Chapter 2
Foundations

2.1 Stability Definitions

Consider a Multiple-Input Multiple-Output (MIMO) nonlinear system:

$$x_{k+1} = F(x_k, u_k) \tag{2.1}$$
$$y_k = h(x_k) \tag{2.2}$$

where $x \in \Re^n$ is the system state, $u \in \Re^m$ is the system input, $y \in \Re^p$ is the system output, and $F \in \Re^n \times \Re^m \to \Re^n$ is nonlinear function.

Definition 2.1 ([5]) System (2.1) is said to be forced, or to have input. In contrast the system described by an equation without explicit presence of an input u, that is

$$x_{k+1} = F(x_k)$$

is said to be unforced. It can be obtained after selecting the input u as a feedback function of the state

$$u_k = \vartheta(x_k) \tag{2.3}$$

Such substitution eliminates u:

$$x_{k+1} = F(x_k, \vartheta(x_k)) \tag{2.4}$$

and yields an unforced system (2.4) [8].

Definition 2.2 ([5]) The solution of (2.1)–(2.3) is semiglobally uniformly ultimately bounded (SGUUB), if for any Ω, a compact subset of $\Re^n$ and all $x_{k_0} \in \Omega$, there exists an $\varepsilon > 0$ and a number $\mathbf{N}(\varepsilon, x_{k_0})$ such that $\|x_k\| < \varepsilon$ for all $k \geq k_0 + \mathbf{N}$.

In other words, the solution of (2.1) is said to be SGUUB if, for any a priory given (arbitrarily large) bounded set Ω and any a priory given (arbitrarily small)

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set Ω_0, which contains $(0, 0)$ as an interior point, there exists a control (2.3) such that every trajectory of the closed loop system starting from Ω enters the set $\Omega_0 = \{x_k | \, \|x_k\| < \varepsilon\}$, in a finite time and remains in it thereafter [13].

Theorem 2.3 ([5]) *Let $V(x_k)$ be a Lyapunov function for the discrete-time system (2.1), which satisfies the following properties:*

$$\begin{aligned} \gamma_1(\|x_k\|) &\leq V(x_k) \leq \gamma_2(\|x_k\|) \\ V(x_{k+1}) - V(x_k) &= \Delta V(x_k) \\ &\leq -\gamma_3(\|x(k)\|) + \gamma_3(\zeta) \end{aligned}$$

where ζ is a positive constant, $\gamma_1(\bullet)$ and $\gamma_2(\bullet)$ are strictly increasing functions, and $\gamma_3(\bullet)$ is a continuous, nondecreasing function. Thus if $\Delta V(x_k) < 0$ for $\|x_k\| > \zeta$, then x_k is uniformly ultimately bounded, i.e., there is a time instant k_T, such that $\|x_k\| < \zeta$, $\forall k < k_T$.

Definition 2.4 ([8]) A subset $S \in \Re^n$ is bounded if there is $r > 0$ such that $\|x\| \leq r$ for all $x \in S$.

Definition 2.5 ([8]) System (2.5) is said to be BIBO stable if for a bounded input u_k, the system produces a bounded output y_k for $0 < k < \infty$

Lemma 2.6 ([17]) *Consider the linear time varying discrete-time system given by*

$$\begin{aligned} x_{k+1} &= A_k x_k + B u_k \\ y_k &= C x_k \end{aligned} \tag{2.5}$$

where A_k, B and C are appropriately dimensional matrices, $x \in \Re^n$, $u \in \Re^m$ and $y \in \Re^p$.
Let $\Phi(k_1, k_0)$ be the state transition matrix corresponding to A_k for system (2.5), $\Phi(k_1, k_0) = \prod_{k=k_0}^{k=k_1-1} A_k$. If $\Phi(k_1, k_0) < 1 \forall k_1 > k_0 > 0$, then system (2.5) is
(1) globally exponentially stable for the unforced system ($u_k = 0$) and
(2) Bounded Input Bounded Output (BIBO) stable [5, 17].

Theorem 2.7 (Separation Principle) *[12]: The asymptotic stabilization problem of the system (2.1)–(2.2), via estimated state feedback*

$$\begin{aligned} u_k &= \vartheta(\widehat{x}_k) \\ \widehat{x}_{k+1} &= F(\widehat{x}_k, u_k, y_k) \end{aligned} \tag{2.6}$$

is solvable if and only if the system (2.1)–(2.2) is asymptotically stabilizable and exponentially detectable.

Corollary 2.8 ([12]) *There is an exponential observer for a Lyapunov stable discrete-time nonlinear system (2.1)–(2.2) with $u = 0$ if and only if the linear approximation*

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k \end{aligned} \tag{2.7}$$
$$A = \left.\frac{\partial F}{\partial x}\right|_{x=0}, \quad B = \left.\frac{\partial F}{\partial u}\right|_{x=0}, \quad C = \left.\frac{\partial h}{\partial x}\right|_{x=0}$$

of the system (2.1)–(2.2) is detectable.

2.2 Discrete-Time High Order Neural Networks

The use of multilayer neural networks is well known for pattern recognition and for modelling of static systems. The neural network (NN) is trained to learn an input-output map. Theoretical works have proven that, even with one hidden layer, a NN can uniformly approximates any continuous function over a compact domain, provided that the NN has a sufficient number of synaptic connections.

For control tasks, extensions of the first order Hopfield model called Recurrent High Order Neural Networks (RHONN) are used, which presents more interactions among the neurons, as proposed in [14, 16]. Additionally, the RHONN model is very flexible and allows to incorporate to the neural model a priory information about the system structure.

Let consider the problem to identify nonlinear system

$$\chi_{k+1} = F(\chi_k, u_k) \tag{2.8}$$

where $\chi_k \in \Re^n$ is the state of the system, $u_k \in \Re^m$ is the control input and $F \in \Re^n \times \Re^m \to \Re^n$ is a nonlinear function.

To identify system (2.8), we use the discrete-time RHONN proposed in [18]

$$x_{i,k+1} = w_i^\top \varphi_i(x_k, u_k), \quad i = 1, \cdots, n \tag{2.9}$$

where $x_k = [x_{1,k}, x_{2,k} \ldots x_{n,k}]^\top$, x_i is the state of the i-th neuron which identifies the i-th component of state vector χ_k in (2.8), $w_i \in \Re^{L_i}$ is the respective on-line adapted weight vector, and $u_k = [u_{1,k} u_{2,k} \ldots u_{m,k}]^\top$ is the input vector to the neural network; φ_i is an L_i dimensional vector defined as

$$\varphi_i(x_k, u_k) = \begin{bmatrix} \varphi_{i_1} \\ \varphi_{i_2} \\ \vdots \\ \varphi_{i_{L_i}} \end{bmatrix} = \begin{bmatrix} \Pi_{j \in I_1} \xi_{i_j}^{d_{i_j}(1)} \\ \Pi_{j \in I_2} \xi_{i_j}^{d_{i_j}(2)} \\ \vdots \\ \Pi_{j \in I_{L_i}} \xi_{i_j}^{d_{i_j}(L_i)} \end{bmatrix} \tag{2.10}$$

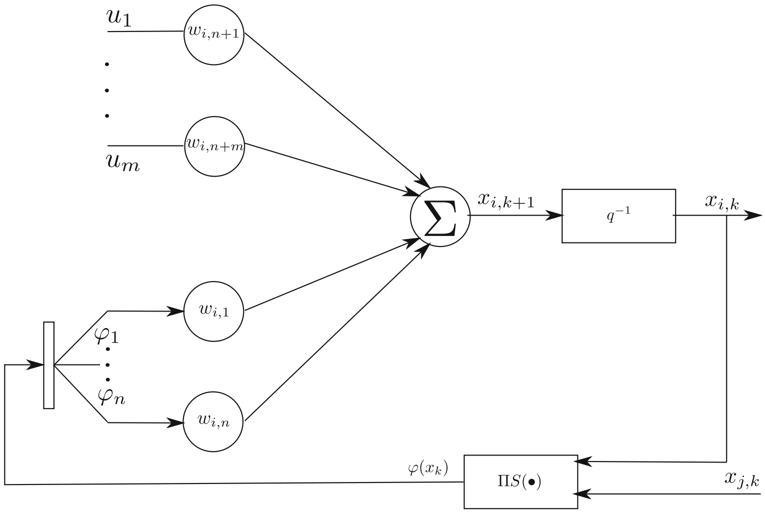

Fig. 2.1 Discrete-Time RHONN

with d_{i_j} nonnegative integers, $j = 1, \ldots, L_i$, L_i is the respective number of high order connections, and $\{I_1, I_2, \cdots, I_{L_i}\}$ a collection of non-ordered subsets of $\{1, 2, \cdots, n+m\}$, n the state dimension, m the number of external inputs, and ξ_i defined as

$$\xi_i = \begin{bmatrix} \xi_{i_1} \\ \vdots \\ \xi_{i_1} \\ \xi_{i_{n+1}} \\ \vdots \\ \xi_{i_{n+m}} \end{bmatrix} = \begin{bmatrix} S(x_{1,k}) \\ \vdots \\ S(x_{n,k}) \\ u_{1,k} \\ \vdots \\ u_{m,k} \end{bmatrix} \tag{2.11}$$

where the sigmoid function $S(\bullet)$ is formulated as

$$S(\varsigma) = \frac{1}{1+\exp(-\beta\varsigma)}, \quad \beta > 0 \tag{2.12}$$

where β is a constant and ς is any real value variable. Figure 2.1 shows the scheme of a discrete-time RHONN.

Consider the problem to approximate the general discrete-time nonlinear system (2.8), by the following modification of the discrete-time RHONN (2.9) [16]:

$$x_{i,k+1} = w_i^{*\top} \varphi_i \left(\chi_k\right) + \varepsilon_{\psi_i} \tag{2.13}$$

where w_i are the adjustable weight matrices, ε_{ψ_i} is a bounded approximation error, which can be reduced by increasing the number of the adjustable weights [16]. Assume that there exists an ideal weights vector w_i^* such that $\|\varepsilon_{\psi_i}\|$ can be minimized on a compact set $\Omega_{\psi_i} \subset \Re^{L_i}$. The ideal weight vector w_i^* is an artificial quantity required for analytical purpose [16]. In general, it is assumed that this vector exists and is constant but unknown. Let us define its estimate as w_i and the respective weight estimation error as

$$\widetilde{w}_{i,k} = w_{i,k} - w_i^* \tag{2.14}$$

Since w_i^* is constant, then $\widetilde{w}_{i,k+1} - \widetilde{w}_{i,k} = w_{i,k+1} - w_{i,k}$. From (2.9) three possible models of RHONN can be derived:

- *Parallel model.* In this configuration, the feedback connections of the NN come from the NN outputs.

$$x_{i,k+1} = w_i^\top \varphi_i(x_k, u_k) \tag{2.15}$$

- *Series-Parallel model.* In this configuration, the feedback connections of the NN are taken from the real plant.

$$x_{i,k+1} = w_i^\top \varphi_i(\chi_k, u_k) \tag{2.16}$$

- *Feedforward model (HONN).* In this configuration, the connections of the NN come from the input signals.

$$x_{i,k+1} = w_i^\top \varphi_i(u_k) \tag{2.17}$$

where x_k is the neural network state vector, χ_k is the plant state vector and u_k is the input vector to the neural network.

2.3 The EKF Training Algorithm

The best well-known training approach for recurrent neural networks (RNN) is the backpropagation through time learning [22]. However, it is a first order gradient descent method and hence its learning speed can be very slow [10]. Recently, Extended Kalman Filter (EKF) based algorithms have been introduced to train neural networks [4, 18], with improved learning convergence [10]. The EKF training of neural networks, both feedforward and recurrent ones, has proven to be reliable and practical for many applications over the past fifteen years [4]. With the EKF based algorithm, learning convergence and robustness are guaranteed as explained in [18].

The training goal is to find the optimal weight values which minimize the prediction error. The EKF-based training algorithm is described for each i-th neuron by [6]:

$$\begin{aligned} K_{i,k} &= P_{i,k} H_{i,k} M_{i,k} \\ w_{i,k+1} &= w_{i,k} + \eta_i K_{i,k} e_{i,k} \\ P_{i,k+1} &= P_{i,k} - K_{i,k} H_{i,k}^{\top} P_{i,k} + Q_{i,k} \end{aligned} \tag{2.18}$$

with

$$M_{i,k} = \left[R_{i,k} + H_{i,k}^{\top} P_{i,k} H_{i,k}\right]^{-1} \tag{2.19}$$

$$e_{i,k} = \chi_{i,k} - x_{i,k} \tag{2.20}$$

$$i = 1, 2, \cdots, n$$

where $P_i \in \Re^{L_i \times L_i}$ is the prediction error associated covariance matrix, $e_{i,k}$ is the respective identification error, $w_i \in \Re^{L_i}$ is the weight (state) vector, L_i is the total number of neural network weights, $\chi_i \in \Re$ is the i-th plant state component, $x_i \in \Re$ is the i-th neural state component, η_i is a design parameter, $K_i \in \Re^{L_i}$ is the Kalman gain matrix, $Q_i \in \Re^{L_i \times L_i}$ is the state noise associated covariance matrix, $R_i \in \Re$ is the measurement noise associated covariance matrix, $H_i \in \Re^{L_i}$ is a matrix, for which each entry (H_{ij}) is the derivative of one of the neural network output, (x_i), with respect to one neural network weight, (w_{ij}), as follows

$$H_{ij,k} = \left[\frac{\partial x_{i,k}}{\partial w_{ij,k}}\right]^{\top}_{w_{i,k}=w_{i,k+1}} \tag{2.21}$$

where $i = 1, ..., n$ and $j = 1, ..., L_i$. As an additional parameter, we introduce the rate learning η_i such that $0 \le \eta_i \le 1$. Usually P_i, Q_i and R_i are initialized as diagonal matrices, with entries $P_i\,(0)$, $Q_i\,(0)$ and $R_i\,(0)$, respectively. We set Q_i and R_i fixed. It is important to note that $H_{i,k}$, $K_{i,k}$ and $P_{i,k}$ for the EKF are bounded [21]. Therefore, there exist constants $\bar{H}_i > 0$, $\bar{K}_i > 0$ and $\bar{P}_i > 0$ such that:

$$\begin{aligned} \|H_{i,k}\| &\le \bar{H}_i \\ \|K_{i,k}\| &\le \bar{K}_i \\ \|P_{i,k}\| &\le \bar{P}_i \end{aligned} \tag{2.22}$$

It is worth to notice that L_i refers to the number of high order connections and the dimension of the weight vector.

Remark 2.9 The EKF algorithm is used only to train the neural network weights which become the states to be estimated by the EKF.

Remark 2.10 The neural network approximation error vector ϵ_{z_i} is bounded. This is a well-known NN property [3].

2.4 Optimal Control Introduction

This section closely follows [19]. First, we give briefly details about optimal control methodology and their limitations. Let consider the discrete-time affine in the input nonlinear system:

$$\chi_{k+1} = f(\chi_k) + g(\chi_k)u_k, \quad \chi(0) = \chi_0 \tag{2.23}$$

where $\chi_k \in \Re^n$ is the state of the system, $u_k \in \Re^m$ is the control input, $f(\chi_k) : \Re^n \to \Re^n$ and $g(\chi_k) : \Re^n \to \Re^{n\times m}$ are smooth maps, the subscript $k \in \mathbb{Z}^+ \cup 0 = \{0, 1, 2, \ldots\}$ stands for the value of the functions and/or variables at the time k. We consider that $\bar{\chi}$ is an isolated equilibrium point of $f(\chi) + g(\chi)\bar{u}$ with $\bar{u}$ constant; that is, $f(\bar{\chi}) + g(\bar{\chi})\bar{u} = \bar{\chi}$. Without loss of generality, we consider $\bar{\chi} = 0$ for an $\bar{u}$ constant, $f(0) = 0$ and $rank\{g(\chi_k)\} = m\ \forall \chi_k \neq 0$.

The following meaningful cost function is associated with system (2.23):

$$\mathscr{J}(\chi_k) = \sum_{n=k}^{\infty}(l(\chi_n) + u_n^\top R(\chi_n)u_n) \tag{2.24}$$

where $\mathscr{J}(\chi_k) : \Re^n \to \Re^+; l(\chi_k) : \Re^n \to \Re^+$is a positive semidefinite[1] function and $R(\chi_k) : \Re^n \to \Re^{m\times m}$ is a real symmetric positive definite[2] weighting matrix. The meaningful cost functional (2.24) is a performance measure [9]. The entries of $R(\chi_k)$ may be functions of the system state in order to vary the weighting on control efforts according to the state value [9]. Considering the state feedback control approach, we assume that the full state χ_k is available.

Equation (2.24) can be rewritten as

$$\begin{aligned}\mathscr{J}(\chi_k) &= l(\chi_k) + u_k^\top R(\chi_k)u_k + \sum_{n=k+1}^{\infty} l(\chi_n) + u_n^\top R(\chi_n)u_n \\ &= l(\chi_k) + u_k^\top R(\chi_k)u_k + \mathscr{J}(\chi_{k+1})\end{aligned} \tag{2.25}$$

where we require the boundary condition $\mathscr{J}(0) = 0$ so that $\mathscr{J}(\chi_k)$ becomes a Lyapunov function [1, 20]. The value of $\mathscr{J}(\chi_k)$, if finite, then it is a function of the initial state χ_0. When $\mathscr{J}(\chi_k)$ is at its minimum, which is denoted as $\mathscr{J}^*(\chi_k)$, it is named the optimal value function, and it could be used as a Lyapunov function, i.e., $\mathscr{J}(\chi_k) \triangleq V(\chi_k)$.

From Bellman's optimality principle [2, 11], it is known that, for the infinite horizon optimization case, the value function $V(\chi_k)$ becomes time invariant and satisfies the discrete-time (DT) Bellman equation [1, 2, 15]

[1]A function $l(z)$ is positive semidefinite (or nonnegative definite) function if for all vectors z, $l(z) \geq 0$. In other words, there are vectors z for which $l(z) = 0$, and for all others z, $l(z) \geq 0$ [9].

[2]A real symmetric matrix R is positive definite if $z^\top Rz > 0$ for all $z \neq 0$ [9].

$$V(\chi_k) = \min_{u_k} \{l(\chi_k) + u_k^\top R(\chi_k) u_k + V(\chi_{k+1})\} \tag{2.26}$$

where $V(\chi_{k+1})$ depends on both χ_k and u_k by means of χ_{k+1} in (2.23). Note that the DT Bellman equation is solved backward in time [1]. In order to establish the conditions that the optimal control law must satisfy, we define the discrete-time Hamiltonian $\mathscr{H}(\chi_k, u_k)$ [7] as

$$\mathscr{H}(\chi_k, u_k) = l(\chi_k) + u_k^\top R(\chi_k) u_k + V(\chi_{k+1}) - V(\chi_k). \tag{2.27}$$

The Hamiltonian is a method to include the constraint (2.23) for the performance index (2.24), and then, solving the optimal control problem by minimizing the Hamiltonian without constraints [11]. A necessary condition that the optimal control law u_k should satisfy is $\frac{\partial \mathscr{H}(\chi_k, u_k)}{\partial u_k} = 0$ [9], which is equivalent to calculate the gradient of (2.26) right-hand side with respect to u_k, then

$$\begin{aligned} 0 &= 2R(\chi_k)u_k + \frac{\partial V(\chi_{k+1})}{\partial u_k} \\ &= 2R(\chi_k)u_k + g^\top(\chi_k)\frac{\partial V(\chi_{k+1})}{\partial \chi_{k+1}} \end{aligned} \tag{2.28}$$

Therefore, the optimal control law is formulated as

$$u_k^* = -\frac{1}{2} R^{-1}(\chi_k) g^\top(\chi_k) \frac{\partial V(\chi_{k+1})}{\partial \chi_{k+1}} \tag{2.29}$$

with the boundary condition $V(0) = 0$; u_k^* is used when we want to emphasize that u_k is optimal. Moreover, if $\mathscr{H}(\chi_k, u_k)$ has a quadratic form in u_k and $R(\chi_k) > 0$, then

$$\frac{\partial^2 \mathscr{H}(\chi_k, u_k)}{\partial u_k^2} > 0$$

holds as a sufficient condition such that optimal control law (2.29) (globally [9]) minimizes $\mathscr{H}(\chi_k, u_k)$ and the performance index (2.24) [11].

Substituting (2.29) into (2.26), we obtain the discrete-time Hamilton-Jacobi-Bellman (HJB) equation described by

$$\begin{aligned} V(\chi_k) &= l(\chi_k) + V(\chi_{k+1}) \\ &\quad + \frac{1}{4}\frac{\partial V^\top(\chi_{k+1})}{\partial \chi_{k+1}} g(\chi_k) R^{-1}(\chi_k) g^\top(\chi_k) \frac{\partial V(\chi_{k+1})}{\partial \chi_{k+1}} \end{aligned} \tag{2.30}$$

which can be rewritten as

$$0 = l(\chi_k) + V(\chi_{k+1}) - V(\chi_k) + \frac{1}{4}\frac{\partial V^\top(\chi_{k+1})}{\partial \chi_{k+1}} g(\chi_k) R^{-1}(\chi_k) g^\top(\chi_k) \frac{\partial V(\chi_{k+1})}{\partial \chi_{k+1}} \tag{2.31}$$

Solving the HJB partial-differential equation (2.31) is not straightforward; this is one of the main disadvantages of discrete-time optimal control for nonlinear systems. To overcome this problem, we propose the inverse optimal control.

Due to the fact that inverse optimal control is based on a Lyapunov function, we establish the following definitions and theorems:

Definition 2.11 A function $V(\chi_k)$ satisfying $V(\chi_k) \to \infty$ as $\|\chi_k\| \to \infty$ is said to be radially unbounded.

Theorem 2.12 *The equilibrium $\chi_k = 0$ of (2.23) is globally asymptotically stable if there is a function $V : \Re^n \to \Re$ such that (I) V is a positive definite function, radially unbounded, and (II) $-\Delta V(\chi_k)$ is a positive definite function, where $\Delta V(\chi_k) = V(\chi_{k+1}) - V(\chi_k)$.*

Theorem 2.13 *Suppose that there exists a positive definite function $V : \Re^n \to \Re$ and constants $c_1, c_2, c_3 > 0$ and $p > 1$ such that*

$$\begin{aligned} c_1 \|\chi_k\|^p \leq V(\chi_k) \leq c_2 \|\chi_k\|^p \\ \Delta V(\chi_k) \leq -c_3 \|\chi_k\|^p, \quad \forall k \geq 0, \quad \forall \chi_k \in \Re^n. \end{aligned} \tag{2.32}$$

Then $\chi_k = 0$ is an exponentially stable equilibrium for system (2.23). Clearly, exponential stability implies asymptotic stability. The converse is, however, not true.

Definition 2.14 Let $V(\chi_k)$ be a radially unbounded function, with $V(\chi_k) > 0$, $\forall \chi_k \neq 0$. and $V(0) = 0$. If for any $\chi_k \in \Re^n$, there exist real values u_k such that

$$\Delta V(\chi_k, u_k) < 0 \tag{2.33}$$

where the Lyapunov difference $\Delta V(\chi_k, u_k)$ is defined as $V(\chi_{k+1}) - V(\chi_k) = V(f(\chi_k) + g(\chi_k)u_k) - V(\chi_k)$. Then $V(\bullet)$ is said to be a "discrete-time Control Lyapunov Function" (CLF) for system (2.23).

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