Chapter 2
A New Surrogate Modeling Method Associating Generalized Polynomial Chaos Expansion and Kriging for Mechanical Systems Subjected to Friction-Induced Vibration

E. Denimal, L. Nechak, J.-J. Sinou, and S. Nacivet

Abstract This study focuses on a hybrid surrogate modelling technique in order to predict parameter-dependent mode coupling instabilities for uncertain mechanical systems subjected to friction-induced vibration. For this purpose, the most common strategy consists in associating a Monte Carlo procedure and/or a scanning technique together with the Complex Eigenvalue Analysis (CEA). This numerical strategy is computationally too prohibitive, particularly in an industrial context such as in the brake systems. To overcome this drawback, a novel approach is proposed. It consists in the combination of the generalized polynomial chaos (GPC) together with the kriging based meta-models. The association of both methods gives rise to a hybrid meta-model allowing taking into account two sets of uncertain parameters in the prediction of mode coupling instabilities. Moreover, it permits avoiding the use of the prohibitive MC and scanning methods. Thereby, this study analyses the feasibility of the proposed meta-model and its potential to be an efficient predictor of squeal propensity under parameter uncertainty.

Keywords Friction-induced vibrations • Kriging • Generalized polynomial chaos • Uncertainty • Meta-modelling

2.1 Introduction

Numerous studies have been proposed the last decades for the prediction of friction-induced instabilities submitted to parameter uncertainties [1–4]. A suitable choice of the predictor depends on models considered for the uncertainty description. For example, the generalized polynomial chaos based predictor is preferred to the MC method when dealing with random uncertainties described by probability density functions [4] while methods based on Kriging are privileged when parameters are defined only by intervals [1–3]. However, in several cases, friction induced instabilities may be simultaneously submitted to random and interval parameters. Thus, it is necessary to develop hybrid predictors capable of taking into account of both uncertainties sets. So, the purpose of this paper is to present a new surrogate modeling method that associate the generalized polynomial Chaos and Kriging formalisms for the prediction of friction-induced instabilities when subjected to random and interval parameters. The Complex Eigenvalue analysis method associated with the Monte Carlo and/or Scanning methods commonly used is unfortunately prohibitive. Hence, the main aim is to propose a surrogate which offers a suitable compromise between the accuracy of predictions and the computation time. The method developed here lies in the superposition of the two methods to namely the generalized polynomial chaos and the Kriging meta-models. The first purpose is to assess the feasibility of the proposed meta-model. A mechanical system with four degrees of freedom is then considered. Otherwise, another objective of this study is to study the impact of the input law on the stability of the system under study.

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N. Dervilis (ed.), *Special Topics in Structural Dynamics, Volume 6*, Conference Proceedings of the Society for Experimental Mechanics Series, DOI 10.1007/978-3-319-53841-9_2

2.2 System Under Study

2.2.1 *Description of the System*

A minimal system of four-degrees-of-freedom is studied and displayed on Fig. 2.1. This phenomenological model is based on the well-known two-degree-of-freedom model proposed by Hulten [5, 6]. It is used in [7] to point out the role of the damping and the destabilization paradox. The system under study is an extension to investigate the case of multi-instabilities. The model consists of two masses m_1 and m_2 linearly coupled and held against moving bands disposed as in Fig. 2.1. Contact between masses and bands are modeled by plates supported by spring and damping. The Coulomb's law with a constant friction coefficient μ is used. A more developed description of the system can be found in [2].

By noting $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ the mass, damping and stiffness matrix respectively and $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$ and $\mathbf{X}$ the acceleration, velocity and displacement vectors respectively, the equation of motion is given by:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{0} \tag{2.1}$$

2.2.2 *The Complex Eigenvalue Analysis to Predict the Stability of the System*

Due to the high computation cost of non-linear transient and stationary responses of the system, methods based on the CEA are often used in order to predict the instabilities of the system in a given frequency range, even if this method can lead to an under- or over-estimation of the unstable modes [8].

The CEA is based on the numerical analysis of the system eigenvalues. Assuming a solution of the form $\mathbf{X}(t) = \mathbf{\Phi} e^{\boldsymbol{\lambda} t}$ where $\boldsymbol{\lambda}$ is a complex eigenvalue of the system and $\mathbf{\Phi}$ the associated eigenvector, the equation of motion becomes:

$$\left(\boldsymbol{\lambda}^2\mathbf{M} + \boldsymbol{\lambda}\mathbf{C} + \mathbf{K}\right)\mathbf{\Phi} = \mathbf{0} \tag{2.2}$$

So the eigenvalue analysis can be performed by solving the characteristic equation:

$$\det\left(\boldsymbol{\lambda}^2\mathbf{M} + \boldsymbol{\lambda}\mathbf{C} + \mathbf{K}\right) = 0 \tag{2.3}$$

According to the Lyapunov theory, the asymptotic stability of the system is given by the sign of the real part of eigenvalues. If at least one eigenvalue has a positive real part, the system is unstable. The corresponding imaginary part is the pulsation of the associated unstable mode.

The set of parameters for the mechanical model under study is given in Table 2.1. Eigenvalues obtained with the CEA by increasing the constant friction coefficient μ from 0 to 1 are presented Fig. 2.2. Three mode-coupling phenomena are

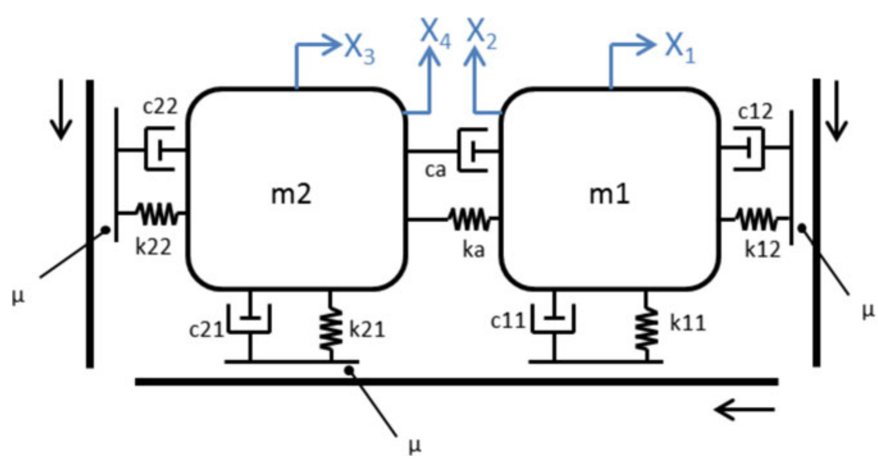

Fig. 2.1 Mechanical system

Table 2.1 Set of considered parameters

Parameters	M1 (kg)	M2 (kg)	K11 (N/m)	K12 (N/m)	K21 (N/m)	K22 (N/m)	Ka (N/m)	C11 (N.s/m)	C12 (N.s/m)	C21 (N.s/m)	C22 (N.s/m)	Ca (N.s/m)
Value	1	1	3000	6000	1000	3000	1000	1	1	1	1	1

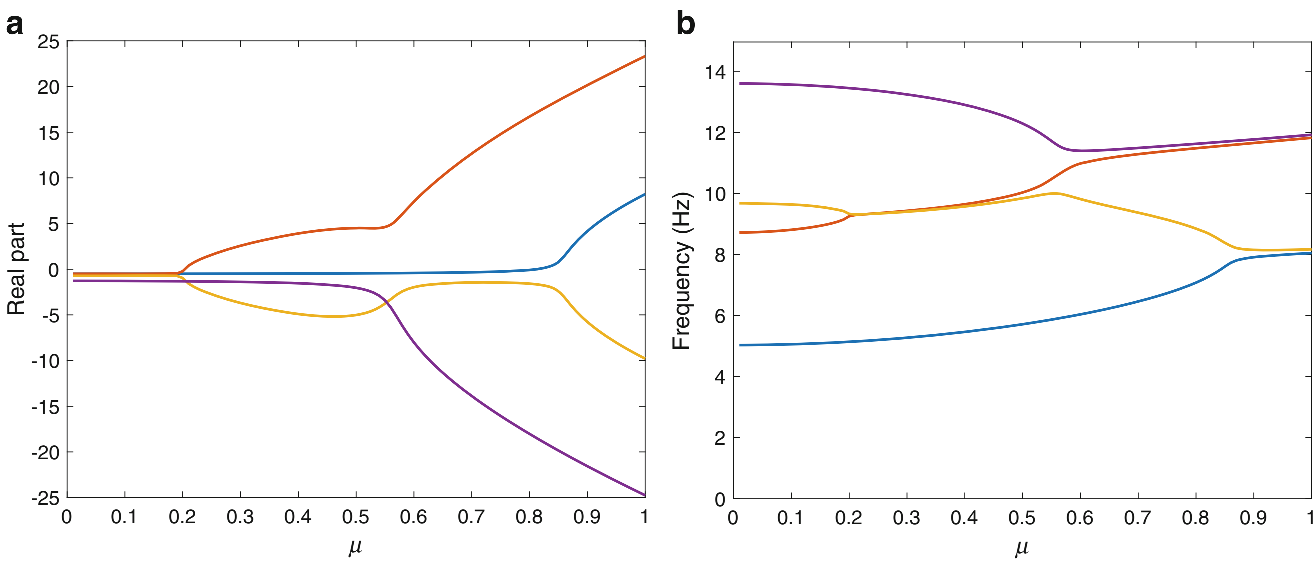

Fig. 2.2 Evolution of real (**a**) and imaginary (**b**) parts of eigenvalues versus the friction coefficient **μ**

observed. The first one appears when $\mu = 0.20$ with the frequency of the unstable mode equal to 9.25 Hz. The other coalescences are detected for $\mu \geq 0.5$ and $\mu \geq 0.81$. Modes are involved in several successive coupling coalescences, a crossing phenomenon between two modes is observed for $\mu = 0.55$.

2.3 Hybrid Surrogate Modeling Method

As mentioned in the introduction, the construction of a hybrid surrogate model is required for the predicting of friction induced instabilities when these depend on two sets of uncertain parameters (random parameters described by their probability density functions and interval parameters). The GPC expansion combined with the Kriging meta-model proposed in recent studies [1–4] for the prediction of friction-induced instabilities subjected to parameter uncertainties, are proposed in this framework. The main idea consists of exploiting a limited number of simulations (CEA solutions) to build the response surfaces of the system's eigenvalues. The taking into account of probabilistic and interval parameters, results in the expression of the stochastic modes of the GPC expansions by kriging functions. Brief descriptions of the two formalisms are given below.

2.3.1 Kriging

The main idea from the kriging theory [9–11] is based on the possibility to estimate a response surface just with a small number of simulations at points generated, randomly or pseudo-randomly, from the design space. Hence, any parameter-dependent eigenvalue $\lambda(\mathbf{p})$ ($\mathbf{p}$ being a vector parameter) can be approximated by a meta-model constructed from a regression defining the average behavior of the function and the stochastic process $Z(\mathbf{p})$ defining its dispersion.

$$\lambda(\mathbf{p}) = \sum_{i=1}^{m} \beta_i f_i(\mathbf{p}) + Z(\mathbf{p}) \tag{2.4}$$

where the first terms is a linear combination of m polynomial functions weighted by regression parameters β_i. $Z(.)$ is a zero-mean random process characterized by a special correlation function which estimates the similarity of two points in the design space, [9–11]. The regression β_i and correlation parameters characterize the kriging meta-model. In this study, they are calculated by using the DACE toolbox [9].

2.3.2 Generalized Polynomial Chaos

The generalized polynomial chaos (GPC) formalism the possibility to approximate the random parameter-dependent eigenvalue $\lambda(\boldsymbol{\xi})$ by a convergent, in the L^2 sense, expansion onto a polynomial basis which is orthogonal with respect to the probability measures associated to the random parameter vector $\boldsymbol{\xi}$, [12, 13].

$$\lambda\left(\boldsymbol{\xi}\right) \cong \sum_{i=0}^{d} a_i \psi_i\left(\boldsymbol{\xi}\right) \tag{2.5}$$

Coefficients a_i are the unknown deterministic coefficients and ψ_i the multivariate polynomial basis. Coefficients a_i can be computed with non-intrusive techniques as the regression or the non-intrusive spectral projection method [14].

2.3.3 Association of Kriging and Generalized Polynomial Chaos Expansion

Based on the both approximations presented previously, a system eigenvalue depending on random and interval parameters can be approximated by the following double expansion:

$$\lambda\left(\mathbf{p},\boldsymbol{\xi}\right) \cong \sum_{j=0}^{d}\left(\sum_{i=1}^{m}\beta_i f_i\left(\mathbf{p}\right) + Z\left(\mathbf{p}\right)\right)\psi_j\left(\boldsymbol{\xi}\right) \tag{2.6}$$

The main task is then to compute GPC coefficients $a_j$that, as they are functions in the vector parameter p, are approximated by kriging functions. Two experimental plans are constructed: the first one concerns the probabilistic parameters while the second concerns the interval parameters. The first plan is used to determine a small number of GPC expansions by using the regression technic [14] while the second plan is used to construct the surface response of the GPC coefficients $(a_j)_{j\in[0,P-1]}$ by the kriging meta-model.

2.4 Application

To assess the proposed hybrid meta-model, the approximating of the eigenvalues depending on the random friction coefficient μ and the two interval stiffnesses parameters k_{11} and k_{22} is realized. The friction coefficient is assumed to follow a Beta distribution, when the two stiffnesses are supposed to be within two known intervals given by 10% around their mean values. Reference results obtained from performing the CEA associated with a scanning method is presented in [2]. As the friction coefficient is governed by a beta-probability density function, the Jacobi polynomial basis is used to propagate and quantify the corresponding uncertainty [13]. A 10th order Jacobi polynomial expansion is considered. The associated coefficients depending on k_{11} and k_{22} are approximated by a kriging function which is constructed by considering a first order regression functions and a cubic correlation function [9]. A μ-experience plan defined by 14 samples (zeros of the 14th order Jacobi polynomial) with a (k_{11}, k_{22})-LHS plan (Latin hypercube sampling) with 50 samples of the couple (k_{11}, k_{22}), define 700 CEA solutions required for the calculation of the hybrid meta-model. Otherwise, to observe the influence of the probability density function on the prediction results, different density functions are considered as indicated in Table 2.2.

Figure 2.3 displays response surfaces of the three first chaos coefficients and the last one corresponding to the real part of the first eigenvalue of the system under study. Results obtained by kriging (in black) are compared to those computed with the regression technique in blue. Points of the design space used to build the kriging surrogate model are represented in red. It can be observed that the kriging meta-mode gives suitable approximations of polynomial chaos coefficients.

Table 2.2 Parameters of beta laws

Parameters	α	β
Law Beta 1	5	5
Law Beta 2	2	8
Law Beta 3	8	2

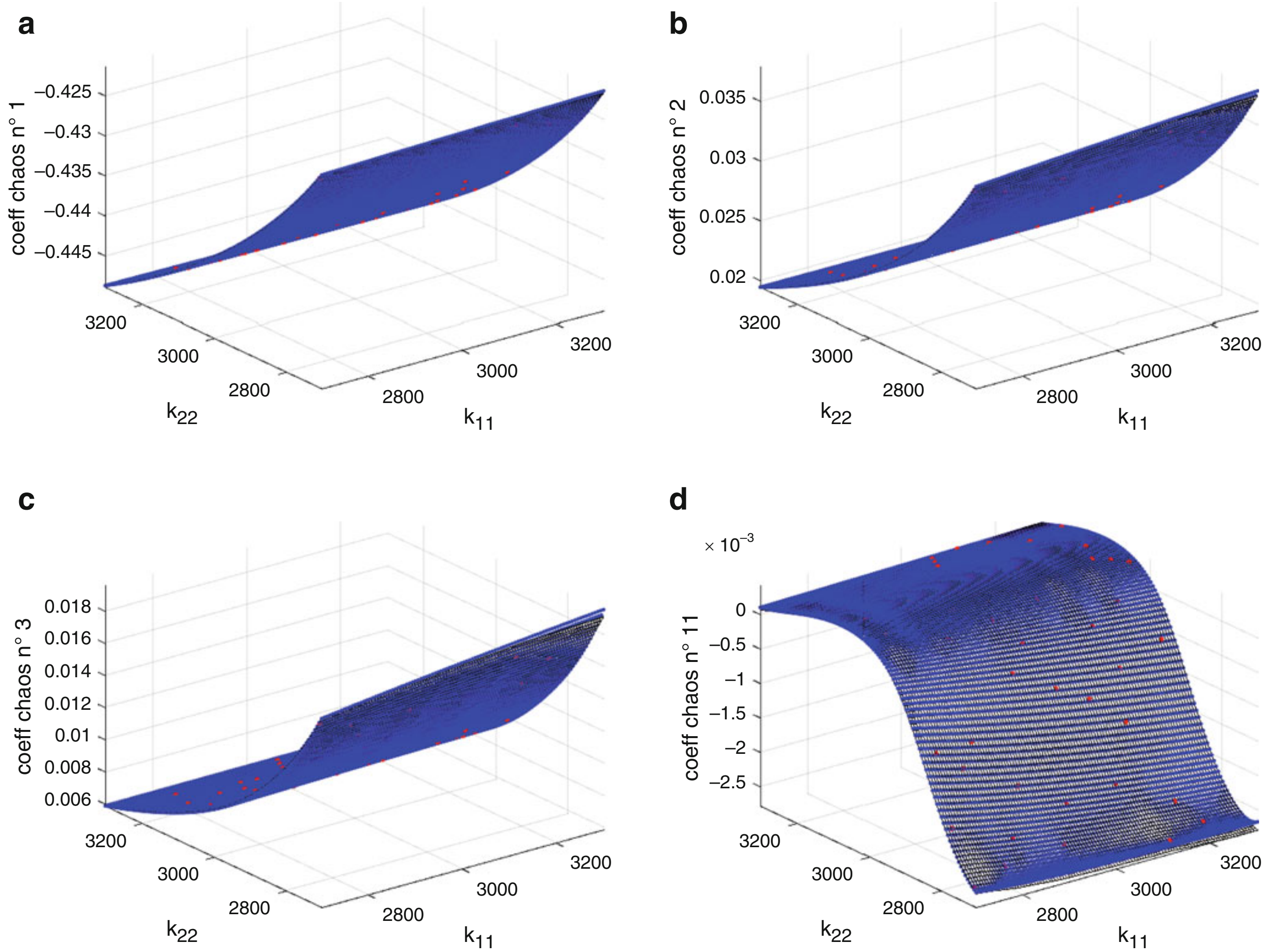

Fig. 2.3 First (**a**), second (**b**), third (**c**) and last (**d**) Polynomial Chaos coefficients: Kriging results (*blue*), experimental design (*red*) and deterministic results (*black*)

The resulting hybrid meta-model is considered to predict friction-induced instabilities. Moreover, to analyze the effect of the density function governing the friction coefficient on the predicted friction-induced instabilities, three beta-distributions with different parameters (Table 2.2) are considered. The parameters α and β are chosen in such a manner to have different repartitions for the friction coefficient. The first couple $\alpha = 5$ and $\beta = 5$ gives rise to a symmetric law centered on 0.5 while the other couples define repartitions with mirror effect.

The hybrid meta-model predictions corresponding to friction coefficient governed by a beta distribution of parameters $\alpha = 5$ and $\beta = 5$ are plotted in Fig. 2.4b. The associated reference, shown in Fig. 2.4a, is obtained by associating the MC and scanning methods with the CEA. The total number of CEA solutions required for the reference was equal to $25 * 10^7$. With only 700 CEA solutions, the calculated hybrid meta-model has allowed accurate repartitions of the real and imaginary parts of the unstable modes. Moreover, the almost-sure unstable character of the system is also well predicted by the hybrid meta-model. Otherwise, it can be observed, from Fig. 2.4b, c, d that the stability properties are strongly impacted by the probability density function governing the friction coefficient. Indeed, the real and imaginary parts of unstable modes obtained with the

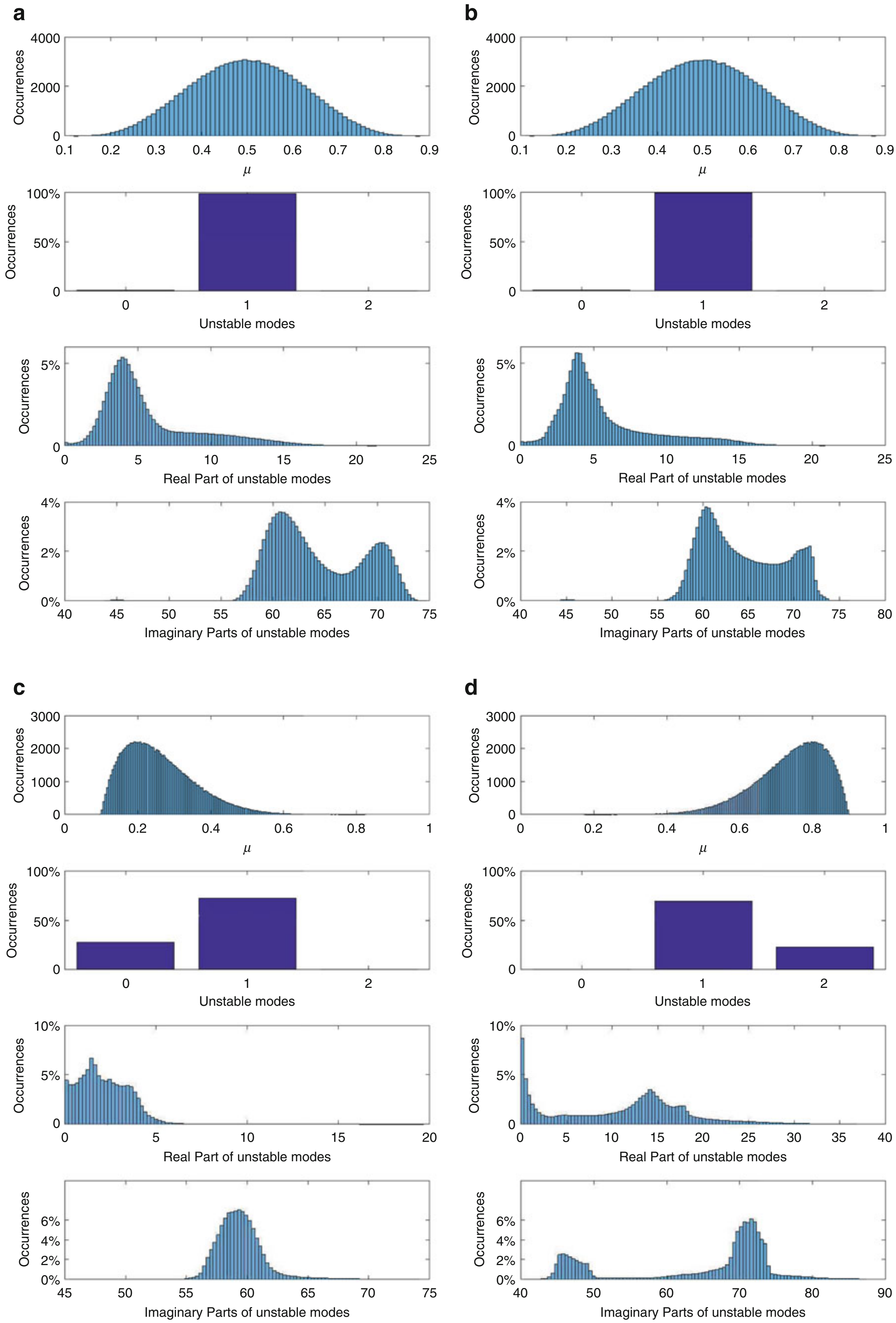

Fig. 2.4 Results of simulation using different Beta laws: (from *top* to *bottom*) the histograms of the random parameter $\boldsymbol{\mu}$, histograms of stable/unstable occurrences, histograms of unstable frequencies, histograms of unstable real parts—Case of a CEA computation for a Beta$_{5,5}$ law (**a**), of using the hybrid surrogate model for a Beta$_{5,5}$ law (**b**), a Beta$_{2,8}$ law (**c**) and a Beta$_{8,2}$ law (**d**)

beta law 3 are spread over larger intervals than the two other laws. For example, by considering frequencies, the $\text{Beta}_{2,8}$ distribution points out a peak at 58 rad/s while the $\text{Beta}_{5,5}$ distribution reveals two peaks at 62 rad/s and 72 rad/s and $\text{Beta}_{8,2}$ presents two peaks around 48 rad/s and 72 rad/s corresponding to the frequencies of the two unstable modes. Otherwise, the impact of the friction law on the stability analysis is non-negligible. Indeed, when for the first Beta law the system is stable 30% of the time, it is almost-surely unstable for the other cases. All these results point out the high sensitivity of the results of the stability analysis of mechanical systems subjected to friction induced vibration toward the probability density functions governing system parameters. These are needed to be accurately identified.

2.5 Conclusion

This study presents a new hybrid meta-model dedicated to the prediction of friction-induced instabilities. The proposed predictor is constructed from the associating of the general polynomial chaos expansion together with the kriging meta-model. Its main interest is related to its capacity to take into account of uncertain parameter defined by different models. The feasibility of the proposed hybrid surrogate model has been shown while its efficiency has been shown to be promising for applications involving squeal prediction under uncertainty. Otherwise, the taking into account of the uncertainty related to the probability density functions describing random parameters, in the stability analysis is carried out.

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http://www.springer.com/978-3-319-53840-2

Special Topics in Structural Dynamics, Volume 6
Proceedings of the 35th IMAC, A Conference and Exposition on Structural Dynamics 2017
Dervilis, N. (Ed.)
2017, VIII, 197 p. 175 illus., 141 illus. in color., Hardcover
ISBN: 978-3-319-53840-2