

Preface

This is the first of a series of three volumes (the other ones being [4] and [5]) devoted to the mathematics of mathematical olympiads. Generally speaking, they are somewhat expanded versions of a collection of six volumes, first published in Portuguese by the Brazilian Mathematical Society in 2012 and currently in its second edition.

The material collected here and in the other two volumes is based on course notes that evolved over the years since 1991, when I first began coaching students of Fortaleza to the Brazilian Mathematical Olympiad and to the International Mathematical Olympiad. Some 10 years ago, preliminary versions of the Portuguese texts also served as textbooks for several editions of summer courses delivered at UFC to math teachers of the Cape Verde Republic.

All volumes were carefully planned to be a balanced mixture of a smooth and self-contained introduction to the fascinating world of mathematical competitions, as well as to serve as textbooks for students and instructors involved with math clubs for gifted high school students.

Upon writing the books, I have stuck myself to an invaluable advice of the eminent Hungarian-American mathematician George Pólya, who used to say that one cannot learn mathematics without *getting one's hands dirty*. That's why, in several points throughout the text, I left to the reader the task of checking minor aspects of more general developments. These appear either as small omitted details in proofs or as subsidiary extensions of the theory. In this last case, I sometimes refer the reader to specific problems along the book, which are marked with an * and whose solutions are considered to be an essential part of the text. In general, in each section, I collect a list of problems, carefully chosen in the direction of applying the material and ideas presented in the text. Dozens of them are taken from former editions of mathematical competitions and range from the almost immediate to real challenging ones. Regardless of their level of difficulty, we provide generous hints, or even complete solutions, to virtually all of them.

This first volume concentrates on real numbers, elementary algebra, and real functions. The book starts with a non-axiomatic discussion of the most elementary properties of real numbers, followed by a detailed study of basic algebraic identities,

equations and systems of equations, elementary sequences, mathematical induction, and the binomial theorem. These pave the way for an initial presentation of algebraic inequalities like that between the arithmetic and geometric means, as well as those of Cauchy, Chebyshev, and Abel. We then run through an exhaustive elementary study of functions that culminates with a first look at implicitly defined functions. This is followed by a second look on real numbers, focusing on the concept of convergence for sequences and series of reals. We then return to functions, this time to successively develop, in detail, the basics of continuity, differentiability, and integrability. Along the way, the text stays somewhere between a thorough calculus course and an introductory analysis one. Lots of interesting examples and important applications are presented throughout. Whenever possible (or desirable), the examples are taken from mathematical competitions, whereas the applications vary from the proof and several applications of Jensen's convexity inequality to Lambert's theorem on the irrationality of π and Stirling's formula on the asymptotic behavior of $n!$. The text ends with a chapter on sequences and series of functions, where, among other interesting topics, we construct an example of a continuous and nowhere differentiable function, develop the rudiments of the generating function method, and discuss Weierstrass' approximation theorem and the rudiments of the theory of Fourier series.

Several people and institutions contributed throughout the years for my effort of turning a bunch of handwritten notes into these books. The State of Ceará Mathematical Olympiad, created by the Mathematics Department of the Federal University of Ceará (UFC) back in 1980 and now in its 36th edition, has since then motivated hundreds of youngsters of Fortaleza to deepen their studies of mathematics. I was one such student in the late 1980s, and my involvement with this competition and with the Brazilian Mathematical Olympiad a few years later had a decisive influence on my choice of career. Throughout the 1990s, I had the honor of coaching several brilliant students of Fortaleza to the Brazilian Mathematical Olympiad. Some of them entered Brazilian teams to the IMO or other international competitions, and their doubts, comments, and criticisms were of great help in shaping my view on mathematical competitions. In this sense, sincere thanks go to João Luiz Falcão, Roney Castro, Marcelo Oliveira, Marcondes França Jr., Marcelo C. de Souza, Eduardo Balreira, Breno Falcão, Fabrício Benevides, Rui Vigelis, Daniel Sobreira, Samuel Feitosa, Davi Máximo Nogueira, and Yuri Lima.

Professor João Lucas Barbosa, upon inviting me to write the textbooks to the Amílcar Cabral Educational Cooperation Project with Cape Verde Republic, had unconsciously provided me with the motivation to complete the Portuguese version of these books. The continuous support of Professor Hilário Alencar, president of the Brazilian Mathematical Society when the Portuguese edition was first published, was also of great importance for me. Special thanks go to professors Abdênago Barros and Fernanda Camargo, my colleagues at the Mathematics Department of UFC, who had made quite useful comments on the Portuguese editions, which were incorporated in the text in a way or another; they had also read the entire English version and helped me in improving it in a number of ways. If it weren't for my editor at Springer-Verlag, Mr. Robinson dos Santos, I almost surely would not

have had the courage to embrace the task of translating more than 1500 pages from Portuguese into English. I acknowledge all the staff of Springer involved with this project in his name.

Finally, and mostly, I would like to express my deepest gratitude to my parents Antonio and Rosemary, my wife Monica, and our kids Gabriel and Isabela. From early childhood, my parents have always called my attention to the importance of a solid education, having done their best for me and my brothers to attend the best possible schools. My wife and kids filled our home with the harmony and softness I needed to get to endure on several months of solitary nights of work while translating this book.

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Antonio Caminha Muniz Neto

An Excursion through Elementary Mathematics, Volume

I

Real Numbers and Functions

Caminha Muniz Neto, A.

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