

# Preface

In standard quantum mechanics, the collapse of the wave function resulting of the interaction of a given system with a measuring apparatus is a clear unsatisfactory assumption. As is well known, this assumption, due to von Neumann, was established around 1932 by means of the so-called reduction (or collapse or projection) postulate. A unitary Schrödinger evolution can never lead to such a collapse and, therefore, at some point, the time evolution of a quantum system undergoing a measurement has to be suspended and replaced by a discontinuous, noncausal, nonlinear, nonunitary (but norm-preserving) and stochastic one. The collapse is postulated to happen in accordance with the well-known Born probability rule, with no dynamical theory behind such a mechanism. It is also very well known that the absence of macroscopic superpositions is at the heart of the measurement problem in quantum mechanics. Another aspect worth mentioning is the emergence of the classical features from the quantum world; in other words, the conversion of a coherent superposition into a mixture. The resulting decoherence process leads to the selection of one of the components of this mixture. This selection of one out of many alternatives still remains obscure. Thus, we still do not understand why the measurement process destroys the linear superposition of the initial states and why macroscopic objects are not found in superposed states. Even more, as asked by Bell, where is the quantum-classical dividing line? Mesoscopic systems are also ideal to address this fundamental question. In brief, too many open questions are even today waiting for some sort of satisfactory response.

Many physicists believe that there is a *measurement problem* in quantum mechanics. Even more, measurement definitely and unavoidably affects the dynamics of the measured system. In the Copenhagen interpretation of quantum theory, the world is divided into *system* and *apparatus*. In the simplest case, the system, which is quantum mechanical, is characterized by a wave function. This interpretation implies that the apparatus that performs and records measurements on the system must be classical. In practice, experimenters are always on the classical side of the line. As pointed out by Bell, the lack of clarity in regard to where the transition between the classical and quantum worlds is located is one aspect of the measurement problem. This problem represents one of the most important

conceptual difficulties in quantum mechanics. The presence of a classical apparatus considerably affects the behavior of the observed quantum system and measurements typically fail to have outcomes of the sort the theory was created to explain. Another conceptual difficulty is that in a system under observation there are many degrees of freedom such that information can be lost in the couplings which may account for dissipation and decoherence. It appears that the measurement problem before alluded is becoming slowly a critical or singular quantum process (converted to a dynamical problem) rather than a conceptual issue. In other words, it is no longer considered as a conceptual or philosophical problem but a typical many-body problem (with all the computational issues involved in its resolution).

A measurement can be seen as a physical process by which the state of a quantum system influences somehow a classical variable of a macroscopic apparatus. This influence is usually described by an interaction between the system of interest (S) and the apparatus (A). Then, the combined quantum system SA has properties that S and A alone do not have and it is said that S and A are entangled by means of quantum correlations due to the quantum nonlocality. It is widely accepted that A can also be considered as an environment. In 1970, Zeh proposed to analyze the measuring process within the context of the theory of open quantum systems and see it as a typical many-body interaction problem. In 1984, Gisin carried out a theoretical connection between them. Furthermore, almost any interaction with an environment produces localization (or reduction) in position space. It is very often alluded that the system can not have a state on its own when interacting with its environment and, therefore, the only description in the standard quantum formalism is by means of the density matrix. The measurement process then implies the conversion of a coherent superposition into a mixture together with the selection of one of the components (or alternatives) of the mixture leading to a definite measurement readout. Thus, this decoherence process leads to the emergence of the classical world (classicality). Decoherence, which is ubiquitous in nature, is extremely effective and virtually impossible to escape (only irreversible when the environment is macroscopic). It is a pure quantum process and has no classical counterpart. Furthermore, entanglement is the key process underlying decoherence. This fundamental process has been and is still being studied from many, many points of view emphasizing its very different aspects and subtleties. The number of articles and books devoted to such an important and fundamental issue is extremely huge; to cite only some of them, we have the Ghirardi–Rimini–Weber model of continuous spontaneous collapse, the entropic-dynamic approach to quantum evolution and the Fisher information based derivation of the fundamental Lagrangians leading to relativistic wave equations. Several stochastic approaches are being used nowadays within the quantum theory which could be classified according to the following scheme: group or semigroup algebra, the influence functional, the restricted path integral, consistent histories, effective wave equations and quantum state diffusion equations. They are mathematically connected. Continuous quantum measurement theory is playing a fundamental role in this context since it provides new aspects and ingredients to the global quantum theory. For example, quantum transitions can be monitored as well as perturbed,

altering the dynamics of the measured system. However, very few studies within the Bohmian framework can be found in literature. This approach should also be considered in the measuring process since the outcome of an experiment is very often the position of a particle. In Bohmian mechanics, particles are assumed to have well-defined positions in configuration space. Moreover, the description provided by this mechanics to the measurement process is entirely compatible with Born's rule in standard quantum mechanics.

Bohmian mechanics, which is also called the de Broglie–Bohm theory, the pilot-wave model, and the causal interpretation of quantum mechanics, was originated in the 1920s by Louis de Broglie, re-discovered and developed in 1952 by David Bohm. Its roots are also in the pioneering works of Madelung's hydrodynamic formulation of quantum mechanics. No one more eloquently than John Bell has championed this theory in recent decades. It should be remembered that Bell's inequalities were inspired by his reaction to the work by Bohm. Bell and Bernstein also argued convincingly that the de Broglie–Bohm interpretation of quantum mechanics should be part of any college curriculum on the subject. Tegmark and Wheeler also considered this theory as one of the most significant elements in the history of quantum mechanics. Comprehensive discussions of Bohmian mechanics can be found everywhere in the literature. This mechanics mainly arose as a result of the unsatisfactory interpretation of standard quantum mechanics, which claimed that the wave function provides the most general and complete physical information about a quantum system. This led to a very exciting, never-ending debate focussed on the completeness of the wave function and the quantum theory of measurement. Within Bohmian mechanics, a quantum system has a well-defined (in space and time) trajectory, namely a quantum or Bohmian trajectory; the evolution of this trajectory is determined by the wave function associated with the system. Quite recently, a revival of the debate about the role played by this mechanics in quantum physics can be found in the specialized literature. There are several groups for whom this theory constitutes the natural framework of quantum mechanics, whereas other groups consider it as an alternative and exact formulation able to characterize, interpret and predict quantum processes, standing on equal footing with the standard theory. In fact, it can also be considered as a picture of quantum mechanics at the same level as those due to Schrödinger, Heisenberg, Dirac (Interaction) and Feynman. As Aharanov says, this theory provides the same experimental predictions than quantum mechanics and it contains additional hidden variables which are nonlocal in time that account for the results of every measurement. In this sense, there is no way to decide from experiment which picture is prevalent. This formulation has also received an important impulse over the last 20 years from different communities, which translates into an impressive and fruitful theoretical development. Furthermore, from our own longstanding experience in the field, Bohmian mechanics can tackle any quantum problem as standard quantum mechanics can do it. Obviously, the amount of time and effort invested by many researchers in standard quantum mechanics completely outweigh that invested in Bohmian mechanics. However, we think that this situation will be corrected in the near future owing to the fact that this theory appears in more and more modern

quantum mechanics books at the introductory level. We are going to deal with quantum or causal trajectories but they must not be confused with the same concept introduced by several authors in optics and quantum state diffusion which can be seen as the realizations of the wave function or state vector considered as an underlying stochastic process in Hilbert space. Unfortunately, the concept of quantum trajectory (stochastic or deterministic) is not unique and there is an important proliferation of quantum stochastic trajectories in different fields of research within the theory of open quantum systems as well as the Bohmian mechanics.

The scope of the book is to tackle such conceptual difficulties mentioned above. In particular, when certain conditions are met, an answer to the alluded challenge posed by Bell on the dividing line between the quantum and classical regimes in a measurement problem is also discussed and proposed. In Bell's opinion, the most important element in quantum theory was that the subjectivity of the orthodox version, the necessary reference to the observer, could be eliminated. The theoretical procedure followed here it is inspired from Mensky's theory about restricted path integrals, combined with the so-called Langevin–Schrödinger equation or Kostin equation (which can be seen as an effective wave equation) within the Bohmian framework. This new approach presents several advantages. First, it provides a causal aspect to the dynamics of the measuring process. Second, the proposed stochastic nonlinear approach is an alternative and complementary view of the continuous quantum measurement. Furthermore, in this context, it is put on equal footing the stochastic character coming from the presence of the environment and the phenomenological contribution coming from the measuring process. Third, the information due to the readout of the measurement is governing the equations of motion of Bohm through the quantum potential which is the nonlocal agent of this theory. Fourth, the correspondence principle, established in terms of the smallness of the quantum potential (that is, when it is approaching a zero value), provides a natural way to the emergence of the classical world. Fifth, the gradual decoherence process due to the interaction with the measuring device is introduced and described in terms of quantum stochastic trajectories; that is, in a continuous way with no collapse of the wave function. And sixth, following a dressing scheme, the quantum (stochastic) trajectory is always split into a classical trajectory plus a quantum or nonlocal term.

With this goal, and in order to be as self-contained as possible, although a minimal background on stochastic processes is assumed, this monograph is divided into four chapters. In Chap. 1, a brief historical description and short and updated account of Bohmian mechanics is provided for closed and open quantum systems; in particular, how open quantum systems and the measurement problem are typically considered in this formalism. This introductory chapter aims at providing the essential tools and background to better understand the rest of the monograph. It is clear that this mechanics is not the only one describing quantum mechanics in terms of trajectories but maybe it is becoming more and more known and widely used.

In Chap. 2, our purpose is to simply show that Bohmian mechanics is a powerful route to bring about new solutions to problems discussed by conventional quantum

mechanical approaches, apart from allowing some striking correspondence between both frameworks. This is not carried out by means of a systematic exposition of conservative quantum processes but following some illustrations of solutions to some key quantum mechanical problems such as, for example, the so-called Ermakov–Bohm invariants, boundary conditions and uncertainty principle in tunneling, the quantum traversal time, Airy wave packets and Airy slits, the detection of inertial and gravitational masses with Airy wave packets, the geometric phase analyzing the Aharonov–Bohm effect and quantum vortices, the reformulation of the Gross–Pitaevskii equation within the hydrodynamical framework and ending with the well-known Caldirola–Kanai Hamiltonian for dissipative processes. In this dissipative scenario, the motion of a free particle, the quantum interference of two wave packets and the dynamics in a linear potential as well as the corresponding of a damped harmonic oscillator (within the underdamped, critically damped and overdamped regimes) are finally analyzed for ulterior references.

In the last decades, many nonlinear extensions of the Schrödinger equation have been proposed in literature either to explore the fundamental aspects of quantum mechanics, with the usual linear theory representing only a limiting case, or to describe open quantum systems. In Chap. 3, one of those nonlinear quantum equations is introduced and widely developed. For the description of nonconservative quantum systems, Kostin formulated in an heuristic way the so-called Schrödinger–Langevin (SL) equation or Kostin equation for the Brownian motion. This equation has been subsequently rederived, improved and extended for its use in numerous applications, mainly without including the noise term. Numerous features of this SL equation can be better revealed within the framework of de Broglie–Bohm (quantum hydrodynamical trajectory formulation) of quantum mechanics resulting the Schrödinger–Langevin–Bohm (SLB) equation. Within this formalism, several dissipative problems are presented and discussed: the Ramsauer–Townsend effect, the tunneling dynamics through a barrier, the plasma fluid formulation and the Lorentz–Abraham (extended electron) equation for a point-charge electron. These two last examples are also discussed in order to see the correspondence between classical and quantum dynamics. Very few applications of this SL equation are devoted to stochastic problems in the literature where the noise term needs to be included. The so-called Bohmian–Brownian motion is introduced in the context of surface diffusion with single adsorbates. An extension to interacting adsorbates is discussed within a simple, phenomenological model. Interestingly enough, this study leads us to quantum anomalous diffusion. Finally, a generalization of the SL equation is proposed for nonlinear dissipation.

Chapter 4 is devoted to the study of continuous or repeated (prolonged in time) measurements on a quantum system. This is a very active field since it is directly connected to the foundations of quantum mechanics, the theory of quantum measurements being one of its main topics. Following a phenomenological approach, a general theory for decoherence in the framework of restricted path integrals (RPI) has been proposed by Mensky. The corresponding propagator is modified according to the information provided by the measurement through the so-called quantum corridors, which correspond to different readouts of the measurement. The

action uncertainty principle is then ruling the corresponding measurement process. The measured system is also considered in this theory as an open system since the back reaction of the environment, considered as a measuring apparatus, is taken into account implicitly. In this Chapter, the RPI formulation is briefly presented and discussed since our approach is inspired from Mensky's procedure. The linear time-dependent Schrödinger equation derived from this formulation is reached for a non-Hermitian (complex) effective Hamiltonian which takes into account the measurement readout. The RPI formulation is equivalent to a master equation, a special case of the Linblad equation, and to a stochastic Schrödinger equation. A nonlinear equation is proposed and discussed, within the Kostin framework, by extending Mensky's approach to be analyzed in terms of Bohmian trajectories, the so-called SLB equation for continuous measurement. When a time-dependent Gaussian shape is assumed for the probability density, soliton-like solutions are first analyzed in order to establish the dividing line between the quantum and classical trajectories, leading to the concept of Bohmian time. Afterwards, the solutions of the SLB equation for continuous measurement are analyzed in terms of mathematical stability for three simple cases, that is, the free particle and the linear and harmonic potentials. In particular, the continuous position measurement is seen that it is no longer governed by a standard continuity equation. The sign of the extra source/sink term appearing in this new equation is critical for the stability of the solutions. Two types of general solutions are found: (i) the stationary wave packet considered becomes unstable (that is, this solution is not a physically acceptable solution for that process since, as time evolves, the width of the corresponding wave packet becomes more delocalized) and (ii) the probability density becomes more and more localized at asymptotic times (the stationary wave packet is then an attractor of the corresponding width dynamics); in terms of trajectories, this analysis indicates that the Bohmian trajectories are not approaching the corresponding classical ones. These conclusions are also corroborated by carrying out an analysis of the entropy. Finally, it is also briefly discussed the question of coupling classical variables (apparatus) to quantum ones (system) in terms of Lyapunov exponents.

In the epilogue of this monograph some concluding remarks and future perspectives towards the establishment of a complete theory of continuous quantum measurement, within the Bohmian mechanics, are briefly presented and discussed.

This monograph is the result of more than 30 years working on trajectory-based formalisms; in particular, on Bohmian mechanics. Concerning citations, we have tried to furnish a historical development of the different topics presented here. However, to provide a selection of the very last references in a very active field is really difficult. We apologize to those who think they should be cited and are not. During this long but exciting time, we have benefitted from discussions with many colleagues from all over the world. In particular, we would like to acknowledge fruitful discussions and collaborations with P.T.S. Alencar, P. Bargueño, J.M.F. Bassalo, I. Besieris, M.S.D. Cattani, H. Peñate-Rodríguez, J. Margalef-Roig, R. Martínez-Casado, G. Rojas-Lorenzo, A.S. Sanz, E. Outerelo, E. Pollak and O. Roncero. ABN would like to thank all members of the Harvard-Westlake Science Department, the UCLA Extension Science Department

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