

Chapter 2

Nuclear Models for High Spin Phenomena

Physics is well known for its strong emphasis on quantitative theory to guide understanding. In nuclear physics we have numerous theoretical models that explain different aspects of nuclear behavior. In many cases these models build upon each other, for instance, the deformed shell model (or Nilsson model) is a generalization of the shell model. In some cases these models are composited together, the output of one calculation serving as part of the input to another calculation. In this chapter the models that contribute to understanding the nucleus at high spin, particularly in the wobbling mode, are discussed.

2.1 Introduction

The atomic nucleus, discovered in 1911 by Rutherford [34], is a tiny concentration of matter at the heart of an atom. The nucleus is approximately 1–10 fm across, contains more than 99.94% of an atom's mass, and is composed of protons and neutrons (collectively called nucleons). The force between nucleons is a powerful short range force that overcomes the Coulomb repulsion to produce a bound system. The range of this force is quite limited, to perhaps nearest neighbors only, as can be seen in the saturation of binding energy per nucleon around $A \sim 60$ at a value close to 8.5 MeV (Fig. 2.1).

Examination of the two proton and two neutron separation energies (Fig. 2.2) shows several distinct discontinuities at specific numbers of protons or neutrons. Further examination of the energies of the first 2^+ (Fig. 2.3) states shows peaks at the same numbers. These “magic numbers” occur at numbers of protons and neutrons where there is a dramatic drop off in the nucleon separation energy with the addition of another nucleon. Further evidence for certain “Magic” values of N and Z can be seen in the increase first excited state energies located at these magic numbers, substantially decreased neutron absorption cross-sections at

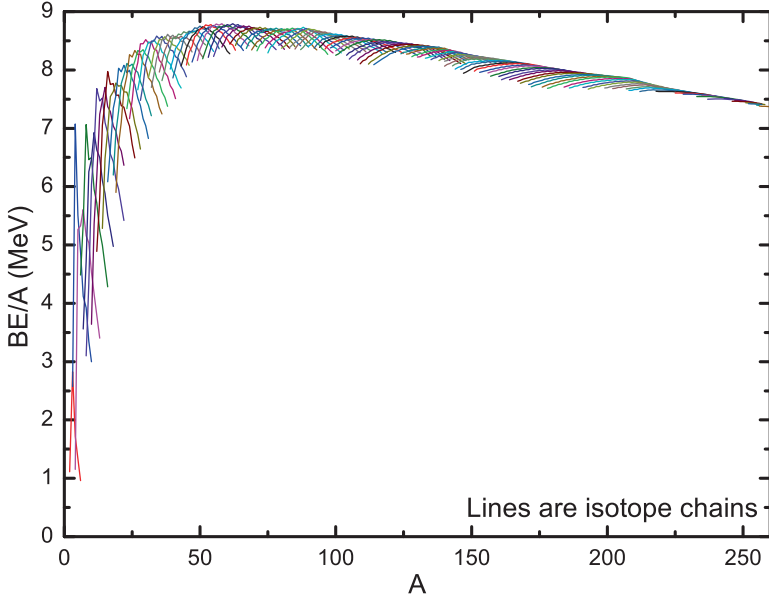


Fig. 2.1 Binding energies per nucleon plotted versus mass number. Values calculated from Ref. [1, 37]

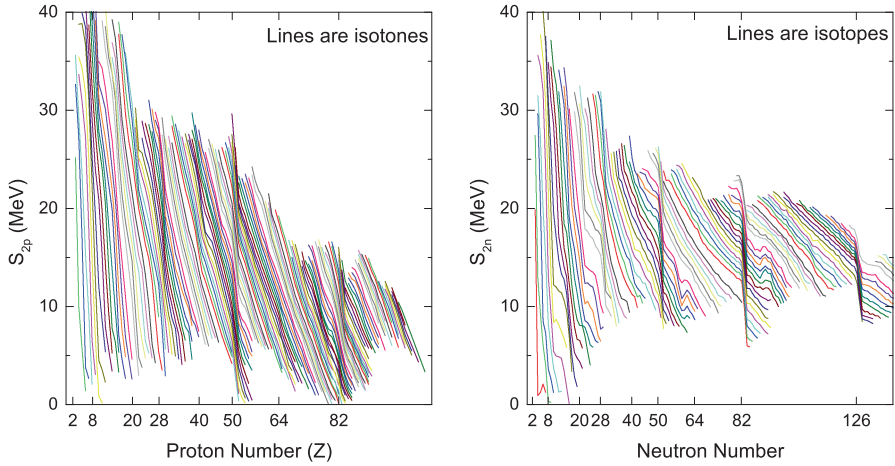


Fig. 2.2 *Left:* Two proton separation energies plotted versus proton number. Each line is a set of isotones. *Right:* Two neutron separation energies plotted versus neutron number. Each line is a set of isotopes. Values calculated from Ref. [1, 37]

neutron magic numbers, and in the enhanced abundance of nuclides where N and Z are magic numbers. The magic numbers are 2, 8, 20, 28, 50, 82, and 126, with 40 and 64 also weakly magic over certain ranges of N and Z .

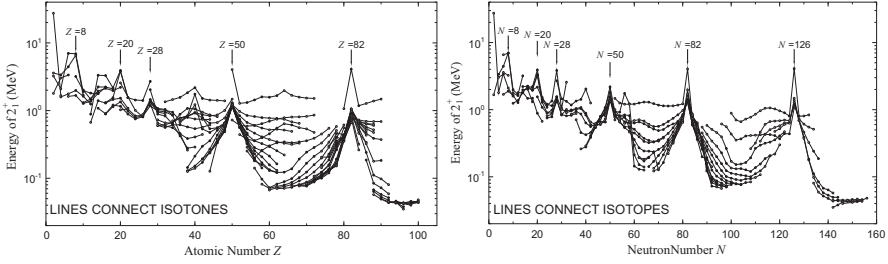


Fig. 2.3 *Left*: First excited 2^+ energies of nuclei with even Z and N , plotted versus proton number. Each line is a set of isotones. *Right*: First excited 2^+ energies of nuclei with even Z and N , plotted versus neutron number. Each line is a set of isotopes. Figures adapted from Ref. [32]

Analogy to atomic theory shows these magic numbers are major shell closures. This leads to the conclusion that the nucleus has shell structure, leading to the shell model of the nucleus. These numbers can be derived from the calculation of a single particle in a mean field potential. Far from stability, quenching of the known shell gaps and the opening of new shell gaps have been observed [23].

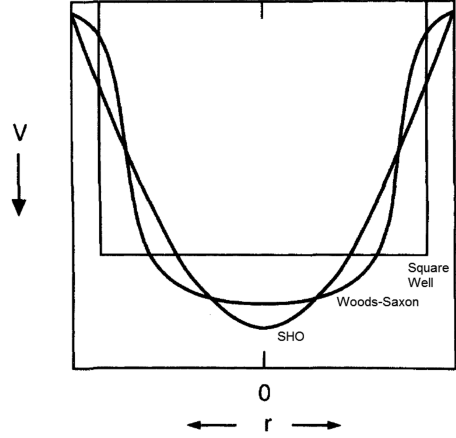
The nucleus also exhibits collective excitations such as rotation and vibration which are described later in the chapter. The crucial differences between an atomic system and the nuclear system are as follows: First, the atom has a nucleus whose Coulomb field guides the electron cloud, preventing collective motions. Second, the nucleus has two types of particles, protons and neutrons, occupying its shells. With this, certain shell occupations can lead to the “single particle behavior” exhibited around closed shells, while different occupations can yield collective phenomena.

2.2 The Shell Model

The nuclear shell model, in its simplest form, seeks to explain the shell structure observed in nuclei by describing the nucleons as independent particles in the mean field potential produced by the other nucleons. While the short range nature of the nuclear force might lead to the use of a square well potential or similar as the mean field, the Simple Harmonic Oscillator (SHO) potential is a reasonable first order approximation (seen in Fig. 2.4), which happens to be much simpler to solve. Placing a single particle in the SHO potential gives the first few magic numbers observed; however, to reproduce all the magic numbers it is necessary to add an $\vec{l}^2$ potential and a strong spin-orbit ($\vec{l} \cdot \vec{s}$) potential. The Hamiltonian for such a potential, called the modified oscillator potential, is as follows:

$$H = \frac{-\hbar^2}{2m} \nabla^2 + \frac{1}{2} m (\omega r)^2 + B \hat{L}^2 + A \hat{L} \cdot \hat{s} \quad (2.1)$$

Fig. 2.4 Schematic of a square well, SHO potential, and a realistic Woods–Saxon potential. Figure adapted from Ref. [13]



The progression of shell gaps from SHO to $\text{SHO} + \vec{l}^2$ to $\text{SHO} + \hat{\mathbf{l}}^2 + \hat{\mathbf{l}} \cdot \hat{\mathbf{s}}$ is shown in Fig. 2.5.

In more realistic shell model calculations a Woods–Saxon potential is adopted. Here the Hamiltonian becomes:

$$\mathbf{H} = \frac{-\hbar^2}{2m} \nabla^2 + \frac{V}{1 + e^{(r-R)/a}} - \alpha(r) \hat{\mathbf{l}} \cdot \hat{\mathbf{s}} \quad (2.2)$$

where $R = r_0 A^{1/3}$, $r_0 \sim 1.2 \text{ fm}$, $a \sim 0.5 \text{ fm}$, and $V \sim 50 \text{ MeV}$. In addition to the central term, a spin-orbit term is necessary to reproduce the observed shell gaps. The central flatness of the Woods–Saxon potential reflects the saturation of the nuclear force better than an SHO potential. This increased realism comes at the cost of increased difficulty in calculations—with this Hamiltonian the Schrödinger equation is no longer analytically solvable and must instead be solved with numerical methods.

Up to now in this chapter, the nucleons were considered as independent particles within a mean field which was the average potential produced by the other particles. This description, while powerful, is not accurate, especially for nuclei away from the closed shells. The missing piece is the interactions between nuclei that are not accounted for in the mean field, often called “residual interactions.” Residual interactions must be accounted for to produce an accurate description of the nucleus.

2.3 The Deformed Shell Model

2.3.1 Parameterization of Deformation

In the standard shell model the potentials are spherically symmetric, depending only on the radius. While this is a successful approach near the shell gaps where nuclei

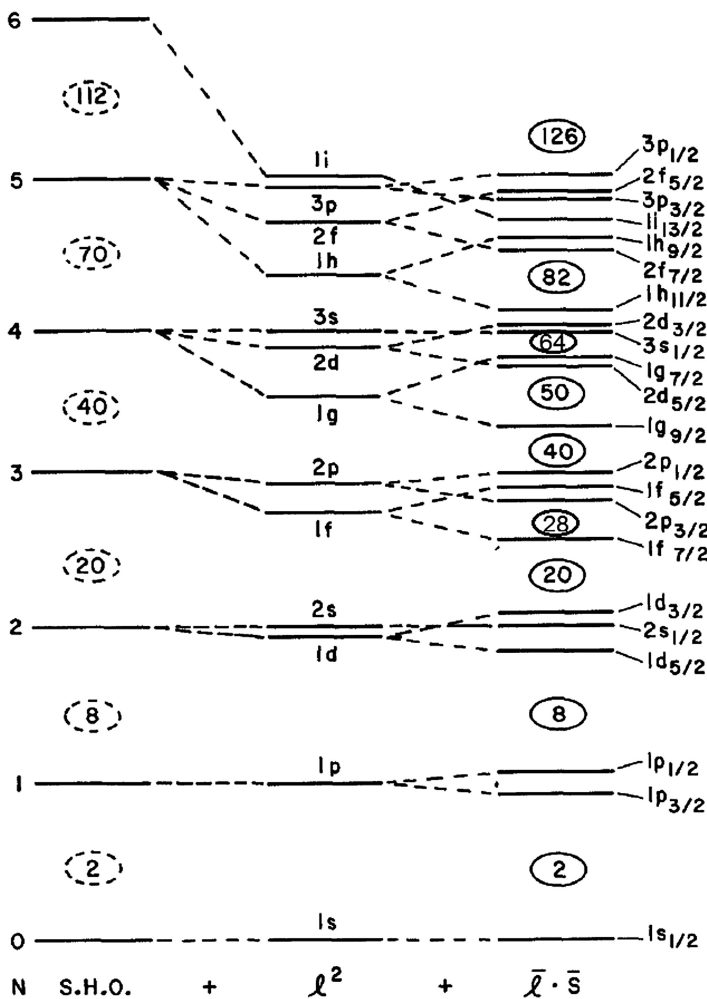


Fig. 2.5 Spectrum of a single nucleon in an SHO potential, $SHO + l^2$, and, $SHO + l^2 + \text{spin-orbit}$. Figure adapted from Ref. [13]

are spherical, or nearly spherical, it breaks down in the deformed regions farther from shell closures. In such nuclei, the long-range correlations experienced by the valence nucleons can lead to deformation as the valence nucleons are arranged into deformed shells with lower energy levels. When the nuclear shape is deformed the surface of the nucleus is described with a parametric function defining the radius as follows:

$$R(\theta, \phi) = R_o \left(1 + \sum_{\lambda=2}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right) \quad (2.3)$$

Here R_0 is the radius of a sphere with the same volume as the deformed nucleus, $Y_{\lambda\mu}(\theta, \phi)$ are the spherical harmonic functions, and $\alpha_{\lambda\mu}$ are the coefficients of the expansion. The expansion starts at $\lambda = 2$ because the $\lambda = 1$ corresponds to the translation of the center of mass, a term easily eliminated by requiring the nuclear center of mass to coincide with the coordinate's origin. The first of the non-trivial terms, $\lambda = 2$, corresponds to the components of quadrupole deformation, $\lambda = 3$ gives octupole deformation, $\lambda = 4$ gives hexadecapole deformation, and so on. As $\lambda = 2$ is the most important component in this work, only quadrupole deformation terms will be considered henceforth.

The quadrupole shapes cover oblate spheroids (two semi-major axes, like a doorknob), prolate spheroids (two semi-minor axes, like an American football), and triaxial ellipsoids (all three axes are different lengths, like a potato). Because the rotation of the nucleus is not fixed, i.e., the nucleus can be rotated such that its principal axes coincide with the axes of the coordinate system, the five quadrupole shape parameters are reduced to two independent parameters a_{20} and a_{22} , with the conditions: $a_{2-2} = a_{22}$ and $a_{21} = a_{2-1} = 0$. Further, it is common to parameterize the remaining expansion coefficients using the Hill–Wheeler variables, β and γ , as follows [39]:

$$a_{20} = \beta \cos(\gamma) \quad a_{22} = a_{2-2} = \frac{1}{\sqrt{2}} \beta \sin(\gamma) \quad (2.4)$$

Expanding the spherical harmonics as:

$$Y_{20}(\theta, \phi) = \frac{1}{4} \sqrt{\frac{5}{\pi}} (-1 + 3 \cos(\theta)^2) \quad (2.5)$$

$$Y_{22}(\theta, \phi) = \frac{1}{4} e^{2i\phi} \sqrt{\frac{15}{2\pi}} \sin(\theta)^2 \quad (2.6)$$

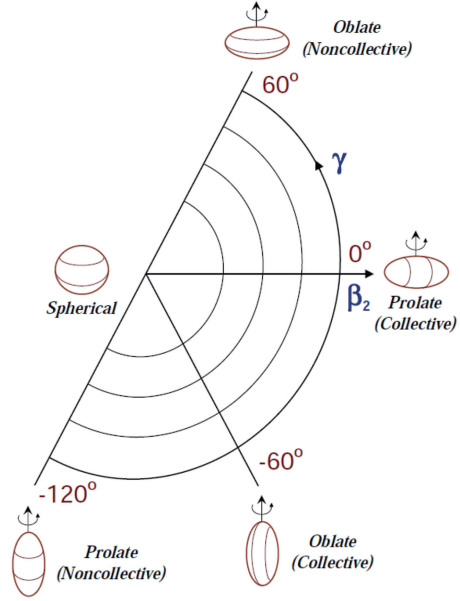
$$Y_{2-2}(\theta, \phi) = \frac{1}{4} e^{-2i\phi} \sqrt{\frac{15}{2\pi}} \sin(\theta)^2 \quad (2.7)$$

and inserting all this into Eq. (2.3) yields:

$$R(\theta, \phi) = R_0 \left(1 + \sqrt{\frac{5}{16\pi}} \beta \left(\cos(\gamma) (3 \cos(\theta)^2 - 1) + \sqrt{3} \sin(\gamma) \sin(\theta)^2 \cos(2\phi) \right) \right) \quad (2.8)$$

The Hiller–Wheeler variables have some redundancy: for $\beta > 0$ the nucleus is prolate for $\gamma = 0^\circ, 120^\circ, 240^\circ$ and oblate for $\gamma = 180^\circ, 300^\circ, 60^\circ$. However, for $\gamma = 0^\circ$ and $\gamma = 180^\circ$ the symmetry axis is the $\hat{z}$ -axis of the intrinsic frame, for $\gamma = 120^\circ$ and $\gamma = 300^\circ$ the symmetry axis is the $\hat{x}$ -axis, and for $\gamma = 240^\circ$ and $\gamma = 60^\circ$ the symmetry axis is the $\hat{y}$ -axis. The Lund convention makes use of this redundancy by selecting a rotational axis according to the following rules [39]:

Fig. 2.6 Schematic of the Lund convention. Figure adapted from Ref. [2]



1. $\beta \geq 0$
2. For rotation about the smallest axis $0^\circ \leq \gamma \leq 60^\circ$
3. For rotation about the longest axis $-120^\circ \leq \gamma \leq -60^\circ$
4. For rotation about the intermediate axis $-60^\circ \leq \gamma \leq 0^\circ$.

A graphical schematic of the Lund convention is in Fig. 2.6.

2.3.2 The Nilsson Model

In 1955, to treat deformed nuclei, S.G. Nilsson introduced a modified version of the shell model in Ref. [29]. This modified shell model, known as the Nilsson model, allows deformation to be taken into account using an anisotropic harmonic oscillator potential as follows:

$$H_{\text{nil}} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + 2\kappa \hbar \omega_o \left[\vec{l} \cdot \vec{s} - \mu (l^2 - \langle l^2 \rangle_N) \right] \quad (2.9)$$

Here $\omega_{x,y,z}$ are the oscillator frequencies in each of the three dimensions, κ controls the strength of the spin-orbit part of the potential, and μ controls the strength of the correction term. The correction term, $l^2 - \langle l^2 \rangle_N$, originally had the form of μl^2 [29]. It served the purpose of suppressing the energy of the higher lying shells; however, it was noted in Ref. [22] that this shift was too large for large N quantum numbers.

Thus the correction term was modified to its current form to compensate. The three oscillator frequencies are chosen to be inversely proportional to the axis lengths of the ellipsoid a_x , a_y , and a_z as follows:

$$\omega_i = \omega_0 \frac{R_0}{a_i} \quad i = x, y, z \quad (2.10)$$

Here ω_0 is the oscillator frequency for the spherical case, defined as $\hbar\omega_0 = (\omega_x\omega_y\omega_z)^{1/3}$. In the case of axial symmetry the oscillator frequencies are defined in terms of the deformation parameter ϵ_2 as follows:

$$\omega_x = \omega_y = \omega_\perp = \omega(\epsilon_2) \left(1 + \frac{1}{3}\epsilon_2\right) \quad (2.11)$$

$$\omega_z = \omega_\parallel = \omega(\epsilon_2) \left(1 - \frac{1}{3}\epsilon_2\right) \quad (2.12)$$

$$\omega(\epsilon_2) = \omega_0 \left(1 - \frac{1}{3}\epsilon_2^2 - \frac{2}{27}\epsilon_2^3\right)^{-1/6} \quad (2.13)$$

Here the deformation β is defined with respect to ϵ_2 using the following series:

$$\beta = \sqrt{\frac{16\pi}{5}} \left(\frac{1}{3}\epsilon_2 + \frac{1}{9}\epsilon_2^2 + \frac{1}{27}\epsilon_2^3 + \frac{1}{81}\epsilon_2^4 \dots \right) \quad (2.14)$$

With these definitions and the Nilsson Hamiltonian, the energy eigenstates ($\epsilon_{\Omega[Nn_z\Lambda]}$), sometimes called Nilsson orbitals, can be extracted from solving the Schrödinger equation.

$$H_{\text{nil}}\psi_i = \epsilon_i\psi_i \quad (2.15)$$

Here i represents the complete set of asymptotic quantum numbers used to specify Nilsson orbitals:

$$\Omega^\pi[Nn_z\Lambda] \quad (2.16)$$

Ω is projection of the particle's total angular momentum onto the symmetry axis, π is the parity defined as $\pi = (-1)^l = (-1)^N$, N is the oscillator quantum number, n_z is the number of oscillator quanta (number of nodes in the wave function), and Λ is the projection of the particle's orbital angular momentum onto the symmetry axis. Some of these quantum numbers are shown schematically in Fig. 2.7.

The deformation dependence of Nilsson orbitals is usually summarized in a Nilsson diagram. In these diagrams energy levels for various sets of asymptotic quantum numbers are plotted versus the deformation parameter ϵ . A Nilsson

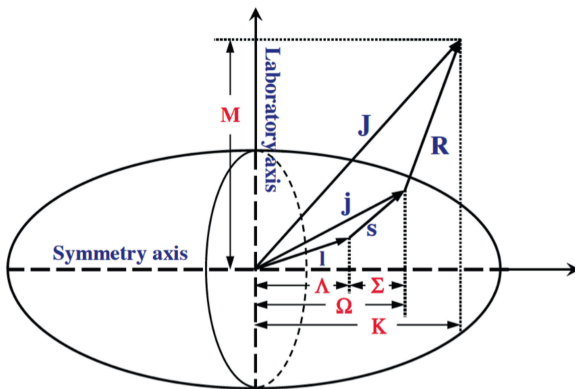


Fig. 2.7 Schematic of an axially symmetric nucleus and the various quantum numbers that are used in the description of the system. Here R is the angular momentum of collective rotation, J is the total nuclear angular momentum, j is the total angular momentum of the particle, l is the orbital angular momentum of the particle, s is the spin of the particle, M is the projection of J onto the non-symmetry axis, K is the projection of J onto the symmetry axis, Ω is the projection of j onto the symmetry axis, Λ is the projection of l onto the symmetry axis, and Σ is the projection of s onto the symmetry axis. Figure adapted from Ref. [2]

diagram for protons in the $40 \leq Z \leq 82$ region is shown in Fig. 2.8. A diagram for neutrons in the $40 \leq Z \leq 82$ region is shown in Fig. 2.9. Together these diagrams cover the $A \sim 130$ region where ^{135}Pr lies.

2.4 Collective Rotation

A model of collective nuclear motion was developed by Bohr and Mottleson [8, 9] due to several observations that could not be explained with single particle motion. Among these are: (1) Fission, many of the features are successfully explained with the liquid drop model [11, 26]. (2) Deformation of the nuclear surface by particle structure [31] giving rise to quadrupole moments which exceed single particle estimates in many nuclei [12, 35]. (3) The occurrence of electric quadrupole gamma-ray transitions with lifetimes much shorter than single particle estimates [21] which is a characteristic feature of the excitation spectra of strong deformed nuclei and most importantly: rotational bands [7].

2.4.1 Rigid Triaxial Rotor Model

One of the simplest forms of excitation a nucleus can experience is rotation. However, in quantum mechanics rotation can only occur about axes that are not symmetry axes (where the symmetry axis is denoted the $\hat{z}$ -axis). This is because

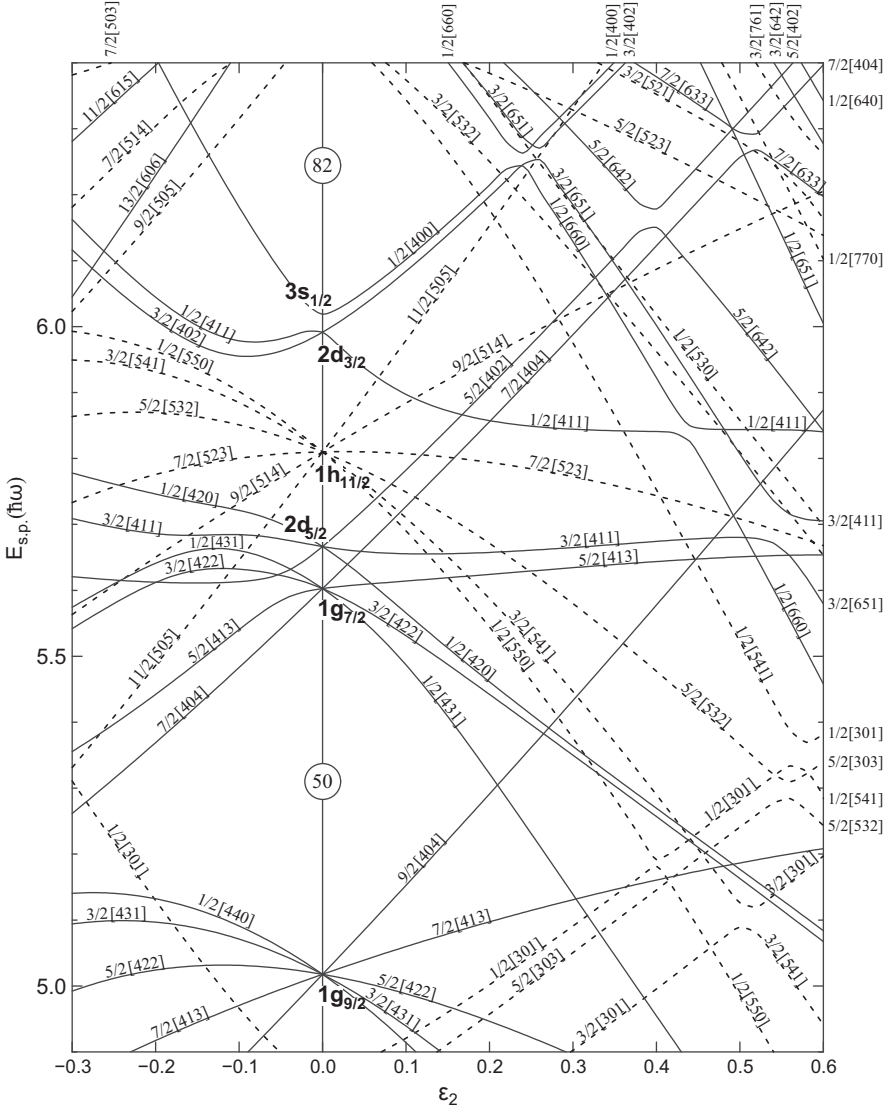


Fig. 2.8 Nilsson diagram for protons in the $50 \leq Z \leq 82$ region with $\epsilon_4 = \epsilon_2/6$. Solid lines represent positive parity orbitals and dashed lines represent negative parity orbitals. Labels follow the $\Omega[N n_z \Lambda]$ convention. Figure adapted from Ref. [30]

the wave function that describes this system is an eigenfunction of $\hat{\mathbf{J}}_z$, and thus, is symmetrical about the $\hat{\mathbf{z}}$ -axis. Due to this symmetry any rotation of this wave function about the symmetry axis generates only phase shift and has a wave function identical to that of the ground state. A system cannot collectively rotate about

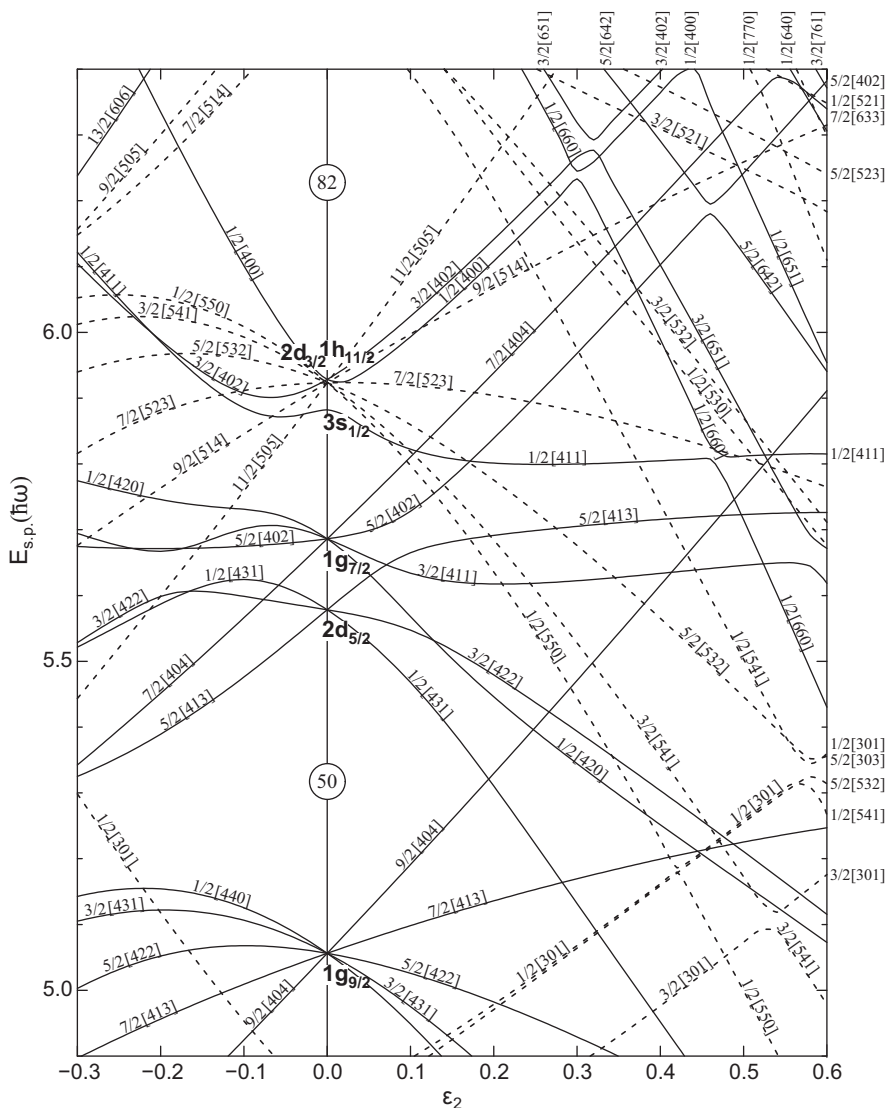


Fig. 2.9 Nilsson diagram for neutrons in the $50 \leq N \leq 82$ region with $\epsilon_4 = \epsilon_2/6$. *Solid lines* represent positive parity orbitals and *dashed lines* represent negative parity orbitals. Labels follow the $\Omega[N n_z \Lambda]$ convention. Figure adapted from Ref. [30]

symmetry axes. Since spherical nuclei are symmetric about all axes, they cannot exhibit collective rotation.

In rotating nuclei there are two primary components of the angular momentum. The first is the rotational angular momentum, $\vec{R}$, generated by the collective motion of many nucleons about some axis. The second component is found when there

are unpaired valence nucleons. In these cases their angular momentum, $\vec{j}$, must be accounted for as well. This presents a situation like that in Fig. 2.7 which schematically illustrates the total angular momentum of the nucleus, $\vec{J}$, as the sum of $\vec{R}$ and $\vec{j}$.

While it is often the case that the shape (and thus the moments of inertia (MOI)) of a nucleus changes somewhat as spin increases. By approximating the system as a rigid rotor with unchanging MOI the problem can be simplified while maintaining descriptive power. Here each axis has a fixed MOI and the rotational component of the Hamiltonian depends only on the MOIs and the components of the spin along each axis. In the case of a triaxial nucleus with no unpaired valence particles (thus $\vec{J} = \vec{R}$) the Hamiltonian can be written as follows [14, 38]:

$$H = H_{\text{rot}} + H_{\text{intr}} \quad (2.17)$$

$$H_{\text{rot}} = A_3 \hat{\mathbf{J}}_3^2 + A_1 \hat{\mathbf{J}}_1^2 + A_2 \hat{\mathbf{J}}_2^2 \quad (2.18)$$

Here H_{intr} is the intrinsic component of the Hamiltonian accounting for the internal state of the nucleus, $A_i = \frac{\hbar^2}{2\mathcal{J}_i}$ where $\mathcal{J}_i$ is the MOI of rotation about the i th axis, and $\hat{\mathbf{J}}_i$ is the operator for the component of the angular momentum along the i th axis. Rewriting the rotational Hamiltonian to a form that uses the operators $\hat{\mathbf{J}}^2$, $\hat{\mathbf{J}}_3^2$, and $\hat{\mathbf{J}}_{\pm}^2$ yields:

$$H_{\text{rot}} = H_r^{\text{Diag}} + H_r^{\text{OffDiag}} \quad (2.19)$$

$$H_r^{\text{Diag}} = \left[\frac{1}{2} (A_1 + A_2) \hat{\mathbf{J}}^2 + \left(A_3 - \frac{1}{2} (A_1 + A_2) \right) \hat{\mathbf{J}}_3^2 \right]$$

$$H_r^{\text{OffDiag}} = \frac{1}{4} (A_1 - A_2) (\hat{\mathbf{J}}_+^2 + \hat{\mathbf{J}}_-^2)$$

Here H_r^{Diag} is the diagonal component of the Hamiltonian, H_r^{OffDiag} is the off-diagonal component, $\hat{\mathbf{J}}^2$ is the total angular momentum operator, and $\hat{\mathbf{J}}_{\pm}^2$ are the angular momentum raising and lowering operators $\hat{\mathbf{J}}_{\pm} = \hat{\mathbf{J}}_1 \pm i\hat{\mathbf{J}}_2$. Taking the basis states of the wave function to be [14]:

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}} [\mathcal{D}_{MK}^J + (-1)^J \mathcal{D}_{-KM}^J] \quad (2.20)$$

where δ_{K0} is the Kronecker delta function and $\mathcal{D}_{MK}^J$ are the Wigner D-functions which are functions of the three Euler angles that determine the orientation of the nucleus' principal axes in space. With these basis states the Hamiltonian matrix can be calculated and the energy eigenvalues and eigenstates for a given spin I are a matter of diagonalizing the matrix. The reduced transition probabilities for $E2$ transitions of the triaxial rotor model are given by Wen-Xian and Qi-Bo [38]:

$$\begin{aligned}
B(E2, I \rightarrow I') &= \sum_{\mu, K, K'} |\langle I' M' K' | \mathcal{M}_{2\mu}^E | I M K \rangle|^2 \\
\mathcal{M}_{2\mu}^E &= \sqrt{\frac{5}{16\pi}} \left[\mathcal{D}_{\mu 0}^{2*} \hat{\mathbf{Q}}'_{20} + \left(\mathcal{D}_{\mu 2}^{2*} + \mathcal{D}_{\mu -2}^{2*} \right) \hat{\mathbf{Q}}'_{22} \right] \\
Q'_{20} &= Q \cos(\gamma) \quad Q'_{22} = \frac{1}{\sqrt{2}} Q \sin(\gamma)
\end{aligned} \tag{2.21}$$

2.4.2 Quasiparticle + Triaxial Rotor

The QTR model is an extension of the particle + rotor model which was put forth in its earliest form in 1952 by Aage Bohr in Ref. [6]. This model allows the calculation of the effect of coupling the odd quasiparticle to the triaxial even-even core. The lab frame Hamiltonian of this model is given in Ref. [19] to be:

$$H = H_{\text{sqp}} + H_{\text{rot}} - H_{\text{int}} \tag{2.22}$$

$$H_{\text{rot}} = A_3 \hat{\mathbf{R}}_3^2 + A_1 \hat{\mathbf{R}}_1^2 + A_2 \hat{\mathbf{R}}_2^2 \tag{2.23}$$

$$H_{\text{int}} = \kappa \sum_{\mu} q_{\mu}^* Q_{\mu} = \kappa f r^2 \left(\epsilon \cos(\gamma) \bar{Y}_0^2 + \frac{\epsilon \cos(\gamma)}{\sqrt{2}} (\bar{Y}_2^2 + \bar{Y}_{-2}^2) \right) \tag{2.24}$$

Here H_{sqp} is the single quasiparticle Hamiltonian accounting for the presence of the central potential and the pairing interaction, H_{rot} is the rotor model Hamiltonian, H_{int} is the interaction of the quasiparticle to the triaxial core, $\hat{\mathbf{J}}_k$ is the total angular momentum of the system along the k th axis, $\hat{\mathbf{j}}_k$ is the angular momentum of the quasiparticle along the k th axis, $\bar{Y}_m^l$ is the result of applying the spherical harmonic operator Y_m^l to the basis state, f is the scaling factor obtained from the following:

$$\kappa f = \frac{2}{3} \sqrt{\frac{4\pi}{5}} \hbar \omega_o \tag{2.25}$$

and κ is the coupling strength, which is related to the deformation of the system by:

$$\kappa \langle 0 | |Q| | 2 \rangle = \hbar \omega_o \epsilon \cos(\gamma) \tag{2.26}$$

Here $\langle 0 || Q || 2 \rangle$ is the reduced matrix element of the core quadrupole operator between the 0^+ ground state and the first 2^+ state. This is related to the reduced quadrupole transition probability as follows:

$$B(E2, J_I \rightarrow J_F) = \frac{|\langle J_F || Q || J_I \rangle|^2}{2J_I + 1} \quad (2.27)$$

Qualitatively, the H_{int} works as follows. A high j quasiparticle with predominantly particle nature will align its $\vec{j}$ to the short axis of the core. This orientation results in the maximum overlap of the torus like density distribution of the quasiparticle and the triaxial core which minimizes the energy of the attractive interaction between the two. For a high j quasiparticle with hole nature, the $\vec{j}$ aligns to the long axis which minimizes the overlap of the torus with the triaxial core and minimizes the energy of the repulsive interaction between the two. Finally a quasiparticle from the middle of the shell, possessing particle and hole nature in roughly equal measure, would align the medium axis of the triaxial core.

The reduced transition probabilities of this system can be extracted similarly to those of the triaxial rotor model (Eq. (2.21)); however, the operator $\mathcal{M}_{2\mu}^E$ will change to account for the quadrupole moment of both the core (Q) and the quasiparticle (q).

2.4.3 Pairing

The pairing interaction is the force that couples two identical nucleons in the time reversed orbits (opposite spin projections). Evidence for this force can be seen in a variety of experimental results, among them: (1) The staggering seen in the single neutron (or proton) separation energies between even and odd neutron (or proton) number (see Fig. 2.10); (2) The ground state of *every* even-even nucleus has $J^\pi = 0^+$; and (3) The ground states of the odd-A nuclei always have J^π determined by the orbital of the last unpaired nucleon.

In 1950, M. Goeppert suggested describing the pairing force as a delta function multiplied by some interaction strength [25]. Calculations were performed using this interaction to explore its effects [4] and then later analogies between nuclear pairing and BCS superconductivity theory [3] were discovered and examined [10, 27]. The Hamiltonian to describe this interaction is usually written as:

$$H_{\text{pair}} = -GP^+P - \lambda \hat{N} \quad (2.28)$$

Here, λ is the chemical potential, $\hat{N}$ is the particle number operator, together these give particle number conservation, and G is the pairing interaction strength, which has been determined to have different values for the protons and neutron. For protons the pairing strength is approximately $G_p = \frac{17}{A}$ MeV and for neutrons the pairing

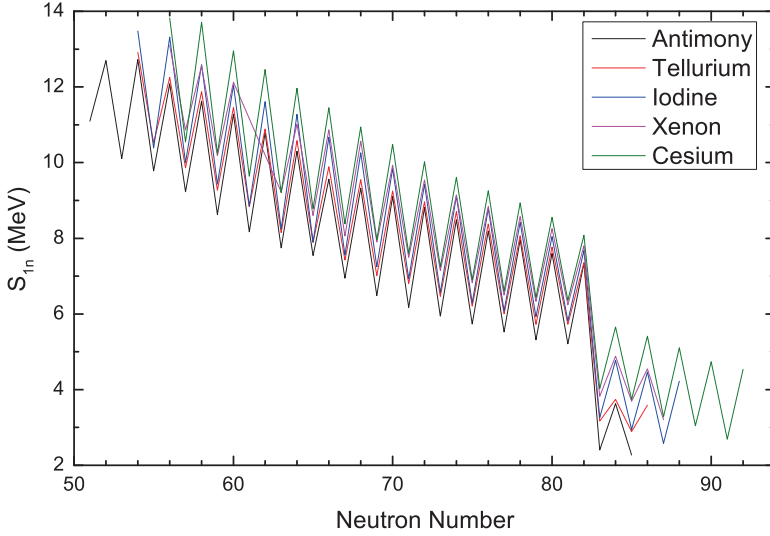


Fig. 2.10 One neutron separation energies vs neutron numbers for five elements from $Z = 51$ to $Z = 55$. The staggering between even and odd neutron number gives strong evidence for the pairing interaction. Additionally the $N = 82$ shell gap can be seen in the abrupt drop of the neutron separation energies

strength is approximately $G_n = \frac{23}{A} \text{ MeV}$. Finally, P^+ is the monopole pair field, expressed by:

$$P^+ = \sum_{k>0} c_k^+ c_{\bar{k}}^+ \quad (2.29)$$

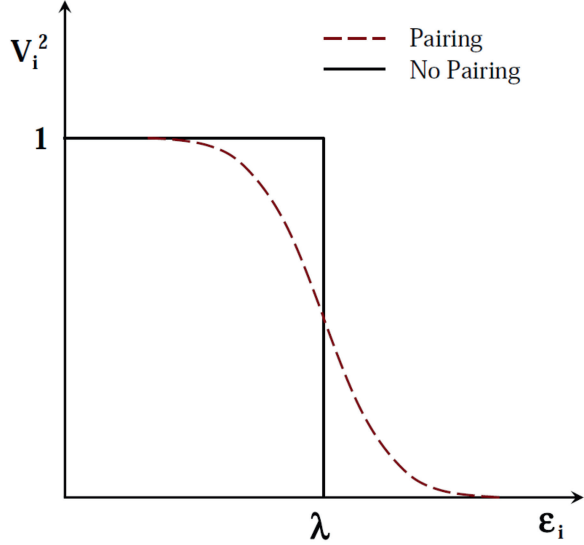
Here $\bar{k}$ represents the time reversed state of k , i.e., spin-up $\rightarrow$ spin-down.

If the spacing between levels near the Fermi surface is small relative to G , then the pairing interaction scatters pairs of nucleons with $J^\pi = 0^+$ from occupied levels to empty levels above the Fermi level. This results in the “smeared” nucleon distribution shown in the dashed line of Fig. 2.11. If the level spacing is large near the Fermi surface, then the pairing interaction cannot scatter pairs into unoccupied levels and the Fermi surface has a sharp cutoff as seen in the solid line of Fig. 2.11.

The energy range that this “smearing” of level occupancies covers is equal to the gap parameter (Δ) which can be defined as follows:

$$\Delta = G \sum_{ij} U_i V_j \quad (2.30)$$

Fig. 2.11 Schematic of the occupancy of single particle states with and without pairing. Figure adapted from Ref. [36]



where U is the emptiness factor (the probability that a state will be occupied by a hole) and V is the fullness factor (the probability that a state will be occupied by a particle). U and V are defined as follows:

$$V_i = \sqrt{\frac{1}{2} \left(1 + \frac{\epsilon_i - \lambda}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right)} \quad (2.31)$$

$$U_i = \sqrt{\frac{1}{2} \left(1 - \frac{\epsilon_i - \lambda}{\sqrt{(\epsilon_i - \lambda)^2 + \Delta^2}} \right)} \quad (2.32)$$

Here ϵ_i represents the single particle energies, and λ is the chemical potential for a given number of particles. The probabilities are normalized such that $U_i^2 + V_i^2 = 1$. Far below the Fermi surface the probability that an orbit will be occupied is $V_i = 1$, while the probability that the orbit is unoccupied $U_i = 0$. This situation reverses itself far above the Fermi surface. Close to the Fermi surface both probabilities will have a finite value. In BCS theory [3] the quasiparticle energy is expressed as:

$$E_{qp}(n, p) = \sqrt{(\epsilon_i - \lambda)^2 + \Delta^2} \quad (2.33)$$

As the nucleus rotates the induced Coriolis force competes with the pairing interaction. The competition attempts to break the pairs and align the individual angular momenta of the nucleons with the rotation axis. In general rotational motion weakens the pairing strength in the nucleus. While some pairs break and align at specific rotational frequencies, all pairs experience the effects. This phenomenon is called the Coriolis anti-pairing effect (CAP) [28].

2.4.4 Tilted Axis Cranking

The tilted axis cranking (TAC) model [16] is a generalization of the cranked shell model introduced by Bengtsson and Frauendorf in [5]. In this model, nucleons are individual particles independently moving in an average potential, which in turn rotates about some axis. As expected of a rotating frame, the Coriolis and centrifugal forces play an important role in the TAC model with consequences for shape, pairing correlations, and quasiparticle orbitals. The Hamiltonian for this model has two terms, the cranking component $-\omega \hat{\mathbf{J}}_x$ which represents the centrifugal and Coriolis forces from the rotating reference frame and a component in the rotating frame, H_0 .

$$H' = H_0 - \omega \hat{\mathbf{J}}_x \quad (2.34)$$

Ref. [16] starts with the pairing plus quadrupole Hamiltonian given by:

$$H_0 = H_{\text{sph}} - \frac{\chi}{2} \sum_{\mu=-2}^2 Q_{\mu}^{+} Q_{\mu} - GP^{+}P - \lambda \hat{\mathbf{N}} \quad (2.35)$$

Here the spherical potential is simply parameterized as by the energy ϵ_k for a state labeled k and then constructed in second quantization as:

$$H_{\text{sph}} = \sum_k \epsilon_k c_k^{+} c_k \quad (2.36)$$

The short range pair correlations are accounted for using the monopole pair operator defined in Eq. (2.29). The quadrupole interaction operator accounts for the long-range particle-hole interaction:

$$Q_{\mu} = \sum_{k,k'} \sqrt{\frac{4\pi}{5}} \langle k | r^2 Y_{2\mu} | k' \rangle c_k^{+} c_{k'} \quad (2.37)$$

Finally, the term $\lambda \hat{\mathbf{N}}$ accounts for the number of particles (N) using the chemical potential λ . As written the Hamiltonian of equation (2.34) only accounts for one type of particles, protons, or neutrons; thus all the expressions should be understood as sums across neutron and proton parts.

To determine the system wave functions, $|\rangle$, the eigenstates of the Hartree-Fock-Bogoliubov (HFB) mean field Routhian are found. The HFB Routhian is:

$$h' = h - \sum_{\mu=-2}^2 (q_{\mu} Q_{\mu}^{+} + q_{\mu}^{*} Q_{\mu}) - \Delta(P^{+} + P) - \lambda N - \omega \hat{\mathbf{J}}_z \quad (2.38)$$

Applying the self-consistency conditions: $q_\mu = \chi \langle Q_\mu \rangle$, $\Delta = G \langle P \rangle$, and conservation of particle number $N = \langle \hat{N} \rangle$ gives the deformed potential h . Further information regarding operator definitions and procedures for self-consistent solutions can be found in References: [16, 17, 33]

As stated in Refs. [17, 24], self-consistent solutions must have their angular frequency and angular momentum vectors parallel, i.e., $\vec{\omega} \parallel \vec{J}$. Additionally, self-consistent solutions to the total Routhian ($E' = \langle H' \rangle$) satisfy the extremum conditions of:

$$\left. \frac{\partial E'}{\partial q_\mu} \right|_\omega = 0 \quad \left. \frac{\partial E'}{\partial \Delta} \right|_\omega = 0 \quad (2.39)$$

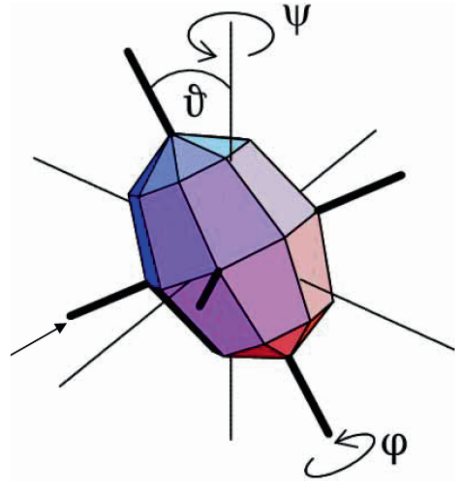
and have total energy and angular momentum:

$$E(J) = E'(\omega) + \omega J(\omega) \quad J(\omega) = \langle \hat{J}_z \rangle \quad (2.40)$$

The orientation intrinsic frame is chosen such that the components of the quadrupole tensor satisfy $q'_{-1} = q'_1 = 0$ and $q'_{-2} = q'_2$. With this, the principal axes of the intrinsic frame coincide with the principal axes of the quadrupole tensor and both are related to the lab frame by the three Euler angles ψ , θ , and ϕ shown in Fig. 2.12. The component quadrupole moments of the lab frame, q_μ , are related to the quadrupole moments of the intrinsic frame, q'_μ , as follows:

$$q_\mu = D_{\mu 0}^2(\psi, \theta, \phi) q'_0 + \left[D_{\mu 2}^2 + D_{\mu -2}^2(\psi, \theta, \phi)(\psi, \theta, \phi) \right] q'_2 \quad (2.41)$$

Fig. 2.12 Orientation of the rotational axis with respect to the principal axes of the deformed density distribution. Figure adapted from Ref. [15]



With the intrinsic quadrupole moments expressed using the standard deformation parameters of the Lund convention:

$$q'_0 = K\beta * \cos(\gamma) \quad q'_2 = -K \frac{\beta \sin(\gamma)}{\sqrt{2}} \quad (2.42)$$

where K sets the energy scale for the deformed potential [16].

As the rotation angle ψ is about the cranking axis, the intrinsic states are invariant with respect to it and it is set to zero. With this, in the intrinsic frame, the mean field Routhian is fixed by β , γ , θ , and ϕ , the two parameters of the deformation and the two parameters of the orientation of $\vec{\omega}$ relative to the principal axes. If we take the angular velocity as:

$$\vec{\omega} = (\omega_1, \omega_2, \omega_3) = \omega(\sin(\theta) \sin(\phi), \sin(\theta) \cos(\phi), \cos(\theta)) \quad (2.43)$$

then the HFB Routhian becomes:

$$h' = h - q'_0 Q'_0 - q'_2 (Q'_2 + Q'_{-2}) - \Delta(P^+ + P) - \lambda \hat{N} \quad (2.44)$$

$$- \omega(J_1 \sin(\theta) \sin(\phi) + J_2 \sin(\theta) \cos(\phi) + J_3 \cos(\theta))$$

With this β and γ are determined from the self-consistency equations

$$q'_0 = \kappa \langle Q'_0 \rangle \quad q'_2 = \kappa \langle Q'_2 \rangle \quad (2.45)$$

The remaining Euler angles θ and ϕ are determined from the self-consistency requirement of $\vec{J} \parallel \vec{\omega}$. These parameter sets correspond to extrema of the total Routhian:

$$\left. \frac{\partial E'}{\partial q'_0} \right|_{\omega} = 0 \quad \left. \frac{\partial E'}{\partial q'_2} \right|_{\omega} = 0 \quad \left. \frac{\partial E'}{\partial \theta} \right|_{\omega} = 0 \quad \left. \frac{\partial E'}{\partial \phi} \right|_{\omega} = 0 \quad (2.46)$$

Of these extrema, only the minima are interpreted as bands.

We can see that there are three possible TAC solutions. They are: the principal axis solution shown in top panel of Fig. 2.13, the planar solution found in the middle panel of Fig. 2.13, and the aplanar solution in the bottom panel of Fig. 2.13. The axial and planar solutions of the TAC model will be discussed further in the subsequent sections. The aplanar solution is of no consequence in this work beyond it giving rise to one of the two unique signatures of nuclear triaxiality *viz.* chirality. Refs. [16] and [20] have good discussions of this solution and its implications.

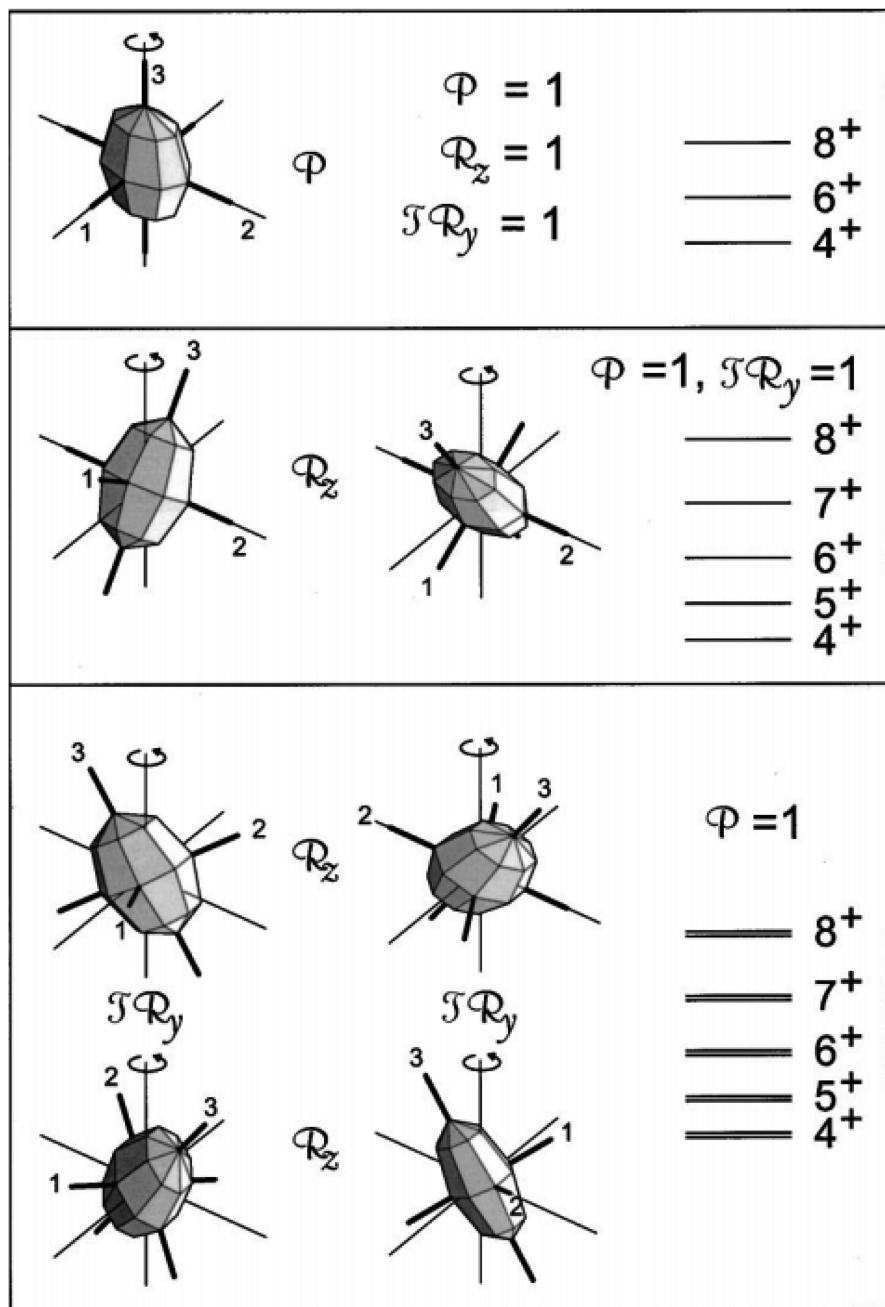


Fig. 2.13 Discrete symmetries of the mean field of a rotating triaxial nucleus for each of the TAC solutions along with schematics of the band structures that arise from each solution. The axis of rotation (z), which coincides with the $\vec{J}$, is marked with a *circular arrow*. The operators are: $\mathcal{P}$: Parity inversion, $\mathcal{R}_{y,z}$ Rotation by π about the y or z axis, and $\mathcal{T}$: Time reversal. Figure adapted from Ref. [16]

2.4.4.1 Rotation About A Principal Axis

If the axis of rotation, (z), coincides with one of the principal axes of the self-consistent solution, then the solution is called Principal Axis Cranking (PAC) and the orientation angles satisfy:

$$\theta = 0, \pi/2 \quad \phi = 0, \pi/2 \quad (2.47)$$

The upper panel of Fig. 2.13 shows the PAC case where $\vec{J}$ has the direction of the principal axis 3. In this case the solution has the following properties:

$$\mathcal{R}_z(\pi) | \rangle = e^{-i\alpha\pi} | \rangle \quad (2.48)$$

Here α is the quantum number called signature. In this situation α is a good quantum number and in even-A nuclei taking on values of 0 or 1, while odd-A nuclei have values of $\pm 1/2$. Signature gives the selection rule that the total spin can only take on the following values:

$$I = \alpha + 2n \quad n \in \mathbb{Z} \quad (2.49)$$

here, $\mathbb{Z}$ represents the set of integers. This selection rule shows that the PAC solution represents a single $\Delta I = 2$ band, as illustrated on the right side of the upper panel of Fig. 2.13.

2.4.4.2 Rotation About A Tilted Axis

If the axis of rotation does not coincide with one of the principal axes but still lies within the planes defined by them, then the solution is called the Planar Tilted Axis Cranking (TAC) solution. In this case the orientation angles satisfy:

$$\theta \neq 0, \pi/2 \quad \phi = 0, \pi/2 \quad (2.50)$$

$$\theta = 0, \pi/2 \quad \phi \neq 0, \pi/2 \quad (2.51)$$

The middle panel of Fig. 2.13 shows a case in which the axis of rotation lies in the principal planes spanned by axes 1 and 3. Invariance with respect to rotation about the z -axis by π is lost, yielding:

$$\mathcal{R}_z(\pi) | \rangle \neq e^{-i\alpha\pi} | \rangle \quad (2.52)$$

With the breaking of this symmetry, α ceases to be a good quantum number and there is no longer a selection rule on the total spin. This can be seen schematically on the right side of the middle panel of Fig. 2.13.

Since 1993, when Frauendorf found a set of self-consistent Hartree–Fock mean field solutions for uniform rotation about a tilted axis [18], TAC has turned out to be a reliable approximation to calculate and investigate the properties of the various types of rotational bands. Details of TAC not covered in this section can be found in Refs. [16–18, 20]

2.5 The Wobbling Mode in Nuclei

2.5.1 Models of the Wobbling Mode

The wobbling mode in nuclei was first predicted by Bohr and Mottelson in the second volume of their textbook on nuclear structure [9]. Their formulation of wobbling was for even-even nuclei using only the triaxial rotor model. Following the nomenclature of Ref. [19], this form of wobbling is called simple wobbling. More recent investigations [19] in wobbling for odd-A nuclei have yielded two more forms of wobbling using the QTR model. These forms are named for the coupling of the quasiparticle. The mode is called “transverse” wobbling if the odd quasiparticle is coupled to an axis perpendicular to the axis of maximum moment of inertia (the intermediate length axis). Contrariwise if the quasiparticle couples to the intermediate axis it is called “longitudinal” wobbling.

2.5.1.1 Simple Wobbling

Simple wobbling, the mode predicted by Bohr and Mottelson in Ref. [9] for even-even nuclei, can be examined entirely within the framework of the TRM, though a semiclassical analysis elucidates the structure of the mode [19]. The rotational kinetic energy of the system can be written as:

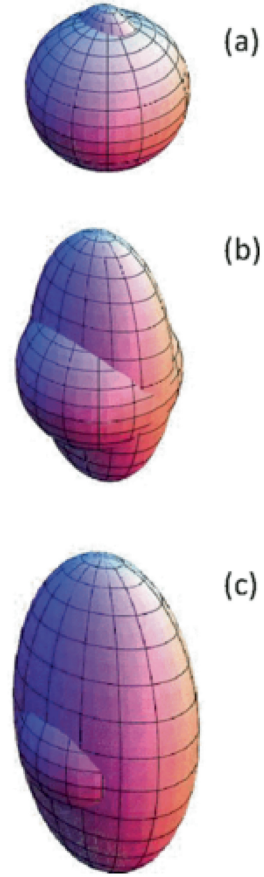
$$E_{\text{rot}} = A_3 J_3^2 + A_1 J_1^2 + A_2 J_2^2 \quad (2.53)$$

Additionally the condition that the squares of the components of the angular momentum sum to the square of the total angular momentum must be met:

$$J^2 = J_1^2 + J_2^2 + J_3^2 = I(I + 1) \quad (2.54)$$

Together these surfaces give the classical orbits of $\vec{J}$ at the intersection of the angular momentum sphere and energy ellipsoid. Assuming the axes are chosen such that $A_1 > A_2 > A_3$ ($\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3$), the surface equations can be plotted for 3 different situations as in Fig. 2.14. Here the classic angular momentum orbits can be seen directly in the intersection between the two surfaces. For Fig. 2.14 the rotor parameters were chosen so that $A_1 = 6 \cdot A_3$ and $A_2 = 3A_3$ and A_3 was used to adjust the energy scale of the ellipsoid between the various panels of the plot. The yrast

Fig. 2.14 Intersections of the angular momentum and energy surfaces for a simple wobbling system with $A_1 = 6 * A_3$ and $A_2 = 3A_3$ for $J = 2$ taken at progressively larger energies from (a) to (c). Figure adapted from Ref. [19]



line then corresponds to the two surfaces touching at $J_3 = J$ with rotational energy $E(J) = A_3 J^2$. This is not plotted in any of the panels of Fig. 2.14 as the energy ellipsoid would be contained within the angular momentum sphere, except for an invisible point of contact at $J_3 = J$. In Fig. 2.14 the top panel is an orbit slightly above the yrast line which is the harmonic wobbling motion described by Bohr and Mottelson [9]. The middle panel is the separatrix, representing the unstable rotation about the axis with intermediate MOI. This orbit is the boundary between rotation about the axis with the maximum MOI (axis 3) and the axis with minimum MOI (axis 1) and has a classical frequency of zero. The bottom panel gives an example of an orbit corresponding to rotation about axis 1.

Simple wobbling excitations are small amplitude oscillations of $\vec{J}$ about the axis with the largest MOI (axis 3 in this coordinate system). Their Hamiltonian can be taken to be [19]:

$$H = A_3 I(I + 1) + (n + \frac{1}{2})\hbar\omega_w \quad (2.55)$$

where n is the number of wobbling phonons and the wobbling frequency is:

$$\hbar\omega_w = 2I\sqrt{(A_1 - A_3)(A_2 - A_3)} \quad (2.56)$$

Within the Harmonic Approximation (HA) the intraband reduced transition probabilities $B(E2)$ [38] are:

$$B(E2; (I, n) \rightarrow (I - 2, n)) \approx \frac{5}{16\pi} e^2 Q_{22}^{\prime 2} \quad (2.57)$$

Here $Q'_{22} = Q \cos(\gamma)$ where Q is the intrinsic quadrupole moment. The interband $B(E2)$ s [38] become:

$$B(E2; (I, n) \rightarrow (I - 1, n + 1)) \approx \frac{5}{16\pi} e^2 \frac{n+1}{J} \left(\sqrt{3} Q'_{20} y + \sqrt{2} Q'_{22} x \right)^2 \quad (2.58)$$

$$B(E2; (I, n) \rightarrow (I - 1, n - 1)) \approx \frac{5}{16\pi} e^2 \frac{n}{J} \left(\sqrt{3} Q'_{20} x + \sqrt{2} Q'_{22} y \right)^2. \quad (2.59)$$

where $Q'_{22} = \frac{1}{\sqrt{2}} Q \sin(\gamma)$ and x and y are defined as follows:

$$\begin{aligned} x &= \sqrt{\frac{1}{2} \left(\frac{A_2 + A_1 - 2A_3}{2\sqrt{(A_1 - A_3)(A_2 - A_3)}} + 1 \right)} \\ y &= \sqrt{\frac{1}{2} \left(\frac{A_2 + A_1 - 2A_3}{2\sqrt{(A_1 - A_3)(A_2 - A_3)}} - 1 \right)} \end{aligned} \quad (2.60)$$

A final note about the simple wobbler is that the wobbling bands have alternating signature. Wobbling bands with an odd number of phonons have signature opposite to that of the yrast band. Contrariwise, wobbling bands with even numbers of phonons have the same signature as the yrast band.

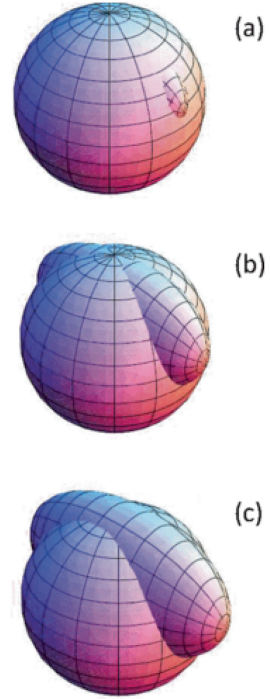
2.5.1.2 Transverse Wobbling

Transverse wobbling, the first of the two new wobbling modes defined by Frauendorf and Dönau in Ref. [19], occurs when the quasiparticle aligns itself perpendicular to the intermediate length axis. The energy ellipsoid for such a system is:

$$E = A_3(J_3 - j)^2 + A_1 J_1^2 + A_2 J_2^2 \quad (2.61)$$

Here the axes are chosen so that axis 3 is the short axis, i.e., the axis with intermediate MOI, and axis 2 is the intermediate axis with maximum MOI, yielding: $A_1 > A_3 > A_2$ ($\mathcal{J}_1 < \mathcal{J}_3 < \mathcal{J}_2$). Taking the same angular momentum sphere in Eq. (2.54) the classical orbits of angular momentum are found in the intersections of

Fig. 2.15 Intersections of the angular momentum and energy surfaces for a transverse wobbling system with $A_1 = 6 * A_2$ and $A_3 = 3A_2$ for $J = 3$ taken at progressively larger energies from (a) to (c). Figure adapted from Ref. [19]



the two surfaces in Fig. 2.15. As with Fig. 2.14, the top panel represents a wobbling state lying a little above the yrast line, the middle panel shows the separatrix, and the bottom panel shows orbits that revolve around axis 3.

Unlike the simple wobbler, in the transverse wobbling case, the separatrix will be encountered here simply with increasing angular momentum. In the simple wobbler increasing J is not enough to encounter the separatrix; the system's stability continues indefinitely. However, with transverse wobbling, the offset of the energy ellipsoid causes the separatrix to be encountered at a critical angular momentum $J_c = jA_3/(A_3 - A_2)$. At this angular momentum the rotational axis of the yrast line changes to point at $j_1 = 0, j_2 = \sqrt{J^2 - J_c^2}, j_3 = J_c$ where the energy ellipsoid touches the angular momentum sphere from the inside.

For $J > J_c$ the rotation is about a tilted axis and as discussed in Sect. 2.4.4.2 this causes signature to cease to be a good quantum number. With this the yrast band collapses into a $\Delta I = 1$ band, destroying the wobbling. Leading up to this collapse is a decrease of the angular frequency.

Applying the frozen alignment approximation (in which it is assumed the particle is firmly coupled to an axis and can be taken simply as a number) and then applying the second order expansion of

$$\hat{\mathbf{J}}_3 = \sqrt{J^2 - \hat{\mathbf{J}}_1^2 - \hat{\mathbf{J}}_2^2} \approx J - \frac{1}{2} \left(\frac{\hat{\mathbf{J}}_1^2}{J} + \hat{\mathbf{J}}_2^2 J \right) \quad (2.62)$$

the QTR Hamiltonian becomes

$$H = A_3(J - j)^2 + (A_1 - \bar{A}_3)\hat{\mathbf{J}}_1^2 + (A_2 - \bar{A}_3)\hat{\mathbf{J}}_2^2 \quad (2.63)$$

$$\bar{A}_3 = A_3(J) = A_3 \left(1 - \frac{j}{J} \right) \quad (2.64)$$

With this the Hamiltonian is in the same form as the TRM Hamiltonian and the same expressions can be used by substituting according to the following pattern $A_3 \rightarrow \bar{A}_3$. This gives Eqs. (2.58) and (2.57) with the following modified expression for the x and y coefficients.

$$\begin{aligned} x &= \sqrt{\frac{1}{2} \left(\frac{A_2 + A_1 - 2\bar{A}_3}{2\sqrt{(A_1 - \bar{A}_3)(A_2 - \bar{A}_3)}} + 1 \right)} \\ y &= -\text{sign}((A_1 - A_2)J) \sqrt{\frac{1}{2} \left(\frac{A_2 + A_1 - 2\bar{A}_3}{2\sqrt{(A_1 - \bar{A}_3)(A_2 - \bar{A}_3)}} - 1 \right)} \end{aligned} \quad (2.65)$$

Additionally, Frauendorf and Dönau [19] extracted reduced transition probabilities for the $M1$ components of the interband transitions. They are as follows:

$$B(M1; (I, n) \rightarrow (I - 1, n + 1)) \approx \frac{3}{4\pi} \frac{n + 1}{J} (j(g_j - g_R)x)^2 \quad (2.66)$$

$$B(M1; (I, n) \rightarrow (I - 1, n - 1)) \approx \frac{3}{4\pi} \frac{n}{J} (j(g_j - g_R)y)^2 \quad (2.67)$$

These semiclassical analyses and expressions are intended to clarify the physics of wobbling. While useful in assisting understanding of these systems, the calculations performed to compare theory with experiment in Sect. 4.2 are done with the full QTR model. Neither the semiclassical approximation nor assumption of frozen alignment is used in the calculations.

2.5.1.3 Longitudinal Wobbling

Longitudinal wobbling is the other possible wobbling mode of an odd- A nucleus. In this case the quasiparticle has aligned itself with the axis of maximum MOI. Using the energy ellipsoid equation of (2.61) but choosing axes such that $A_1 > A_2 > A_3$ ($\mathcal{J}_1 < \mathcal{J}_2 < \mathcal{J}_3$) and finding the classical orbits of the angular momentum gives us the conclusion that, similar to the simple wobbler, this system will remain stable with increasing angular momentum indefinitely. The orbits will not shift with increasing angular momentum preventing the separatrix from being encountered by increasing angular momentum.

Except for a lack of collapse of the wobbling band and the increasing wobbling frequency, longitudinal wobblers behave very similarly to transverse wobblers. Equations (2.58), (2.57), and (2.65) apply to them as well; the only difference is the exact assignment of the axes.

2.5.2 Signatures of Wobbling

There are several signatures of wobbling. These signatures can be broken into two broad categories: signatures that require lifetime information and those that do not.

There are two observables that do not require a lifetime measurement. The first observable is the measurement of the experimental wobbling frequency which is defined as:

$$\hbar\omega_w = E(I, n = 1) - (E(I + 1, n = 0) - E(I - 1, n = 0))/2 \quad (2.68)$$

These wobbling frequencies, while not a unique observable of wobbling, will allow differentiation between longitudinal and transverse wobbling modes in odd-A nuclei. The second observable is unique to wobbling bands. This signature is that $n_w = x \rightarrow n_w = x - 1$ interband transitions must be of $\Delta I = 1, E2$ nature. This is measurable using angular distributions.

In the lifetime measurement category there are two observables. Measuring the lifetime of a state allows the $B(E2)$ values to be extracted. As seen in Eq. (2.58) the $B(E2, (I, n) \rightarrow (I - 1, n - 1)) \propto n_w/I$. Coupled with the mixing ratio, this gives us two observables. Observing the $B(E2, (I, n) \rightarrow (I - 1, n - 1))$ of a wobbling band as spin increases allows the $1/I$ proportionality to be verified. If a two phonon band is identified, the n dependence of the proportionality can be verified as well.

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Exotic Nuclear Excitations: The Transverse Wobbling
Mode in ^{135}Pr

Matta, J.T.

2017, XV, 112 p. 57 illus., 35 illus. in color., Hardcover

ISBN: 978-3-319-53239-4