/ y_1, \ldots, x_n /\!\!/ y_n]$ is valid, given that each variable y_i is free for every variable x_j. It is *quasi-extensional* if it is both existence-entailing and *pro mundo*. $\Box$

Note that a predicate $\phi(x)$ is existence-entailing if a world line $\mathbf{I}$ cannot satisfy $\phi(x)$ in a world w unless $\mathbf{I}$ is realized in w. It is *pro mundo* if its satisfaction in w by a world line depends only on the realization (if any) of the world line in w: if $\mathbf{I}(w) = \mathbf{J}(w)$ and $\mathbf{I}$ satisfies $\phi(x)$ in w, then also $\mathbf{J}$ satisfies $\phi(x)$ in w.

Fact 2.1 *(a) Being existence-entailing and being* pro mundo *are mutually independent properties of predicates. (b) The set of existence-entailing predicates is not closed under applications of the operators $\neg$ or $\Box$. (c) The set of* pro mundo *predicates is not closed under applications of the operator $\Box$.*

Proof Let us begin with item (a). If P is unary and atomic, the predicate $x = x \wedge \Diamond P(x)$ is trivially existence-entailing, but it is not *pro mundo*: world lines $\mathbf{I}$ and $\mathbf{J}$ can coincide in w while only one of them satisfies $\Diamond P(x)$ in w. In order to see that, conversely, being *pro mundo* does not guarantee existence-entailment, note first that the predicate $\neg P(x)$ is *pro mundo*: if $\mathbf{I}(w) = \mathbf{J}(w)$, then $\mathbf{I}(w) \notin Int(P, w)$ iff $\mathbf{J}(w) \notin Int(P, w)$. Yet, it fails to be existence-entailing: $\neg P(x)$ is satisfied in w by any world line not realized in w. For items (b) and (c), observe that $P(x)$ is both

[13] We say that y is *free for* x in ϕ iff x does not occur free in the scope of the quantifier $\exists y$ in ϕ. If y is not free for x in ϕ, substituting y for a certain free occurrence of x in ϕ results in a formula in which that occurrence of y is bound.

existence-entailing and *pro mundo*. Yet, $\neg P(x)$ is not existence-entailing. Neither is $\Box P(x)$. For having $w, x := \mathbf{I} \models \Box P(x)$, it suffices that $\mathbf{I}$ is realized and satisfies P in all worlds v accessible from w; $\mathbf{I}$ need not be realized in w unless the accessibility relation corresponding to $\Box$ happens to be reflexive. Finally, $\Box P(x)$ also fails to be *pro mundo*: having $\mathbf{I}(w) = \mathbf{J}(w)$ and $w, x := \mathbf{I} \models \Box P(x)$ does not entail $w, x := \mathbf{J} \models \Box P(x)$. The fact that $\mathbf{I}$ and $\mathbf{J}$ coincide locally, in w, does not generally guarantee that they behave similarly in all worlds accessible from w. □

A predicate ϕ is *non-modal* if it contains no occurrences of $\Box$. It is *positive* if all its atomic subformulas occur in the scope of an even number of negation-signs. All non-modal predicates are *pro mundo*. Not all of them are quasi-extensional. For example $\neg P(x)$ is not, as it fails to be existence-entailing. By contrast, all positive non-modal predicates are quasi-extensional. A case in point is $Q(x) \wedge \neg(\neg R(x, y) \wedge \neg P(y))$. Also some modal predicates are quasi-extensional. An example is $P(x) \wedge \theta$ if θ is any modal sentence, no matter how many nested boxes it contains. It is characteristic of quasi-extensional predicates that their semantic values can be encoded by interpretations of extensional predicates in the following sense. Suppose $\phi(x_1, \ldots, x_n)$ is a quasi-extensional predicate of vocabulary τ. Let Q_ϕ be an n-ary predicate symbol with $Q_\phi \notin \tau$. For every model M of vocabulary τ, expand its interpretation function *Int* by setting $Int(Q_\phi, w) := \{\langle a_1, \ldots, a_n\rangle \in dom(w)^n$: there are $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$ such that $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle \in |\phi(x_1, \ldots, x_n)|^M$ and $a_1 = \mathbf{I}_1(w)$ and...and $a_n = \mathbf{I}_n(w)\}$. Now, the set $Int(Q_\phi, w)$ provides enough information for us to tell whether a given tuple $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle$ belongs to $|\phi|^M$. Namely, for all $\mathbf{J}_1, \ldots, \mathbf{J}_n \in \mathrm{WL}(M)$, we have:

$$\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M \text{ iff } \langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w).$$

For the direction from left to right, note that if $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M$, then each $\mathbf{J}_i$ is, indeed, realized in w. This is because ϕ is existence-entailing. It follows by the definition of $Int(Q_\phi, w)$ that $\langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w)$. Conversely, if $\mathbf{J}_1, \ldots, \mathbf{J}_n$ are world lines such that $\langle \mathbf{J}_1(w), \ldots, \mathbf{J}_n(w)\rangle \in Int(Q_\phi, w)$, there are, by the definition of the set $Int(Q_\phi, w)$, world lines $\mathbf{I}_1, \ldots, \mathbf{I}_n$ such that $\mathbf{J}_j(w) = \mathbf{I}_j(w)$ for all $1 \leq j \leq n$ and $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n\rangle \in |\phi|^M$. Because ϕ is *pro mundo*, it follows that $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n\rangle \in |\phi|^M$.

In metaphysical literature, especially in connection with four-dimensionalism, a distinction between sortal and non-sortal predicates is made.[14] Among predicates of the former variety, there are '— is a soccer ball', '— is an apple', and '— is a dog', while '— is spherical', '— is red', and '— barks' are predicates of the latter kind. Four-dimensionalism is a metaphysical view according to which individuals are not wholly present at any moment at which they exist. They have temporal parts, and those parts are what we encounter at specific instants. These temporal parts are not objects of a terribly extraordinary variety. As Wasserman [123] puts it, if you want to know what a temporal part looks like, just look in the mirror: there, one sees one's

[14] For sortals, see, e.g., Grandy [35], Lowe [81], Wiggins [126].

current temporal part. World lines give rise to a view of individuals generalizing four-dimensionalism: the four-dimensionalist's individuals are world lines defined over a set of instants within one and the same structured world. There are different varieties of four-dimensionalism—perdurantism, stage theory. These views differ in how they construe the semantics of sortal predicates. Perdurantists hold that sortal predicates are applicable to material things themselves, not to their temporal parts. Using the terminology above, this would mean that sortal predicates are not *pro mundo*: they are irreducibly intensional predicates. Stage theorists, again, are happy to let sortal predicates apply to *stages* of material things (their brief temporal parts). A stage theorist would not object to ascribing sortal predicates to realizations of world lines or treating them as quasi-extensional predicates, provided that the contexts over which world lines are defined are of a suitable kind—they should be instantaneous, or in any event, they should not themselves involve change.[15]

My discussion is primarily driven by logical considerations. I wish to remain as neutral as I can in metaphysical matters. Also, I do not aim at an all-englobing logical analysis of modal phenomena. I wish to keep my formalism relatively simple. It will always be possible to extend a well-understood formalism, while a sketchy account of a messy formalism serves no purpose. I deliberately confine attention to extensional predicates at the atomic level. From the perdurantist viewpoint, this decision blocks the possibility of representing ascriptions of sortal predicates. Then again, if '— is a lion' does not apply to a local object, '— is a realization of a lion' will. Alternatively, we can save the spirit of the perdurantist view on sortal predicates by distinguishing two types of extensional predicates: those that apply to local objects unconditionally and those that apply to a local object b in a structured world w at an instant t only on condition that there is an individual $\mathbf{I}$ with $b = \mathbf{I}(w, t)$ and a more or less large temporal interval X with $t \in X$ such that the predicate applies to $\mathbf{I}(w, t')$ for all $t' \in X$. The former predicates correspond to non-sortal predicates, the latter to sortal predicates. Viewed in this way, a sortal predicate is an extensional predicate whose applicability in one context presupposes its applicability to the realizations of a fixed world line over a whole set of contexts.

2.5 Contents

I define a general concept of *content*; such contents may but need not be propositional. Further, using the notion of semantic value defined in the previous section, I specify what it means for a content to (locally or uniformly) *support* a formula.

Definition 2.3 (*Content, situated content, internal modal margin*) Let M be a model. Let $V \subseteq dom(M)$ and $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$. The structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ is an *n-ary content* over M. The set V is its *propositional component*, and the $\mathbf{I}_j$ are its *world line components*. A content is *propositional* if $n = 0$, otherwise it is said

[15] For variants of four-dimensionalism, see, e.g., Lewis [76], Hawley [38, 39], Sider [112].

to have a propositional and a non-propositional aspect. If R is a binary relation on $dom(M)$, $w^* \in dom(M)$, and $V = R(w^*)$, the structure $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ is an *R-situated n-ary content*. The set $V \cap marg(\mathbf{I}_j)$ is the *internal modal margin* of $\mathbf{I}_j$. □

In the definition above, the world w^* may but need not belong to the set V. Further, it is allowed that $V \nsubseteq marg(\mathbf{I}_j)$, and it is likewise allowed that $marg(\mathbf{I}_j) \nsubseteq V$. Note that contents as defined above are indeed *structures* and not sets: distinct orders of the world line components $\mathbf{I}_1, \ldots, \mathbf{I}_n$ give rise to distinct contents. I define the notion of content in this way, because I wish to be able to utilize contents when talking about the evaluation of formulas. If in formulas we use variables indexed by positive integers, it will be understood that the variable with the index i takes as its value the i-th world line in the list $\mathbf{I}_1, \ldots, \mathbf{I}_n$.

Let $Cont_n[M]$ be the set of all n-ary contents over M. The semantic value of a formula $\phi(x_1, \ldots, x_n)$ in M gives rise to a subset $Cont(\phi, M)$ of $Cont_n[M]$ as follows.

Definition 2.4 (*Contents generated by a formula*) Let M be a model. The set $Cont(\phi, M)$ of *contents generated by* $\phi(x_1, \ldots, x_n)$ *in* M is the smallest subset of $Cont_n[M]$ satisfying the following condition: if V is non-empty and $\langle w, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$ for all $w \in V$, then $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

If $\mathbf{J}_1, \ldots, \mathbf{J}_n$ are world lines over M, write $W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n}$ for the set of worlds w such that $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in |\phi|^M$. We may note that $Cont(\phi, M)$ is the set of all tuples $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle$ such that $\mathbf{I}_1, \ldots, \mathbf{I}_n \in \mathrm{WL}(M)$ and V is a non-empty subset of $W^{\phi}_{\mathbf{I}_1 \ldots \mathbf{I}_n}$. In particular, $\langle \{w\}, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in Cont(\phi, M)$ whenever $\langle w, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle \in |\phi|^M$. Further, $\langle W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n}, \mathbf{J}_1, \ldots, \mathbf{J}_n \rangle$ belongs to $Cont(\phi, M)$ if $W^{\phi}_{\mathbf{J}_1 \ldots \mathbf{J}_n} \neq \varnothing$. The sets $Cont(\phi, M)$ and $|\phi|^M$ provide two ways of encoding the same information—the former way being more complex than the latter. The set $Cont(\phi, M)$ is empty iff ϕ is contradictory. Whenever $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$, the set V is non-empty.

I will use situated contents to model agents' intentional states. The following notions facilitate discussing how formulas may be used for describing such states.

Definition 2.5 (*Formulas supported by a content*) Let $C = \langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ be a situated n-ary content over M, and let $\phi(x_1, \ldots, x_n)$ be an L_0-formula. *C locally supports* ϕ (in symbols $C \Vdash_{loc} \phi$) if $\langle w^*, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in |\phi|^M$. It *uniformly supports* ϕ (in symbols $C \Vdash_{uni} \phi$) if $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$. □

The following fact is a straightforward consequence of Definition 2.5.

Fact 2.2 *If $\phi(x_1, \ldots, x_n)$ is an L_0-formula and $C = \langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, w^* \rangle$ is an R-situated n-ary content over M, the following four conditions are pairwise equivalent:*

(a) $C \Vdash_{uni} \phi$
(b) $C \Vdash_{loc} \Box\phi$
(c) $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n, v \rangle \Vdash_{loc} \phi$ *for all* $v \in V$

(d) $V \subseteq W^{\phi}_{\mathbf{I}_1 \ldots \mathbf{I}_n}$.

Proof Note that $V = R(w^*)$. Let $g(x_j) = \mathbf{I}_j$ for all $1 \leq j \leq n$. Now, (a) holds iff $\big(M, v, g \models \phi$ for all $v \in V\big)$ iff $\big(M, v, g \models \phi$ for all $v \in R(w^*)\big)$ iff $M, w^*, g \models \Box\phi$ iff (b) holds. Further, (c) is a roundabout way of expressing the condition (b), and (d) means that $\langle V, \mathbf{I}_1, \ldots, \mathbf{I}_n \rangle \in Cont(\phi, M)$—that is, it means that (a) holds. □

2.6 Systems of Modal Unities

In the semantics of the language L_0, values of variables are partial functions mapping worlds to local objects. This is a way of representing world lines for semantic purposes. Individuals are a paradigmatic case of values of variables: I take them to be world lines semantically represented by such partial functions. Whether we adopt a realist or idealist viewpoint on individuals, we will certainly not wish to say that they *are* functions. They are world lines. Unlike functions, world lines are not abstract entities. Yet, for every world line, there is a unique partial function that serves to model the world line.[16] That non-abstract objects can be correlated with abstract ones is hardly controversial. The form of my desk can be represented as a subset of $\mathbb{R}^3$, even if my desk is definitely not a set of triples of reals. The whereabouts of a person over a timespan can be represented by a function from instants to points in space, even if the subject of change is not a function but a person. In fact, as soon as a feature of one type depends on how a feature of another type is, this dependence relation induces a function in the mathematical sense. Not every relation of functional dependence among features corresponds to a non-abstract entity, but some do. In the case of world lines, the relevant dependence relation is between worlds and local objects, and the induced function indeed corresponds to a non-abstract entity—namely, a world line whose realizations those various local objects are.

The specific character of the position I am putting forward consists of claiming that in modal settings, individuals and worlds are related in a certain way: individuals are world lines. Somewhat surprisingly, perhaps, my framework can be reformulated in a way that allows viewing worlds and individuals as objects of the *same* general type—as interrelated objects that live in distinct dimensions, so to say. As hinted at in Sect. 1.3, they can both be modeled as *sets of local objects*; cf. Tulenheimo [120]. Individuals are extended in the 'dimension of worlds': one and the same set corresponding to an individual can intersect with several sets, each of which corresponds to a world. Worlds are extended in the 'dimension of individuals': one and the same set corresponding to a world can intersect with several sets, each of which corresponds to an individual. Both worlds and individuals are *modal unities*. Contents in the sense of Definition 2.3 emerge as *systems of modal unities*. Let us formulate these ideas more precisely.

[16] As already noted in footnote 15 in Sect. 1.3, this means that I take world lines to be related to partial functions in the same way as variable embodiments are related to principles of variable embodiment in Fine's metaphysics (see Sect. 2.7.3).

Let **A** be a non-empty set. Its elements are referred to as local objects. These are things for which the notion of local identity is, by definition, utterly simple and unproblematic and for which the question of cross-world identity cannot be posed. Elements of **A** are purely 'extensional objects'. There are different ways in which they can be grouped together. Any non-empty subset of **A** is a *modal unity*. Worlds and individuals are naturally modeled as covers of the set **A**.[17] Indeed, let **B** and **C** be covers of **A**. We declare that elements of **B** are *worlds* and that those of **C** are *individuals*. Consequently, both **B** and **C** are sets of modal unities—i.e., sets of sets of elements of **A**. The very idea that the elements of **A** are *local* objects entails that **B** must be a partition (and not an arbitrary cover) of **A**: distinct elements of **B** cannot have common elements. Unless we wish to enter into a discussion on the material constitution of physical objects, we may assume that an element of **A** cannot be shared by several individuals, either—i.e., we may also take **C** to be a partition of **A**. Regarding the interrelations of **B** and **C**, the following is required:[18]

$$\text{if } \mathbf{b} \in \mathbf{B} \text{ and } \mathbf{c} \in \mathbf{C}, \text{ then } |\mathbf{b} \cap \mathbf{c}| \leq 1.$$

In other words, either a world and an individual do not intersect at all or their intersection consists of a single element. If $\mathbf{b} \cap \mathbf{c} = \{\mathbf{a}\}$, then **a** is said to be the *realization* of **c** in **b**. If, again, the set $\mathbf{b} \cap \mathbf{c}$ is empty, we say that **c** is *not realized* in **b**.[19] The two partitions **B** and **C** give rise to two 'dimensions': a fixed individual (element of **C**) can be realized in several worlds (elements of **B**), and a fixed world (element of **B**) can serve to realize several individuals (elements of **C**). In order to obtain an alternative description of the notion of a model of vocabulary τ as defined in Sect. 2.3, the structural information provided by sets **A**, **B**, and **C** must be complemented by a binary relation R on set **B** and an interpretation function *Int* assigning to every n, every n-ary predicate Q, and every $\mathbf{b} \in \mathbf{B}$ a subset $Int(Q, \mathbf{b})$ of the set $\mathbf{b}^n$. Elements of $Int(Q, \mathbf{b})$ are n-tuples of elements of **A**—more specifically, n-tuples of elements of **b**. The quintuple $\langle \mathbf{A}, \mathbf{B}, \mathbf{C}, R, Int \rangle$ is a *system of modal unities*. Systems of modal unities give us a symmetric notion of content: if $\mathcal{S} = \langle \mathbf{A}, \mathbf{B}, \mathbf{C}, R, Int \rangle$ is a system of modal unities, $\mathbf{B}_0 \subseteq \mathbf{B}$, and $\mathbf{C}_0 \subseteq \mathbf{C}$, then the pair $\langle \mathbf{B}_0, \mathbf{C}_0 \rangle$ is a *content* over $\mathcal{S}$.

Systems of modal unities defined as above correspond to models $\langle W, R, \mathcal{I}, Int \rangle$ in the special case that all world lines are available in all worlds ($\mathcal{I}_w = \mathcal{I}_{w'}$ for all $w, w' \in W$) and no two world lines overlap (for all $w, w', u \in W$ and all $\mathbf{I} \in \mathcal{I}_w$, $\mathbf{I}' \in \mathcal{I}_{w'}$, if $\mathbf{I}(v) = \mathbf{I}'(v)$ for some $v \in W$, then *either* neither **I** nor **I**′ is realized in u *or else* $\mathbf{I}(u) = \mathbf{I}'(u)$). Systems of modal unities can, however, be defined more generally as structures $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$, where $\mathfrak{C} = \{\mathbf{C_b} : \mathbf{b} \in \mathbf{B}\}$ is a collection of subcovers of **A** indexed by elements of **B**, satisfying $|\mathbf{b} \cap \mathbf{c}| \leq 1$ for all $\mathbf{b}, \mathbf{b}' \in \mathbf{B}$

[17] Let X be a set, κ a cardinal number, and $\mathcal{C} = \{C_i : i < \kappa\}$ a collection of non-empty, not necessarily pairwise disjoint subsets of X. If $X = \bigcup_{i<\kappa} C_i$ and $X \subseteq Y$, then $\mathcal{C}$ is a *cover* of X and a *subcover* of Y. If $\mathcal{C}$ is a cover of X and the elements of the collection $\mathcal{C}$ are pairwise disjoint, then $\mathcal{C}$ is a *partition* of X and a *subpartition* of Y.

[18] If S is a set, I write $|S|$ for its cardinality.

[19] We could opt for a symmetric concept of **b** and **c** being 'co-realized'. However, I prefer to view **B** as providing the contexts in which elements of **C** may or may not be realized.

and $\mathbf{c} \in \mathbf{C}_{\mathbf{b}'}$.[20] In the general setting, contents must be relativized to elements of $\mathbf{B}$, a content over $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$ relative to $\mathbf{b}$ being a pair $\langle \mathbf{B}_0, \mathbf{C}_0 \rangle$, where $\mathbf{B}_0 \subseteq \mathbf{B}$ and $\mathbf{C}_0 \subseteq \mathbf{C}_{\mathbf{b}}$.

The transcendental interpretation of world lines poses the availability of individuals in the sense of modal unities as a necessary condition of our talking and thinking about individuals. The modal unities we call 'worlds' provide a medium relative to which we may consider the modal unities we call 'individuals'. One way of viewing the proposal the transcendental interpretation puts forward is along the lines of Kant's transcendental idealism. It was noted in Sect. 1.5 that taking our modal thoughts to be structured in terms of world lines can be compared with Kant's view, according to which appearances are structured in terms of pure intuitions of space and time. However, in order to motivate an idealist construal of the transcendental interpretation, we should find—over and above such a formal analogy—a role for world lines in relation to the cognitive operations that can be considered as rendering the experience of objects possible. In Kant's framework, there is actually a candidate that fits the bill in connection with sense experience: synthetic unities resulting from the combination of a manifold of intuition. According to Kant, sense experience gives us a plurality of representations. By themselves, such manifolds do not give rise to an experience of objects; Kant maintains that the concept of object is not derived from experience. At the same time, the concept of object cannot have the status of a category (a pure concept of the understanding): categories are only applicable to an experience structured in terms of objects. It is the cognitive operation of synthesis that puts different representations together and comprehends their manifoldness in one cognition [B 103]. What results from a synthesis is a 'synthetic unity', and it is such a synthetic unity that enables us to say that we have an experience of an object [A105, B130]. As examples of synthetic unities, Kant mentions a house viewed from different angles—the process of viewing takes time but gives rise to a spatial unity [A190/B235]—and a line in space that is cognized by drawing it, thereby synthetically bringing about a determinate combination of a given manifold [B138].

If anyone wanted to formally develop what Kant calls transcendental logic—proceeding from the manifold of sensibility and moving toward the applicability of the pure concepts of the understanding [A77/B102]—then the way in which *objects* should be represented in such a logic would be precisely in terms of world lines defined over the relevant manifolds. They constitute unities of the appropriate type

[20] We could go much further in generalizing the notion of system of modal unities, but the type of language to be used for talking about such systems imposes limits to what are reasonable generalizations. Modal operators do not carry syntactic variables, and they are evaluated in terms of binary relations among worlds, which is why we cannot end up evaluating a formula relative to n worlds for $n \geq 2$. Neither is the set of worlds accessible at a given world dependent on values of first-order variables. Thus, it would be pointless to replace $\mathbf{B}$ by a collection of subcovers of $\mathbf{A}$ indexed by elements of $\mathbf{C}$ or to replace the relation R (or, more generally, a collection of binary relations) by a collection of relations on $\mathbf{B}$ with arbitrary arities. When predicate symbols are interpreted extensionally, it suffices to define the function *Int* as above. Otherwise, its values should be defined as sets of tuples of elements of the set $\bigcup_{\mathbf{b} \in \mathbf{B}} \mathbf{C}_{\mathbf{b}}$.

in as straightforward a sense as one can hope for. This does not mean that Kant equates objects with unities of representations; it only means that this is how he takes objects to emerge in our experience (cf. Gardner [32, p. 101]). According to him, the manifold of an intuition is united in the concept of an object [B137], and it is only in terms of the concept of an object that experience of objects is possible.

2.7 Relation to Other Views

As indicated in Sect. 2.2, in order to defend the position according to which individuals are world lines, we must compare it with widely discussed ways of understanding individuals in modal settings. I will briefly comment on Kripke's view on the one hand and Lewis's view on the other hand. Further, I relate the transcendental interpretation of world lines to Fine's metaphysical theory of variable embodiments. Finally, I discuss different notions taken up in the literature that superficially resemble the concept of world line but should not be confounded with it.

2.7.1 Kripke's Stipulative Account

According to Kripke [69, 71], for any actual individual, we can envisage different scenarios in which this very individual appears—by *stipulating* that we are speaking of what might have happened to it. Kripke takes it that once we fix attention to an individual in this world, nothing prevents us from asking how *it* would behave in counterfactual situations. This is how he puts it [71, pp. 52–3]:

> I have the table in my hands, I can point to it, and when I ask whether *it* might have been in another room, I am talking, by definition, about *it*. . . . If I am talking about it, I am talking about *it*, in the same way as when I say that our hands might have been painted green, I have stipulated that I am talking about greenness.

Kripke contrasts his view with the idea that we may only speak of individuals as inhabitants of several worlds in terms of some sort of criteria of transworld identity—means of recognizing the same individual in different circumstances. He takes such an opposing view to suggest that counterfactual scenarios can only be considered purely qualitatively, so that speaking of an actual individual in a counterfactual situation w would require possessing means to locate it among the inhabitants of w on the basis of its properties. In Kripke's view, we talk about the table *directly*. When we reflect on the possibility for the table to be in another room, properties of the table need not be used for identifying it in counterfactual situations, nor need they be used to identify it in the actual world.

If information about the local features of a world is systematically insufficient for establishing those cross-world links that allow us to speak of an individual relative to a number of worlds (indeed, if world lines are independent of worlds), then the seemingly innocent idea of fixing attention to a locally manifested individual in a world and considering *it* elsewhere conceals the fact that such a stipulation has a conceptual precondition that is not met merely by having fixed attention to a local inhabitant of a world: a suitable world line must be given. Assuming the perspective on identity in modal settings formulated in this book, Kripke's account, with its recourse to the counterfactual behavior of stipulatively fixed individuals, presupposes what it is meant to clarify. As Hintikka stresses on many occasions, the conceptual confusion is deepened by phrasing the idea in linguistic terms, with reference to rigid designation (cf. [49, pp. 27–8]).

The very idea of rigid designation—the idea that such expressions as proper names refer to the same object in all worlds in which the object exists, unmediated by a sense representing properties of the object—requires that the referent be a sort of entity that can be found in several worlds.[21] As Hintikka points out, exponents of the idea of rigid designation appear to mix questions of reference with questions of identity (cf. [49, pp. 24–34], [51, 52]). The problem with the idea of rigid designation is not the suggestion that proper names refer directly without ascribing properties to their bearers. The problem is that the very notion of referring to the *same object* in distinct worlds presupposes the notion of cross-world identity. Yet, Kripke wishes to use the notion of rigid designation to clarify the meaning of statements involving individuals that appear in several possible worlds. That is, his strategy is question-begging: he attempts to employ a notion that presupposes the possibility of speaking of cross-world sameness to clarify the meaning of statements about cross-world sameness. The notion of reference is linguistic and world-relative—it is about interpreting non-logical symbols world by world. The notion of cross-world identity, again, does not presuppose recourse to language, and it is about world lines determining which objects in which domains realize the same individual. Correlating objects of distinct worlds is not a matter of reference—not a linguistic matter to begin with.

[21] In any event, this is the most natural way of understanding what Kripke says. Admittedly, if the referents of rigid designators were world-bound objects, there would by hypothesis be no issue of cross-world identity concerning them. Since Kripke rejects the domain constraint, he could speak of ascribing predicates to a world-bound object of world w relative to a distinct world w'. However, in fact, Kripke does not assume that referents of rigid designators are world-bound but allows them to be objects that exist in several worlds. Independently of this interpretive issue, we may note, as Kaplan [61, pp. 492–3] does, that Kripke characterizes his notion of rigid designator in mutually incoherent ways. At times, he says that a rigid designator refers to the same object in *all* worlds [71, p. 48]. At other times, he takes a rigid designator to refer to the same object in all *those* worlds in which the object exists ([68, p. 146], [71, p. 49]). The two formulations are equally problematic from the viewpoint of cross-world identity adopted in this book. Kripke's notion of rigid designator was anticipated in the work of Marcus [82]. She speaks of proper names as 'identifying tags' whose descriptive meaning is lost or ignored.

The notion of cross-world identity must, then, be secured before we can even attempt to define the notion of rigid designation.[22] This fact has repercussions on the possibility of the substitutional interpretation of quantifiers in modal logic (cf. Kripke [70]). One cannot explicate the semantics of a formula like $\exists x \Box P(x)$ by saying that its truth in w amounts to there being a rigid designator n such that $P(n)$ is true in all worlds accessible from w. No singular term can *be* a rigid designator unless it makes sense to speak of its referent in several worlds. The semantics of $\exists x \Box P(x)$ must clarify what renders it meaningful to speak of cross-world identity; this cannot be accomplished by resorting to conceptualizations that simply presuppose the meaningfulness of such a discourse. Therefore, quantification into modal contexts cannot be accounted for in terms of rigid designators.[23] Further, it would not help to turn attention to local objects. If quantification is understood objectually, the truth of $\exists x \Box P(x)$ in w does not amount to there being an object b of the domain of w such that this same object b, when considered in any world v accessible from w, is P in v. According to the analysis I am propagating, such transportation of objects to other worlds is impossible, and quantification into modal contexts must rely on world lines. In sum, the question of whether the 'horizontal' requirement of cross-world identity is satisfied cannot be approached in terms of the 'vertical' question of world-relative reference, nor by simply fixing attention to a local object.

The criticism levelled against Kripke's view on the above grounds is appropriate in the context of the transcendental interpretation of world lines. By contrast, the epistemic interpretation is indeed vulnerable to Kripke's critique against the qualitative view of individuals in modal settings. I have argued in Sect. 1.6 that Hintikka confuses the conceptual issue of individuation with the epistemic issue of reidentification and ends up speaking as if the epistemic capacity of recognition itself were a transcendental precondition of modal talk. This leaves the problem of cross-world identity unanswered or amounts to declaring that there is no such problem. Due to the ambiguity of Hintikka's motivations, it is understandable that his case against Kripke's position has not been perceived as being particularly strong.

Formally, Kripke's semantics of quantified modal logic can be seen as the result of denying the distinction between local objects and world lines and giving up the domain constraint adopted in world line semantics. Here is how Kripke's semantic framework can be obtained by modifying systems of modal unities as defined in Sect. 2.6. Given a set $\mathbf{A}$ of objects, let $\mathbf{B}$ be a cover of $\mathbf{A}$. Elements of $\mathbf{A}$ will be individuals in Kripke's sense, while elements of $\mathbf{B}$ will be worlds. It will precisely

[22] As was explained above, already the language-independent idea of considering how *this* object behaves in counterfactual circumstances presupposes the notion of cross-world identity. A fortiori, then, this same presupposition is involved in the idea of taking the actual referent of a linguistic expression as one's starting point and considering how this referent behaves in counterfactual circumstances. The linguistic detour cannot remove the heart of the problem, though it can serve to hide it. In particular, the identity of a proper name does not translate into the identity of its actual referent: it may be unproblematic to say that two occurrences of 'Hesperus' are two occurrences of the same name, but this linguistic fact has no bearing on the issue of whether it makes sense to say that 'Hesperus' refers to one and the same non-linguistic entity on the two occasions.

[23] For a critique of the substitutional interpretation applied in modal logic, see [49, p. 28], [51].

not be assumed that $\mathbf{B}$ is a *partition* of $\mathbf{A}$: distinct worlds may have elements of $\mathbf{A}$ in common. (Consequently, elements of $\mathbf{A}$ are not substantially speaking *local* objects.) Further, if $\mathbf{b} \in \mathbf{B}$, let $\mathbf{C_b}$ consist of singletons of elements of $\mathbf{b}$. That is, $\mathbf{C_b} = \{\mathbf{c} :$ there is $\mathbf{a} \in \mathbf{b}$ such that $\mathbf{c} = \{\mathbf{a}\}\}$. Consequently, each $\mathbf{C_b}$ is a very particular sort of subpartition of $\mathbf{A}$—namely, a subpartition whose cells are the singleton sets of elements of $\mathbf{b}$. We may consider elements $\{\mathbf{a}\}$ of $\mathbf{C_b}$ and elements $\mathbf{a}$ of $\mathbf{b}$ as two equivalent ways of representing Kripkean individuals existing in the world $\mathbf{b}$. Like in world line semantics, also here every individual has at most one realization in a given world: if $\mathbf{b}, \mathbf{b}' \in \mathbf{B}$ and $\mathbf{c} \in \mathbf{C_{b'}}$, then $|\mathbf{b} \cap \mathbf{c}| \leq 1$. Indeed, $|\mathbf{b} \cap \mathbf{c}| = 1$ iff $\mathbf{a} \in \mathbf{b}$, where $\mathbf{a}$ is the unique element of the set $\mathbf{c}$. In particular, every individual belonging to the domain $\mathbf{C_b}$ of the world $\mathbf{b}$ is realized in the world $\mathbf{b}$: if $\mathbf{c} \in \mathbf{C_b}$, then $\mathbf{c} = \{\mathbf{a}\}$ for some $\mathbf{a} \in \mathbf{b}$, and therefore, $|\mathbf{b} \cap \mathbf{c}| = 1$. On the other hand, in Kripke's framework, the same object $\mathbf{a} \in \mathbf{A}$ can be the realization of an individual in several worlds: if $\mathbf{a} \in \mathbf{b} \cap \mathbf{b}'$, then $\{\mathbf{a}\} \in \mathbf{C_b} \cap \mathbf{C_{b'}}$, and indeed, $\mathbf{a}$ is the realization $\{\mathbf{a}\}$ in both worlds $\mathbf{b}$ and $\mathbf{b}'$. Predicate symbols are interpreted relative to worlds, but it is not required that the interpretation $Int(Q, \mathbf{b})$ of an n-ary predicate Q in $\mathbf{b}$ be a subset of $\mathbf{b}^n$. It is merely required that $Int(Q, \mathbf{b})$ be a subset of $\mathbf{A}^n$. Thereby, the domain constraint is given up. Kripke's semantics can be obtained as a variant of world line semantics based on systems of modal unities $\langle \mathbf{A}, \mathbf{B}, \mathfrak{C}, R, Int \rangle$ whose components are specified as above, with $\mathfrak{C} = \{\mathbf{C_b} : \mathbf{b} \in \mathbf{B}\}$. By contrast, world line semantics cannot be obtained as a variant of Kripke's semantics, since the latter does not make the requisite distinction between extensional identity and cross-world identity and correspondingly fails to distinguish between local objects and world lines. In particular, Kripke's framework cannot simulate the distinction between availability and realization, essential for my analysis of intentional objects understood as intentionally individuated world lines that can lie in the range of intentional quantifiers in a world without being realized in that world.[24]

2.7.2 Lewis on Counterparts and Humean Supervenience

In this book, I do not explore ways of construing the notion of world line epistemically: my focus is neither on the epistemic nor on the anti-realist interpretation of world lines. Still, on the face of it, at least, thinking of individuals as world lines *need* not be motivated by the transcendental interpretation. It was already hinted at in Sect. 2.4 that nothing prevents us from considering world lines *metaphysically*. Thus understood, they give rise to a view of individuals generalizing four-dimensionalism.

Lewis [76] is a four-dimensionalist who takes the basic entities to be temporally extended objects with temporal parts. In his analysis, the quantifiers 'something' and 'everything' range over world-bound cross-temporal hybrids whose identity over time is taken to be utterly simple and unproblematic. Concerning the similarities between Lewis's view and the approach I adopt, it was already noted in Sect. 1.3

[24] Cf. the discussion in Sect. 1.3. For details, see Chap. 3.

that *local objects* in my sense are world-bound, like Lewis's *individuals*. If attention is confined to a single temporally extended world—so that the 'contexts' to be considered are pairs of structured worlds and times—*individuals* in my sense are four-dimensional, like *individuals* in Lewis's sense, and *local objects* are bound not only to a structured world but also to a time, like *temporal parts* of individuals are according to Lewis.

Lewis's account of statements pertaining to temporally extended individuals within one and the same world is categorically different from his analysis of statements that seem to be about one and the same individual in distinct possible worlds. He has one thing to say about cross-temporal identity and another story to tell about seeming cases of cross-world identity. He takes the former relation to be utterly simple and unproblematic but denies the reality of the latter. To compensate for the denial, Lewis develops his modal counterpart theory, which is supposed to provide an analysis of representation *de re*—an account of how the modal properties of a temporally extended individual of one world can be represented in terms of other worlds [75, 76]. Lewis's individuals remain within one world and may merely have *counterparts* in other worlds. According to my view, individuals have a modal margin, comprising not only various spatiotemporal locations within a single structured world but also locations in distinct structured worlds. I take cross-context identity to be a notion in need of analysis, and the analysis I give is uniform: modal behavior is generally inbuilt within the individual itself, not just its temporal behavior. Cross-temporal identity within a fixed structured world is no less problematic than identity across structured worlds. The preservation of identity over any sorts of contexts is uniformly analyzed in terms of world lines.[25]

Kripke [71, p. 45] criticizes Lewis's counterpart theory, according to which the sentence 'Humphrey might have won the election' is true in the actual world if in a counterfactual world a certain numerically distinct individual—namely, a counterpart of Humphrey—indeed wins the election. Kripke finds this a doubtful analysis, as according to him, Humphrey presumably could not care less that another person would have been victorious in another possible world. Kripke takes it that the qualitative similarity between Humphrey and his counterpart would not render the success of this counterpart any more interesting from Humphrey's viewpoint. This type of critique is not available against world line semantics. In my analysis, the sentence 'Humphrey might have won the election' is about a certain (physically individuated) world line—Humphrey the physical individual. Writing $\mathbf{J}$ for Humphrey, the sentence means that $\mathbf{J}$ is realized not only in the actual world w_0 but in another world w, as well, and the realization $\mathbf{J}(w)$ of $\mathbf{J}$ satisfies the predicate '— wins the election'. Winning the election is a contingent property of Humphrey himself, a property *he* has in some but not all worlds. Humphrey (the world line $\mathbf{J}$) must absolutely not be

[25] Hawley [39] calls attention to the fact that in Lewis's analysis, the analog between time and modality is not complete (ordinary objects are world-bound but not time-bound) and discusses the possibility of achieving uniformity by explicating not only sameness across worlds but also sameness over time in counterpart-theoretic terms. In Sider's stage theory [111], statements about identity over time are indeed accounted for in terms of temporal counterpart relations.

confused with this or that realization of the world line **J**—in particular, not with the realization $\mathbf{J}(w_0)$.[26]

Lewis [77, 78] defends the thesis of Humean supervenience. One way of attempting to concisely formulate this thesis would be as follows: all facts, other than facts about spatiotemporal distance, supervene upon local matters of particular fact.[27] This thesis leads to what Lewis calls anti-haecceitism [76, p. 221]: counterpart relations between denizens of distinct worlds are supervenient on features internal to worlds.[28] Indeed, any world-bound individuals that qualitatively resemble one another are thereby counterparts of one another. *All facts* include modal facts—in particular, facts about the modal behavior of actual individuals. Now, these modal facts rely on representations *de re* articulated in terms of counterpart relations. If these facts also supervene on local matters of fact (i.e., matters of fact concerning each world taken by itself), it follows that counterpart relations among worlds are determined as soon as a set of worlds and the geometrical arrangement of their space-time points are fixed: counterpart relations supervene on local features of the worlds. Were this not so, modal facts could vary while the distribution of qualitative properties and relations within worlds remains the same, which would contradict anti-haecceitism. Lewis says that he intends the thesis of Humean supervenience to be *contingent* [78, p. 474]. (For a discussion, see Hall [37].) Indeed, if in some worlds, though perhaps not in the actual one, we must take into account irreducible *non-spatiotemporal relations* as constitutive of the structure of the world, then Humean supervenience, as formulated above, can at best be true of worlds in which, by chance, such relations are not instantiated.

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[26]For contingently satisfied predicates, see Sect. 4.2. The fact that Kripke's Humphrey-objection is not applicable against my analysis does not depend on how we choose to deal with proper names in our semantics. In this book, I opt for construing proper names as standing for local objects relative to a world: for every world w in the modal margin of **J**, the interpretation of 'Humphrey' in w equals $\mathbf{J}(w)$. Another option would be to construe proper names intensionally, letting 'Humphrey' to stand for the world line **J**. For a discussion, see Sect. 3.4 and footnote 29 in Sect. 6.6.

[27]See Bricker [7, p. 287]. Lewis considered different versions of the thesis. Arguably, his final formulation [78] does not even count as a supervenience thesis (cf. Weatherson [124]).

[28]Lewis defines *haecceitism* as the doctrine that there are at least some cases of 'haecceitistic difference' between worlds: there are worlds that do not differ qualitatively in any way but differ in what they 'represent *de re*' concerning some individual. According to *anti-haecceitism*, there are no cases of haecceitistic difference. Haecceitism is, for instance, compatible with there being the worlds w_0, w_1, and w_2 satisfying the following conditions: (1) each world w_i has exactly two inhabitants (a_i and b_i); (2) individual a_1 is P but b_1 is not P, while a_2 is not P though b_2 indeed is P; and (3) both individuals a_1 and a_2 are counterparts of a_0. The worlds w_1 and w_2 are qualitatively exactly alike, but w_1 represents *de re* concerning a_0 that it is P, while w_2 represents *de re* concerning a_0 that it is not P. Lewis [76, p. 225] maintains that a haecceitist need not accept that an individual is distinguished from all other individuals by a *haecceity*—an unanalyzable non-qualitative property that this individual has and all other individuals lack. In the example, haecceitism without haecceities would mean that not only is the fact that a_1 and a_2 are counterparts of a_0 not triggered by qualitative properties of the three worlds w_0, w_1, and w_2, but this is a primitive fact not determined by any property at all that would belong to the individuals a_0, a_1, and a_2. It must be noted that Lewis's characterization of haecceitism is not neutral but depends on the idea of *de re* representation conceptualized in terms of counterparts of world-bound individuals.

The position I defend is utterly antithetical to Humean supervenience—and, more generally, to any view entailing that all facts about a structured world supervene on properties of world-bound objects or on relations among world-bound objects within a fixed world, no matter what the specific nature of those relations is. There are two conceptually independent factors to consider: worlds and world lines. Precisely because these factors are mutually independent, it will not be generally possible to provide an account of what unifies world lines by making use of world-internal features—just like it will not be generally possible to analyze in terms of features of world lines what unifies worlds.[29] My view does not exclude the possibility that world lines indeed supervene on local properties: this is what would happen if the remarkable metaphysical coincidence referred to in Sect. 1.3 was actualized and, as a matter of contingent fact, world lines were not independent from worlds. Assuming Humean supervenience as a *fait accompli* would in any event be theoretically harmful. It would lead attention away from the fact that worlds and world lines are conceptually independent of each other.

Lewis's counterpart theory can be seen as the result of supposing that world lines are in fact generated by world-internal characteristics—by qualitative resemblance of world-bound objects. Formally, here is how Lewis's semantic framework can be simulated in the context of world line semantics.[30] Consider a system of modal unities $\langle \mathbf{A}, \mathbf{B}, \{\mathbf{C_b} : \mathbf{b} \in \mathbf{B}\}, R, Int\rangle$, where $\mathbf{B}$ is a partition of $\mathbf{A}$. While this assumption has merely interpretative but not formal consequences, let us suppose that it so happens that the sets $\mathbf{c} \in \mathbf{C_b}$ are supervenient on elements of $\mathbf{B}$. We suppose that distinct local objects $\mathbf{a}, \mathbf{a}' \in \mathbf{c}$ resemble each other qualitatively; world-internal qualitative features of local objects determine the elements of the sets $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$. It will be further assumed that every local object $\mathbf{a}$ of world $\mathbf{b}$ is the realization of some world line $\mathbf{c} \in \mathbf{C_b}$ available in $\mathbf{b}$, so that $\mathbf{c} \cap \mathbf{b} = \{\mathbf{a}\}$. Let us use the sets $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$ for defining a relation CP on the set $\mathbf{A}$ as follows: if $\mathbf{a} \in \mathbf{b}$ and $\mathbf{a}' \in \mathbf{b}'$, where $\mathbf{b}$ and $\mathbf{b}'$ may but need not be distinct worlds, let $CP(\mathbf{a}, \mathbf{a}')$ iff there is $\mathbf{c} \in \mathbf{C_b}$ such that

[29] World lines should not be viewed as generated by any sorts of properties—in particular, not by anything qualifiable as 'haecceities'; cf. the comments on essences in Sect. 2.7.4. My position is certainly closer in spirit to haecceitism without haecceities than to anti-haecceitism—although literally, my position is anti-haecceitist by Lewis's criteria, since I maintain that there are no cases of haecceitistic difference between worlds, for the simple reason that I maintain that there are no Lewisian *de re* representations in the first place. World lines are not supervenient on world-internal facts, so there could be a world line $\mathbf{I}$, worlds w_1 and w_2, and local objects $a_1 \in dom(w_1)$ and $a_2 \in dom(w_2)$ such that as to their internal qualitative features, w_1 and w_2 are exactly alike, the local object a_1 is P while a_2 is not P, and yet a_1 and a_2 could both be realizations of the world line $\mathbf{I}$. (For 'exact likeness', cf. the notion of internal indistinguishability discussed in Sect. 4.5.) Even though my view resembles haecceitism, I do not subscribe to haecceitism in *Lewis's sense*: the world w_2 does not involve a representation *de re* concerning the local object a_1 in virtue of the fact that $a_2 = \mathbf{I}(w_2)$ and $a_1 = \mathbf{I}(w_1)$. Local objects or the worlds in which they are located do not in any sense represent local objects to be found in other worlds. What may but need not happen is that local objects of two worlds are *realizations* of one and the same world line.

[30] In Sect. 5.7, I explain how my modal language L_0 (and its extension L, to be introduced in Sect. 3.4) can be translated into first-order logic. This translation makes it particularly easy to observe the formal similarities between world line semantics and Lewis's counterpart theory—the latter being standardly presented in terms of first-order logic, cf. [75].

$\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$. (Note that if $\mathbf{b} = \mathbf{b}'$, we can have $\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$ only if $\mathbf{a} = \mathbf{a}'$.) The terms of the relation *CP* are, then, local objects belonging to one and the same world line available in $\mathbf{b}$—the same element $\mathbf{c}$ of $\mathbf{C_b}$. The relation *CP* is reflexive: if $\mathbf{a} \in \mathbf{b}$, there is $\mathbf{c} \in \mathbf{C_b}$ such that $\mathbf{c} \cap \mathbf{b} = \{\mathbf{a}\}$, whence we have $CP(\mathbf{a}, \mathbf{a})$. The relation *CP* is not symmetric: having $CP(\mathbf{a}, \mathbf{a}')$ for local objects $\mathbf{a} \in \mathbf{b}$ and $\mathbf{a}' \in \mathbf{b}'$ merely requires that there be $\mathbf{c} \in \mathbf{C_b}$ such that $\mathbf{a} \in \mathbf{c}$ and $\mathbf{a}' \in \mathbf{c}$, but it need not happen that $\mathbf{c}$ is available in $\mathbf{b}'$, as well: we may have $\mathbf{c} \notin \mathbf{C_{b'}}$. The relation *CP* is not transitive, either—again, because distinct sets of world lines may be available in distinct worlds.[31] These observations are enough to show that the world line framework allows accommodating counterpart-theoretic ideas.[32] By contrast, world line semantics cannot be obtained as a variant of Lewis's counterpart semantics, since the latter assumes that counterpart relations are generated by qualitative resemblance among world-bound individuals, while in world line semantics, worlds and world lines are taken to be, generally, mutually independent and cross-world identity is taken to be a notion that defies all attempts of analysis in world-internal terms. In particular, counterpart theory cannot simulate the distinction between availability and realization. In Lewis's account, individual-valued quantifiers evaluated relative to a world w always take as their values world-bound individuals of the world w. Counterparts enter the picture only insofar as modal claims concerning those inhabitants of the world w are being made. By contrast, in my analysis, intentionally individuated world lines can lie in the range of intentional quantifiers in a world w without being realized in the world w.

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[31] Lewis resorts to non-transitive counterpart relations in his reply to Chisholm's identity paradox (see [14], [76, pp. 243–8]). The paradox can be presented as follows. Take two individuals existing in w_0—say, Adam and Noah. Suppose Adam has properties $P_1, \ldots, P_n$ and Noah the properties $Q_1, \ldots, Q_n$ in w_0. (For simplicity, let us suppose that the properties $P_1, \ldots, P_n, Q_1, \ldots, Q_n$ are pairwise independent.) Presumably, at least some of Adam's and Noah's properties could get exchanged. If so, by repeated exchanges, we arrive at a sequence of worlds $w_0, \ldots, w_n$ such that in w_i, Adam has the properties $Q_1, \ldots, Q_i, P_{i+1}, \ldots, P_n$ and Noah the properties $P_1, \ldots, P_i, Q_{i+1}, \ldots, Q_n$ (for all $1 \leq i \leq n$). In w_n, Adam's properties are those of Noah in w_0, and vice versa. Supposing there are no other individuals in w_0 and w_n, these worlds are qualitatively exactly alike, so individuals appear to have a 'bare identity' entirely unrelated to their properties. This suggests that for any property and any individual, there is a world in which the individual has this property. Lewis blocks this reasoning by appealing to the idea of world-bound individuals. Adam and Noah are located in w_0. Even if there were individuals a_1 of w_1 and a_2 of w_2 such that a_1 is a counterpart of Adam and a_2 is a counterpart of a_1—with a_1 having the properties of Adam except for P_1, and a_2 having the properties of Adam save for P_1 and P_2—we cannot infer that a_2 is a counterpart of Adam, since the counterpart relation need not be transitive. From my viewpoint, Adam is a world line. He is realized in w_0. He has a fixed modal margin, which may or may not contain the worlds $w_1, \ldots, w_n$. World lines are not supervenient on qualitative world-internal features. Whatever (extensional) predicates a local object of a world w_i satisfies, it need not be the realization of Adam in w_i. This is why Chisholm's reasoning cannot be carried out when world line semantics is assumed.

[32] Note that Lewis does not preclude the possibility of an individual having several counterparts in a given world. Unless it is required that every $\mathbf{C_b}$ with $\mathbf{b} \in \mathbf{B}$ be a partition of $\mathbf{A}$, we can indeed have $CP(\mathbf{a_0}, \mathbf{a_1})$ and $CP(\mathbf{a_0}, \mathbf{a_2})$ with $\mathbf{a_1} \neq \mathbf{a_2}$, where $\mathbf{a_1}, \mathbf{a_2}$ belong to the same element $\mathbf{b}'$ of $\mathbf{B}$. Namely, there can be $\mathbf{b} \in \mathbf{B}$ with $\mathbf{b} \neq \mathbf{b}'$, and distinct $\mathbf{c_1}$ and $\mathbf{c_2}$ in $\mathbf{C_b}$, such that $\mathbf{a_0} \in \mathbf{c}_1 \cap \mathbf{c}_2 \cap \mathbf{b}$, while $\mathbf{a_1} \in \mathbf{c}_1 \cap \mathbf{b}'$ and $\mathbf{a_2} \in \mathbf{c}_2 \cap \mathbf{b}'$.

2.7.3 Fine's Notion of Variable Embodiment

In Lewis's metaphysics, it is the postulation of Humean supervenience that allows him to avoid recognizing world lines as an independent component of modal reality. Surveying recent metaphysical literature, there is one proposal that is formally analogous to the idea of construing individuals as world lines: Kit Fine's theory of *variable embodiments* [25]. Fine formulates his view in a temporal setting relative to a single world, but he definitely rejects Humean supervenience.

Fine's goal is to sketch a theory of the general nature of material things. He proposes a novel way of thinking about mereology. According to him, there are two operations by means of which wholes can be formed from parts: one operation produces *rigid embodiments* and the other *variable embodiments*. The former operation is supposed to explain the mereological structure of an object at a time. The latter is meant to account for the variation of an object over time. A rigid embodiment is a special sort of hylomorphic entity consisting of a number of objects (its matter) interrelated according to a fixed relation (its form, a principle of rigid embodiment). The matter is given independently of the form. An example is a ham sandwich: a piece of ham and two slices of bread spatiotemporally arranged in a certain way. *Principles of variable embodiment* are of special interest here. Such a principle F is a function mapping times to things, and it determines an object $/F/$, referred to as a *variable embodiment*. Values of F are *manifestations* of $/F/$. Typically, such manifestations are rigid embodiments.[33] The variable embodiment $/F/$ does *not* supervene on its manifestations. In fact, the matter of a variable embodiment (the plurality of its manifestations) is specified by its form (the principle of variable embodiment) instead of being available independently. An example is a car with a varying constitution: at each time, it consists of an engine, chassis, and body related in a certain way, but neither the matter nor the form of one manifestation need be transferred to another manifestation: at different times, the constituents of the car may be differently related, and besides, the constituents may vary. For example, the carburetor of the car at t_1 may be replaced by a new one at t_2.

Fine develops his position as a response to problems he detects in traditional mereology. He maintains that standard mereology is incapable of accounting for the notions of timeless part and temporary part. First, suppose a ham sandwich has two slices of bread (s_1 and s_2) and a piece of ham (h) as its *timeless parts*. Each of the three components is itself a temporally extended object. According to standard mereology, the sum $s_1 + s_2 + h$ exists, at all instants at which at least one of its components exists. It would follow, absurdly, that the sandwich exists as soon as the relevant piece of ham does, even if the slices of bread are yet to come into existence. Second, supposing that objects have time-slices (instantaneous timeless parts), it could be proposed that x is a *temporary part* of y at t if the time-slice x_t of x at t is a timeless part of the time-slice y_t of y at t. It does not help the mereologist to restrict attention to instants

[33] Fine [25, p. 73] allows even variable embodiments as values of F (cf. Koslicki [63, p. 78]).

at which all three parts of the ham sandwich exist simultaneously. Namely, consider the sum $h + c$ of the piece of ham and Cleopatra (c). The sum exists according to the standard mereology, which subscribes to unrestricted mereological composition. Not only is h a temporary part of the sandwich at t—even $h + c$ is one, because $(h+c)_t = h_t$ is a timeless part of the time-slice of the sandwich at t. The conclusion appears absurd: among the temporary parts that the ham sandwich has at t, there is a certain object with Cleopatra as one of its parts. Similar absurdities result if, instead of a temporally relatively stable object, like a ham sandwich, we consider an object with a variable constitution, such as a car with one carburetor at one time and another one at another time.

Formally, variable embodiments are related to their manifestations exactly as world lines are related to their realizations. Principles of variable embodiment are related to variable embodiments as partial functions representing world lines are related to world lines themselves. Like variable embodiments, world lines also have both a formal and a material aspect. The matter of a world line is what gets realized in various worlds, its form being the fact that the relevant local objects jointly constitute a single world line. Unlike a principle F, the corresponding variable embodiment $/F/$ is not an abstract object. It can have properties of the sort concrete objects have. Fine stipulates that the *pro tem* properties of $/F/$ at a time t are those of its manifestation $F(t)$ at t. This sort of 'transfer principle' is likewise built into the semantic clause for atomic formulas of the language L_0: saying that a world line satisfies a predicate P in w means that its realization in w satisfies P in w. Formally, the main difference between variable embodiments and world lines is that the manifestation $F(t)$ of a variable embodiment $/F/$ at t need not be local in any way: typically, it is a rigid embodiment composed of suitably arranged temporally extended objects (and it may even itself be a variable embodiment), whereas realizations of a world line at t are temporally limited to the instant t. I do not enter into systematic mereological discussions in this book. However, my framework could be generalized by allowing local objects to have a mereological structure.[34] Parts of local objects would themselves be local.

Fine's notions of composition are supposed to avoid absurdities of the type described above. The ham sandwich as a rigid embodiment consists of the two slices of bread and a piece of ham arranged in a certain way, and this structured object exists only when s_1, s_2, and h are thus related; it does not suffice that one or more of these objects exist. In his theory of variable embodiment, Fine formulates a somewhat elaborate definition of temporary part (in terms of manifestations and a relation of simultaneity among parts), which allows him to disqualify the combination of Cleopatra and the piece of ham as a temporary part of the ham sandwich, retaining, however, the piece of ham as its temporary part. In my setting, the ham sandwich is a world line $\mathbf{I}$ over an interval of time, and there are three further world lines involved: slices of bread $\mathbf{J}_1$ and $\mathbf{J}_2$ and a piece of ham $\mathbf{J}_3$. If my framework was generalized so as to allow speaking of timeless parts of local objects, it would become possible to

[34]For further remarks in this direction, see Sect. 4.6, esp. footnote 17.

analyze the interrelations of the four world lines: each of the $\mathbf{J}_i$ is realized at every instant t at which $\mathbf{I}$ is realized, the local object $\mathbf{J}_i(t)$ being a timeless part of $\mathbf{I}(t)$. As Fine's analysis, this view does not suggest that the world lines $\mathbf{J}_1$, $\mathbf{J}_2$, and $\mathbf{J}_3$ being realized at t is a sufficient condition for the realization of $\mathbf{I}$ at t. Concerning the case of monster objects like Cleopatra combined with a carburetor, I think we had better deny their existence altogether. Indeed, if there exists a variable embodiment manifested initially as Cleopatra and later on as a carburetor of a car—this much Fine grants without hesitation—it should not be particularly shocking that the car has the hybrid of Cleopatra and the carburetor as its temporary part during the life span of the carburetor. Fine takes the absurdity of the latter claim as a reason to reject traditional mereology. However, it is already absurd to countenance the existence of some one thing that is partially Cleopatra and partially a carburetor.

Kathrin Koslicki [63] criticizes Fine's view for proliferation of *sui generis* relations of composition—those producing rigid embodiments and variable embodiments, plus arbitrary hierarchical combinations of compositions of these two types. She also criticizes his theory for its commitment to a superabundance of objects—objects generated from already available objects by ever more involved compositions.[35] I think Koslicki's criticism is justified. At any point in space, there could be an object, but that does not mean there is one at every point. A function over points in space could describe the spatial form of an object in space, but that does not mean there is an object thus described. I do *not* assume that for every selection of local objects, one object per world, there is a corresponding world line (unrestricted cross-world composition of local objects), *nor* that for every selection of local objects within a fixed world, there is a further local object having those objects as parts (unrestricted world-internal composition of local objects), *nor* that every selection of world lines with pairwise disjoint modal margins can be conjoined into a single world line (unrestricted composition of world lines). The question of which individuals are available to be talked about in which world cannot be settled on a priori grounds—not even conditionally to a prior specification of the set of all relevant worlds. This is why each model of L_0 has a domain of world lines as a separate component. Additional world lines over the same set of worlds are perfectly conceivable, but this does not mean that all conceivable world lines should be present in every model.

Rejecting unrestricted composition of world lines is compatible with world lines being divisible into component world lines. In particular, one world line could be divisible in many ways. Whenever there are such divisions, there is of course the corresponding composition. If so, some world lines can be obtained by composition from other world lines. Further, rejecting the above-mentioned two forms of unrestricted composition of local objects is compatible with local objects being divisible.

[35] Priest [95, pp. 46–7] takes 'objects' to be functions from worlds to 'identities' (see Sect. 3.8 below) and notes that in his framework, there is a risk of a similar superabundance problem. If d is an object and $d(w)$ is its identity in w, can $d(w)$ be viewed as a *part* of d at w? Priest discusses this idea but dismisses it because he sees it as giving metaphysical priority to identities, and he takes it that this would lead to an uncontrolled proliferation to objects: any function from worlds to identities would count as one.

Koslicki proposes to restrict allowed compositions and divisions by postulating an ontology of kinds. This is a reasonable approach if one interprets world lines metaphysically, as Fine does. I prefer to understand world lines in accordance with the realist version of the transcendental interpretation. According to this position, if the external objects we can meaningfully talk about are temporally extended and have modal properties, we must think of them as world lines. This view does not lead us to postulate any more world lines than those we employ in thinking about the reality. Since our doing so need not involve arbitrary compounds of world lines, nor arbitrary compounds of local objects, this position gives no support to any form of unrestricted composition.

2.7.4 What World Lines Are Not

I have explained how the transcendental interpretation of world lines formulated in Sects. 1.5 and 2.2 differs from alternative ways of construing this idea: epistemic (Sect. 1.6), anti-realist (Sect. 1.6), and metaphysical (Sects. 2.7.2 and 2.7.3) interpretation. There are certain notions that are much discussed in the literature and bear some resemblance to the notion of world line but should not be confounded with it. Since the risk of confusion is real, it is worthwhile to spend some words on what world lines are *not*.

It was noted in Sect. 1.3 that in semantics, world lines (individuals) can be represented by partial functions whose arguments are worlds and whose values are world-bound local objects. Note that the values of these functions are not individuals but local objects—possible realizations of individuals. Now, it is neither necessary nor sufficient for cross-world identity that cross-world identical local objects satisfy a given descriptive condition. Consequently, it would be a mistake to suppose that a partial function induced by a world line must be defined by some descriptive condition that all values of the function satisfy. Conversely, if we take an arbitrary partial function from worlds to corresponding local objects, there is no reason to think that this function is induced by some world line. In particular, even if all values of such a function satisfy some descriptive condition, this does not guarantee that the values (which are local objects) are realizations of one and the same world line.[36]

Let us proceed to consider concrete examples of much-discussed notions that one might misconstrue as being capable of taking up the conceptual role of world lines.

[36] For a blatant example, the condition expressed by the definite description ‘the president of the US’ picks out, in every world in which it is applicable at all, the realization of a unique individual, but these different realizations do not belong to any one individual: no world line is first manifested as (a realization of) Bill Clinton and later on as (a realization of) George Bush Jr. This definite description defines a certain partial function from worlds to local objects, but there is no reason to assume that this function could be a value of a quantified variable—i.e., that it corresponds to a world line. Cf. footnote 38 in Sect. 6.7.

Following Carnap [12], the expression 'individual concept' is often used in the philosophy of language for a function that selects for every context (out of some relevant class of contexts) an individual as the referent of a given singular term, such as a proper name or a definite description. In the special case of proper names, such functions are usually taken to be constant functions—that is, they are taken to select the same individual as the referent of the name in each relevant context. One might be tempted to think that world lines in my sense are such individual concepts. This would be erroneous for several reasons. First, unlike the notion of individual concept (individual concept *of* a singular term), the notion of world line is not language-relative. Intensions of singular terms are of interest for semantic reasons, but the mere fact that someone introduces a novel singular term into our language does not mean that thereby *there are* novel things to talk about—i.e., does not create new values for (physical or intentional) quantifiers to range over. Second, in order for an individual concept of a proper name to be able to assign the *same* individual as the referent of the name in two contexts, the domains of these contexts must have elements in common—this being a nonsensical idea, according to my analysis. Third, while it is indeed formally correct that a world line, like an individual concept, is a function assigning to any context on which it is defined an element of the domain of this context, it is just as much correct formally that values of individual concepts are always individuals, whereas values of world lines (that is, their realizations) are never individuals. For construing world lines as individual concepts, see, e.g., Kraut [65].[37]

World lines are not essences of any kind.[38] If $\phi(x)$ is a unary intensional predicate, let us say that a world line $\mathbf{I}$ satisfies $\phi(x)$ *necessarily*, if for every world w in which $\mathbf{I}$ is realized, $\mathbf{I}$ satisfies $\phi(x)$ in w.[39] There are at least two ways in which we may use the expression 'essence' in my framework if we so wish, but essences in neither sense have any role in clarifying the notion of cross-world identity. First, we might take the essence of $\mathbf{I}$ to be the totality of those predicates that $\mathbf{I}$ satisfies necessarily. A more refined idea would be to identify the essence of a world line $\mathbf{I}$ with the function that assigns to every world w in $marg(\mathbf{I})$ the set of predicates that $\mathbf{I}$ satisfies in w.[40] If for all worlds w in which a physical object is realized, we have available full information about the predicates it satisfies in w, then we can read off its essence in either sense. It must be noted that essences of either sort can by no means be substitutes for

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[37] Also, Kracht and Kutz [66] assimilate world lines to what they call individual concepts, but in the sense in which they take world lines to be individual concepts (world lines being extracted from counterpart relations), these individual concepts could not be constant functions.

[38] A haecceity can be viewed as a *trivial* individual essence, as opposed to an informative or *non-trivial* individual essence—a set of qualitative properties whose possession by the individual would be a necessary and sufficient condition for its being the individual it is. It was remarked in footnote 29 of this chapter that world lines are not haecceities. See also footnote 28 in this chapter.

[39] In the special case that $\phi(x)$ is atomic, this amounts to a condition concerning the *realizations* of $\mathbf{I}$. The world line $\mathbf{I}$ is necessarily $P(x)$, if for all worlds w in the modal margin of $\mathbf{I}$, the realization $\mathbf{I}(w)$ of $\mathbf{I}$ belongs to the interpretation of P in w.

[40] This is basically how Plantinga [94, pp. 76–7] defines the notion of essence.

world lines. They cannot in any way generate or determine world lines. There is no guarantee that a given set of (world-indexed) predicates is the essence of a world line, although conversely, a given world line automatically has an essence in both senses. Kripke [71, p. 114] understands the essence of a physical object in terms of material constitution and causal continuity, in particular with reference to the way in which the material constitution of the object has evolved over time from a specific origin. It might well turn out that essences of physical objects (in one or both of the above senses of 'essence') can be profitably discussed in terms of the notions of material constitution and spatiotemporal continuity. However, while these notions might indeed play an important role in the characterization of the *epistemic* task of recognizing that two local objects are realizations of the same physically individuated world line, they cannot be of the slightest use in explicating the notions of world line and cross-world identity. Namely, in order to make sense of the very ideas of the origin of a physical object and its evolution over time, the notion of cross-temporal sameness must already be presupposed. Generally, nothing we can say about local objects and their possible mereological structure at this or that time, will be enough to deliver the requisite notion of cross-contextual sameness. Piecewise information about worlds cannot, generally, generate information about world lines. World lines are independent of worlds.

It may be useful to stress separately that world lines must not be equated with Fregean senses of singular terms. A Frege-inspired philosopher may speak of an agent as thinking of the planet Venus under the sense (mode of presentation) associated with the description 'the morning star'. Understood as an individual concept, this sense picks out the unique morning star from each context in which one is available. There is no valid analogy to the case of world lines: there is no reason whatsoever to think that one and the same description would uniformly apply to each and every realization of a world line.

World lines are not criteria of identity in the epistemic sense: their role is not to provide us means to recognize an individual in different circumstances. In searching for an answer to the question 'What is a criterion of identity?', E. J. Lowe [80, pp. 12–3] similarly stresses that what he calls criteria of identity are not epistemic or heuristic principles for settling questions of identity concerning individuals. His positive view is that criteria of identity are *semantic* principles governing the meaning of certain sorts of *general* terms.[41] By contrast, world lines are not semantic in character; they are language-independent. Besides, the linguistic items whose semantics can be explicated with reference to world lines are not predicates but quantifiers.

[41] Lowe makes much of Frege's discussion [29, Sects. 62–9] of identity criteria (*Kennzeichen*) in connection with the mathematical practice of defining abstract entities (like directions) as equivalence classes of somewhat less abstract entities (like lines). This leads Lowe to postulate that various sortal terms Φ have an associated 'criterial relation' R so that whenever x and y satisfy Φ, we have $x = y$ iff $R(x, y)$. This is a dubious generalization, since here, identity is applied to entities of the same logical type as those to which the criterial relation is applied, while Frege applies identity to *sets* of lines (directions) and the criterial relation—parallelism—to lines themselves.

Given that world lines cannot be construed as Carnapian individual concepts or as descriptive modes of presentation, one might insist that actually the most natural notion of individual concept is the notion of a *de re* sense formulated by Evans [24] and McDowell [84]. Such *de re* senses are *non-descriptive* modes of presentation, but they count as Fregean senses by being constituents of thoughts and by determining an object as their denotation. However, world lines cannot be analyzed as *de re* senses either, because the mere fact that objects o_1 and o_2 are determined as denotations of one and the same *de re* sense on two occasions does not guarantee that they are one and the same object. Rather, the very idea of determining the same object in different circumstances presupposes the notion of cross-world identity. It is a part of the notion of *de re* sense that it determines the same individual on a variety of occasions—in this respect, *de re* senses behave just like Carnapian individual concepts associated with proper names. It is not up to a mode of presentation—descriptive or not—to constitute the notion of cross-world identity. Modes of presentation and senses are means of having access to entities that are already there. They cannot provide an analysis of what it means to be the same individual. They can only provide means of accessing individuals whose cross-world identity is independently secured.

Given that I am not putting forward a metaphysical understanding of world lines, it is useful to make clear in what sense this viewpoint is and in what sense it is not ontologically committing. As will be seen in Chap. 3, we must distinguish between two ways of individuating world lines: the physical and the intentional. Talking about physically individuated world lines (physical objects) is ontologically committing, whereas speaking of intentionally individuated world lines (intentional objects) is not. What is more, speaking of certain objects as physical ascribes to them a certain sort of objectivity, whereas no such objectivity is assumed in connection with intentional objects. Physically individuated world lines are constrained by objective regularities articulated in physical theories. As for intentionally individuated world lines, they are severely conditioned by an agent, and yet, the agent cannot choose them at will. As remarked in Sect. 1.5, the fact that I take it to be a part of the *form* of our modal cognition that our modal thoughts are structured in terms of world lines does not mean that it is up to us to create the specific objects we are thinking of. Analogously, it is not Kant's view that the pure intuitions and the pure concepts of understanding alone render it possible for us to have experience of objects. The appearances must have a *matter* and not only form.

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