

Unified Logical Analysis in Robots' CNS Based on N-Tuple Algebra

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Abstract *Objective* A robot's behavior in a complex environment is largely determined by the ability to perform complex logical analysis of the current situation. Such analysis uses both deductive and non-deductive methods (defeasible reasoning, hypothesis, abductive conclusion, and so on). The aim of this publication is to describe approaches to mathematical modeling of the unified logical analysis within the central nervous system of a robot taking into account peculiarities of human reasoning where imaginative schemas play a significant role. *Results* A mathematical model is proposed for a generalized logical analysis on the basis of n -tuple algebra. This model is distinguished with representation of inference as a cognitive image-schematic structure "container", which mathematically corresponds to the inclusion relation between structures of n -tuple algebra involved in reasoning. *Practical significance* The proposed methods and algorithms of generalized logical analysis can be used for formation of intelligent robot control systems.

Keywords Deductive and defeasible reasoning • Abductive conclusion • Central nervous system of a robot • Cognitive semantics • N -tuple algebra

1 Introduction

A generalized logical analysis means a combination of deductive and non-deductive techniques to analyze systems and reasoning, considered from some unified theoretical positions. Below, it is supposed that a model of logical human thinking is

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used for constructing a mathematical model for logical analysis in the central nervous system (CNS) of a robot.

Usually in the literature on artificial intelligence, non-deductive methods (defeasible reasoning, argumentation, hypothesis, abduction, etc.) are presented as something loosely related to deduction. As a rule, their theoretical foundation is based on special (non-classical) logics and a specifically introduced terminology [1–4], which sometimes makes it difficult to understand the essence of logical analysis and even narrows possibilities and areas of its applications. Closer ties can be determined between the objectives of deductive and non-deductive analysis by using some schemas of the cognitive approach [5]. One of them is the “container” schema that provides a good example of Euler/Venn diagrams. In mathematics, this schema corresponds to the inclusion relation upon sets. In this chapter, we consider a possibility to use such a schema for building not only a logical inference theory, but also some methods to analyze defeasible reasoning, including abduction.

Researches on cognitive semantics [6, 7] have shown that a person think with the help of imaginative schemas rather than according to the laws of formal logic. As a result, some of his obtained proofs become much shorter than those presented in the style of predicate logic. This is not surprising when you consider that “imaginative schemas” provide for some seemingly primitive, but at the same time sufficiently rigorous, methods to analyze reasoning. This refers to the “naive” set theory (called algebra of sets in modern interpretation) based on the inclusion relation. Started in the early XX century, the debate about foundations of mathematics has undermined the confidence of many mathematicians to this theory because some paradoxes were discovered in its basics. Though it became clear later that the paradoxes resulted not from objective reasons, but from errors of reasoning (in particular, from the ambiguity of understanding the membership relation in the formulation of Russell’s paradox) [8], the credibility of algebra of sets was never fully restored.

Another argument against algebra of sets as an instrument of logical analysis was that its apparatus (in particular, Euler/Venn diagrams) was used mainly to analyze relatively simple reasoning such as syllogisms and propositional calculus problems [9, 10]. That is why opportunities to apply it for many of the more complex models and for artificial intelligence systems were not even discussed. This difficulty has been overcome with the development of n -tuple algebra (NTA). On the one hand, NTA is isomorphic to algebra of sets, but at the same time NTA is applicable for processing sophisticated structures. Moreover, NTA occurred to be a useful tool for modeling and analysis of defeasible reasoning that includes abduction as well [11].

NTA is an algebra of arbitrary n -ary relations based on properties of the Cartesian product of sets. Connections between NTA and logical calculi are reflected in the fact that NTA objects model truth scopes of logical formulas, NTA operations correspond to the logical connectives, and the inclusion relation upon NTA structures is fully consistent with respect to deducibility relation in logical calculi.

Traditionally, deductive analysis and the logical inference in particular is considered from the point of view of the theory of formal systems, where concluding a consequence or verifying its correctness is done by means of serial application of

certain inference rules to a set of logical formulas that play the role of premises (or axioms) for a reasoning system. NTA can implement deductive analysis not only by using the inference rules (including the resolution method), but also by checking correctness of the inclusion relation between NTA structures, which represent logical formulas. In some cases, this method not only allows to simplify the task of obtaining consequences with certain properties (for example, a consequence with a specific composition of meaningful variables), but also to find a closer connection between deductive and non-deductive methods of analysis. In particular, abductive conclusion can be obtained in NTA by running certain algorithms to adjust structures, in which the inclusion relation is violated. In NTA, generation and validation of hypotheses can also be implemented by checking inclusion relations or by constructing projections of NTA structures. These projections have the property to be a superset of the source structures.

NTA structures also comply well with the problem of logical-mathematical implementation of the behavioral processes formation based on analysis of sensations in a robots' CNS [12]. There it was proposed to integrate robots sensors into some groups forming the robots humanlike sense organs: sight, hearing, smell, taste, etc. in the form of sets $X, Y, Z, U, \dots$. In each of the introduced sets, we can distinguish some subsets, which characterize the properties of an observed or studied object. Let they be $X_i \subset X, Y_i \subset Y, Z_i \subset Z, U_i \subset U, \dots$. The totality of such subsets depends on the set of sensors employed in the senses of a particular robot. For instance, the following subsets can be assigned to its sight: X_1 —the contour of image; X_2 —the size of the image; X_3 —the brightness of the image; X_4 —the color of the image; X_5 —the distance to the object. Also, some sets can represent the situations within which the robots will interact, as well as their possible responses or actions. Then the formalization task for building a CNS of robots' senses can be considered in terms of NTA, if we use the mentioned sets as attributes, and the interconnections among them will be displayed as relations [13].

In NTA, arbitrary n -ary relations can be expressed as four types of matrix-like structures called **NTA objects**. Every NTA object is immersed into a certain space of **attributes**. **Domain** is a set of values of an attribute.

Names of NTA objects contain an identifier followed by a sequence of attributes names in square brackets; these attributes determine the **relation diagram**, in which the NTA object is defined. For example, $R[XYZ]$ denotes an NTA object defined within the space of attributes X, Y, Z , and $R[XYZ] \subseteq X \times Y \times Z$.

2 Deductive Analysis from the Perspective of the Cognitive Schema “Container”

Deductive analysis (DA) includes solving the following two problems.

The first task of DA is validating a specific consequence B for some given pre-mises A_i ;

The second task of DA is deriving possible consequences from some given premisses A_i with consideration of certain semantic constraints, for instance, presence of given variables or their combinations in a consequence, deriving a consequence with the minimal number of significant variables, etc.

In propositional and predicate calculi, the first task is mainly used, but the second task is no less important in terms of practical applications. They are solved by means of inference rules. Systems of these rules vary in different scientific schools, but anyway, it is difficult to predict the optimal procedure for applying these rules in advance. In many cases, non-optimal order of application of the rules can lead to an ample increase in algorithm execution time.

Solving such problems in NTA is not based on inference rules only, but also on certain types of algorithms that use some generalized operations and verify inclusions of sets into other ones [11, 13]. In many cases, these algorithms are more efficient than conventional algorithms based on the resolution method.

Formulas of classical logic, representing premises and consequences, are expressed as NTA objects, with which you can perform generalized operations and check generalized relations of equality and inclusion. Consider an example.

Example 1 Suppose a robot must find an object located in one of the three rooms, with one of the rooms it may not enter (there is a danger), and one empty room. The following conditions (premises) are given that can be used to solve the problem.

- (1) the first room is not dangerous, the second room is not empty;
- (2) the third room is not dangerous, the second room is not empty.

We also know that one of the rules is true, and the other rule is false. Is it possible to locate the desired object on the basis of these data?

Let A denotes the statement “this room contains the desired object”, B is the phrase “the room is empty”, and C means “there is a danger in the room”. To model this problem by NTA, let us use names of rooms R_1, R_2, R_3 as attributes; each of the rooms may be in one of the three permitted situations from the set $\{A, B, C\}$. Then conditions can be expressed by C - n -tuples:

- (1) $T_1[R_1R_2] = [\{A, B\}\{A, C\}]$;
- (2) $T_2[R_2R_3] = [\{A, C\}\{A, B\}]$.

To solve the problem, it is necessary to consider two cases: (1) the first condition is true, and the second is false; (2) the first condition is false, and the second is true.

Let us consider the first case. If T_2 is false, then $\overline{T_2} =]\{B\}\{C\}[$ is true. The solution of the problem can be found by calculation the generalized intersection of true conditions:

$$\begin{aligned} T_1 \cap_G \overline{T_2} &= [\{A, B\}\{A, C\}] \cap_G]\{B\}\{C\}[\\ &= [\{A, B\}\{A, C\}] *]\{B\}\{C\}[\\ &= [\{A, B\}\{A, C\}\{C\}]. \end{aligned}$$

The resulting C - n -tuple contains 4 possible placement options (it becomes clear by calculating the Cartesian product $\{A, B\} \times \{A, C\} \times \{C\}$), but the conditions of the problem require for different situations in different rooms, so the only option (B, A, C) is correct. Thus, the desired object is located in the second room. Similar results were obtained when testing the second case.

The proposed transition to algebraic representations becomes clear when you consider that NTA objects model the truth domains of logical formulas. Suppose that some given premises A_i and an expected consequence C are possible to express as NTA structures. Then the validation algorithm for the consequence C from the given premises A_i has to calculate their generalized intersection and check the generalized inclusion [13]:

$$(A_1 \cap_G \dots \cap_G A_n) \subseteq_G C \quad (1)$$

It was found that all the inference rules in mathematical logic and natural calculus after Gentzen comply with (1) after converting premises and conclusions into NTA structures. The validation algorithm based on the relation (1) is proved to be correct [13]. This relation implements a “container” schema and results in some statements unusual for the traditional theory of logical inference.

- (1) There exists a **minimal consequence** for a given set of premises. It is equal to the generalized intersection of the premises: $A = A_1 \cap_G \dots \cap_G A_n$. The concept of the minimal consequence allows to calculate the number of all possible consequences in systems with finite ranges of variables values [11]. It is equal to 2^N where $N = |U| - |A|$, $|U|$ is the number of elementary n -tuples in the universe, and $|A|$ is the number of elementary n -tuples in the minimal consequence. NTA provides a method to calculate quantitative characteristics of NTA objects.
- (2) Any superset of a minimal consequence A (i.e. any relation $A \cap_G R$ where R is an arbitrary relation) is a consequence from the given system of premises. Thus, in many cases, **formal logical inference significantly increases the degree of uncertainty comparing to the statements contained in the minimal consequence**. As a result, if a minimal consequence contains a statement C that has no alternatives, it is possible to find such a statement R in another correct consequence that will contain an alternative to C . For example, if a minimal consequence A contains the statement “This obstacle is insurmountable by bypassing it from the left” and does not contain the alternative statement “This obstacle is surmountable by bypassing it from the left with difficulty”, the statement “This obstacle is insurmountable by bypassing it from the left or surmountable with difficulty” will be a correct consequence from A according to the rules of logical calculi.
- (3) The reasonability of increasing uncertainty in a consequence by addition arbitrary relations to A is justified only in cases where the given system of premises results in a formula with a reduced number of variables compared to the overall composition of the variables in the premises. It is proved in NTA

that this way we obtain an **inductive generalization of a minimal consequence** [11]. Thus, a consequence with a reduced variables composition is formed by adding some combination of values for “disappearing” variables, which were missed in a minimal consequence. The solution to this problem in NTA is obtained by using a simple algorithm to calculate projections of the resulted formula. Conventional inference engines propose no simple algorithms to solve this task. Such NTA algorithms can be used to solve the second problem of DA as well, namely to derive possible consequences with specific properties from a given set of premises A_i . Consider an example.

Example 2 Suppose that one of the two rooms contains an object that can be a danger to a robot or to be safe. It is necessary to find out whether the robot can enter at least one of these rooms, if the following is known: (1) if the first room is empty, the object is safe; (2) if the object is dangerous, the second room is empty; (3) if the first room is not empty, then the second room is empty. To solve the problem, you need to answer the question whether these conditions infer that the object is safe.

We introduce the notation as follows. Let x means “the first room is empty”, y stands for “the second room is empty”, z denotes “the object is safe”. Then the premises A_1, A_2, A_3 can be expressed by the logical formulas:

$$A_1 : x \rightarrow z; \quad A_2 : \neg z \rightarrow \neg y; \quad A_3 : \neg x \rightarrow y.$$

The intended consequence of these premises should be z . To verify this by using NTA methods, we will consider attributes as variables X, Y, Z with two values: 1 (literal itself) or 0 (negation of a literal). After substituting, we obtain expressions for the premises and the consequence C :

$$A_1[XZ] =]\{0\} \{1}\}; A_2[YZ] =]\{0\} \{1}\}; A_3[XY] =]\{1\} \{1}\}; C[Z] =]\{1}\}.$$

Using the NTA methods, we calculate the minimal consequence:

$$A[XYZ] = A_1[XZ] \cap_G A_2[YZ] \cap_G A_3[XY] = \begin{bmatrix} \{1\} & * & \{1\} \\ \{0\} & \{1\} & \{1\} \end{bmatrix}.$$

By adding dummy attributes, $C[Z]$ is possible to rewrite as occurs when the relation

$$C[XYZ] = [* * \{1\}].$$

It is easy to verify that $A[XYZ] \subseteq C[XYZ]$, which proves the correctness of the consequence and, accordingly, the possibility for the robot to enter into any room.

The same answer can be obtained after solving this problem by the method of calculating projections of the minimal consequence. In this case, when the minimal

consequence $A[XYZ]$ is represented as a C -system, it is sufficient to eliminate attributes X and Y from the matrix representation in order to get the same result.

Let us consider the second task of DA. According to (1), a totality of consequences $\{C_j\}$ can be inferred from the premises A_i by using the following relationship: any NTA object C_j , which satisfies the relation $A \subseteq C_j$ where A is the minimal consequence of the premises A_i , is their correct consequence. It is clear that it makes no sense to infer all possible consequences from a given system of premises; besides, their number was proved to grow exponentially [11]. Only consequences having some pre-defined properties are of interest actually. In particular, the shortened variables composition as compared with the minimal consequence can be one of such properties. Example 2 illustrates this case.

For constructing such consequences, NTA proposes calculating projections of the minimal consequence. Let us consider this method in more detail. Suppose a minimal consequence A was obtained in the space of attributes $X_1, \dots, X_n$. In NTA, any projection of an NTA object is proved to be its superset. Therefore, an arbitrary projection of the minimal consequence A is a correct consequence. Such projections are easy to calculate, if the NTA object is a C - n -tuple or a C -system [13]. Then it is enough to eliminate the corresponding attributes from this NTA object, i.e. to remove these attributes from the relation diagram and delete the columns with these attributes from the matrix representation.

In some cases, the minimal consequence is easier to obtain as a D -system [11]. To calculate its projection then, it is necessary to convert it to the C -system. There are NTA algorithms designed for such transformations. Their disadvantage is that they have exponential computational complexity in general. Although for systems of small dimension and some special cases investigated in [13], their complexity is relatively low.

Calculations of projections for a minimal consequence do not always lead to a positive result since many NTA objects have projections that do not contain useful information. These are projections represented by a complete set of all combinations of attributes. For the C -system $C[XYZ] = \begin{bmatrix} \{1\} & \{0\} & * \\ * & \{1\} & \{1\} \end{bmatrix}$ as an example, such projections are all projections with a single attribute. At the same time, all projections with two attributes do not include complete sets of elementary n -tuples and therefore they can be used as consequences. For instance, the projection $[XY]$ (first and second columns) does not contain a complete set of elementary n -tuples and corresponds to the logical formula $x \vee y$.

The literature on mathematical logic mainly uses the first problem of DA in examples of logical inference when it is needed to validate a given consequence, and such consequences usually contain a reduced composition of variables. However, there are no effective methods to obtain such consequences, since it is only possible for a matrix representation of logical formulas.

3 Analysis of Hypotheses and Abduction Based on Calculation of Projections

Abduction can be represented as a search of explanations for “unexpected” results, which traditionally should not take place in a given standard situation [14]. In a reasoning system, abduction can be considered as an explanation for appearance of not clearly formulated facts or arguments, which refute a formally correct consequence of a given premises system. Hypotheses are used as explanations, whereby the abduction is a variant of revised reasoning. In many publications, in [1–3] for instance, revised reasoning is associated with non-classical logics, in particular, with non-monotonic logic. Thus, modeling of revised reasoning allows violations of the laws of classical logic and, in particular, the laws of algebra of sets.

To model deductive and revised reasoning in NTA, we have proposed a general theoretical approach that, firstly, does not violate any laws of algebra of sets, and, secondly, uses relation (1) obtained in the framework of NTA. However, while in deductive analysis, the relation (1) serves a measure of formal correctness for a consequence, violations of (1) indicate a necessity to proceed to non-deductive analysis during analysis of hypotheses and revised reasoning. Moreover, NTA-characteristic matrix properties provide an effective search for necessary consequences or hypotheses with predetermined features in the course of applying both deductive and non-deductive methods of analysis.

Analysis of the arguments, on which many authors offer to abandon classical logic when modeling revised reasoning, has shown that so-called violations of the monotony of logical inference in practical reasoning result actually from violations of some implicitly given constraints [11]. In the theoretical foundations of NTA, such violations of constraints revealed by using certain methods are called **collisions** [13].

An example of a collision is as follows. Suppose, from certain given premises, we concluded two statements: (a) “all objects in this room are dangerous” and (b) “all objects in this room are not dangerous”. There is no formal contradiction in them, since a formal negation of the first statement is the statement “some of objects in this room are not dangerous” rather than the statement (b). At the same time, the statements (a) and (b) infer that no objects exist in the room. From a formal point of view, this is not a contradiction, but a situation in which a (sometimes implicit) constraint is violated. In this case, the constraint is the existence of any objects in the room. Violation of this constraint results in a collision during reasoning. Let us consider some types of collisions.

The term “collision” was initially used by Kulik [8] for analysis of syllogistics-like reasoning, where two kinds of formal collisions were defined, namely.

a **paradox collision** arises if premises infer a statement like “No A are A ” ($A \supset \bar{A}$), that is, the volume of the term A is empty;

a **cycle collision** occurs when the relation $A \subseteq B \subseteq \dots \subseteq A$ can be deduced from a system of sets; this means that the terms contained in the cycle are equal.

The collisions listed above can be detected without taking the subject domain into account; this is why we named them **formal collisions**.

The third kind of collisions is not a formal one; it features a situation when some consequences do not match some indisputable facts or justified statements. We call this collision an **inadequacy collision**.

Unlike a logical contradiction which expresses an absolute degeneration of premises, collisions can have opposite interpretations in different cases. In other words, a collision, as opposed to a contradiction, is semantically dependable. For example, within one system, the equality $A = \emptyset$ means an absence of the object A that is necessary for existence of the system, and in another system this equality specifies a status of the object A . The first case requires changing the premises while the second case provides a new useful datum and amends our knowledge.

Let us adduce an example of collisions upon NTA structures.

Example 3 A closed box contains articles described by their shapes (a sphere or a cube), colors (white or red) and materials (plastic or wood) It is necessary to determine which types of objects can be in a box, if it is known: (1) all balls are red; (2) all the wooden objects are painted white; (3) all plastic items are not balls.

Let S stands for balls, W denotes white items, and P means plastic items. We write the conditions of the problem in the language of propositional calculus:

- (1) $A_1 = S \rightarrow \bar{W}$;
- (2) $A_2 = \bar{P} \rightarrow W$;
- (3) $A_3 = P \rightarrow \bar{S}$.

Now we express premises in NTA terms, matching variables S, W, Z with attributes X_S, Y_W, Z_P respectively:

$$A_1[X_S Y_W] =]\{0\}\{0}[; A_2[X_P Y_W] =]\{1\}\{1}[; A_3[X_P Y_S] =]\{0\}\{0}[$$

$$A[X_S Y_W Z_P] = A_1 \cap_G A_2 \cap_G A_3 = \begin{bmatrix} \{0\} & \{0\} & \emptyset \\ \emptyset & \{1\} & \{1\} \\ \{0\} & \emptyset & \{0\} \end{bmatrix}.$$

After converting A into the C -system we find: $A[X_S Y_W Z_P] = \begin{bmatrix} \{0\} & \{1\} & * \\ \{0\} & \{0\} & \{1\} \end{bmatrix}.$

The first column of the resulting C -system contains only a singleton component $\{0\}$. This is the sign of the paradox collision $S \supset \bar{S}$. Indeed, the expression $(S \supset \bar{S}) = \bar{S} \vee \bar{S} = \bar{S}$ corresponds to the C - n -tuple $R[X_S Y_W Z_P] = [\{0\} * *]$ in NTA, and therefore, the collision $S \supset \bar{S}$ is a consequence of the initial system of premises.

Identification of the paradox collision in the analyzed reasoning system means that there are no balls in the box, and all of the items have a cube shape.

Collisions can model the following cases.

- (1) When new knowledge is input, some different attributes become identical in composition of their elements, which contradicts to the semantics of the system.

- (2) A discrepancy occurs between the obtained results and some restrictions that are hard to formalize and they are described in task settings. For instance, a modeled system can contain some limitations expressed as relations, which must not appear in consequences.

It is not easy to foresee all possible kinds of collisions; they can well be unique for some logical systems. We propose the following brief definition for the term “collision” in NTA.

Collisions are situations recognized as violations of some formally expressed rules and/or limitations, which control consistency and meaning content of a logical system. In many cases, the collisions detection means semantic incorrectness of reasoning.

The concept of collisions allows for a formal definition of hypotheses in NTA. Let us suppose that a system of premises expressed as NTA objects $A_1, \dots, A_n$ is given and the NTA object $A = A_1 \cap_G \dots \cap_G A_n$ is calculated.

A certain formula H is called a **hypothesis**, if $A \subseteq_G H$ is false. Otherwise, H is a consequence according to (1).

The hypothesis is **correct**, if:

- (1) $A \setminus_G H \neq \emptyset$, i.e., the new system of premises is consistent;
- (2) the object $H \cap_G A$ contains no collisions.

Processing of hypotheses is an intrinsic part of abduction. Let us now formally define abduction.

If B is an estimated consequence of the premises $A_1, \dots, A_n$ and the statement $A \subseteq_G B$ is known to be false (once again, $A = A_1 \cap_G \dots \cap_G A_n$), then a hypothesis H is an admissible **abductive conclusion** when the two following conditions are met:

- (1) H is a correct hypothesis (i.e. $A \subseteq_G B$ is false) and $H \cap_G A$ is not empty;
- (2) $(H \cup_G A) \subseteq_G B$, that is, adding H into the system of premises results in deducibility of the estimated consequence B .

Let us consider the condition

$$(H \cup_G A) \subseteq_G B \quad (2)$$

If the anticipated consequence B is known, then the need for a hypothesis arises, if $A \setminus B = R \neq \emptyset$, i.e. the formal condition for a consequence (namely, $A \subseteq_G B$) is not met. On the basis of these relations, we can construct the following algorithm.

3.1 Search Algorithm for Abductive Conclusions

- Step 1 Calculate the “remainder” $R = A \setminus_G B$;
- Step 2 Build an intermediate object R_i , for which $R \subseteq_G R_i$ is true;
- Step 3 Calculate $H_i = \bar{R}_i$ (R_i can now be denoted by $\bar{H}_i$);

Step 4 Calculate $H_i \cap_G A$ and check it for presence of collisions; if they are detected, return to Step 2, otherwise *End*.

The Step 2 of this algorithm allows to obtain R_i by means of methods to calculate projections of the object R discussed in the previous section. Thus we can form hypotheses with predetermined properties, in particular, the hypothesis with a certain pre-defined composition of attributes.

Let us give an example.

Example 4 There are 3 rooms, the first and the third ones may be empty or contain some objects, the second room is not empty, but a dangerous object can be inside it. Then opening of this room leads to a big trouble. However, this room can contain an object that a robot needs in accordance with its job. It is also given that: (1) if the first room is not empty, then the second room contains a useful (desired) object, or the first room contains a dangerous object; (2) if the second room contains a dangerous object, the first room is empty and the third room is not empty; (3) if the third room is empty, then either the object in the second room is useful or the object in the first room is dangerous. Can we conclude that the robot can find the desired object in the second room?

To start solving, we express the given statements in the language of propositional calculus. Denotations are as follows: A is “the first room is empty”, B means “the object in the second room is useful”, C states that “the first room contains a dangerous object”, and D stands for “the third room is empty”. By transforming every premise and the question into DNFs, we obtain:

- for the first premise: $A \vee (B \wedge \neg C) \vee (\neg B \wedge C)$;
- for the second premise: $B \wedge (\neg A \wedge D)$;
- for the third premise: $\neg D \vee (B \wedge \neg C) \vee (\neg B \wedge C)$;
- for the question under investigation: B .

To present these formulas as NTA objects, let us use the universe $X_A \times X_B \times X_C \times X_D = \{0, 1\}^4$ where $A = B = C = D = 1$ and $\neg A = \neg B = \neg C = \neg D = 0$. Then premises become C -systems:

$$P_1 = \begin{bmatrix} \{1\} & * & * & * \\ * & \{1\} & \{0\} & * \\ * & \{0\} & \{1\} & * \end{bmatrix}; \quad P_2 = \begin{bmatrix} * & \{1\} & * & * \\ \{0\} & * & * & \{1\} \end{bmatrix};$$

$$P_3 = \begin{bmatrix} * & * & * & \{0\} \\ * & \{1\} & \{0\} & * \\ * & \{0\} & \{1\} & * \end{bmatrix},$$

and the estimated consequence will look like the C - n -tuple $S[X_B] = [\{1\}]$.

Solution of the problem can be found by checking the relation $(P_1 \cap_G P_2 \cap_G P_3) \subseteq_G S[X_B]$. To do this, we calculate

$$P[X_A X_B X_C X_D] = P_1 \cap_G P_2 \cap_G P_3 = \begin{bmatrix} \{1\} & \{1\} & * & \{0\} \\ * & \{1\} & \{0\} & * \\ \{0\} & \{0\} & \{1\} & \{1\} \end{bmatrix}$$

For testing the inclusion, we add the missing attributes into S :

$$S[X_A X_B X_C X_D] = [* \{1\} * *].$$

Then testing shows that the third C -tuple of P is not included in S , so the estimated consequence (the object in the second room is useful) is not deducible yet. To confirm or deny correctness of this consequence, it is required to clarify certain circumstances. The search for such circumstances can be formulated as a search for an abductive conclusion.

Assume that the consequence S is correct. Then we need to find additional hypotheses suitable for addition to the given premises. Let us write intermediate results in the new notation.

$$\text{Intersection of premises equals to } A[X_A X_B X_C X_D] = \begin{bmatrix} \{1\} & \{1\} & * & \{0\} \\ * & \{1\} & \{0\} & * \\ \{0\} & \{0\} & \{1\} & \{1\} \end{bmatrix},$$

and the expected consequence is: $B[X_A X_B X_C X_D] = [* \{1\} * *].$

Next, we use the algorithm.

$$R = A \setminus_G B = \begin{bmatrix} \{1\} & \{1\} & * & \{0\} \\ * & \{1\} & \{0\} & * \\ \{0\} & \{0\} & \{1\} & \{1\} \end{bmatrix} \cap_G [* \{0\} * *] = [\{0\} \{0\} \{1\} \{1\}].$$

You can select any projection of R as R_i . It is reasonable to check the third room, since it cannot contain a dangerous object. Then the projection will be $R[X_D]$ and we obtain $R_i = [* * * \{1\}]$. So, $H_i = \overline{R_i} = [* * * \{0\}]$.

Thus, we can conclude that the visit to the second room will be safe, if inspection of the third room shows it is not empty.

4 Conclusion

Proposed methods of constructing deductive and defeasible reasoning are based on cognitive semantics; they use an algebraic approach to logical inference as well as to generation and validation of hypotheses. Due to the matrix representation of logical structures, it becomes possible to form a set of admissible hypotheses considering some predefined properties. This approach is expected to increase the transparency of reasoning mechanism in the course of modeling of unified logical analysis within a robot's CNS together with providing flexibility in adjusting the reasoning system for solving various practical problems.

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