Preface

Sequences of matrices with increasing size naturally arise in several contexts and especially in the discretization of continuous problems, such as integral and differential equations. The theory of generalized locally Toeplitz (GLT) sequences was developed in order to compute/analyze the asymptotic spectral distribution of these sequences of matrices, which in many cases turn out to be GLT sequences. In this book we present the theory of GLT sequences together with some of its main applications. We will also refer the reader to the available literature for further applications that are not included herein.

It normally happens in mathematics that ideas are better conveyed in the univariate setting and then transferred to the multivariate setting by successive generalizations. This is the case with mathematical analysis, for example. Any first course in mathematical analysis focuses on the theory of continuous/differentiable/integrable functions of *one variable*, while concepts like multivariate continuous functions, partial derivatives, multiple integrals, etc., are introduced only later (usually in a second course, not in the first one). Something similar occurs here. The present volume is the analog of a first course in mathematical analysis; it addresses the theory of what we may call *univariate* GLT sequences (or *unilevel* GLT sequences according to a more traditional terminology). As we shall see, univariate GLT sequences arise in the discretization of *unidimensional* integral and differential equations. The analog of a second course in mathematical analysis is [62], which deals with *multivariate/multilevel* GLT sequences, a kind of sequence arising from the discretization of *multidimensional* integral and differential equations. The generalization to the multivariate setting offered by [62] is certainly fundamental, especially for the applications, but it is essentially a technical matter whose purpose is to implement appropriately the ideas we are already informed of by the present volume. In short, this volume already contains all the ideas of the theory of GLT sequences, just as a first course in mathematical analysis contains all the ideas of integro-differential calculus.

The book is conceptually divided into two parts. The first part (Chaps. 1–8) covers the theory of GLT sequences, which is finally summarized in Chap. 9. The second part (Chap. 10) is devoted to the applications, corroborated by several numerical illustrations. Some exercises are scattered in the text and their solutions are collected in Chap. 12. Each exercise is placed at a particular spot with the idea that the reader at that stage possesses all the elements to solve it.

The book is intended for use as a text for graduate or advanced undergraduate courses. It should also be useful as a reference for researchers working in the fields of linear algebra, numerical analysis, and matrix analysis. Given its analytic spirit, it could also be of interest for analysts, primarily those working in the fields of measure and operator theory.

The reader is expected to be familiar with basic linear algebra and matrix analysis. Any standard university course on linear algebra covers all that is needed here. Concerning matrix analysis, an adequate preparation is provided by, e.g., [16] or [67]; we refer in particular to [16, Chaps. 1–3, Sects. 1–3 of Chap. 6, and Sects. 1–8 of Chap. 7] and [67, Chap. 2, Sects. 5.5, 7.1–7.2, and 8.1]. In addition, the reader who knows Chaps. 1–4 of Bhatia's book [12] will certainly take advantage of this. Some familiarity with real and complex analysis (especially, measure and integration theory) is also necessary. For our purposes, Rudin's book [95] is more than enough; actually, Chaps. 1–5 of [95] cover almost everything one needs to know. Finally, a basic knowledge of general topology, functional analysis, and Fourier analysis will be of help.

Assuming the reader possesses the above prerequisites, most of which will be addressed in Chap. 2, there exists a way of reading this book that allows one to omit essentially all the mathematical details/technicalities without losing the core. This is probably "the best way of reading" for those who love practice more than theory, but it is also advisable for theorists, who can recover the missing details afterwards. It consists in reading carefully the introduction in Chap. 1 (this is not really necessary but it is recommended), the summary in Chap. 9, and the applications in Chap. 10.

To conclude, we wish to express our gratitude to Bruno Iannazzo, Carla Manni, and Hendrik Speleers, who awakened our interest in the theory of GLT sequences and ultimately inspired the writing of this book. We also wish to thank all of our colleagues who worked in the field of "Toeplitz matrices and spectral distributions", and contributed with their work to lay the foundations of the theory of GLT sequences. We mention in particular Bernhard Beckermann, Albrecht Böttcher, Fabio Di Benedetto, Marco Donatelli, Leonid Golinskii, Sergei Grudsky, Arno Kuijlaars, Maya Neytcheva, Debora Sesana, Bernd Silbermann, Paolo Tilli, Eugene Tyrtyshnikov, and Nickolai Zamarashkin. Finally, special thanks go to Giovanni Barbarino and Dario Bini, who agreed to read this book and provided useful advice on how to improve the presentation.

Based on their research experience, the authors propose a reference textbook in two volumes on the theory of generalized locally Toeplitz sequences and their applications. This first volume focuses on the univariate version of the theory and the related applications in the unidimensional setting, while the second volume, which addresses the multivariate case, is mainly devoted to concrete PDE applications.

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