

Chapter 2

A Certain Interest in Pure Science

Theoretical Research

As soon as Ettore Majorana moved to Fermi's group, he distinguished himself not only as an expert calculator but also by his extraordinary abilities as a researcher. In the Rome group some seminars were organized periodically on specific topics, these being held in rotation by the different members of the group. When it came to Majorana's turn, although everybody was very attentive, only Fermi was able to understand what was presented, and sometimes even showed his "irritation" towards Ettore, accusing him of "not saying everything", and would invite him to complete his presentation.¹ Actually, according to accounts by some of those present, the only one in the group to hold his own with Majorana was Fermi. From the start, the relationship between them was as peer to peer, also because of the small age gap. An episode recalled by the chemist Oscar d'Agostino, who also joined the *Via Panisperna* group (although later), and who made notable contributions to some of their research, is illuminating. One day some students, who were coming back to the Institute in the afternoon, after the lunch break, found Fermi and Majorana in a lecture hall in front of blackboards full of calculations, shouting at each other in a lively argument. "They were calling each other fools. The argument had started at midday and they had been there several hours in a passionate discussion, forgetting of course to eat lunch".²

Well before changing to physics, while he was still attending the Engineering School, Majorana often used to adopt a "personal" view about what was discussed in those university lectures that particularly struck him. Examples of this habit can be found in certain manuscripts, in particular, the already mentioned "Volumetti" (five notebooks totalling about 500 written pages), now kept in Pisa. It is clear that some of the topics presented in the lessons served as a starting point for personal

¹This story was provided by the engineer Gabriele Paparo, one of Amaldi's former collaborators; he used to tell his collaborators anecdotes about his friend Ettore during breaks at work.

²See the article (Ferrieri and Magnano 1972).

research later on. Anyway, Majorana's interests were not only of the academic kind, and a nice example is given by the anecdote in the previous chapter about the meeting between Majorana and Fermi. Indeed, in the Pisa manuscripts we can find the pages where Ettore wrote down the solution to the Thomas-Fermi equation. This served as a basis for the study of many atomic problems, one of the main topics of physics research from that period to which Fermi's group also contributed.

And Fermi himself, noticing the results achieved by Majorana in their discussions, invited him to present them (Esposito 2005a) in a talk at the XXII General Assembly of the Italian Physical Society (which took place at the Institute of Physics in Rome in 29–30 December 1928), long before he graduated in physics (and on a different topic to the one he discussed at his graduation). Even then, however, the physicist from Catania had a tendency not to publish the results of his research, when these could be considered premature, or at least not until they had reached an adequate form, according to his overcritical judgement, of course. As a matter of fact, Majorana actually gave a report at the said conference on December 29, between two of Fermi's speeches, but he did not regularly publish his work: *"the research carried out so far is still insufficient to appreciate the full value of these results"*, he said at the end of his talk.³

The question of the Thomas-Fermi atomic model must have particularly fascinated Ettore, if it is true that he actually applied "his" solution to other particular problems (only in a few cases did Majorana later return to specific topics in his *Volumetti*, as indeed he did on this occasion), sometimes with the clear purpose of re-deriving results already obtained by Fermi. An application of the statistical method by Thomas-Fermi to atoms was also the object of the study reported in Majorana's first paper, published with Giovannino Gentile a year before graduation. This is the only proof of the collaboration with his friend Gentile, who frequented the Fermi group in Rome at that time (for the other papers, Majorana was the only author). However, a close friendship developed between the two of them, as attested in their exchange of letters during the period when they were not working in the same establishment, and Majorana never missed an opportunity to help his friend with particular questions of physics and mathematics.

As we have seen, Majorana's way of promptly answering requests from friends and colleagues who were experiencing difficulties in solving certain scientific problems dates back to his university years in engineering. However, it remained a feature during the following years in research as well. Segrè provides a typical example of this: in 1931–2 he was on a fellowship in Hamburg (Segrè 1993), where he was working with Otto Robert Frisch in Otto Stern's group, carrying out experiments on atoms in magnetic fields. To interpret the results of the experiment they needed to solve some theoretical problems and, perhaps during a holiday break in Rome,⁴ Segrè involved his friend Majorana (Amaldi 1968). The latter duly came

³See what is quoted in *Il Nuovo Cimento*, 6, XIV–XVI.

⁴T.S. Kuhn's interview with E. Segrè, *loc. cit.*

up with the appropriate theory for the phenomenon being studied experimentally by Frisch and Segrè, and it became his sixth published article.

Another revealing example of Majorana's approach is the following. Toward the end of 1931, after a few months in Leipzig (where he had been working with Peter Debye), Edoardo Amaldi was conducting some experiments on the spectrum of the ammonia molecule together with George Placzek, who was then a visiting scientist at the Institute of Physics in Rome. In those days, after spending a few hours of the afternoon in the library, Majorana would join Amaldi in the laboratory, and when work was over, they would go back home together (the two friends lived quite close to each other). During these walks, besides making friends with the Czech visitor, Ettore learned about the research they were doing in the lab. And these investigations certainly caught his interest, so much so that he solved the problem of determining the ammonia oscillation frequencies on his own, relating them to the geometrical structure of the molecule, which has a tetrahedral shape. This study was never published by Majorana (Di Grezia 2006), but was stored in his personal notebooks, and it was probably never revealed to his friends, because they make no mention of it.

We also have the observations of a newcomer to the group, Gian Carlo Wick, who had just joined Fermi in the autumn of 1932, and who remembers this of the few contacts he had with Majorana:

[They] were enough to understand the extraordinary versatility and quickness of the workings of his mind. I also noticed his hypercritical but slightly ironical attitude towards, not only the others, but also his own work. Still he was too kind and fair to exercise his irony on someone who was inexperienced, and on the contrary he showed his interest in what I was doing with encouraging words – it concerned the magnetic moment of a hydrogen molecule in its various states of rotation. He also gave me some useful advice about some numerical calculations required by the problem, suggesting the use of a function different from the Heitler-London function I had been using. After hearing what value I had got from the first calculations, he observed: “Yes, this may be an upper limit; the other function will give a lower limit, which will be remarkably smaller”. I was quite surprised: how could he know that? That was Majorana's famous intuition! But when he explained his reasons, I became persuaded that they were totally rational. Needless to say, the conjecture proved to be perfectly right (Wick 1981).

He was equally open, not only towards his own friends, but also with the young international guests who came and worked for a while with Fermi. For example, one of these guests, Rudolph E. Peierls, remembers:

I surely received a lot of useful ideas and explanations from Fermi and the others, as Wick and Majorana [...]. Majorana was quite strange and retiring; he was a Sicilian [...]. He made his reputation on two important things. One the exchange nature of the nuclear forces, where he basically corrected an oversight in Heisenberg's ideas. And then the other was the neutrino theory.⁵

There were also times when Majorana's precious contribution was put to good use by Fermi himself in his research, as happened in 1932 when he and Segrè were elaborating the theory of the so-called hyperfine structures of atomic spectra. In the

⁵T.S. Kuhn's interview with R.E. Peierls, 18 June 1963.

work published by Fermi and Segrè in 1933 (Fermi and Segrè 1933), they explicitly thank “*Dr. Majorana for several discussions about calculations*” needed for their theory; examples of Majorana’s calculations on this topic can be found in the pages of his *Quaderni*, kept in Pisa.

For this reason another episode reported by Amaldi⁶ is particularly enlightening. On Sundays during the summer, Fermi and the other members of his group used to go to Fregene (a popular seaside resort) and Majorana would join them. However, he was always late, as he went there by bus from Rome, unlike the others. Once Majorana told his friends that the night before he had been working on some problem of quantum electrodynamics (in his personal notebooks there are, in fact, many pages devoted to this topic), but that he had “not quite figured it out”. However, he informed them that he had overcome the theoretical difficulties right that morning, on the bus to Fregene, and had written down the related calculations on a cigarette packet. He then proceeded to show the theory to his friends (who may well not have understood it at all, as Amaldi recalled), and wrote the formulae with his finger in the sand. Anyway Majorana never went back to it again, and the work was finally carried out by Fermi.

Ettore’s theoretical contribution to Fermi’s group in Rome was therefore substantial, but most of his research and the most relevant part was what he carried out *motu proprio*, and this can be found in his personal study notes. Particularly important was Majorana’s discovery of Hermann Weyl’s book on the mathematical theory of groups applied to quantum mechanics (Weyl 1928). This was in 1928–9 and it deeply affected all the later work of the physicist from Catania. To his friend Gentile, who was in Germany, he wrote on 22 December 1929:

As for me, I am not doing anything sensible In fact, I am studying the theory of groups with the firm intention of learning it, which makes me a bit like that hero from Dostoyevsky who, one fine day, started saving loose change, in the hope that he would soon be as rich as Rothschild.⁷

The concise, clear, and general view given by the theory of groups was, as a matter of fact, well appreciated by Majorana (and by very few others in the world, until its eventual rediscovery during the 1950s–1960s), and would form the basis for all his most important work.

The “Neutral Proton” and the Turning Point in 1932

James Chadwick’s discovery of the neutron, the companion particle of the proton in the atomic nucleus, was made at the beginning of 1932. It proved to be a turning point for nuclear physics around the world, and hence also for Fermi’s group in Rome. The actual experimental evidence came in the wake of several intuitions which later turned out to be false: these tried to explain certain strange observations

⁶This anecdote too was recalled by the engineer Gabriele Paparo, already mentioned.

⁷Letter MGR/1 of 22 December 1929 in (Recami 1987).

of nuclear reactions without postulating the existence of an electrically neutral subnuclear particle (De Gregorio 2007), namely, what we now know as the neutron.

When Joliot’s paper arrived in Rome in January (1932) with the *Comptes Rendus* and the paper of Chadwick had not yet appeared, we all were very much interested in it. But Majorana said: ‘How stupid of Joliot! They have not understood that this is the neutron’. [...] “This is obvious. These gamma rays make no sense in a nucleus. They should be neutral particles”. Well, we used the word ‘neutron’ because, you remember maybe, the idea of a neutron was suggested by Rutherford a few years before in order to explain the anomalous scattering of alpha particles. Gentile, another young man of our group, a great friend of Majorana’s, had done some calculations trying to introduce in quantum mechanics the effect of neutral particles moving around the nucleus. He had taken the idea of Rutherford and he had done some calculations. So the idea of a neutral particle, suggested by Rutherford, was discussed in the Institute. [...] The comments of Majorana were really quite interesting. He would say, “Well, how stupid⁸ they are!” This was not because they were stupid, because they were very intelligent people – it was just his way of expressing himself. “They don’t understand! This should be a neutral particle. There’s no sense to think of gamma rays of 50 MeV. There is no sense to talk of gamma rays of 50 MeV. This should be neutral”. [...] No, nobody took Majorana’s suggestion seriously. [...] Majorana said it then, but everybody said, “Ha, ha”. [...] When Chadwick’s paper came out, we were all convinced. And we said, “Look how quick Ettore is – he understood before Chadwick”.⁹

We have proof of Majorana’s intuition, though indirectly, from R.E. Peierls, an international guest in Fermi’s group:

This of course was just the time when the artificial radioactivity had been discovered, and when there were the experiments beginning to come out which led to the discovery of the neutron. Fermi always had a slightly peculiar attitude to that. I think he felt that the Paris group, the Joliot, should really have seen the existence of the neutron from their experiments which were later pointed out by Chadwick. I had the impression that he knew what the experiments meant, but hadn’t got round to publishing it, or felt he must leave it to the experimenters. I don’t know. This is only a hunch.¹⁰

Majorana’s happy intuition, which remained unknown outside the Rome circle, was not a mere improvisation! As usual, it was based on a deep knowledge of nuclear physics. This proficiency dated back to his time at university (his graduation thesis dealt with the mechanics of radioactive nuclei) and it continued in the following years, as we can see from the pages of his *Volumetti* (Esposito et al. 2003) and *Quaderni* (Esposito et al. 2008).

As Segrè recalls, the group at the *Via Panisperna* Institute had in fact long been discussing the possibility of changing tack and focusing on nuclear rather than atomic physics (Segrè 1993). While Fermi was quite ready to begin seriously

⁸Amaldi himself tells us that: [Majorana] felt everybody was stupid, himself included. The only people who were not completely stupid according to him were Dirac and Weyl. He greatly respected Weyl. Heisenberg was stupid. Everyone else was stupid. Even himself. He would not say that the others were stupid and he was clever. No, he was convinced that he was a fool. [...] Pauli was rather stupid. But Majorana was stupid as well, he was convinced of that. This gave him a feeling of emptiness.

⁹T.S. Kuhn’s interview with E. Amaldi, 8 April 1963.

¹⁰T.S. Kuhn’s interview with R.E. Peierls, *loc. cit.*

studying the nucleus, his young colleagues were doubtful about abandoning the research they had by then become quite expert about, especially since they had only recently mastered its most advanced experimental techniques. However the general orientation of the group gradually changed, even though they did not completely abandon the study of atomic phenomena;¹¹ as a “bridge” between the two different fields of physics, they started to investigate the problem of hyperfine structures, as mentioned above.

Chadwick’s discovery, announced in February 1932, shook the international community, and many physicists devoted themselves to develop theories on the structure of nuclei which would also take the neutron into account as a constituent. The aim was to interpret the mounting experimental evidence in a satisfactory (and correct) way. Amaldi remembers that “before Easter that same year, Ettore Majorana had already tried to develop a theory of light nuclei assuming that they were made up only of protons and neutrons (or “neutral protons” as he used to say then) and that the former interacted with the latter through exchange forces (Amaldi 1968). Such forces are of a purely quantum origin and are responsible for the existence of certain kinds of molecules, as had been discovered by Heitler and London at the end of the 1920s; Majorana already had a good knowledge of them, as attested by some of his papers published in the previous years and in his personal study notes.

Immediately after Chadwick’s discovery [...] Majorana wrote down the expression for all possible forms of the exchange forces. All of them. Then he started to make a shell model; this is not known. He started to calculate the [energies of the] shells, but when he reached carbon, he was unable to go any further. He had started to do the calculations for helium [...] but when he reached carbon, or immediately after carbon – and these are very light elements – it suddenly became difficult. There is nothing astonishing about that. But he said that was stupid. It was obviously complete nonsense.¹²

Majorana “had told his friends at the Institute about this rough theory” and, as recalled by Eugene Feenberg, who was in Rome with the others at the time:

[...] he actually gave a seminar on his forces, nuclear forces. I remember Uhlenbeck and Inglis were there at that time and they thought it was a very big thing.¹³

“Fermi, who immediately understood their importance, suggested that he should publish the results as soon as possible, even if they were incomplete. But Ettore did not want to hear about it, as he considered his work incomplete” (Amaldi 1968). Needless to say, what Majorana’s overcritical eye viewed as “incomplete” was not

¹¹Another example of the incipient shift in interest in Italy is provided by the Institute of Physics in Naples, directed by Antonio Carrelli. Perhaps drawn by Fermi’s euphoria, Carrelli originally took an interest in the physics of the nucleus, but the main topic of research conducted in his institute (and later, the only one) remained atomic and molecular spectroscopy. This decision, possibly due to the limited means available, did not take the same happy turn as the Rome group.

¹²T.S. Kuhn’s interview with E. Amaldi, *loc. cit.*

¹³C. Weiner’s interview with E. Feenberg, 13 April 1973, kept at the Niels Bohr Library of the American Institute of Physics.

at all considered as such, even by a first-class physicist like Fermi, and this is once again confirmed by the notes kept in Pisa (the *Volumetti* and particularly the *Quaderni*). As so rightly pointed out (De Gregorio 2007), Fermi and his group, with the clear and decisive support of Majorana, contributed substantially within the international scientific community to the emergence of the idea (which later proved to be true) of the neutron as an independent and fundamental particle, a “neutral proton” and not a combination of a proton and an electron.

Despite Fermi’s encouragements, Majorana never published anything about his theory on the structure of nuclei in terms of just protons and neutrons. When Fermi was invited to attend an important conference in Paris on 7 July 1932, he chose to take stock of the situation in the physics of the atomic nucleus, and again decided to put pressure on Majorana, sure as he was of the relevance of the theory Ettore had developed. So, according to Amaldi, “he asked Majorana’s permission to sketch his ideas on nuclear forces. Majorana forbade Fermi to speak about it, saying that, if he did want to, he might do so, but in that case he should say they were the ideas of a famous professor of electrical engineering,¹⁴ who happened to be at the Paris Conference and whom Majorana considered a living example of how not to do scientific research. The condition imposed by Majorana had the clear purpose of preventing Fermi from quoting his “incomplete” theory. His allusion to the “famous professor of electrical engineering” relates to the personal vicissitudes of a cousin of his, with whom Ettore was in close contact, and who was then struggling to pass his own electrical engineering exam at the *Scuola degli Ingegneri* in Rome (his cousin eventually gave up his studies).¹⁵

Some weeks after Fermi came back from the conference, Werner Heisenberg’s first paper on nuclear theory appeared: it was based on the idea of an “exchange force” among nuclear constituents, already successfully introduced in the theory of the molecular bond, and made it possible to solve many theoretical difficulties in the understanding of nuclear structure. The international scientific community soon understood that, even though it was incomplete and there were some undeniable imperfections in Heisenberg’s theory, they were at last on the right track. “In the Institute of Physics at the University of Rome, everybody was extremely interested and full of admiration for Heisenberg’s results, but at the same time they were disappointed that (Majorana) had neither wanted to publish, nor allowed Fermi to speak of his ideas at an international conference. Fermi did his best once again to convince Majorana to publish something, but every effort of his and of ours, his friends and colleagues, was useless. Ettore would answer that Heisenberg had already said everything that could possibly be said and that, furthermore, he had probably said too much”.

¹⁴The “professor of electrotechnics”, as Amaldi recalls, was Giovanni Giorgi, known for his system of physical units.

¹⁵This piece of news was referred to us by A. De Gregorio.

This last caveat clearly refers to the flaws in the model proposed by Heisenberg and which prevented him from going further, flaws that were not present in Majorana's theory, as we will see in the next chapter. As a matter of fact, the idea of an "exchange force" which held together protons and neutrons inside the nuclei, as conceived by Heisenberg in close analogy with what happens between two hydrogen atoms H held together in the H_2^+ molecular ion by the exchange of an electron, actually contained within it the idea that the neutron was somehow made up of a proton and an electron. In contrast, Majorana's idea of a "neutral proton" led to the consequence that the nuclear bond was not due to the actual exchange of an electron among protons and neutrons in the nucleus, but rather to the exchange (according to quantum mechanics) in the mutual positions of the protons and neutrons. The consequences of Majorana's theory, unlike those of Heisenberg's, were well supported by experimental facts, and Majorana was clearly aware of this in his hint that Heisenberg might have said "too much".

A Trip to Leipzig

Shortly after what happened in July 1932, Fermi insisted once again: on the one hand he urged Majorana to obtain a lecturing post in theoretical physics (to teach in Italian universities), while on the other he encouraged him to go abroad, perhaps to visit Heisenberg himself. This time, in contrast to his other attempts, Fermi succeeded on both fronts: Ettore competed for, and as expected, obtained a lecturing post—*libera docenza*—in November 1932 (as we will see later on), and in the meanwhile he was persuaded to make a journey abroad, perhaps following the example of other members of the Rome group (Ettore's friends such as Giovannino Gentile, Edoardo Amaldi, Emilio Segrè, and others). As soon as he got his approval, Fermi managed to obtain a grant for this trip from the *Consiglio Nazionale delle Ricerche* (C.N.R.).

The "programme" for his trip abroad is clearly stated by Majorana himself in his letter dated 9 January 1932 to the vice president U. Bordoni of the National Committee for Astronomy, Physics and Applied Mathematics of the C.N.R. (Recami 1987):

It is my intention to leave around the 15th of this month for Leipzig and stay there the whole month of June to carry out, under the guidance of professor W. Heisenberg, theoretical research focusing mainly on the structure of nuclei and the relativistic formulation of the new quantum theory. During the holiday period between the winter and summer terms I will be returning to Italy for about fifteen days. I will spend the rest of the time attending conferences and scientific meetings which traditionally take place during that period in Germany and Denmark.

Once he got the C.N.R. grant, Majorana eventually left for Leipzig, where Heisenberg had brought together a group of young and very talented physicists,

making the Institute of Theoretical Physics one of the major world centres for the study of modern physics. The productive environment that welcomed Ettore in Germany is well described by his colleague Giovannino Gentile, who had been there a couple of years before, and who probably described it to his friend, as Ettore's first two letters¹⁶ to Giovannino seem to attest:

From Leipzig, where I am staying for my studies, I am going back to Berlin after almost a year. This time I am leaving with my colleagues. Heisenberg is with us too and we are here to meet our colleagues in Berlin. These meetings are interesting. We talk, we discuss things, we get to know each other; that is the way we work together. [...] In Leipzig, in our Institute of Theoretical Physics, there is a bunch of people, from different countries, who work around a man who has brought brand new ideas and methods to science. This man is Werner Heisenberg. Very young, about twenty-nine or thirty, fair haired, he almost looks like an Englishman: but in fact he is German, a rather typical German, with a typical mentality and refined culture. [...]

But it is mainly in the "colloquia" – weekly meetings held in the various physics institutes – that one feels the greatness of Berlin scientific life. After all this is an institution common to all the faculties of every German university. [...]

What I am saying is that it is hard to put across how productive such conversations, and such collaboration, can be. There is no such thing as an isolated scientist here; everywhere he will hear voices echoing his own, and we are easily motivated to scientific research and critique, which is the first condition for carrying out any scientific work. Because any science not yet crystallised in a research paper will pulsate and shake, and a new thought can arise spontaneously in such luxuriant surroundings.

This is one of the reasons why here in Germany one is soon convinced that the conditions for scientific research are extremely favourable, precisely because there is such a wealth of men and equipment.¹⁷

Ettore arrived in Leipzig on the night of 19 January 1933,¹⁸ and two days later he was already talking about¹⁹ "the work to be done" with Heisenberg. To his mother he wrote (on January 22):

At the Institute of Physics I have been warmly welcomed. I have had a long conversation with Heisenberg, who is an extraordinarily polite and friendly person. I get on very well with everybody, especially with the American Inglis whom I had met in Rome and who now often keeps me company and shows me around.²⁰

During his stay abroad, Majorana maintained "personal relations with various eminent people";²¹ among others, he met D.R. Inglis, E. Feenberg, F. Bloch, P. Ehrenfest, F. Hund, P. Debye, B.L. van der Waerden, G. Wataghin, G. Placzek, N. Bohr, C. Møller, V. Weisskopf, H. Kopfermann, W. Pauli, H.A. Bethe, L.

¹⁶Letters MG/R1 of 22 December 1929 and MG/R2 of 15 May 1930 in Recami (1987).

¹⁷G. Gentile jr.'s letter to his family of 9 February 1931, in Gentile (1942).

¹⁸Letter MF/L1 of 20 January 1933 in Recami (1987).

¹⁹Letter MB/L1 of 21 January 1933 in Recami (1987).

²⁰Letter MF/L2 of 22 January 1933 in Recami (1987).

²¹Letter MF/L4 of 14 February 1933 in Recami (1987).

Rosenfeld, and G. Beck. He soon felt the difference in the working conditions as compared to the Institute of Physics in Rome, these being clearly better suited to his research in theoretical physics. As a matter of fact, where Fermi had failed with Majorana, Heisenberg now easily succeeded just a few weeks after his arrival: “I am writing some papers in German. The first one is ready”.²² And, even more explicitly: “I have written a paper on the structure of nuclei which Heisenberg much appreciates, though it contained some corrections to a theory of his”.²³ We already dealt with these remarkable corrections in the previous paragraph, but it is quite surprising that Heisenberg himself (who would receive the Nobel prize right at the end of 1933) stressed the importance of Majorana’s theory on many occasions, as Ettore had already noted in a letter to his family:

In our last “colloquium”, a weekly meeting among more or less a hundred physicists, mathematicians, chemists, etc., Heisenberg spoke about the theory of nuclei and sponsored one of the works I have written here.²⁴

On this occasion, Heisenberg also invited Majorana to present his theory to those present, but he refused:

I remember a conversation when Heisenberg was talking about nuclear forces. Majorana was among the listeners and Heisenberg asked him to stand up and explain his version of exchange forces.

But I also remember that Majorana was too shy or too aware of his lack of knowledge of the German language, whence I am quite sure that he declined the invitation and said nothing.²⁵

Even after this episode, as for example at the VII International Solvay Conference of Physics held in Brussels in the autumn of 1933, during his lecture on the theory of nuclear forces, Heisenberg “*almost always* uses sentences like “according to Majorana”, “following the example of Majorana”, “as Majorana has stressed”, “we will adopt, as did Majorana”, mentioning his personal contribution only on rare occasions and in a very understated way. This reference to the Sicilian physicist is so persistent that he seems to want to express a debt of gratitude towards the colleague who had made some corrections to his theory” (De Gregorio 2007).

The general impression Ettore gave of himself in the Leipzig group is reported by Heisenberg himself during an interview of 1963 archived in the Oral History of Quantum Physics:

There was a very good physicist in my laboratory in Leipzig with the name Majorana. You know the so-called Majorana representation of the Dirac particle. He came as a young Italian physicist to Leipzig. He was a very brilliant man and at the same time a very nervous type of a man. He did excellent work. He was always extremely pessimistic about physics. I tried always to induce him to write papers and so he did finally write a very good paper.²⁶

²²Letter MF/L4, *loc. cit.*

²³Letter MF/L5 of 18 February 1933 in Recami (1987).

²⁴Letter MF/L6 of 22 February 1933 in Recami (1987).

²⁵C. Weiner’s interview with E. Feenberg, *loc. cit.*

²⁶T.S. Kuhn’s interview with W. Heisenberg, 28 February 1963, Session no. 10.

Heisenberg was clearly struck by Ettore's pessimistic character:

I would say that he was perhaps not pessimistic about physics especially, but rather about life in general. He was that kind of difficult fellow. Well, sometimes I thought perhaps he had had very difficult experiences in his life with other people, perhaps with girls or something like that. I don't know. Anyway, I couldn't make out why he, being such a young man, and such a brilliant young man, could always be so pessimistic. He was a very attractive fellow, so I liked him in our Leipzig group. I tried to see him frequently, and we had him with us for our ping pong games. Then I would sit down with him and ask him, not only about physics but more personal things, and so on. So I tried to keep in touch with him. He was a very attractive fellow but very nervous, so he would get into a state of some excitement if you talked to him. So he was a bit difficult.²⁷

And again:

People tried to talk to him and he was always very kind and very polite and very shy. It was very difficult to get something out of him. But still, one could see at once that he was a very good physicist. When he made a remark, it was always to the point.²⁸

The relationship between Majorana and Heisenberg struck many of those who witnessed their conversations, as one of these remembers, A. Recknagel:

I was very much impressed to see how Majorana would discuss with Professor Heisenberg as an equal. At the time I was still a student, and what a professor said was the truth for me, all the more so if that professor had been awarded a Nobel prize. Only gradually have I understood that you can criticize a professor the same way you criticize a student. But at the time if a young man like Majorana was able to discuss freely or even criticize a professor, that seemed astonishing to me. And that is the reason why I still remember Majorana today.²⁹

The theory of nuclei, however, was not the only topic Ettore was developing during his stay abroad. For instance, Victor Weisskopf tells us that he had "a discussion with Majorana about the latest developments in quantum electrodynamics",³⁰ a topic which was to become one of Ettore's favourites, as we will see later on. However, the most interesting news concerned the pioneering paper on the "relativistic theory of particles with arbitrary intrinsic momentum", published before he left for Leipzig and which we have already discussed.

In this paper there is an important mathematical discovery, as I have checked with professor van der Waerden, a Dutchman who teaches here, one of the greatest authorities on the theory of groups.³¹

²⁷T.S. Kuhn's interview to W. Heisenberg, *loc. cit.*

²⁸T.S. Kuhn's interview to W. Heisenberg, 5 July 1963, Session no. 11.

²⁹Interview to A. Recknagel in F. and D. Dubini, *La scomparsa di Ettore Majorana*, television programme aired in 1987 by Swiss TV.

³⁰Testimony T4 in Recami (1987).

³¹Letter MF/L5, *loc. cit.*

The next episode, told by Heisenberg, is also revealing and it involves B.L. van der Waerden:

van der Waerden's role in Leipzig was very important because he had a tremendous ability to understand quickly what people were talking about [concerning group theory] and then he knew all these things so well, so with just a few sentences of explanation, he could immediately clarify a complicated situation arising at one of our seminars. [...] I feel that I have really learned a large part of my mathematical training from van der Waerden, just by discussing with him. [...] Well, we spoke about the Weyl spinor business. The Dirac spinor was the thing everybody was talking about, but then there was the Weyl spinor business, which van der Waerden knew. The others did not know about it, but then there was Majorana. He was in Leipzig and Majorana found his Majorana particle, which has no charge but still has spin $1/2$, and that, of course, had to be represented by Weyl's spinor.³²

Clearly encouraged to go further in this direction (unlike the theory of nuclear structure, Fermi and the other members of the Rome group had not understood the importance of this work which was, without doubt, among the author's most theoretical), Majorana wrote to professor Bordoni (C.N.R.) to inform him that "the manuscript for a new relativistic theory of elementary particles is ready".³³ This paper is now lost (perhaps, according to the above quote from Heisenberg, it contained the theory Majorana published some years later on the so-called "Majorana neutrino"), and it was certainly not published in the German journal as had been Ettore's intention, probably under pressure from C.N.R. bureaucrats who wanted papers to be published first in Italian journals (Recami 1987).

Anyway, his personal studies continued in this field too, and the subject matter became one of those he spent most time investigating, even after his return to Italy, as is clear from the *Quaderni* kept in Pisa, where many pages are devoted to it. Here his interest in purely theoretical questions (but those with a noted experimental relevance), linked both to quantum electrodynamics and in particular to Dirac's theory of the electron, is quite evident. Although it had already developed some time earlier during his personal studies in the library of the Institute of Physics in Rome (where Ettore would spend his time reading papers by Heisenberg, Dirac, and others), this "passion" really flourished in the favourable theoretical environment in Leipzig. As a matter of fact he often hinted at that in his letters to friends and colleagues (such as Segrè and Gentile):

They take Dirac's theory of positive electrons quite seriously. Heisenberg studies the properties of relativistic invariance and the possibility of having other applications, besides Dirac's calculation of the lifetime of positive electrons. Of particular importance is the calculation, already attempted by Beck, of the probability that a high-energy light quantum produce a pair of oppositely charged electrons when scattering from a heavy nucleus.³⁴

The winter term in Leipzig finished at the end of February, after which there were almost two months of holiday; Majorana took advantage of this to go to

³²T.S. Kuhn's interview with W. Heisenberg, 5 July 1936, *loc. cit.*

³³Letter MB/L2 of 3 March 1933 in Recami (1987).

³⁴Letter MG/L1 of 7 June 1933 in Recami (1987).

Copenhagen “until April 15 in the company of professor N. Bohr”,³⁵ as in the “schedule” he had presented some months earlier. However, to begin with, perhaps inspired by Heisenberg, on February 7 he wrote to his family:

I am going to stay in Leipzig until the end of February. In March and April, there are holidays here. I will probably take advantage of this by going to Zurich work with Pauli, one of the greatest living scientists.³⁶

As Majorana himself would state when he wrote to his friend Gentile from Copenhagen,³⁷ “there does not seem to be much choice for theoretical physicists outside Leipzig, Zurich, Copenhagen, and Rome”. These were of course the places where Heisenberg, Pauli, Bohr, and Fermi were working. It is curious to note that, though keen to move to Copenhagen for a few weeks, Ettore found it hard to leave the stimulating German environment that he had so recently joined:

I am really sorry I have to leave Leipzig where I was warmly welcomed, and I shall be glad to return in two months.³⁸

And again:

I may stay in Leipzig for two or three days more because I must talk with Heisenberg. His company is unique and I wish to take advantage as long as he is here.³⁹

Ettore arrived in Copenhagen on March 4 and fitted in “perfectly from the first moment”.⁴⁰ Even if his meeting with Niels Bohr, one of the founding fathers of quantum theory, and the other guests at Bohr’s *Institut for Teoretisk Fysik*, was pleasant enough, it is easy to tell from Majorana’s letters that the new experience in Denmark was not so fruitful as the one in Leipzig, to which he enthusiastically returned on May 5, after a short trip to Italy for the Easter holidays.

He soon got back to his old study schedule, but with less intensity, owing to a persistent gastritis whose symptoms were hard to bear: “my activity in the last month has been somewhat reduced due to my poor state of health”.⁴¹ This illness would have certain consequences on Ettore’s future activity as well, but in the final reckoning his stay abroad could only be considered highly positive.

³⁵Letter MB/L2, *loc. cit.*

³⁶Letter MF/L3 of 7 February 1933 in Recami (1987).

³⁷Letter MG/C1 of 12 March 1933 in Recami (1987).

³⁸Letter MF/L6, *loc. cit.*

³⁹Letter MF/L7 of 28 February 1933 in Recami (1987).

⁴⁰Letter MG/C1, *loc. cit.*

⁴¹Letter MG/L1, *loc. cit.*

Back in Rome

Once back in Italy in the summer of 1933, Majorana entered several “gloomy years” of isolation. Amaldi, for instance, says (Amaldi 1968):

He started attending the Institute in *Via Panisperna* only occasionally and, as the months went by, he stopped coming at all; he would spend his days at home, immersed in his studies for hours on end.

And again:

More than one attempt made by Giovanni Gentile Jr, Emilio Segrè, and myself to bring him back to a normal life was useless. [...] None of us was able to discover whether he was still keen on theoretical physics; I think so, but have no evidence.

We shall not dwell further on the possible reasons why he acted this way: it might have been his poor state of health since the last part of his trip abroad or, later on, his father’s death in 1934. We just don’t have enough information (but the reader is free to consult the available bibliography).

It is far more interesting to linger on his possible scientific and/or even educational research. Even to a witness like Amaldi, it sounded strange that Majorana would so suddenly abandon theoretical physics to devote himself to topics such as “politics, the navies of different countries and their balance of power, and ship-building characteristics” (Amaldi 1968), or indeed to philosophy—although that was something he had always cultivated—and even medicine. Indeed, he had always had a deep interest in theoretical physics over the previous years (even before his graduation), a fact proved not so much by his publications as by his many personal notes. Nevertheless, all of these and other “strange” topics were among the subjects Ettore took up in those years, as we can see by the presence of the Italian Nautical Almanac of July 1937 in his otherwise scanty personal library (only 29 volumes, later looked after by his sister Maria). And the “attempts” Amaldi refers to are indirectly documented by the presence of at least two or three books in his library: James Jeans’s *I nuovi orizzonti della scienza* [*The New Background of Science*] (1934), containing a handwritten inscription by his friend Giovannino Gentile, who was the Italian editor of the book; Franco Rasetti’s *Il nucleo atomico* [*The atomic nucleus*] published in 1936; and *Fisica nucleare* [*Nuclear Physics*] written by Gentile and published in 1937, also autographed by the author. His friend Giovannino’s two volumes were probably not given to Ettore in person, as there are two thankyou letters, the first dated 27 July 1934, and the second 20 June 1937. Clearly, such letters seem to contradict the idea that Majorana had abandoned physics, as they testify to a non-superficial reading of the two texts. Even the circumstance that these letters were written at Monteporzio Catone, where the Majoranas used to spend the occasional holiday breaks, tends to contradict the idea of Ettore’s living in “isolation”: they were both written in summertime, while other letters he wrote testify to his presence in Rome at various times of the year. It was perhaps rather the serenity of the country house that favoured his reading the cited texts.

In his first letter of 27 July 1934 Majorana wrote:

I thank you very much for sending me Jeans's book in your nice edition (and translation?). It arrived at a perfect time to distract me from my country idleness. I admire the thoughtful preface, well suited to Italian readers, with appropriate references to the prevailing current of thought here. I think that the greatest quality of this book is that it anticipates the psychological reactions the recent advances in physics will necessarily produce once it is generally understood that science has stopped being a justification for vulgar materialism. I do think your translation can seriously contribute to reviving an interest in scientific problems here in Italy. I will be in Rome in a few days and I hope we will have the chance to meet.⁴²

The hint about the “materialistic” use of science is clear evidence of Majorana's interest for philosophical questions, but his main concern nevertheless always seems to be an “interest for scientific problems”.

Likewise, but in an even more direct tone, in the letter dated 20 June 1937:

I do thank you for your fine book *Fisica nucleare*. It is a truly perfect educational work, and for the amount and quantity of information, it is indeed an agreeable read, extremely interesting for anyone with a minimum of technical competence. I hope your publisher can “launch” it properly, because there has not been anything like that in Italy for ages, and nor will there be any time soon. It really should be in everybody's hands.⁴³

Finally, it is worth noting that, at least according to the documents we currently have in our possession (above all Rasetti's book, as Gentile's second book might simply have been a gift from a friend), Majorana had not apparently spent much time doing theoretical physics since his return from Leipzig. So does that mean we must assume that Majorana abandoned theoretical physics for four years, only to take it up again at the end of 1937 when he entered the selection for a university chair? Some letters to his uncle Quirino do show Majorana going through a “peculiar” period. Indeed, in a letter dated 6 September 1933, when he was just back from Leipzig, we read:

I have been in Rome for a month. My folks have come in dribs and drabs as well, and will probably go to Monteporzio in a short time. I will stay in Rome and you should write letters home as I seldom go to the Institute. I am not going back to Leipzig this year. I believe dad has stopped doing research. In Rome I have only found Rasetti; Fermi is coming back from his trip to America in a short while.⁴⁴

And then, in a letter dated 20 February 1935:

I thank you for your wishes regarding my work which I hope will soon be back to a normal level (that's not saying much).⁴⁵

⁴²Letter MG/R4 of 27 July 1934 in Recami (1987).

⁴³Letter MG/R5 of 20 June 1937 in Recami (1987).

⁴⁴Letter MQ/R4 of 6 September 1933 in Recami (1987).

⁴⁵Letter MQ/R5 of 20 September 1935 in Recami (1987).

In other letters to his uncle Quirino, there is no mention of Ettore's scientific activity, except for a letter dated 16 January 1936, which we will bring up in the next section.

However, for the years we are concerned with here, there is clear evidence that Ettore's "dark period" was not at all a time of abstention from research in physics. A first fascinating part of this evidence is to be found in the letters between Ettore and his uncle Quirino, a noted experimental physicist of the day. They testify to a "collaboration" between the two scientists, even though they did not deal with the kind of frontier research which Ettore had always preferred.

From 1925 to about 1940, Quirino Majorana was involved in experimental research on the phenomenon of photoresistance in thin metal films; he studied the increase in electrical resistance of such films when they are exposed to light under particular conditions. This effect was usually ascribed to the thermal action of light that heats up the film and thus alters its electrical properties. However, Quirino Majorana thought he had identified a new effect which could not be explained through the thermal action of light, but instead through a kind of photoelectric phenomenon differing from the classic effect discovered by Heinrich Rudolf Hertz in 1886. It was the latter whose theoretical interpretation in 1905 had earned Albert Einstein the Nobel Prize (the photoelectric effect refers to the emission of electrons from certain metals when these are illuminated by light of a particular type). Clearly, in order that such an interpretation could even be entertained, it was essential that the thermal origin of the observed effect should be excluded with reasonable certainty; and this could only come out of an appropriate theoretical study, whereupon accurate predictions of the said thermal action could be followed by experiments that would exclude it. Such theoretical calculations were indeed presented by Quirino Majorana in 1938, in what can be considered the conclusive paper on the topic (Majorana Q. 1938). However, a quick comparison between this paper and the contemporary correspondence between Ettore and his uncle⁴⁶ clearly shows that the author of the theoretical study was not uncle Quirino at all! It was in fact his nephew Ettore who had provided the appropriate theory for the observed phenomena.⁴⁷ Several times uncle Quirino tried, quite rightly, to acknowledge his nephew's work, urging him to publish his theory, or at least let mention his contribution in the 1938 paper; but Ettore, as usual, refused and both the theoretical calculations and the interpretation of the experiments appeared as Quirino Majorana's work.

What may seem like a simple piece of help Ettore gave his uncle to confirm or formulate some mathematical predictions actually hides a deeper and more complex kind of cooperation between the two scientists, one which ignored the considerable age gap between them, but which appears clearly from their private

⁴⁶See, in particular, Ettore's letter to Quirino Majorana dated 14 May 1935, kept at the Museum of Physics of the University of Bologna.

⁴⁷An introductory, but particularly interesting, study can be found in (Dragoni 2006). The reference book is (Dragoni 2008).

correspondence. Ettore's interest in his uncle's research dates back at least to the time he was preparing his graduation thesis.⁴⁸ At first, Ettore served simply as a "bridge" between his uncle and Fermi; Quirino Majorana really wanted the opinion of the young and brilliant scientist in Rome, and Fermi was quite ready to make suggestions about possible interpretations of the newly observed effect. Such was Ettore's role until his departure for Leipzig. From the end of 1933, Ettore's "remote" involvement in his uncle's research evolved steadily from simple and occasional comments towards active and interested participation. And he did not limit himself to providing the underlying theory for the experiments. Indeed, he suggested technical improvements or identified possible causes of error or secondary effects, so as to control the phenomenon as far as possible.

The collaboration with uncle Quirino extended to other things as well. In this respect it is perhaps significant that Ettore wrote a whole speech his uncle gave as President of the Italian Physical Society (SIF), at the 1937 conference in Bologna to celebrate the 200th anniversary of the birth of Luigi Galvani, attended by many Nobel prize winners and famous scientists from the world over. It is likely that, as the event was approaching, Quirino Majorana (who had had surgery earlier) had asked for help, and his nephew answered right away:

I am so glad to hear that you are recovering. [...] I am only disappointed that I was not so well myself and was not able to visit you during your stay at the hospital. [...]

Dalla Noce has informed me about the conference for Galvani, which I would like to attend, but may have to skip in order to avoid too long a journey from Sicily. I have improvised an opening speech according to your wishes. You may consider it too generic, but it has not been easy to say much more without exceeding the imposed limits. Anyway, if you do not like it, send it back with your comments. [...] Concerning the speech, the set opening and closing are not models of eloquence, but I have included them for their possible links with the rest.⁴⁹

Quirino Majorana's speech turned out to be really a great success, and Ettore made clear his satisfaction: "I have received also *Sapere*. Much admired, and by many, your speech".⁵⁰

⁴⁸In this regard, compare the title of one of his oral theses, *Su un effetto fotoelettrico constatato negli "audion"* ("On a photoelectric effect observed in audions") with that of Q. Majorana's article "Su di un fenomeno fotoelettrico constatabile con gli audion" ("On a photoelectric effect observable with audions"), in *Rendiconti della R. Accademia dei Lincei*, vol. 7 (1928), p. 801.

⁴⁹Letter MQ/R28 of 1 September 1937 in Recami (1987).

⁵⁰Letter MQ/R29 of 16 November 1937 in Recami (1987).

Libero docente

On 12 November 1932, Majorana became a lecturer (*libero docente*) in theoretical physics, although it only became official on 21st January of the following year, soon after his departure for Leipzig. Up until now, little importance has been given to this qualification, probably because none of those close to Majorana ever made sufficient mention of it. On the one hand, everyone took it for granted that he would eventually become a professor without any effort, even though we do not know whether he had been encouraged by Fermi, or by someone else, to do so, or whether it was an idea of his own. On the other hand, what happened later on might have led some people to think that Majorana disliked teaching.

The importance to Majorana of becoming a university professor of theoretical physics has only recently come to light, thanks to the unexpected discovery of three documents in the archives of the University of Rome.⁵¹ These documents, undoubtedly signed by Majorana himself, are his official request to give lectures in the University. The first carries the authorisation of the director O.M. Corbino “to use the classrooms and the library of the Institute of Physics for his lessons”. Let us examine this in more detail.

The first request dates from May 1933 and concerns the course in *Mathematical methods of quantum mechanics* which Majorana would have liked to give during the academic year 1933–34. It contains the detailed list of the topics to be dealt with in the course, at a rate of three hours a week. It is worth noting that this request was made during the short period Ettore was back in Rome from abroad, from April 12 to May 5, whence it indicates a strong interest on his part, no matter how else one may assess this.

The second request is dated 30 April 1935. The title of the course to be held in the academic year 1935–36 was *Mathematical methods of atomic physics*; here too we find a list of the topics to be dealt with in the course, these differing significantly from those of the previous course.

Finally, the third request was made on 28 April 1936, and concerns the course on quantum electrodynamics to be given in the academic year 1936–37. Obviously, in this case the topics in the scheduled two-hour weekly lessons are totally different from those of the other two courses.

The fact that all three courses were never given is basically certain; not only because his friends and colleagues (Amaldi, Segrè, and others) never mentioned them, but most of all because in the cited documents it is explicitly stated that Majorana “did not deliver courses in the past”. Of course, we cannot be completely sure of this as far as the last course is concerned, as we have no subsequent statement. However, it seems perfectly reasonable to extend what has just been said to the third course as well. It is nevertheless extremely interesting that, in the alleged “dark years”, Majorana would repeatedly put forward his request to give lectures pretty much every year. The only time he did not was in the year 1934–35

⁵¹This discovery is due to Alberto De Gregorio. See (De Gregorio and Esposito 2006).

for the obvious reason of his father's death, and we know he was very close to his father, or possibly also due to a loss or theft.⁵² Though his first request in 1933 was quite possibly made in a period of euphoria due to the experience he was living abroad, where he may well have found working conditions better suited to his personality and abilities than they were in Rome, it is appropriate to remember that he became a lecturer before his stay in Leipzig. The later requests, made while Majorana was living permanently in Rome, thus suggest a sincere, self-determined, and self-motivated interest in teaching, independently of external causes (most likely, a lack of students) which might have prevented the course from taking place.

Another surprising conclusion regarding those years comes from even a superficial analysis of the contents of the courses Majorana had drawn up. These reveal a thorough evaluation of the chosen topics, based on an absolute knowledge and mastery of the contemporary scientific literature, quite different from those dealt with in similar courses (theoretical physics, mathematical physics, and others) of the same period. Different even from those given by Fermi, which were among the most cutting-edge courses in Italy.

There were also significant echoes of these topics in the lessons given later at the University of Naples. The contents of *each* course were drawn up in an amazingly accurate way, given the unprecedented nature of the arguments they dealt with, although they would later become the norm in theoretical physics courses, both in Italy and abroad. This in no way supports the idea that Majorana did no physics at all through the period from 1933 to 1937, just because he no longer frequented his friends and colleagues at the Institute of Physics in Rome. In particular, his second request of 30 April 1935 (right after the gap in the previous year) sounds like a clear reaction to the letter Ettore wrote to his uncle Quirino on February 20, in which he wished for a return to "normal" activity. So, while there is no reason to doubt a succession of one or more "difficult" periods (whose causes we may only speculate about, but which do not seem to be in any way related to his scientific activity), it would nevertheless be an exaggeration to claim that this period lasted for four years, with Majorana shut up in his house as far away as possible from the physics environment. Instead, this seems rather to testify to a change in attitude: he continued to visit the university every now and then (at least to hand in his lecturing requests!), but he no longer took part in the activities of Fermi's group. And this may have a simple explanation if we consider the research being done in Rome during this period, when Fermi and his partners were engaged full time in

⁵²This last conjecture, which might easily be described as fanciful, has nevertheless one verifiable premise. Anyone interested in this may, for example, take a look at the courses given by Fermi (or others) at the University of Rome, paying a visit to the university archive, where they are still kept. Anyone who does this will not fail to notice the lack of a syllabus for Fermi's earth physics course, given in 1928, lectures that were also followed by Majorana, as we have seen. Such an omission would not be surprising at all if it had been known that the aforementioned document was to be found among Majorana's papers deposited by Amaldi at the *Domus Galileana* in Pisa, as observed by both De Gregorio and myself in 2005. For the moment, we do not have any plausible explanation for this.

fundamental studies of nuclear physics. For while these studies were regarded with respectful interest by the international scientific community, they were of an essentially experimental nature... Majorana may simply have distanced himself from something that did not appeal to his tastes and interests, thus continuing his own theoretical research independently.

Let us end with another alleged “enigma” we hinted at the end of the last section. In a letter to uncle Quirino on 16 January 1936, Ettore wrote: “I have been studying quantum electrodynamics for a while now”.⁵³ In relation to this, it has been noted that “given Ettore’s modest way of expressing himself, this means that—and it is of the greatest importance—in the year 1935, Majorana devoted himself deeply to original research in the field of *quantum electrodynamics*. Unfortunately, up to now, there is no trace of his notes, with all the new results he must surely have produced” (Recami 1987). If one studies the 18 *Quaderni* of Majorana’s personal notes (Esposito et al. 2008), it is clear that much attention is devoted to several questions of quantum electrodynamics. Unfortunately, in contrast to the *Volumetti* (Esposito et al. 2003), almost none of these *Quaderni* are dated, whence it is difficult to say for sure that his uncle Quirino’s letter is referring to the contents of these notes. However, it does seem highly plausible for at least some of the topics Majorana wanted to deal with in his third course. And moreover the choice of such a course was not a matter of chance: it is totally different from the other two courses, and it was chosen prior to 28 April 1936. Once again, it seems to correspond to the letter to his uncle written only three months earlier.

⁵³Letter MQ/R14 of 16 January 1936 in Recami (1987).



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