

Preface

We are proud to introduce, as the scientific organisers, the 2013 CIME Course *Vector-valued Partial Differential Equations and Applications*, which took place at Cetraro (Cosenza, Italy) from July 8 to 12, 2013, with the following speakers and courses of lectures:

Bernard Dacorogna (École Polytechnique Fédérale de Lausanne, Switzerland), *The pullback equation*.

Nicola Fusco (Università degli Studi di Napoli Federico II, Italy), *The stability of the isoperimetric inequality*.

Stefan Müller (Universität Bonn, Germany), *Mathematical problems in thin elastic sheets: scaling limits, packing, crumpling and singularities*.

Vladimir Šverák (University of Minnesota, USA), *Aspects of PDEs related to fluid flows*.

The programme included a special session to celebrate the 60th birthday of Bernard Dacorogna, with lectures by Gianni Dal Maso, Carlo Sbordone, Giovanni Cupini, Emanuele Paolini and Giovanni Pisante.

That the meeting was such a success was a consequence of the distinction of the speakers and the high level of their lectures, as evidenced by the quality of the notes in this volume, as well as the participation and active involvement of the participants, who numbered well over 100.

We now briefly describe the course notes included in this set of Lecture Notes, starting with the course of Bernard Dacorogna on the *pullback equation*. A map $\varphi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ solves the *pullback equation* $\varphi^*(g) = f$ if it is a diffeomorphism which satisfies the equation with f, g differential k -forms with $0 \leq k \leq n$. For instance, in the case $k = n$, the equation takes the form $g(\varphi(x)) \det \nabla \varphi(x) = f(x)$. *Local existence* is analysed, as well as *global existence* in the Hölder space $C^{r,\alpha}$.

In his course, Nicola Fusco considered the stability of the *isoperimetric inequality*. Once we know that, for a given volume, balls are the unique area minimisers,

the next natural question is to understand what happens when a set E has the same volume of a ball B and a slightly bigger surface area. Precisely, one would like to show that in this case E must be close in a proper sense to a translation of B . The *stability of the isoperimetric inequality* for general sets of *finite perimeter* is analysed in detail, the proof being based on a suitable symmetrisation argument aimed at reducing a general set of finite perimeter to an axially symmetric bounded set with a centre of symmetry.

Stefan Müller presented in his course an outline of the theory of *thin elastic sheets*; in particular, he considers the *limiting behaviour* of thin elastic objects as the thickness h goes to zero. Mathematically one can distinguish two types of problems: either where the solution has a *well-defined limit* as $h \rightarrow 0$, when the natural goal is to characterise the limit, or where the solution develops *increasing complexity*.

The course of Vladimir Šverák concerned two main themes. The first deals with the *long-time behaviour* of solutions of the $2D$ incompressible Euler equations and other Hamiltonian equations. The second theme is related to the *problem of uniqueness* of the Leray–Hopf weak solutions with L^2 initial data.

We are pleased to express our appreciation to the speakers for their excellent lectures and to the participants for contributing to the success of the CIME Course. We had in Cetraro an interesting, rich and friendly atmosphere, created by the speakers, by the participants and by the CIME Organisers, in particular *Pietro Zecca* (*CIME Director*) and *Elvira Mascolo* (*CIME Secretary*). At the date of publication, *Elvira* now has the role of CIME Director, while the CIME Secretary is *Paolo Salani*. We thank all of them warmly.

Acknowledgements CIME activity is carried out with the collaboration and financial support of: INdAM (Istituto Nazionale di Alta Matematica)—MIUR (Ministero dell’Istruzione, dell’Università e della Ricerca)—Ente Cassa di Risparmio di Firenze.

Oxford, UK
Firenze, Italy

John Ball
Paolo Marcellini

Vector-Valued Partial Differential Equations and
Applications

Cetraro, Italy 2013

Dacorogna, B.; Fusco, N.; Müller, S.; Sverak, V. - Ball, J.;
Marcellini, P. (Eds.)

2017, VII, 250 p. 21 illus., 1 illus. in color., Softcover

ISBN: 978-3-319-54513-4