

## Chapter 2

# Simple Horizontal and Vertical Curves

In the geometric design of highways, circular curves as horizontal curves (Sect. 2.1) and parabolic curves as vertical curves (Sect. 2.2) are the most widespread. Apart from this type of curves, so-called transition curves are traditionally used as a geometric elements between the straight and the circular arc or between two circular arcs with different radii. The most popular transition curve is a clothoid (also known as spiral curve). Apart from the clothoid other solutions of transition curves are also known. They will be presented in the following sections of this work.

### 2.1 Circular Horizontal Curve

In the geometric design of horizontal curves a circular curves are very widespread (Brockenbrough 2009, Easa 2003, Lamm et al. 1999, Meyer and Gibson 1980, Rogers 2008, Wolhuter 2015). Figure 2.1 shows a circular curve (with radius  $R$  and centre  $O$ ) joining two straights  $P'P$  and  $K'K$  with intersect at point  $W$ , where:

$P$  and  $K$  tangent points,

$U$  angle of intersection of straights  $P'P$  and  $K'K$ .

The individual geometric elements occupy the following location:

- point  $S$  is the mid-point of the circular arc and the mid-point of the tangent line  $LM$ ,
- point  $S$  lies on the line  $OW$ ,
- $Q$  is the mid-point of the chord  $PK$  and lies on the line  $OW$ ,
- radii  $OA$  and  $OB$  intersect the straights  $P'P$  and  $K'K$  at right angles,

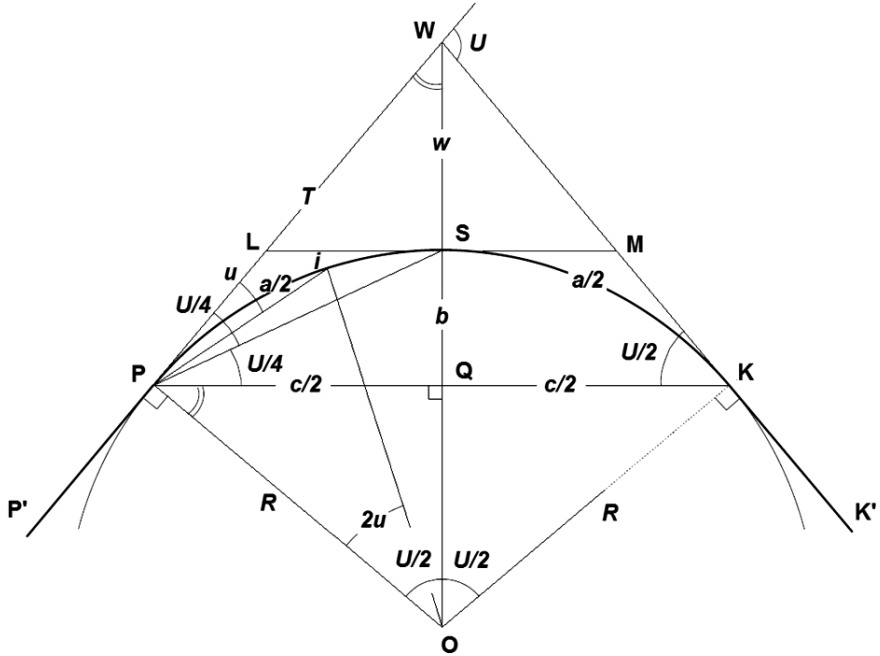


Fig. 2.1 Circular curve

- tangent line  $LM$  and the chord  $PK$  are parallel,
- the chord  $PK$  is perpendicular to the straight line  $OW$ .

The following formulae may be deduced from Fig. 2.1:

- tangent length ( $PW = WK$ )

$$T = R \tan \frac{U}{2} \quad (2.1)$$

- arc length ( $\overline{PK}$ )

$$a = R \cdot U \quad (2.2)$$

- chord length ( $PK$ )

$$c = 2R \sin \frac{U}{2} \quad (2.3)$$

- mid ordinate distance (QS)

$$b = R \left( 1 - \cos \frac{U}{2} \right) \quad (2.4)$$

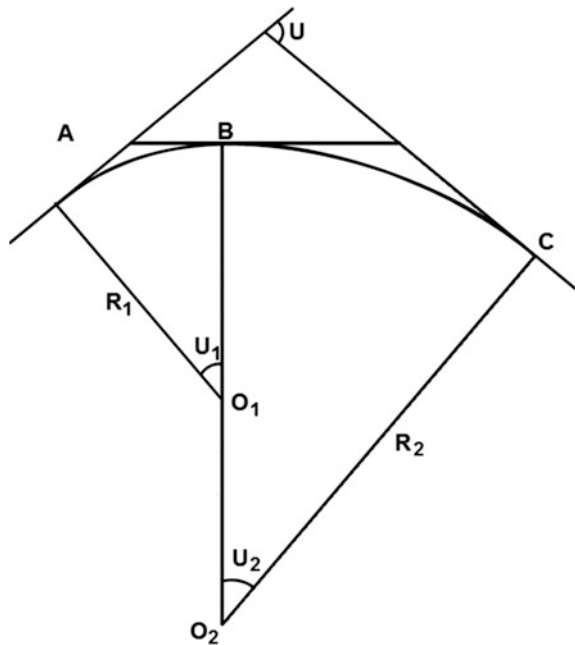
- secant distance (SW)

$$w = R \left( \sec \frac{U}{2} - 1 \right) \quad (2.5)$$

Circular curves can be used not only to design curvilinear transitions between two straights, as shown in Fig. 2.1. They can also be used for designing complex geometric systems, such as: compound circular curves, reverse circular curves and combined curves. Such systems should be understood as follows:

- compound circular curves—two or more consecutive circular curves with different radii (Fig. 2.2),
- reverse circular curves—two or more consecutive circular curves, with the same or different radii which centres lie on different sides of a common tangent point (Fig. 2.3),
- combined curves—geometric systems consisting of consecutive transition and circular curves (Fig. 2.4).

**Fig. 2.2** Compound circular curves (with permission from ASCE)



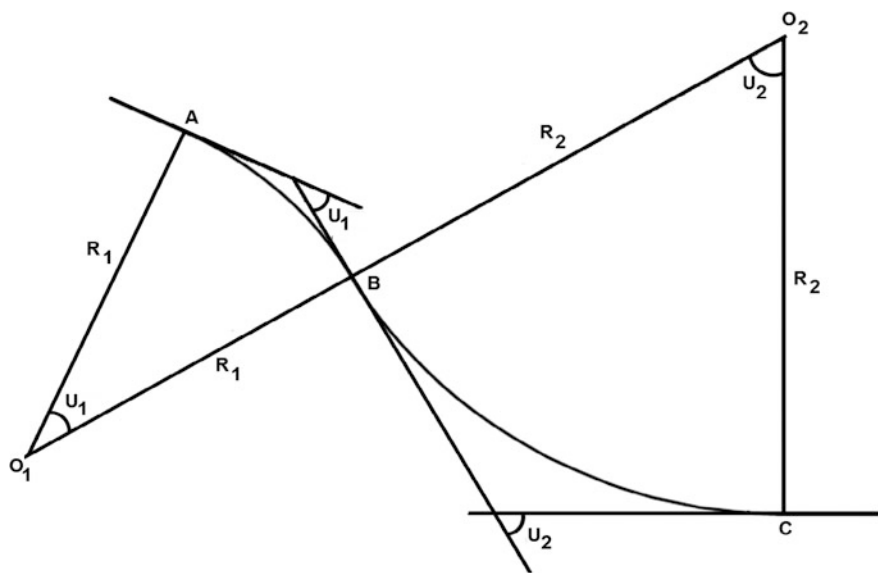


Fig. 2.3 Reverse circular curves (with permission from ASCE)

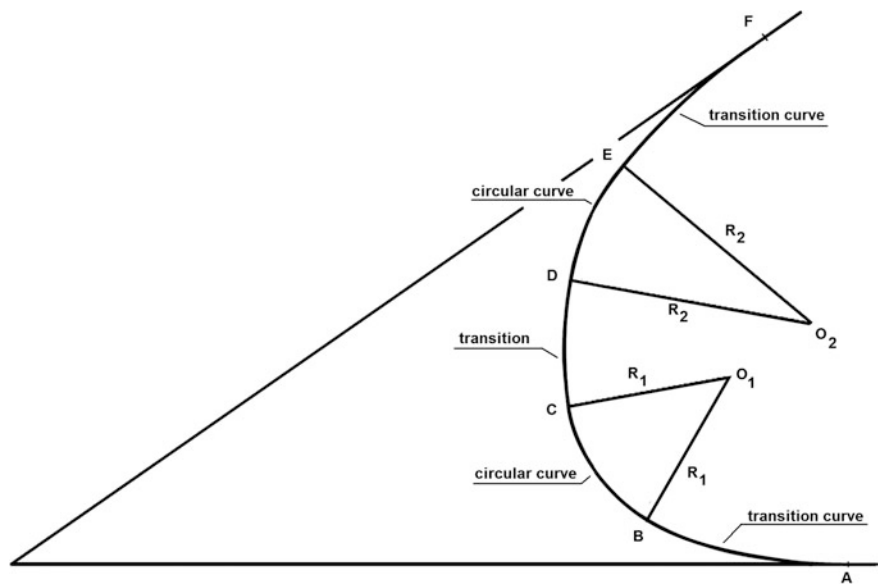


Fig. 2.4 Combined curves (with permission from ASCE)

## 2.2 Parabolic Vertical Curve

Vertical curves are used for to smoothly connection two straight lines with different gradients in the longitudinal profile (Brockenbrough 2009, Easa 2003, Lamm et al. 1999, Meyer and Gibson 1980, Rogers 2008, Wolhuter 2015). In sectional view (Figs. 2.5 and 2.6), the gradient to the left of the vertical curve will be denoted by  $p[\%]$  and the gradient to the right will be denoted by  $q[\%]$ . The vertical curves are generally unsymmetrical and can be crest or sag. It depends on the total change in gradient of two consecutive straight lines:

- crest curves—vertical curves where the total change in gradient is negative,
- sag curves—vertical curves where the total change in gradient is positive.

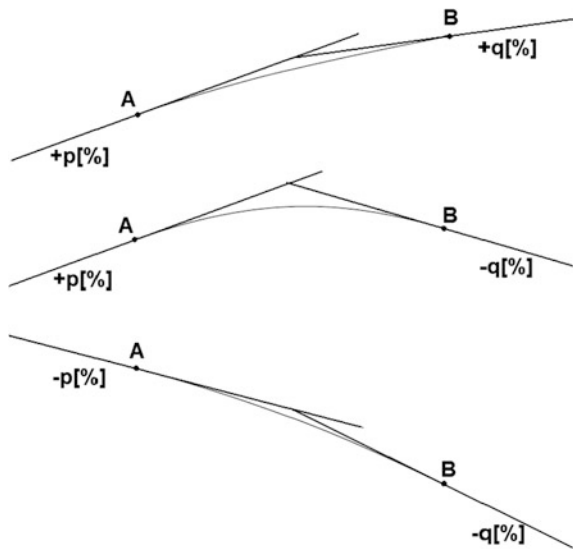
For some roads (high-speed roads), a cubic parabola is sometimes used as the vertical curve whose rate of change of gradient increases or decreases with the length of the curve. In other cases, a quadratic parabola is generally used as the vertical curve.

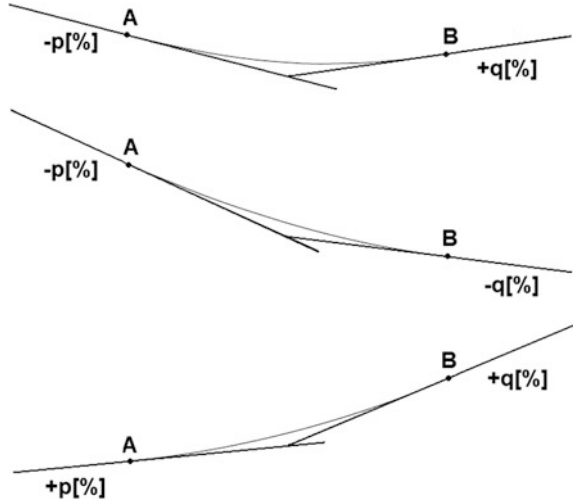
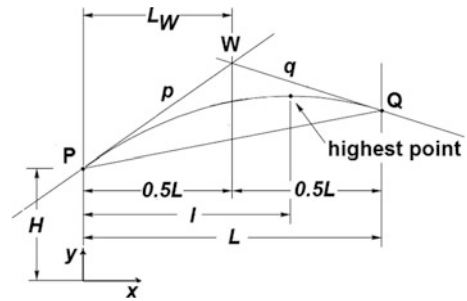
In Fig. 2.7 the vertical parabolic curve between two grades  $p$  and  $q$  which intersect at point W is shown. In this figure are adopted following designations:

- P and Q tangent points,  
 $H$  the reduced level of P,  
 $L$  the horizontal length of the curve,  
 $l$  distance of the highest point of the curve from the point P

The  $x$ - $y$  coordinate origin is vertically below P with the  $x$ -axis being the datum for reduced levels  $y$ .

Fig. 2.5 Vertical crest curves



**Fig. 2.6** Vertical sag curves**Fig. 2.7** Parabolic vertical curve

The basic requirement for the vertical curve is that the rate of change of gradient (with respect to horizontal distance) should be constant. The equation of the vertical curve is

$$y = \left( \frac{q - p}{2L} \right) x^2 + px + H \quad (2.6)$$

The distance of the point W from the point P is

$$L_W = \frac{1}{2}L \quad (2.7)$$

The horizontal distance to the high point (for crest curve) or low point (for sag curve) is

$$l = \frac{-p}{q - p} L \quad (2.8)$$

The reduced level of point Q is

$$H_q = H + pL_w + q(L - L_w) \quad (2.9)$$

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