

Preface

Since Zadeh proposed the concept of fuzzy set in 1965, the fuzzy set theory has been rapidly developed and vastly applied in many fields. Over the last decades, a variety of generalizations of classical fuzzy set have been derived from various angles, one of which is the intuitionistic fuzzy set, which was given by Atanassov in 1983. It depicts the vagueness and uncertainty of things more comprehensively by introducing a membership function and a non-membership function. Later on, Xu and Yager defined the basic elements of an intuitionistic fuzzy set as intuitionistic fuzzy numbers, which are essentially pairs of non-negative numbers belonging to the closed unit interval $[0,1]$. Intuitionistic fuzzy calculus, which is analogous to the calculus of the real numbers and the complex numbers in the classical mathematical analysis, is established by regarding IFNs as the basic elements.

The main purpose of this book is to give a thorough and systematic introduction to the latest research results on intuitionistic fuzzy calculus, which essentially focus on two issues, one of which is to build the calculus theory under intuitionistic fuzzy environment (denoted by Q1 in the structure diagram), the other is about how to aggregate the continuous intuitionistic fuzzy data or information (denoted by Q2 in the structure diagram). The book is constructed into six chapters that deal with the related issues, which are listed as follows:

Chapter 1 mainly introduces the fundamental knowledge related to IFNs. We first introduce the concepts of fuzzy sets and intuitionistic fuzzy sets. Later on, we present the concept of the IFN, and its two representation methods, one of which describes the IFNs as some points in two-dimensional plane, another expresses them as the closed subintervals in the unit interval. Then, the operational laws of IFNs, namely: addition, subtraction, multiplication, division, scalar-multiplication and power operation, are provided in this chapter. Moreover, the geometrical and algebraic properties of these operations are analyzed in detail. Also, we define the change region and the non-change region of IFNs. Last but not least, we show three kinds of order relations and utilize them to compare and rank IFNs, and then reveal the relationships among the several orders.

Chapter 2 first gives the definition of intuitionistic fuzzy functions (IFFs), which is just the object to be studied in the intuitionistic fuzzy calculus. Then, we

introduce the monotonically increasing IFFs and the continuous IFFs. By taking the limit values of difference quotients of IFFs, the derivatives of IFFs are acquired. Moreover, we make further efforts to give a criterion of differentiability of IFFs, and research its important properties, including the chain rule of the derivatives of compound IFFs. After getting the derivatives of IFFs, we define the differentials of IFFs, and provide the relationship between the increment of one IFF and its differential. In addition, the form invariance of differential in the intuitionistic fuzzy calculus is revealed in this chapter.

Chapter 3 is devoted to the indefinite integrals of IFFs, which are essentially the inverse operations of derivatives of IFFs. Then, some properties of the indefinite integrals of IFFs are discussed, including the substitution rules. Afterwards, we define the definite integrals of IFFs by utilizing two different methods, one is developed based on a novel concept (intuitionistic fuzzy integral curves (IFICs)), the other is introduced based on a closed interval of IFNs, and these two definitions of the definite integrals of IFFs are completely equivalent to each other. There only exists one difference between them, which is that the integrals of complex functions along a curve are in the complex plane, while the other is more similar to the integrals of real functions in a closed interval of real number axis. By building the definite integral of IFF with the variable upper limit, we establish the fundamental theorem (Newton–Leibniz formula) in the intuitionistic fuzzy calculus. Finally, the definite integrals of IFFs are successfully utilized to aggregate information and data in intuitionistic fuzzy environment.

Chapter 4 focuses on the methods aggregating continuous intuitionistic fuzzy information. The study of this issue is essential and meaningful, which likes that the probability theory and the mathematical statistics not only need to research the discrete-type random variables, but also concern the continuous-type random variables. We define the integral aggregating value of the region of IFNs, which contains the aggregated IFNs. Moreover, many properties about it are provided. The concept of integral aggregating value is utilized to generate a novel aggregation technique (IFIA), which is able to deal with the continuous intuitionistic fuzzy information. Lastly, we prove the idempotency, boundedness and monotonicity of the IFIA, and utilize the given operator to handle some practical problems.

Chapter 5 mainly investigates the relationships among the definitions and concepts proposed in the previous chapters. We show that there are closed connections among the IFWA operator (a common aggregating technique utilized to aggregate discrete IFNs), the IFIA operator and the definite integral of IFFs, and figure out that the IFWA operator is only the integral of some specific IFFs. In addition, the IFIA operator is the integral of a special IFF, which is essentially the continuous form of the IFWA. In other words, the IFWA is the discrete form of the IFIA.

Chapter 6 proposes the complement theory of the intuitionistic fuzzy calculus built in the previous chapters, and studies the relationship between the calculus of IFFs and its complement theory. Firstly, we study the complements of fundamental knowledge of IFNs, and prove the closed connections among these operations of IFNs and their complements. Secondly, we give the complements of derivatives, differentials, indefinite integrals and definite integrals of IFFs. Then, the

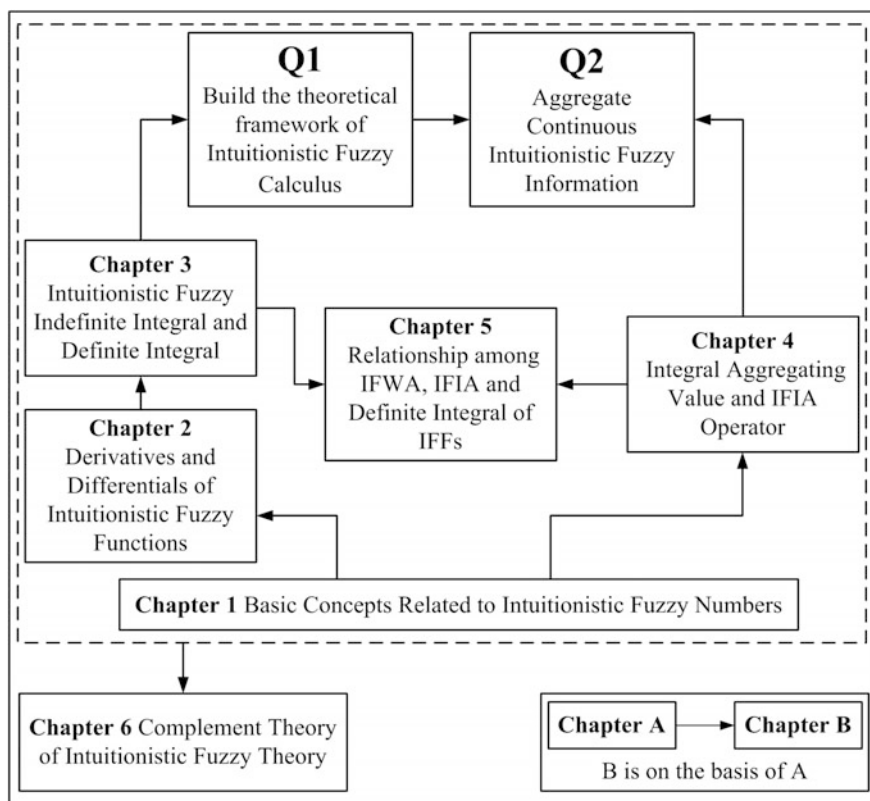


Diagram of the organizational structure of this book

aggregation operators in the previous chapters are investigated based on the concept of complement. In brief, this chapter manages to reveal the fact that any statement or conclusion in the intuitionistic fuzzy calculus must have a counterpart in its complement theory.

A diagram of the organizational structure of this book is provided to manifest the structure of this book more clearly:

This book can be used as a reference for researchers and practitioners working in the fields of fuzzy mathematics, operations research, information science, management science, engineering, etc. It can also be used as a textbook for postgraduate and senior-year undergraduate students.

This work was supported by the National Natural Science Foundation of China (No. 71571123).

Nanjing, China
Chengdu, China
July 2016

Qian Lei
Zeshui Xu

Intuitionistic Fuzzy Calculus

Lei, Q.; Xu, Z.

2017, XI, 168 p. 74 illus., 21 illus. in color., Hardcover

ISBN: 978-3-319-54147-1