# Chapter 2
# Analysis Methods

## 2.1 Nodal Analysis

**Problem 2.1.1** Two current sources with equal internal resistances feed a load as shown in Fig. 2.1.

$$I_a = 200\,\text{A}, \quad I_b = 100\,\text{A}, \quad R = 200\,\Omega, \quad R_L = 100\,\Omega.$$

(a) Find the current through the load resistor $R_L$.
(b) Find the node voltage value.

**Solution**

(a) Parallel-connected current sources, KCL applies, $200\,\text{A}||(-100\,\text{A}) \rightarrow 100\,\text{A}$
By current division,

$$I_{RL} = 100 \times \frac{200||200}{100 + 200||200} = 100 \times \frac{100}{100 + 100} = 50\,\text{A}.$$

(b) Applying Ohm's law,

$$V_x = 100\,\Omega \times 50\,\text{A} = 5000\,\text{V}.$$

**Problem 2.1.2** Find the values of currents and voltages in the circuit shown in Fig. 2.2, for $R = 2\ \Omega$.



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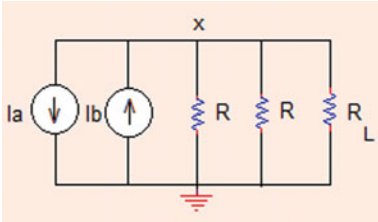



**Fig. 2.1** The circuit for Problem 2.1.1

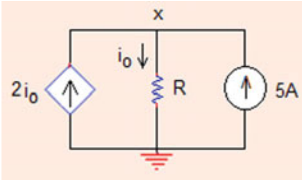



**Fig. 2.2** The circuit for Problem 2.1.2

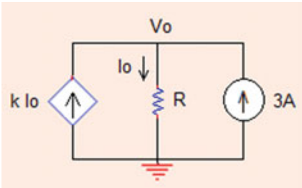



**Fig. 2.3** The circuit for Problem 2.1.3

**Solution**

By Kirchhoff's current law, $2i_0 + 5 - i_0 = 0 \rightarrow i_0 = -5\,\text{A}$

$$v_x = i_0 R = -5 \times 2 = -10\,\text{V}.$$

**Problem 2.1.3**

(a) Find the value of $V_0$ in the circuit shown in Fig. 2.3.
(b) If the gain constant of dependent source is $k$, what are the limiting values of $k$, if $I_0$ has always a positive value? Resistor value is 4 Ω.

**Solution**

(a) KCL: $3 + 0.2I_0 - I_0 = 0 \rightarrow I_0 = 3.75\,\text{A}$

$$V_0 = 4I_0 = 4 \times 3.75 = 15\,\text{V}$$

(b) $3 + kI_0 - I_0 = 0 \rightarrow I_0 = -3/(k-1)\,\text{A}, \quad 0 \le k < 1.$

**Problem 2.1.4** In the circuit shown in Fig. 2.4, the coefficient of current-controlled current source is 2 A/A. If the node voltage is 1 V, find the value of the voltage source, the current through $R_1$ = 1 Ω, and current through $R_2$ = 2 Ω.

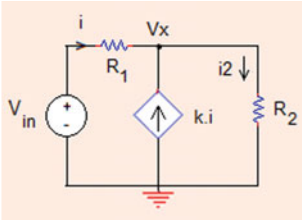



**Fig. 2.4** The circuit for Problem 2.1.4

**Solution**

Applying KCL at the node, and using given component values,

$$\frac{V_{\text{in}} - V_x}{R_1} + ki - \frac{V_x}{R_2} = 0$$

$$V_{\text{in}} - V_x + 2(V_{\text{in}} - V_x) - \frac{V_x}{2} = 0 \to V_{\text{in}} - 1 + 2(V_{\text{in}} - 1) - \frac{1}{2} = 0$$

$$V_{\text{in}} = 1.167\,\text{V}$$

$$i_2 = \frac{V_x}{2} = \frac{1}{2} = 0.5\,\text{A}$$

$$i = \frac{V_{\text{in}} - V_x}{1} = V_{\text{in}} - V_x = 1.167 - 0.5 = 0.667\,\text{A}$$

**Problem 2.1.5** Determine the node voltages in the circuit shown in Fig. 2.5. Use Cramer's rule, if necessary.

(a) For $I_1 = 1\,\text{A}, I_2 = 2\,\text{A}, \quad R_1 = 1/2\,\Omega, \quad R_2 = 1/8\,\Omega, \quad R_3 = 1/4\,\Omega$
(b) For $I_1 = 2\,\text{A}, I_2 = 4\,\text{A}, R_1 = 5\,\Omega, R_2 = 2\,\Omega, R_3 = 10\,\Omega$.

**Solution**

(a) For $I_1 = 1\,\text{A}, I_2 = 2\,\text{A}, R_1 = 1/2\,\Omega, R_2 = 1/8\,\Omega, R_3 = 1/4\,\Omega$

Node equations are

$$1 - 2V_1 - 4(V_1 - V_2) = 0$$
$$2 - 8V_2 + 4(V_1 - V_2) = 0$$

or

$$6V_1 - 4V_2 = 1$$
$$-4V_1 + 12V_2 = 2\,.$$

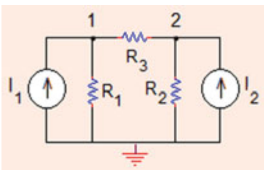



**Fig. 2.5** The circuit for Problem 2.1.5

In matrix form,

$$\begin{bmatrix} 6 & -4 \\ -4 & 12 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Applying Cramer's rule to solve this matrix equation yields

$$\begin{aligned} &\Delta = 72 - 16 = 56, \quad \Delta_1 = 12 + 8 = 20, \quad \Delta_2 = 12 + 4 = 16 \\ &V_1 = \frac{\Delta_1}{\Delta} = \frac{5}{14} = 0.357\,\text{V}, \quad V_2 = \frac{\Delta_2}{\Delta} = \frac{4}{14} = 0.286\,\text{V} \end{aligned}.$$

(b) For $I_1 = 2\,\text{A}, I_2 = 4\,\text{A}, R_1 = 5\,\Omega, \quad R_2 = 2\,\Omega, \quad R_3 = 10\,\Omega.$

Nodal equations in matrix form can be formed using "analysis by inspection";

$$\begin{bmatrix} \frac{1}{5} + \frac{1}{7} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{10} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{6}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}.$$

Applying Cramer's rule to solve this matrix equation yields

$$\begin{aligned} \Delta &= \frac{3}{10} \cdot \frac{6}{10} - \frac{1}{100} = \frac{18}{100} - \frac{1}{100} = \frac{17}{100}, \\ \Delta_1 &= 2 \cdot \frac{6}{10} + 4 \cdot \frac{1}{10} = \frac{12}{10} + \frac{4}{10} = \frac{16}{10} \\ \Delta_2 &= \frac{3}{10} \cdot 4 + \frac{1}{10} \cdot 2 = \frac{12}{10} + \frac{2}{10} = \frac{14}{10} \end{aligned}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\frac{16}{10}}{\frac{17}{100}} = \frac{160}{17} = 9.411\,\text{V}, \quad V_2 = \frac{\Delta_2}{\Delta} = \frac{\left(\frac{14}{10}\right)}{\frac{17}{100}} = \frac{140}{17} = 8.235\,\text{V}.$$

**Problem 2.1.6** Determine the value of current I in the circuit shown in Fig. 2.6. Use Cramer's rule, when necessary $R_1 = R_2 = R_3 = R_4 = 1/2\,\Omega$, $R_5 = 1/4\,\Omega$, $I_1 = 1\,\text{A}, I_2 = 2\,\text{A}$.

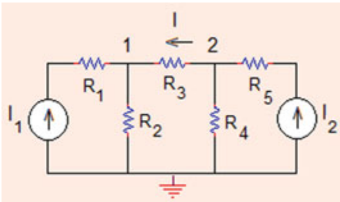



**Fig. 2.6** The circuit of Problem 2.1.6

**Solution**

Nodal matrix equation of the circuit is obtained by applying "analysis by inspection" method,

$$\begin{bmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_4+G_5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 2+2+2 & -2 \\ -2 & 2+2+4 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Applying Cramer's rule to solve this matrix equation yields

$$\begin{aligned} &\Delta = \begin{vmatrix} 6 & -2 \\ -2 & 8 \end{vmatrix} = 44, \quad \Delta_1 = \begin{vmatrix} 1 & -2 \\ 2 & 8 \end{vmatrix} = 12, \quad \Delta_2 = \begin{vmatrix} 6 & 1 \\ -2 & 2 \end{vmatrix} = 14 \\ &V_1 = \frac{\Delta_1}{\Delta} = \frac{12}{44} = 0.273\,\text{V}, \quad V_2 = \frac{\Delta_2}{\Delta} = \frac{14}{44} = 0.318\,\text{V} \\ &I = \frac{V_2 - V_1}{R} = \frac{0.318 - 0.273}{\frac{1}{2}} = 0.09\,\text{A} \end{aligned}.$$

**Problem 2.1.7** Find the values of voltages at the nodes of the circuit shown in Fig. 2.7.

$$G_1 = 0.5\,\text{S}, \quad G_2 = \frac{1}{4}\,\text{S}, \quad G_3 = 0.4\,\text{S}, \quad G_4 = \frac{1}{5}\,\text{S}, \quad G_5 = 1\,\text{S}, \quad I_1 = 5\,\text{A}, \quad I_2 = 4\,\text{A}.$$

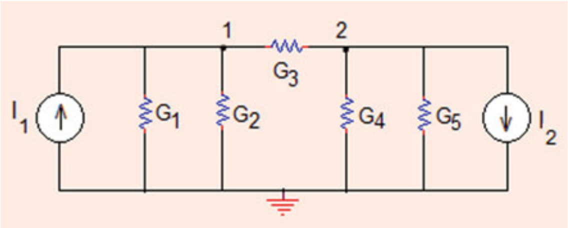
**Solution**

$$\begin{aligned} &G_A = G_1 + G_2 = 0.5 + 0.25 = 0.75\,\text{S}, \quad G_B = G_4 + G_5 = 0.2 + 1 = 1.2\,\text{S} \\ &\begin{bmatrix} G_A+G_3 & -G_3 \\ -G_3 & G_B+G_3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 1.15 & -0.4 \\ -0.4 & 1.6 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix} \end{aligned}.$$

Solution of this matrix equation for unknown voltages yields

$$V_1 = 3.8095\,\text{V}, \quad V_2 = -1.5476\,\text{V}.$$

**Fig. 2.7** The circuit of Problem 2.1.7



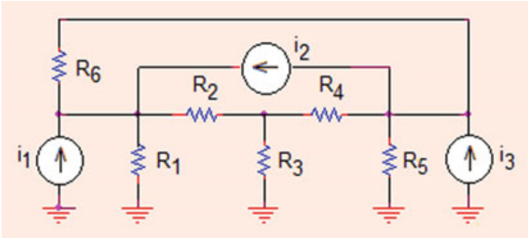



**Fig. 2.8** The circuit of Problem 2.1.8

**Problem 2.1.8** What is the voltage across resistor $R_3$ (in mV)? Use analysis by inspection and Cramer's rule if necessary ($i_1 = i_2 = i_3 = 1$ A, $R_1 = R_3 = R_5 = 1\ \Omega$, $R_2 = ½\ \Omega$, $R_4 = R_6 = 1/6\ \Omega$). Check the result using SPICE (Nodal 2.cir) (Fig. 2.8).

**Solution**

$$[G][V] = [I],$$

$$\begin{bmatrix} G_1+G_2+G_6 & -G_2 & -G_6 \\ -G_2 & G_2+G_3+G_4 & -G_4 \\ -G_6 & -G_4 & G_4+G_5+G_6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1+1 \\ 0 \\ 1-1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+2+6 & -2 & -6 \\ -2 & 2+1+6 & -6 \\ -6 & -6 & 6+1+6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 9 & -2 & -6 \\ -2 & 9 & -6 \\ -6 & -6 & 13 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

$$\Delta = 1053 + (-72) + (-72) - [324 + 324 + 52] = 906 - 700 = 209$$

$$\Delta_2 = \begin{vmatrix} 9 & 2 & -6 \\ -2 & 0 & -6 \\ -6 & 0 & 13 \end{vmatrix} = (2)(-6)(-6) - [(13)(-2)(2)] = 72 - (-52) = 124$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{124}{209} = 0.593\text{ V} = 593\text{ mV}$$

SPICE file:

```
*Operating point anaysis Nodal 2.cir
I1 0 1 1
I2 3 1 1
I3 0 3 1
R1 1 0 1
R2 1 2 0.5
R3 2 0 1
R4 2 3 0.1667
R5 3 0 1
R6 1 3 0.1667
```

**Problem 2.1.9** Use node voltage method to find the values for the voltage at node C (=$V_c$) and the current through the resistor $R_6$ (=$i$). ($R_1 = R_2 = R_3 = 1\ \Omega$, $R_4 = R_5 = R_6 = 4\ \Omega$, $i_S = 2$ A) (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx) (Fig. 2.9).

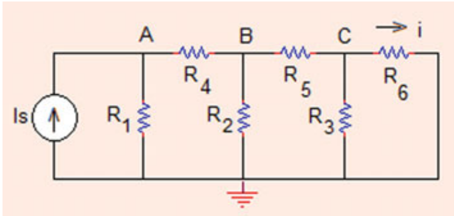



**Fig. 2.9** The circuit of Problem 2.1.9

### Solution

$$\begin{bmatrix} G1+G4 & -G4 & 0 \\ -G4 & G2+G4+G5 & -G5 \\ 0 & -G5 & G3+G5+G6 \end{bmatrix} \begin{bmatrix} V_{\mathrm{A}} \\ V_{\mathrm{B}} \\ V_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+\frac{1}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1+\frac{1}{4}+\frac{1}{4} & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 1+\frac{1}{4}+\frac{1}{4} \end{bmatrix} \begin{bmatrix} V_{\mathrm{A}} \\ V_{\mathrm{B}} \\ V_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1.25 & -0.25 & 0 \\ -0.25 & 1.50 & -0.25 \\ 0 & -0.25 & 1.50 \end{bmatrix} \begin{bmatrix} V_{\mathrm{A}} \\ V_B \\ V_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

Solution of this matrix equation by either manually using Cramer's rule or by employing available software (see, MATLAB m file or EXCEL file) yields

$$V_{\mathrm{A}} = 1.657\,\mathrm{V}, \quad V_{\mathrm{B}} = 0.284\,\mathrm{V}, \mathrm{V_C} = 0.04734\,\mathrm{V},$$

$$i = \frac{V_{\mathrm{C}}}{R_6} = \frac{0.04734}{4} = 0.011834\,\mathrm{A}.$$

### Problem 2.1.10

(a) In the circuit shown in Fig. 2.10, find the voltage gain, i.e., $V_2/V_1$ = ?
(b) If $R_1$ = 2.5 kΩ, $R_o$ = 10 kΩ, $R_L$ = 10 kΩ, $k$ = 50, find the numerical value of ($V_2/V_1$). Check the result using SPICE (cccs8.cir).

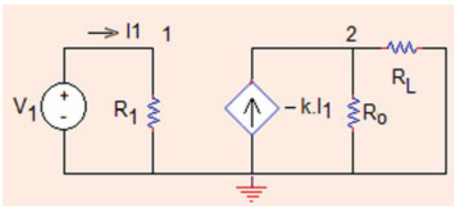



**Fig. 2.10** The circuit of Problem 2.1.10

**Solution**

(a) $I_1 = \frac{V_1}{R_1}, \quad V_2 = -k \cdot I_1 \cdot (R_o//R_L) = -k \cdot \frac{V_1}{R_1} \cdot (R_o//R_L)$

$$\frac{V_2}{V_1} = -k \cdot \frac{R_o//R_L}{R_L} = k \cdot \frac{R_o//R_L}{R_o + R_L} \cdot R_1$$

(b) $$\frac{V_2}{V_1} = -50 \cdot \frac{10//10}{2.5} = -50 \cdot \left(\frac{5}{2.5}\right) = -100\,\mathrm{V/V}$$

SPICE netlist (cccr8.cir);

```
*DC Operating point analysis
*current controlled current source-nodal analysis
*fx N+ N- Vy Value
*Parameters:
*x  Name of the source
*N+ : Name of positive node
*N- : Name of negative node. Current flows from the + node
*   through the source to the - node
*Vr : Name of the voltage source where the controlling current flows.
*     The direction of positive control current is
*     from + node through the source to the - node of Vr=0
*Value: Current gain
v1 1 0 1m
f1 2 0 Vr 50
Vr 1 3 0
R1 3 0 2.5k
R0 2 0 10k
RL 2 0 10k
```

**Problem 2.1.11** In the circuit shown in Fig. 2.11, $f = 2$, $R_1 = R_2 = R_3 = 1\,\Omega$, $i_S = 1\,\mathrm{A}$. $V_1 = ?, V_2 = ?, i = ?$

(Use node voltages method.) Check the result using SPICE (cccs7.cir).

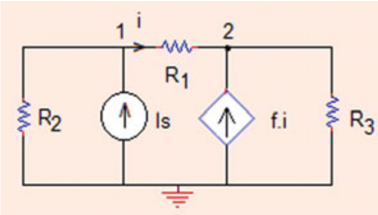



**Fig. 2.11** The circuit of Problem 2.1.11

**Solution**

$$i_S - \frac{V_1}{R_2} - \frac{V_1 - V_2}{R_3} = 0$$

$$\frac{V_1 - V_2}{R_1} - \frac{V_2}{R_3} + \frac{2(V_1 - V_2)}{R_1} = 0.$$

Substituting given values of components into these equation yields

$$\begin{aligned} 1 - 2V_1 + V_2 &= 0 \\ 3V_1 - 4V_2 &= 0 \end{aligned}.$$

Simplifying,

$$\begin{aligned} 2V_1 - V_2 &= 1 \\ 3V_1 - 4V_2 &= 0 \end{aligned}.$$

Solving this simultaneous set of linear equations for unknown voltage values yields

$$\begin{aligned} V_1 = 0.8V,& \quad V_2 = 0.6V, \\ i = \frac{V_1 - V_2}{R_1} &= 0.8 - 0.6 = 0.2\,\mathrm{A} \end{aligned}.$$

```
SPICE Netlist,cccs7.cir
Analysis: DC Operating Point
*fx N+ N- Vy Val
* x  Name of the source, N+ : positive node
*N- : Name of negative node. Current flows from the + node
*  through the source to the - node
*Vy : Name of the voltage source
*The direction of positive control current is
*from + node through the source to the - node of Vy=0
*Val: Current gain
i1 0 1 1
f1 0 2 Vy 2
Vy 3 2 0
R1 1 3 1
R2 1 0 1
R3 2 0 1
```

**Problem 2.1.12** In the circuit shown in Fig. 2.12, $I_S = 1$ A, $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$, $f = 4$ A/A. Find the values of currents flowing through resistors $R_3$ and $R_4$.

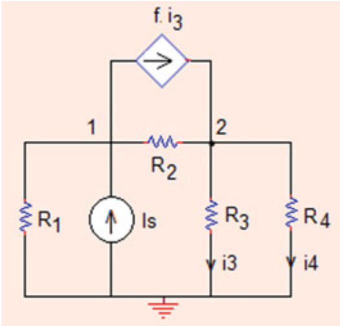



**Fig. 2.12** The circuit of Problem 2.1.12

**Solution**

$$i = \frac{V_2}{R_3} = \frac{V_2}{1} = V_2.$$

KCL at nodes 1 and 2:

$$1 - 4V_2 - V_1 - (V_1 - V_2) = 0 \tag{2.1}$$

$$4V_2 + (V_1 - V_2) - V_2 - V_2 = (0). \tag{2.2}$$

Simplifying (2.1) and (2.2),

$$2V_1 + 3V_2 = 1 \tag{2.3}$$

$$V_1 + V_2 = 0. \tag{2.4}$$

Solving these equations for unknowns yields $V_1 = -1\,\text{V}, V_2 = 1\,\text{V}$,

$$i_4 = \frac{V_2}{R_4} = 1\,\text{A}, \quad i_3 = \frac{V_2}{R_3} = 1\,\text{A}.$$

**Problem 2.1.13** In the circuit shown in Fig. 2.13, $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$, $Is = 1$ A, and $f = 4$ A/A. $V_1 = ?$, $V_2 = ?$, $V_3 = ?$, $i = ?$ Use node voltages method. Check your results using SPICE (cccs4).

**Solution**
KCL at nodes 1 and 2 with $i = V_2/R_3$:

$$-\frac{V_1}{R_1} - f \cdot \frac{V_2}{R_3} - \frac{V_1 - V_2}{R_2} = 0 \tag{2.5}$$

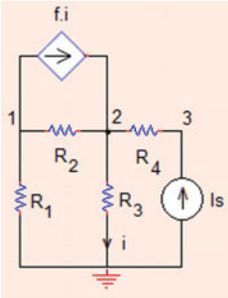



**Fig. 2.13** The circuit of Problem 2.1.13

$$f \cdot \frac{V_2}{R_3} + \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = -1. \tag{2.6}$$

Using given values, these equations become

$$-V_1 - 4V_2 - V_1 + V_2 = 0 \tag{2.7}$$

$$4V_2 + V_1 - V_2 - V_2 = -1. \tag{2.8}$$

Simplifying,

$$2V_1 + 3V_2 = 0 \tag{2.9}$$

$$V_1 + 2V_2 = -1. \tag{2.10}$$

Solution of this set of simultaneous equations yields $V_1$ = 3 V, $V_2$ = −2 V.

$$\begin{aligned} V_{R4} &= (-1\,\text{A}) \cdot R_4 = (-1) \cdot (1) = -1\,\text{V} \\ V_3 &= V_2 - V_{R4} = -2 - (-1)(1) = -2 + 1 = -1\,\text{V} \\ i &= \frac{V_2}{R_3} = -\frac{2}{1} = -2\,\text{A} \end{aligned}$$

Netlist for SPICE check:

```
*cccs4.cir
*Analysis: DC Operating Point
   current controlled current source-nodal analysis
   i1 0 3 1
   f1 1 2 vref 4
   vref 4 0 0
   R1 1 0 1
   R2 1 2 1
   R3 2 4 1
   R4 2 3 1
```

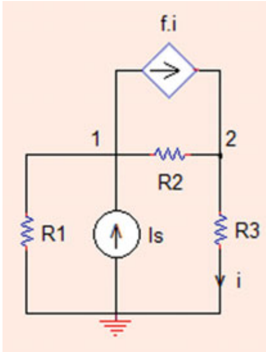



**Fig. 2.14** The circuit of Problem 2.1.14

**Problem 2.1.14** In the circuit shown in Fig. 2.14, $R_1 = R_2 = R_3 = 1\ \Omega$, $I_S = 1$ A, $f = 4$ A/A. $V_1 = ?$, $V_2 = ?$, $i = ?$, $i_{R2} = ?$ Use nodal analysis method.

**Solution**

$$i_s - f \cdot \frac{V_2}{R_3} - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_2} = 0 \tag{2.11}$$

$$f \cdot \frac{V_2}{R_3} + \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = 0. \tag{2.12}$$

Substituting the values,

$$1 - 4V_2 - V_1 - V_1 + V_2 = 0 \tag{2.13}$$

$$4V_2 + V_1 - V_2 - V_2 = 0. \tag{2.14}$$

Simplifying,

$$1 - 2V_1 - 3V_2 = 0 \tag{2.15}$$

$$V_1 + 2V_2 = 0. \tag{2.16}$$

In matrix form,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}. \tag{2.17}$$

Solution of this matrix equation yields

$$V_1 = 2\,\text{V}, \quad V_2 = -1\,\text{V}, \quad i = \frac{V_2}{R_3} = -\frac{1}{1} = -1\,\text{A}$$

$$i_{R2} = \frac{V_1 - V_2}{R_2} = \frac{2-(-1)}{1} = 3\,\text{A}.$$

**Problem 2.1.15**

(a) Determine the voltage-to-current ratio (the input resistance) in the circuit shown in Fig. 2.15:

$$R_x = \frac{V_x}{I_x} = ?$$

(b) Determine the value of current through the voltage source, if

$$R_1 = R_2 = 1\,\Omega, \quad R_3 = 2\,\Omega, \quad R_4 = \frac{1}{2}\,\Omega, \quad V_x = \frac{1}{4}\,\text{V}, \quad k = 0.5\,\text{S}.$$

**Solution**

(a)
$$V_1 = I_x \cdot \left(\frac{(R_1 + R_2)\cdot R_3}{R_1 + R_2 + R_3}\right) \tag{2.18}$$

$$V_x = V_1 - V_2. \tag{2.19}$$

Node 2:

$$-I_x - kV_1 - \frac{V_2}{R_4} = 0 \tag{2.20}$$

$$V_2 = -R_4 \cdot I_x - kV_1R_4 \tag{2.21}$$

Equation (2.21) → (2.19)

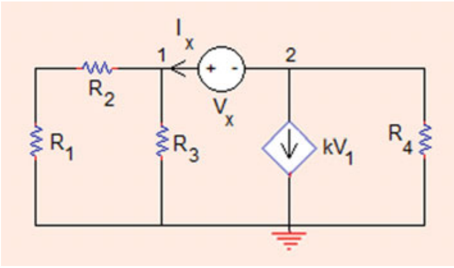



**Fig. 2.15** The circuit of Problem 2.1.15

$$V_x = V_1 + R_4 \cdot I_x + kV_1R_4 = V_1(1 + kR_4) + R_4 \cdot I_x \tag{2.22}$$

divide all terms of (2.22) by $I_x$

$$\frac{V_x}{I_x} = R_x = \frac{V_1}{I_x}(1 + kR_4) + R_4. \tag{2.23}$$

Replace $V_1/I_x$ by (2.18):

$$R_x = \left(\frac{(R_1 + R_2) \cdot R_3}{R_1 + R_2 + R_3}\right)(1 + kR_4) + R_4. \tag{2.24}$$

(b) Substituting given component values into (2.24) yields

$$R_x = \frac{(1+1) \cdot 2}{1+1+2}(1 + 0.5 \times 0.5) + 0.5 = 1.75\,\Omega$$

$$I_x = \frac{V_x}{R_x} = \frac{0.25}{1.75} = \frac{1}{7}A = 0.142857\,\text{A}.$$

**Problem 2.1.16**

(a) Use node voltage method and find the voltage drop across $R_2$ (in mV).
(b) Verify the solution using SPICE and print SPICE netlist (vccs2.cir).
($I_s$ = 1 A, $R_1$ = 1 Ω, $R_2$ = 1/2 Ω, $R_3$ = 1/4 Ω) (Fig. 2.16).

**Solution**
KCL at 1:

$$1 - V_1 - 2(V_1 - V_2) = 0$$

KCL at 2:

$$-\frac{V_1}{2} - 4V_2 + 2(V_1 - V_2) = 0$$

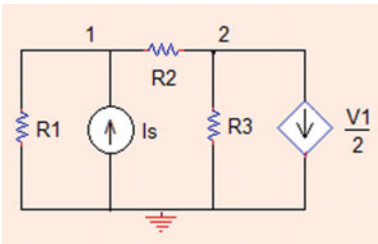



**Fig. 2.16** The circuit of Problem 2.1.16

$$-3V_1 + 2V_2 = -1$$

$$-\frac{V_1}{2} + 2V_1 - 4V_2 - 2V_2 = 0$$

$$3V_1 - 2V_2 = 1 \tag{2.25}$$

$$3V_1 - 12V_2 = 0. \tag{2.26}$$

From (2.26) $3V_1 + 12V_2 \rightarrow$ put into (2.25)

$$12V_2 - 2V_2 = 1 \quad \rightarrow \quad V_2 = 0.1\,\mathrm{V}$$

$$3V_1 = 1.2 \quad \rightarrow \quad V_1 = 0.4\,\mathrm{V}$$

$$V_1 - V_2 = 0.4 - 0.1 = 0.3\,\mathrm{V} = 300\,\mathrm{mV}.$$

(b) SPICE netlist: Operating point analysis, vccs2.cir

```
*gx  N+ N- NC+ NC- VALUE
*x     Name of the source
*N+    Name of positive node
*N-    Name of negative node
*NC+   Name of positive controlling node
*NC-   Name of negative controlling node
*VALUE  Transconductance in S
I1 0 1 1
G1 2 0 1 0  0.5
*Specifies that the current through G1 flowing from node 2 to ground
*is 0.5 times the potential difference between node 1 and ground.
R1 1 0 1
R2 1 2 0.5
R3 2 0 0.25
```

**Problem 2.1.17** A *R*–2*R* ladder circuit is shown in Fig. 2.17.

(a) Find the node voltages and shunt branch currents.
(b) Find the current supplied by the voltage source.
(c) Compute numerical values if $V_i = 5$ V, $R = 1$ kΩ.

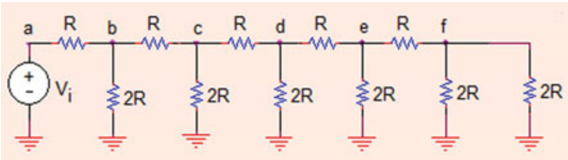



**Fig. 2.17** The circuit for Problem 2.1.17

**Solution**

Start at the rightmost node of the circuit (node f), looking to the right of each node,

$$R_f = 2R//2R = R, \quad R_e = 2R//(R+R) = R, \quad R_d = 2R//(R+R) = R$$

$$R_c = 2R//(R+R) = R, R_b = 2R//(R+R) = R.$$

Node voltages:

$$\begin{aligned}
V_a &= V \\
V_b &= \frac{R_b \cdot V}{R+R_b} = \frac{R \cdot V}{2R} = \frac{V}{2}V \\
V_c &= \frac{R_c}{R+R_c} \cdot \frac{V}{2} = \frac{R}{2R} \cdot \frac{V}{2} = \frac{V}{4}V \\
V_d &= \frac{R_d}{R+R_d} \cdot \frac{V}{4} = \frac{R}{2R} \cdot \frac{U}{4} = \frac{V}{8}V \\
V_e &= \frac{R_e}{R+R_e} \cdot \frac{V}{8} = \frac{R}{2R} \cdot \frac{V}{8} = \frac{V}{16}V \\
V_f &= \frac{R_f}{R+R_f} \cdot \frac{V}{16} = \frac{R}{2R} \cdot \frac{V}{16} = \frac{V}{32}V
\end{aligned} \quad .$$

The shunt branch currents are calculated by Ohm's law:
The right part branch departing from (f) carries a current of

$$I_o = \frac{V_f}{2R} = \frac{\frac{V}{32}}{2R} = \frac{V}{64R}.$$

This is the same current through the left branch departing from (f).
The shunt branch current departing from node (e) is

$$I_{eo} = \frac{V_e}{2R} = \frac{\frac{V}{16}}{2R} = \frac{V}{32} = 2I_o.$$

The shunt branch current departing from node (d) is

$$I_{do} = \frac{V_d}{2R} = \frac{\frac{V}{8}}{2R} = \frac{V}{16R} = 2(2I_o) = 4I_o.$$

The shunt branch current departing from node (c) is

$$I_{co} = \frac{V_c}{2R} = \frac{\frac{V}{4}}{2R} = \frac{V}{8R} = 8I_o.$$

Similarly,

$$I_{\mathrm{bo}} = \frac{V_{\mathrm{b}}}{2R} = \frac{\frac{V}{2}}{2R} = \frac{V}{4R} = 16I_{\mathrm{o}}.$$

Overall current supplied by the voltage source is

$$I = I_{\mathrm{bo}} + I_{\mathrm{co}} + I_{\mathrm{do}} + I_{\mathrm{eo}} + I_{\mathrm{fo}} + I_{\mathrm{fo}} = I_{\mathrm{o}}(16+8+4+2+1+1) = 32 \cdot I_{\mathrm{o}}$$
$$I = 32 \cdot \frac{V}{64R} = \frac{V}{2R}$$
$$V = 5\,\mathrm{V}, \quad R = 1\,\mathrm{k\Omega}; \quad V_{\mathrm{b}} = 2.5\,\mathrm{V}, \quad V_{\mathrm{c}} = 1.25\,\mathrm{V},$$
$$V_{\mathrm{d}} = 0.625\,\mathrm{V}, \quad V_{\mathrm{e}} = 0.3125\,\mathrm{V}, \quad V_{\mathrm{f}} = 0.15625\,\mathrm{V},$$
$$I_{\mathrm{o}} = \frac{V}{64R} = \frac{5}{64}\,\mathrm{mA} = 0.078125\,\mathrm{mA}$$
$$I = \frac{V}{2R} = \frac{5}{2}\,\mathrm{mA} = 2.5\,\mathrm{mA}$$

**Problem 2.1.18** In the circuit shown in Fig. 2.18, find the voltage at node 1 (=$V_1$). Use node voltage method and Cramer's rule for the solution of matrix equations. $R_1 = R_2 = R_3 = R_4 = 1\,\mathrm{k\Omega}$, $I_1 = I_2 = I_3 = I_4 = 1\,\mathrm{mA}$.

**Solution**

$$[I] = [G][V], \quad G = R^{-1}$$

$$G_1 = G_2 = G_3 = G_4 = 10^{-3}\,\mathrm{S}$$

$$10^{-3} \times \begin{bmatrix} I_3 \\ I_{1-}(I_3+I_4) \\ I_2 + I_4 \end{bmatrix} = \begin{bmatrix} G_1+G_2 & -G_2 & 0 \\ -G_2 & G_2+G_3 & -G_3 \\ 0 & -G_3 & G_3+G_4 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \times 10^{-3}$$

$$\begin{bmatrix} 1 \\ 1-(1+1) \\ 1+1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1 & 0 \\ -1 & 1+1 & -1 \\ 0 & -1 & 1+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}.$$

Using Cramer's rule for the solution of matrix equation,

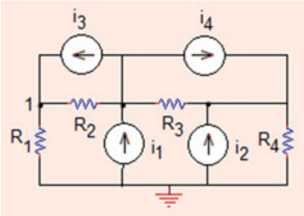



**Fig. 2.18** The circuit for Problem 2.1.18

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\left|\begin{array}{ccc:cc} 1 & -1 & 0 & 1 & -1 \\ -1 & 2 & -1 & -1 & 2 \\ 2 & -1 & 2 & 2 & -1 \end{array}\right|}{\left[\begin{array}{ccc:cc} 2 & -1 & 0 & 2 & -1 \\ -1 & 2 & -1 & -1 & 2 \\ 0 & -1 & 2 & 0 & -1 \end{array}\right]} = \frac{4+2+0-(0+1+2)}{8+0+0-(0+2+2)} = \frac{3}{4}$$
$$= 0.75\,\text{V}.$$

**Problem 2.1.19** In the circuit shown in Fig. 2.19, find the values of node voltages $V_1$ and $V_2$. Use Cramer's rule when necessary $R_1 = R_3 = 2\,\Omega, R_2 = 4\,\Omega, I_1 = I_3 = 1\,\text{A}$, $I_2 = I_4 = I_5 = 2\,\text{A}$.

**Solution**

$$\frac{V_1}{2} + \frac{V_1 - V_2}{4} - 5 = 0 \quad \rightarrow \quad \frac{3}{4}V_1 - \frac{1}{4}V_2 = 5$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{2} + 1 = 0 \quad \rightarrow \quad -\frac{1}{4}V_1 + \frac{3}{4}V_2 = -1$$

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\Delta_1 = \begin{vmatrix} 5 & -\frac{1}{4} \\ -1 & \frac{3}{4} \end{vmatrix} = \frac{14}{4}, \quad \Delta_2 = \begin{vmatrix} \frac{3}{4} & 5 \\ -\frac{1}{4} & -1 \end{vmatrix} = \frac{1}{2}, \quad \Delta = \begin{vmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{vmatrix} = \frac{1}{2}$$

$$V_1 = \frac{\Delta_1}{\Delta} = 7\,\text{V}, V_2 = \frac{\Delta_2}{\Delta} = 1\,\text{V}.$$

**Problem 2.1.20** In the circuit shown in Fig. 2.20, $I_1$ = 1 A, $I_2$ = 1/2 A, $R_1$ = 1/2 Ω, $R_2$ = 1/4 Ω, $R_3$ = 1/8 Ω.

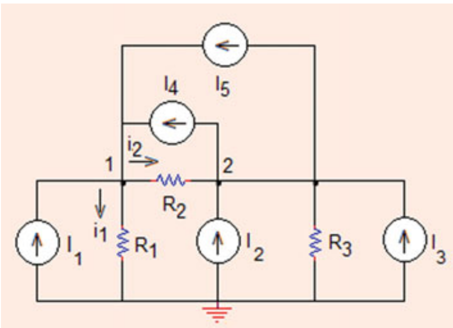



**Fig. 2.19** The circuit for Problem 2.1.19

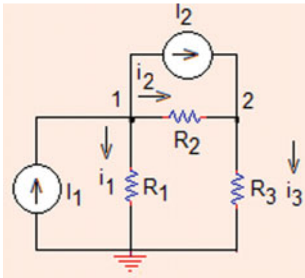



**Fig. 2.20** The circuit for Problem 2.1.20

(a) Find the node voltages,
(b) Find the currents flowing in the circuit (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**

(a)
$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + I_2 - I_1 = 0 \tag{2.27}$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} - I_2 = 0 \tag{2.28}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{V_2}{R_2} = I_1 - I_2 \tag{2.29}$$

$$-\frac{V_1}{R_2} + V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = I_2. \tag{2.30}$$

Using last two equations,

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -4 \\ -4 & 12 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.$$

Solution of this set of simultaneous linear equations by either manually using Cramer's rule or by substitution methods or by employing available software (see, MATLAB m file or EXCEL file referenced in the statement) yields

$$V_1 = 0.14\,\text{V}, \quad V_2 = 0.09\,\text{V}.$$

(b) $$i_1 = \frac{V_1}{R_1} = 0.28\,\text{A}, \quad i_2 = \frac{V_1 - V_2}{R_2} = 0.2\,\text{A}, \quad i_3 = \frac{V_2}{R_3} = 0.72\,\text{A}.$$

**Problem 2.1.21** Use node voltages method and find the values of currents and voltages in the circuit shown in Fig. 2.21.

$$R_1 = \frac{1}{2}\,\Omega, \quad R_2 = \frac{1}{4}\,\Omega, \quad R_3 = R_4 = 1\,\Omega, \quad I_1 = I_2 = 1\,\text{A}, \quad I_3 = 2\,\text{A}.$$

**Solution**

Applying KCL at node 1,

$$I_1 + I_2 - i_1 - i_2 = 0 \quad \rightarrow \quad 1 + 1 - \frac{v_1}{R_1} - \frac{v_1 - v_2}{R_2} = 0$$

$$2 - \frac{v_1}{\frac{1}{2}} - \frac{v_1 - v_2}{\frac{1}{4}} = 0 \quad \rightarrow \quad 2 - 6v_1 + 4v_2 = 0$$

$$-6v_1 + 4v_2 = -2. \tag{2.31}$$

Applying KCL at node 2,

$$i_2 - i_3 - i_4 = 0 \quad \rightarrow \quad \frac{v_1 - v_2}{R_2} - \frac{v_2}{R_3} - \frac{v_2 - v_3}{R_4} = 0 \quad \rightarrow$$
$$\frac{v_1 - v_2}{\frac{1}{4}} - \frac{v_2}{1} - \frac{v_2 - v_3}{1} = 0$$

$$4v_1 - 6v_2 + v_3 = 0. \tag{2.32}$$

Applying KCL at node 3,

$$I_3 - I_2 + i_4 = 0 \quad \rightarrow \quad 2 - 1 + \frac{v_2 - v_3}{R_4} = 0 \quad \rightarrow \quad 1 + \frac{v_2 - v_3}{1} = 0$$

$$v_2 - v_3 = -1. \tag{2.33}$$

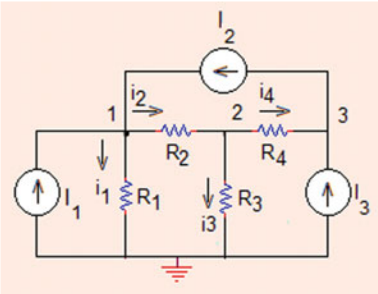



**Fig. 2.21** The circuit for Problem 2.1.21

Combining Eqs. (2.31)–(2.33) into matrix form,

$$\begin{bmatrix} -6 & 4 & 0 \\ 4 & -6 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}.$$

The solution of this matrix equation yields

$$v_1 = 1\,\text{V}, \quad v_2 = 1\,\text{V}, \quad v_3 = 2\,\text{V}$$

$$i_1 = \frac{v_1}{R_1} = 2\,\text{A}, \quad i_2 = \frac{v_1 - v_2}{R_2} = 0\,\text{A}, \quad i_3 = \frac{v_2}{R_3} = 1\,\text{A}, \quad i_4 = \frac{v_2 - v_3}{R_4} = -1\,\text{A}.$$

**Problem 2.1.22** Determine the node voltages in the circuit shown in Fig. 2.22. $R = 1\Omega, \quad I_1 = I_2 = I_3 = 1\,\text{A}$.

**Solution**
Analysis by inspection,

$$G = \frac{1}{R} = 1\,\text{S}$$

$$\begin{bmatrix} G+G & -G & 0 \\ -G & G+G+G & G \\ 0 & -G & G+G \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1+I_2+I_3 \\ -I_2 \\ -I_3 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix}.$$

This matrix equation is solved for unknown voltages and yields the following voltage values:

$$V_1 = 1.5\,\text{V}, \quad V_2 = 0\,\text{V}, \quad V_3 = -0.5\,\text{V}.$$

**Problem 2.1.23**

(a) Use node voltages and Cramer's methods to find the values of currents and voltages in the circuit shown in Fig. 2.23. Use SPICE for checking the results. Print the SPICE netlist (cccs5.cir).

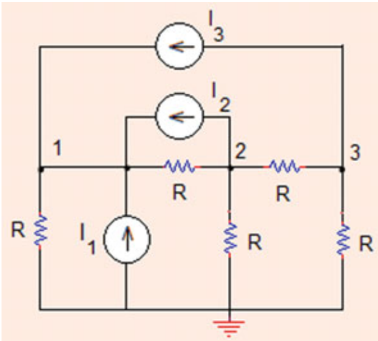
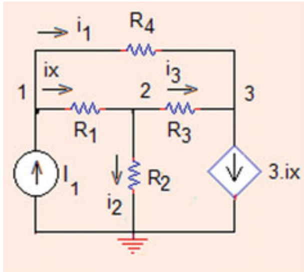



**Fig. 2.22** The circuit for Problem 2.1.22

**Fig. 2.23** The circuit for Problem 2.1.23




$$R_1 = R_4 = \frac{1}{2}\,\Omega, \quad R_2 = R_3 = 1\,\Omega, \quad I_1 = 2\,\text{A}, \quad f = 3\,\text{A/A}.$$

(b) Determine the node voltages using the following component values. Use SPICE for checking the results. Print the new SPICE netlist.

$$R_1 = 1\,\Omega, \quad R_2 = 4\,\Omega, \quad R_3 = 8\,\Omega, \quad R_4 = 4\,\Omega \quad I_1 = 3\,\text{A}, \quad f = 2\,\text{A/A}.$$

**Solution**

(a) Applying KCL at node 1,

$$I_1 - i_1 - i_x = 0 \rightarrow 2 - \frac{v_1 - v_3}{R_4} - \frac{v_1 - v_2}{R_1} = 0 \rightarrow 2 - \frac{v_1 - v_3}{\frac{1}{2}} - \frac{v_1 - v_2}{\frac{1}{2}} = 0$$

$$2 - 2(v_1 - v_3) - 2(v_1 - v_2) = 0$$

$$-4v_1 + 2v_2 + 2v_3 = -2. \tag{2.34}$$

Applying KCL at node 2,

$$i_x - i_2 - i_3 = 0 \rightarrow \frac{v_1 - v_2}{R_1} - \frac{v_2}{R_2} - \frac{v_2 - v_3}{R_3} = 0 \rightarrow \frac{v_1 - v_2}{\frac{1}{2}} - \frac{v_2}{1} - \frac{v_2 - v_3}{1} = 0$$

$$2(v_1 - v_2) - v_2 - (v_2 - v_3) = 0$$

$$2v_1 - 4v_2 + v_3 = 0. \tag{2.35}$$

Applying KCL at node 3,

$$-3i_x + i_1 + i_3 = 0 \rightarrow -3\frac{v_1 - v_2}{R_1} + \frac{v_1 - v_3}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

$$-3\frac{v_1 - v_2}{\frac{1}{2}} + \frac{v_1 - v_3}{\frac{1}{2}} + \frac{v_2 - v_3}{1} = 0 \rightarrow -3[2(v_1 - v_2)] + 2(v_1 - v_3) + (v_2 - v_3) = 0$$

$$-4v_1 + 7v_2 - 3v_3 = 0. \tag{2.36}$$

Combining Eqs. (2.34)–(2.36) into matrix form,

$$\begin{bmatrix} -4 & 2 & 2 \\ 2 & -4 & 1 \\ -4 & 7 & -3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}.$$

The solution of this matrix equation yields

$$v_1 = 0.5\,\mathrm{V}, \quad v_2 = 0.2\,\mathrm{V}, \quad v_3 = -0.2\,\mathrm{V}$$

$$i_1 = \frac{v_1 - v_3}{R_4} = 1.4\,\mathrm{A}, \quad i_2 = \frac{v_2}{R_2} = 0.2\,\mathrm{A},$$

$$i_3 = \frac{v_2 - v_3}{R_3} = 0.4\,\mathrm{A}, \quad i_x = \frac{v_1 - v_2}{R_1} = 0.6\,\mathrm{A}.$$

```
*SPICE Netlist cccs5.cir
*Analysis: DC Operating Point
  current controlled current source-nodal analysis 5
  i1 0 1 2
  f1 3 0 vref 3
  vref 4 2 0
  R1 1 4 0.5
  R2 2 0 1
  R3 2 3 1
  R4 1 3 0.5
```

(b) Using new data set,

KCL at node 1, $12 = 5v_1 - 4v_2 - v_3$
KCL at node 2, $0 = 8v_1 - 11v_2 + v_3$
KCL at node 3, $0 = 14v_1 - 17v_2 + 3v_3$.
Using three nodal equations, one obtains the following matrix equation of the circuit:

$$\begin{bmatrix} 5 & -4 & -1 \\ 8 & -11 & 1 \\ 14 & -17 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -4 & -1 \\ 8 & -11 & 1 \\ 14 & -17 & 3 \end{vmatrix} = -58, \quad \Delta_1 = \begin{vmatrix} 12 & -4 & -1 \\ 0 & -11 & 1 \\ 0 & -17 & 3 \end{vmatrix} = -192$$

$$\Delta_2 = \begin{vmatrix} 5 & 12 & -1 \\ 8 & 0 & 1 \\ 14 & 0 & 3 \end{vmatrix} = -120, \quad \Delta_3 = \begin{vmatrix} 5 & -4 & 12 \\ 8 & -11 & 0 \\ 14 & -17 & 0 \end{vmatrix} = 216$$

$$v_1 = \frac{\Delta_1}{\Delta} \cong 3.31\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} \cong 2.06\,\text{V}, \quad v_3 = \frac{\Delta_3}{\Delta} \cong -3.72\,\text{V}.$$

Renewed SPICE Netlist;

```
* DC operating point analysis cccs5.cir
   i1 0 1 3
   f1 3 0 vref 2
   vref 4 2 0
   R1 1 4 1
   R2 2 0 4
   R3 2 3 8
   R4 1 3 4
```

Advantages of using SPICE are apparent here. It can be easily used for many different component variations of a circuit, rather than performing tedious calculations.

**Problem 2.1.24** Determine the ratio of node voltages $V_1/V_2$ in the circuit shown in Fig. 2.24. Use Cramer's rule when necessary.

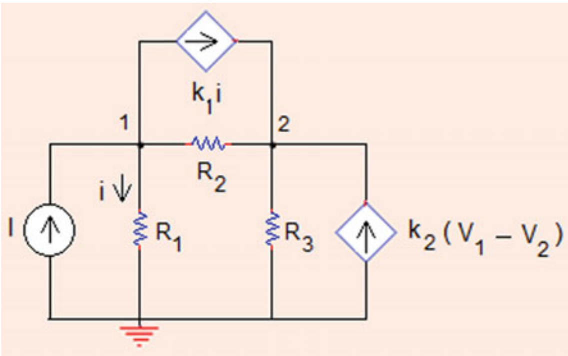



**Fig. 2.24** The circuit for Problem 2.1.24

$$I = 1\,\text{A}, \quad R_1 = 10\,\Omega, \quad R_2 = 1\,\Omega, \quad R_3 = 5\,\Omega, \quad k_1 = 5\,\text{A/A}, \quad k_2 = 2\,\text{A/V}.$$

**Solution**
KCL at node 1:

$$I - i - 5i - i_{R_2} = I - \frac{V_1}{R_1} - 5\frac{V_1}{R_1} - \frac{V_1 - V_2}{R_2} = I - 6\frac{V_1}{R_1} - \frac{V_1}{R_2} + \frac{V_2}{R_2} = 0$$

$$I - 7V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_2}{R_2} = 0. \tag{2.37}$$

KCL at node 2:

$$5i + i_{R_2} - i_{R_3} + 2(V_1 - V_2) = 5\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} - 2(V_1 - V_2) = 0$$

$$5\frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} - \frac{V_2}{R_3} + 2V_1 - 2V_2 = 0. \tag{2.38}$$

From Eqs. (2.37) and (2.38),

$$\begin{bmatrix} 7\left(\frac{1}{R_1} + \frac{1}{R_2}\right) & -\frac{1}{R_2} \\ \frac{5}{R_1} + \frac{1}{R_2} + 2 & -\left(\frac{1}{R_2} + \frac{1}{R_3} + 2\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$R_1 = 10\,\Omega, \quad R_2 = 1\,\Omega, \quad R_3 = 5\,\Omega$$

$$\begin{bmatrix} 7(0.1 + 1) & -1 \\ 0.5 + 1 + 2 & -(1 + 0.2 + 2) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 7.7 & -1 \\ 3.5 & -3.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad \frac{V_1}{V_2} = \frac{\frac{\Delta_1}{\Delta}}{\frac{\Delta_2}{\Delta}} = \frac{\Delta_1}{\Delta_2}$$

$$\Delta_1 = \begin{vmatrix} 1 & -1 \\ 0 & -3.2 \end{vmatrix} = -3.2, \quad \Delta_2 = \begin{vmatrix} 7.7 & 1 \\ 3.5 & 0 \end{vmatrix} = -3.5$$

$$\frac{V_1}{V_2} = \frac{-3.2}{-3.5} = 0.9143(\text{V/V}).$$

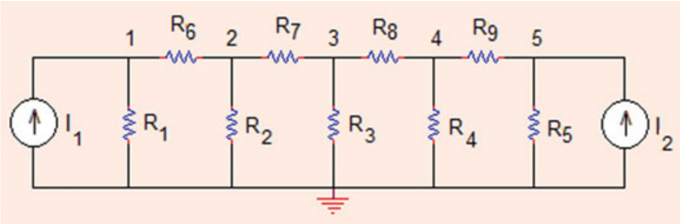



**Fig. 2.25** The circuit for Problem 2.1.25

**Problem 2.1.25** Determine currents flowing through each resistor in the circuit shown in Fig. 2.25 (ladder_node.xlsx).

$R_1 = R_2 = R_3 = R_4 = R_5 = 10\,\Omega, R_6 = R_7 = 5\,\Omega, R_8 = R_9 = 20\,\Omega, I_1 = I_2 = 2\,\text{A}.$

**Solution**

$$[G][V] = [I] = [2\,0\,0\,0\,2]^{\mathrm{T}} \tag{2.39}$$

where T stands for transpose operation.

$$G_{11} = \frac{1}{10} + \frac{1}{5}, \quad G_{22} = \frac{1}{10} + \frac{2}{5}, \quad G_{33} = \frac{1}{10} + \frac{1}{5} + \frac{1}{20}, \quad G_{44} = \frac{2}{20} + \frac{1}{10},$$

$$G_{55} = \frac{1}{10} + \frac{1}{20}, \quad G_{12} = G_{21} = -\frac{1}{5}, \quad G_{32} = G_{23} = -\frac{1}{5}, \quad G_{43} = G_{34} = -\frac{1}{20},$$

$$G_{45} = G_{54} = -\frac{1}{20},$$

$$[G] = \begin{bmatrix} 0.3 & -0.2 & 0 & 0 & 0 \\ -0.2 & 0.5 & -0.2 & 0 & 0 \\ 0 & -0.2 & 0.35 & -0.05 & 0 \\ 0 & 0 & -0.05 & 0.2 & -0.05 \\ 0 & 0 & 0 & -0.05 & 0.15 \end{bmatrix}.$$

Solution of Eq. (2.39) using these numerical values yields the node voltages:

$$V_1 = 10.545\,\text{V}, \quad V_2 = 5.818\,\text{V}, \quad V_3 = 4\,\text{V}, \quad V_4 = 4.727\,\text{V}, \quad V_5 = 14.909\,\text{V}.$$

Current values through resistors are obtained using Ohm's Law:

$$i_1 = \frac{V_1}{R_1} = 1.054545\,\text{A}, i_2 = \frac{V_2}{R_2} = 0.581818\,\text{A}$$
$$i_3 = \frac{V_3}{R_3} = 0.4\,\text{A}, i_4 = \frac{V_4}{R_4} = 0.472727\,\text{A}$$
$$i_5 = \frac{V_5}{R_5} = 1.490909\,\text{A}, i_6 = \frac{V_1 - V_2}{R_6} = 0.945459\,\text{A} \qquad .$$
$$i_7 = \frac{V_2 - V_3}{R_7} = 0.363636\,\text{A}, i_8 = \frac{V_3 - V_4}{R_8} = -0.03636\,\text{A},$$
$$i_9 = \frac{V_4 - V_5}{R_9} = -0.50909\,\text{A}$$

**Problem 2.1.26** Find the node voltage values for the circuit shown in Fig. 2.26.
$I_1 = I_3 = 1$ A, $I_2 = 2$ A, $I_4 = -1$ A, $R_1 = R_2 = R_3 = R_5 = R_6 = R_7 = R_8 = 1\ \Omega$, $R_9 = 0.1\ \Omega$ (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
The conductance values are

$$G1 = G2 = G3 = G4 = G5 = G6 = G7 = G8 = 1\,\text{S}, G9 = 10\,\text{S}$$

$$[G][V] = [I].$$

Solution of this matrix equation in MATLAB or EXCEL platform yields

$$V_1 = 1.795\,\text{V}, \quad V_2 = 0.590\,\text{V}, \quad V_3 = -1.081\,\text{V},$$

$$V_4 = -0.024\,\text{V}, \quad V_5 = -0.581\,\text{V}, \quad V_6 = -0.139\,\text{V}.$$

**Problem 2.1.27**

(a) Find the node voltage values in terms of current gain of the CCCS for the circuit shown in Fig. 2.27.
(b) Verify the solution using SPICE and print SPICE netlist (cccs1.cir).

$$I_S = 1\,\text{A}, \quad R_1 = R_2 = R_3 = 1\,\Omega.$$

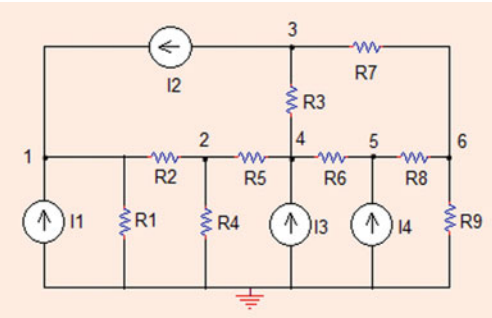



**Fig. 2.26** The circuit for Problem 2.1.26

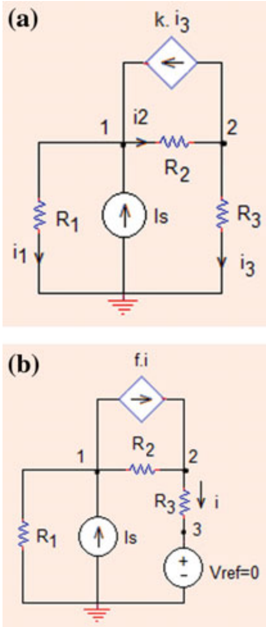



**Fig. 2.27** The circuit for Problem 2.1.27

**Solution**

KCL at node 1: ($G_1 = G_2 = G_3 = 1$ S)

$$I_S - V_1 - (V_1 - V_2) - fi = 0 \quad \rightarrow \quad 1 - V_1 - V_1 + V_2 - f \cdot V_2 = 0$$

$$1 - 2V_1 + V_2(1 - f) = 0$$

$$2V_1 - (1 - f) \cdot V_2 = 1. \tag{2.40}$$

KCL at node 2:

$$fi + (V_1 - V_2) - i = 0 \quad \rightarrow \quad fV_2 + V_1 - V_2 - V_2 = 0$$

$$fV_2 + V_1 - 2V_2 = 0$$

$$V_1 + V_2(f - 2) = 0. \tag{2.41}$$

From (2.41),

$$V_1 = V_2(f - 2) = (2 - f) \cdot V_2. \tag{2.42}$$

**Table 2.1** Circuit voltages and current as a function of current gain

| f | $V1$ (V) | $V2$ (V) | $i$ (A) |
|---|---|---|---|
| 0.5 | 0.6 | 0.4 | 0.4 |
| 1 | 0.5 | 0.5 | 0.5 |
| 4 | 2 | −1 | −1 |

Substitute into (2.40):

$$2V_2(2-f)-(1-f)V_2=1 \rightarrow V_2[2(2-f)-(1-f)]=1$$

$$V_2(4-2f-1+f)=1 \rightarrow V_2(3-f)=1$$

$$V_2=\frac{1}{3-f}. \tag{2.43}$$

From (2.42),

$$V_1=\frac{2-f}{3-f}. \tag{2.44}$$

Note that $f \neq 3$. As a check in SPICE, Table 2.1 displays the results.
Note that $f = 3$ yields a SPICE error message.
The circuit used in SPICE including the CCCS is shown in Fig. 2.27b.
SPICE Netlist (cccs1.cir):

```
*fx N+ N- Vy Value
*x  Name of the source
*N+ : Name of positive node
*N- : Name of negative node. Current flows from the + node
*   through the source to the - node
*Vref : Name of the voltage source through the controlling
*current flows.
* The direction of positive control current is
* from + node through the source to the - node of Vref=0
*Value: Current gain
i1 0 1 1
f1 1 2 Vref 4
Vref 3 0 0
R1 1 0 1
R2 1 2 1
R3 2 3 1
*.op
```

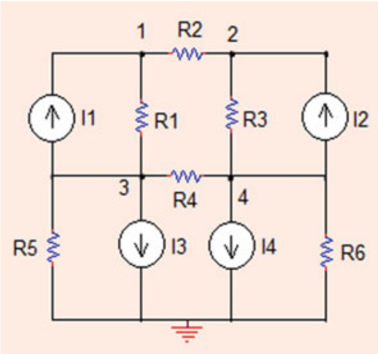



**Fig. 2.28** The circuit for Problem 2.1.28

**Problem 2.1.28** Find the node voltage values in the circuit shown in Fig. 2.28. All resistors are 1 Ω and $I_1$ = 4 A, $I_2$ = 1 A, $I_3$ = 1 A, $I_4$ = 4 A (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**

$$I_1 = \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_1} \quad \rightarrow \quad 2V_1 - V_2 - V_3 = 4$$

$$I_2 = \frac{V_2 - V_1}{R_2} + \frac{V_2 - V_4}{R_3} \quad \rightarrow \quad -V_1 + 2V_2 - V_4 = 1$$

$$I_1 + I_3 + \frac{V_3}{R_5} + \frac{V_3 - V_4}{R_4} + \frac{V_3 - V_1}{R_1} = 0 \quad \rightarrow \quad -V_1 + 3V_3 - V_4 = -5$$

$$I_2 + I_4 + \frac{V_4}{R_6} + \frac{V_4 - V_2}{R_3} + \frac{V_4 - V_3}{R_4} = 0 \quad \rightarrow \quad -V_2 - V_3 + 3V_4 = -5.$$

From these four equations, the following matrix equation is obtained:

$$[G] \cdot [V] = [I] \quad \rightarrow \quad \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -1 \\ -5 \end{bmatrix}.$$

Solution of this matrix equation by employing any available software yields the voltage values as

$$V_1 = 2.727\,\text{V}, \quad V_2 = 1.273\,\text{V}, \quad V_3 = 0.182\,\text{V}, \quad V_4 = -1.182\,\text{V}.$$

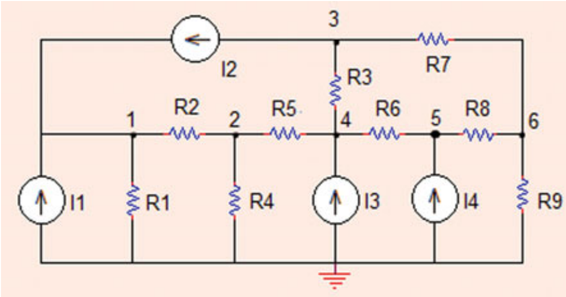



**Fig. 2.29** The circuit for Problem 2.1.29

**Problem 2.1.29** In the circuit shown in Fig. 2.29,

$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = 1\,\text{k}\Omega, \quad R_9 = 100\,\Omega,$$

$$I_1 = I_2 = I_3 = I_4 = 1\,\text{mA}.$$

(a) Find the conductance matrix for the circuit.
(b) Compute the node voltages (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**

(a) $[G][V] = [I]$

$$G_1 = G_2 = G_3 = G_4 = G_5 = G_6 = G_7 = G_8 = 1\,\text{mS}, \quad G_9 = 1 \times 10^{-2}\,\text{mS}.$$

Then, the conductance matrix is

$$[G] = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & -1 \\ 0 & -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & -1 & 12 \end{bmatrix} \times 10^{-3}\,\text{S} \quad (= \text{mS})$$

$$G_7 + G_8 + G_9 = 10^{-3} + 10^{-3} + 10^{-2} = \left(2 \times 10^{-3} + 10^{-2}\right)\text{S} = 12 \times 10^{-3}\,\text{S}.$$

(b) $I = [2 \quad 0 \quad -1 \quad 1 \quad 1 \quad 0]^{\text{T}}$mA, (T: transpose operator), $[G][V] = [I]$. Solution of this matrix equation for voltage vector (e.g., using MATLAB or EXCEL) yields

$$[V] = [1385.5 \quad 771.1 \quad 6.0 \quad 927.7 \quad 1006.0 \quad 84.3]^{\text{T}}\,\text{mV}.$$

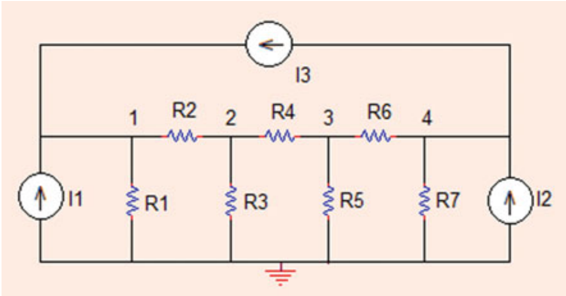



**Fig. 2.30** The circuit for Problem 2.1.30

**Problem 2.1.30** Find the node voltage values in the circuit shown in Fig. 2.30. $R1 = R6 = R7 = 1\ \Omega$, $R2 = R3 = R5 = 2\ \Omega$, $R4 = 4\ \Omega$, $I1 = 2$ A, $I2 = 1$ A, $I3 = 3$ A (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
Analysis by inspection,

$$G_1 = 1\,\mathrm{S}, \quad G_2 = 0.5\,\mathrm{S}, \quad G_3 = 0.5\,\mathrm{S}, \quad G_4 = 0.25\,\mathrm{S}, \quad G_5 = 0.5\,\mathrm{S},$$

$$G_6 = 1\,\mathrm{S}, \quad G_7 = 1\,\mathrm{S}$$

$$[G][V] = [I], \quad \begin{bmatrix} 1.5 & -0.5 & 0 & 0 \\ -0.5 & 1.25 & -0.25 & 0 \\ 0 & -0.25 & 1.75 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3+2 \\ 0 \\ 0 \\ -3+1 \end{bmatrix}.$$

Solution of this matrix equation (e.g., using EXCEL or MATLAB tools) gives node voltages:

$$v_1 = 3.806\,\mathrm{V}, \quad v_2 = 1.419\,\mathrm{V}, \quad v_3 = -0.516\,\mathrm{V}, \quad v_4 = -1.258\,\mathrm{V}.$$

**Problem 2.1.31** In the circuit shown in Fig. 2.31a, $R_1 = R_2 = R_3 = 1\ \Omega$, $i_S = 1$ A.

(a) Use SPICE to find the values of the node voltages $V_1$, $V_2$, and the current $i$ for current-controlled current source constants of $f = 2$, $f = 4$, and $f = 8$.
(b) Plot $i$ versus $i_S$ curve, $-1 \le i_S \le 1$ A, if the CCCS constant is 8 A/A. Include net list.

**Solution**

(a) The circuit used in SPICE analysis is shown in Fig. 2.31b: (Table 2.2)
(b) Fig. 2.31c displays the current sweep.

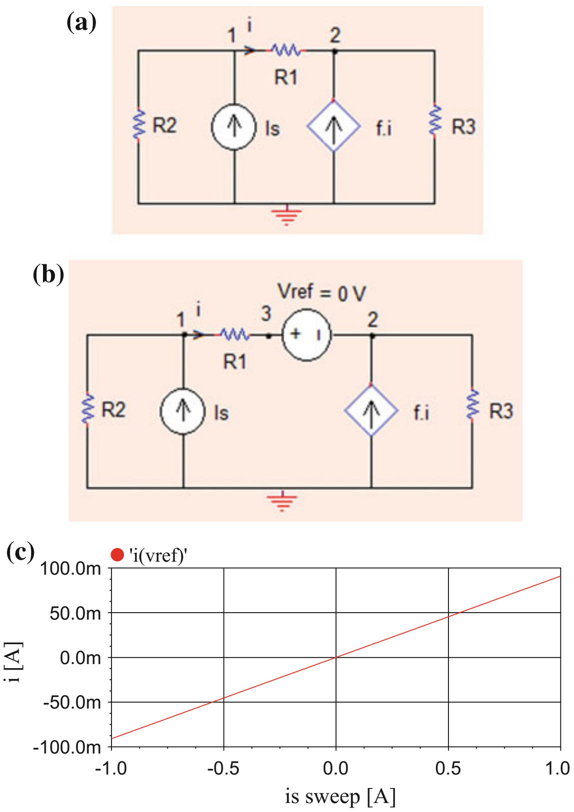



**Fig. 2.31** **a** The circuit for Problem 2.1.31, **b** The circuit for Problem 2.1.31 for SPICE analysis, **c** SPICE analysis result for the Circuit of Problem 2.1.31. The current sweep

**Table 2.2** Voltage and current values against current gain

| $f$ | $V_1$ (V) | $V_2$ (V) | i (A) |
|---|---|---|---|
| 2 | 0.800 | 0.600 | 0.200 |
| 4 | 0.857 | 0.714 | 0.143 |
| 8 | 0.910 | 0.820 | 0.091 |

```
*SPICE Netlist for current sweep with f=8:
*Analysis: DC Transfer Curves
cccs2
is 0 1 dc 1
R1 1 3 1
R2 1 0 1
R3 2 0 1
f1 0 2 vref 8
vref 3 2 0
.dc is -1 1 .1
```

**Problem 2.1.32** Use node voltages method and determine all currents (mA) and $V_2(\text{mV})$ in the circuit shown in Fig. 2.32. $V_1 = 2\,\text{V}, V_3 = 1\,\text{V}, R_1 = 5\,\Omega, R_2 = 3\,\Omega, R_3 = 4\,\Omega, R_4 = 2\,\Omega$.

**Solution**

$$V_1 = 2\,\text{V}, \quad V_3 = 1\,\text{V}, \quad i_3 = \frac{V_1 - V_3}{R_4} = \frac{2-1}{2} = 500\,\text{mA}.$$

KCL at node 2:

$$i_1 - i_2 - i_4 = 0 \rightarrow \frac{2-V_2}{5} - \frac{V_2 - 1}{3} - \frac{V_2}{4} = 0 \quad \rightarrow$$
$$12(2 - V_2) - 20(V_2 - 1) - 15V_2 = 0$$

$$V_2 = \frac{44}{47} = 0.93617\,\text{V} = 936.17\,\text{mV}$$

$$i_1 = \frac{2 - 0.93617}{5} = 212.766\,\text{mA}, \quad i_2 = \frac{0.93617 - 1}{3} = -21.277\,\text{mA}$$

$$i_4 = \frac{0.93617}{4} = 234.043\,\text{mA}, \quad i_{V_1} = i_1 + i_3 = 212.766 + 500 = 712.766\,\text{mA}$$

$$i_{V_2} = -(i_2 + i_3) = -(-21.277 + 500) = -478.723\,\text{mA}.$$

**Problem 2.1.33** Determine the node voltages in the circuit shown in Fig. 2.33.

$$V_1 = 12\,\text{V}, \quad V_2 = 6\,\text{V}, \quad R_1 = 4\,\Omega, \quad R_2 = 2\,\Omega, \quad R_3 = 2\,\Omega, \quad R_4 = 6\,\Omega.$$

**Solution**

There is a voltage source $(V_2)$ connected between two nonreference nodes (2,3). These nodes form a supernode. KCL and KVL can be applied to obtain the node voltages in this circuit.

**Fig. 2.32** The circuit for Problem 2.1.32

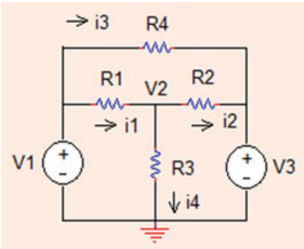
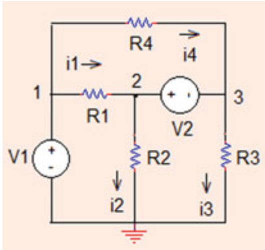


**Fig. 2.33** The circuit for Problem 2.1.33




On the other hand, $V_1$ is connected between node 1 and ground. Thus, the voltage at node 1 equals to $v_1 = V_1 = 12\,\text{V}$.

At the supernode,

$$i_1 + i_4 - i_2 - i_3 = 0 \tag{2.45}$$

$$\frac{v_1 - v_2}{R_1} + \frac{v_1 - v_3}{R_4} - \frac{v_2}{R_2} - \frac{v_3}{R_3} = 0. \tag{2.46}$$

But constraint equation is

$$v_2 - v_3 = V_2 = 6\,\text{V} \quad \rightarrow \quad v_2 = V_2 + v_3 = 6 + v_3 \tag{2.47}$$

$$\frac{v_1 - v_2}{4} + \frac{v_1 - v_3}{6} = \frac{1}{2}(v_2 + v_3) = \frac{1}{2}(6 + v_3 + v_3) = \frac{6 + 2v_3}{2}. \tag{2.48}$$

Since $v_1 = V_1 = 12\,\text{V}$,

$$\frac{12 - (6 + v_3)}{4} + \frac{12 - v_3}{6} = \frac{6 + 2v_3}{2} \tag{2.49}$$

$$36 - 16 - 3v_3 + 24 - 2v_3 = 36 + 12v_3 \tag{2.50}$$

$$-17v_3 = -8 \quad \rightarrow \quad v_3 = \frac{8}{17}\,\text{V}.$$

From (2.47),

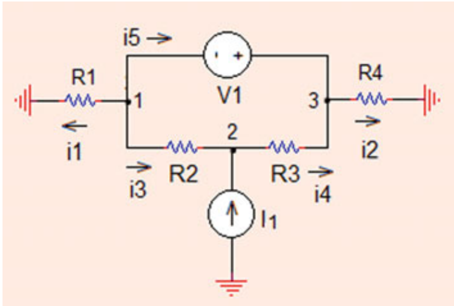
$$v_2 = 6 + \frac{8}{17} = \frac{110}{17}\,\text{V}.$$

Summary: $v_1 = 12\,\text{V}, \quad v_2 = 6.471\,\text{V}, \quad v_3 = 0.471\,\text{V}$.

**Problem 2.1.34** Determine the voltage at node 2 and the current flowing through the voltage source in the circuit shown in Fig. 2.34. Prove the latter result by applying KCL at node 3.

$I_1 = 2\,\text{A}, V_1 = 2\,\text{V}, R_1 = 4\,\Omega, R_2 = 8\,\Omega, R_3 = 8\,\Omega, R_4 = 2\,\Omega$ (supernode1.cir).

**Fig. 2.34** The circuit for Problem 2.1.34




**Solution**

Since independent voltage source is connected between nodes (1, 3), these nodes form a supernode. Node 2 is included in this supernode. Thus,

$$-i_1 - i_2 + 5 = 0 \rightarrow -\frac{v_1}{R_1} - \frac{v_3}{R_4} + 2 = 0 \rightarrow -\frac{v_1}{4} - \frac{v_3}{2} + 2 = 0$$

$$v_1 + 2v_3 = 8. \tag{2.51}$$

The constraint is

$$v_3 - v_1 = 2\,\mathrm{V} \quad \rightarrow \quad v_3 = 2 + v_1. \tag{2.52}$$

Substituting (2.52) into (2.51),

$$v_1 + 2(2 + v_1) = 8 \quad \rightarrow \quad v_1 + 4 + 2v_1 = 8 \quad \rightarrow \quad 3v_1 = 4$$

$$v_1 = \frac{4}{3} = 1.3333\,\mathrm{V}.$$

From (2.52)

$$v_3 = 2 + 1.3333 = 3.3333\,\mathrm{V}.$$

At node 2:

$$i_3 + I_1 - i_4 = 0 \rightarrow \frac{v_1 - v_2}{R_2} + I_1 - \frac{v_2 - v_3}{R_3} = 0. \tag{2.53}$$

Substituting the values for $v_1$ and $v_2$ into (2.53),

$$\frac{1.3333 - v_2}{8} + 2 - \frac{v_2 - 3.3333}{8} = 0 \quad \rightarrow \quad 4.6667 - 2v_2 = -16$$

$$-v_2 = \frac{-16 - 4.6667}{2} \quad \rightarrow \quad v_2 = \frac{20.6667}{2} = 10.3333\,\text{V}$$

$$i_3 = \frac{v_1 - v_2}{R_2} = \frac{1.3333 - 10.3333}{4} = -2.25\,\text{A}.$$

The current flowing through the voltage source is calculated by applying KCL at node 1:

$$-i_1 - i_5 - i_3 = 0 \quad \rightarrow \quad -\frac{v_1}{R_1} - i_5 - \frac{v_1 - v_2}{R_2} = 0$$

$$-\frac{1.3333}{4} - i_5 - \frac{1.3333 - 10.3333}{8} = 0 \quad \rightarrow \quad -0.3333 - i_5 + 1.125 = 0$$

$$i_5 = 0.79167\,\text{A}.$$

*Proof* KCL at node 3,

$$i_5 + i_4 - i_2 = 0 \quad \rightarrow \quad 0.79167 + \frac{v_2 - v_3}{R_3} - \frac{v_3}{R_4} = 0$$

$$0.79167 + \frac{10.3333 - 3.3333}{8} - \frac{3.3333}{2} = 0$$

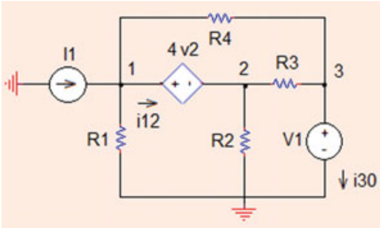
$$0.79167 + \frac{7}{8} - \frac{3.3333}{2} = 0 \quad \rightarrow \quad 0.79167 + 0.875 - 1.66667 = 0 \quad \text{Q.E.D.}$$

SPICE netlist:

```
supernode1
*OP analysis
R1 1 0 4
R2 1 2 8
R3 2 3 8
R4 4 0 2
V1 3 1 2
I1 0 2 2
V3 3 4 0
```

**Problem 2.1.35** Determine the current through dependent source and current though independent voltage source in the circuit shown in Fig. 2.35. Here, $v_2$ is the node voltage at node 2.

**Fig. 2.35** The circuit for Problem 2.1.35




$$I_1 = 4\,\text{A}, \quad V_1 = 5\,\text{V}, \quad R_1 = R_4 = 2\,\Omega, \quad R_2 = 4\Omega, \quad R_3 = 1\,\Omega.$$

**Solution**

Consider the supernode consisting of nodes (1,2). Applying KCL,

$$I_1 - \frac{v_1}{R_1} - \frac{v_1 - v_3}{R_4} - \frac{v_2 - v_3}{R_3} - \frac{v_2}{R_2} = 0$$

$$4 - \frac{v_1}{2} - \frac{v_1 - v_3}{2} - \frac{v_2 - v_3}{1} - \frac{v_2}{4} = 0$$

$$16 - 2v_1 - 2(v_1 - v_3) - 4(v_2 - v_3) - v_2 = 0$$

$$4v_1 + 5v_2 - 6v_3 = 16 \tag{2.54}$$

But,

$$v_1 = v_2 + 4v_2 = 5v_2 \tag{2.55}$$

$$v_3 = V_1 = 5\,\text{V}. \tag{2.56}$$

Substituting (2.55), (2.56), into (2.54),

$$4(5v_2) + 5v_2 - 6 \times 5 = 16 \quad \rightarrow \quad 20v_2 + 5v_2 - 30 = 16 \quad \rightarrow \quad v_2 = 1.84\,\text{V}.$$

From (2.55),

$$v_1 = 5 \times 1.84 = 9.2\,\text{V}.$$

KCL at node 1:

$$i_{12} = I_1 - \frac{v_1}{R_1} - \frac{v_1 - v_3}{R_4} = 4 - \frac{9.2}{2} - \frac{9.2 - 5}{2} = -2.7\,\text{A}.$$

KCL at node 3:

$$i_{30} = \frac{v_2 - v_3}{R_3} + \frac{v_1 - v_3}{R_4} = \frac{1.84 - 5}{1} + \frac{9.2 - 5}{2} = -1.06\,\text{A}.$$

Following is the SPICE netlist (supernode2.cir) for the operating point analysis of this circuit:

```
supernode2
*OP analysis
R1 1 0 2
R2 2 0 4
R3 2 3 1
R4 1 3 2
V1 3 0 5
I1 0 1 4
*VCVS: e{name} {+node} {-node} {+cntrl} {-cntrl} {gain}
e1 1 2 2 0 4
```

## 2.2 Mesh Analysis

**Problem 2.2.1** Find the values of $V_x, V_0$ in the circuit shown in Fig. 2.36. $U = 35$ V.

**Solution**
KVL around the loop, $U - V_x - 2V_x + V_0 = 0$

$$V_x = 10i, \quad V_0 = -5i$$

$$35 - 10i - 20i - 5i = 0$$

$$i = 1\,\text{A}, \quad V_0 = -5i = -5\,\text{V}, \quad V_x = 10i = 10\,\text{V}.$$

SPICE netlist (mesh01):

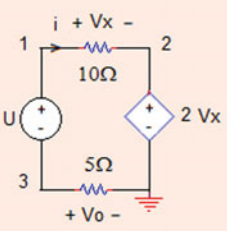



**Fig. 2.36** The circuit for Problem 2.2.1

```
mesh01
*OP ANALYSIS
VU  1 3 35
R1  1 2 10
R2  3 0 5
*VCVS:  Ex N+ N- NC+ NC- VALUE
E1  2 0  1  2  2
```

### Problem 2.2.2

(a) Determine the current $i_{ab}$ in the circuit shown in Fig. 2.37.
(b) If $U1$ = 10 V, $U2$ = 6 V, $R$ = 1 kΩ what is the value of $i_{ab}$? (mA)
(c) If $U1$ = $U2$ = 10 V, $R$ = 1 kΩ what is the value of $i_{ab}$? (mA)
(d) If $U1$ = $U2/2$ = 10 V, $R$ = 1 kΩ what is the value of $i_{ab}$? (mA)

### Solution

(a) Current through the left mesh (in CW direction),

$$i_1 = \frac{U1}{R}$$

Current through the right mesh, (in CW direction)

$$i_2 = -\frac{U2}{2R}$$

$$i_{ab} = i_1 + i_2 = \frac{U1}{R} - \frac{U2}{2R} = \frac{1}{R}\left(U1 - \frac{U2}{2}\right)$$

(b)
$$i_{ab} = \frac{\left(10 - \frac{6}{2}\right)}{10^3} = \frac{7}{10^3} = 7\,\text{mA}$$

(c) If $U1$ = $U2$ = 10 V,

$$i_{ab} = \frac{5}{10^3} = 5\,\text{mA}$$

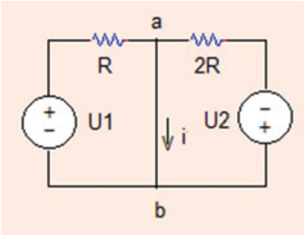



**Fig. 2.37** The circuit for Problem 2.2.2

(d) If $U1 = U2/2 = 10$ V, R = 1 kΩ,

$$i_{ab} = \frac{\left(10 - \frac{20}{2}\right)}{10^3} = 0\,\mathrm{mA}.$$

**Problem 2.2.3** In the circuit shown in Fig. 2.38, use mesh currents method and find the value of voltage $V_x$. What is the voltage drop across $R_2$?. $R_1 = R_3 = 2R_2 = 4\,\Omega$, $V_1 = 3\,\mathrm{V}$, $V_2 = 5\,\mathrm{V}$ (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
By applying mesh currents and analysis by inspection, the governing equation of the circuit is

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} \mathrm{i}_1 \\ \mathrm{i}_2 \end{bmatrix} = \begin{bmatrix} V_1 - V_2 \\ V_2 \end{bmatrix}$$

or

$$\begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}.$$

From this matrix equation, $i_1$ and $i_2$ can be obtained as $i_1 = -0.0625\,\mathrm{A}$; $i_2 = 0.8125\,\mathrm{A}$.

The voltage $V_x$ is calculated as $V_x = i_2 \cdot R_3 = 0.8125 \cdot 4 = 3.25\,\mathrm{V}$.

The voltage drop across $R_2$, $V_{R2} = V_x - V_2 = 3.25 - 5 = -1.75\,\mathrm{V}$.

**Problem 2.2.4** Apply mesh analysis method to find the values of currents $i_1$ and $i_2$ and the node voltage in the circuit shown in Fig. 2.39 ($V_1$ = 2 V, $V_2$ = 1 V, $R_1$ = 1 Ω, $R_2$ = 1 Ω, $R_3$ = 2 Ω) (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
Using mesh analysis for the circuit,

$$-V_1 + i_1 \cdot R_1 + (i_1 - i_2) \cdot R_3 = 0$$

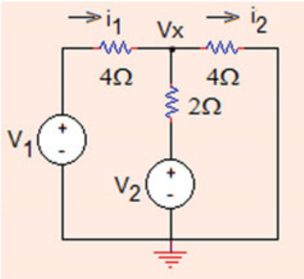



**Fig. 2.38** The circuit for Problem 2.2.3

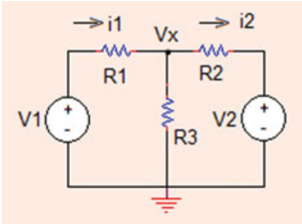



**Fig. 2.39** The circuit for Problems 2.2.4 and 2.2.5

$$(i_2 - i_1) \cdot R_3 + i_2 \cdot R_2 + V_2 = 0.$$

Substituting $V_1$, $V_2$, $R_1$, and $R_2$ values in these equations,

$$-2 + 3i_1 - 2i_2 = 0$$

$$3i_2 - 2i_1 - 1 = 0.$$

Solution of this set of simultaneous linear equations for unknown current values yields

$$i_1 = 0.8\,\text{A}, i_2 = 0.2\,\text{A}$$

$$V_x = (i_1 - i_2).R_3 = 0.6 \times 2 = 1.2\,\text{V}.$$

**Problem 2.2.5** In the circuit shown in Fig. 2.39, use mesh currents method and Cramer's rule to find the values for $V_x$, $V_{R1}$, $V_{R2}$. ($R_1 = R_2 = 2\ \Omega$, $R_3 = 1\ \Omega$, $V_1 = 2$ V, $V_2 = 1$ V) (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**

Using analysis by inspection and mesh currents method,

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix}.$$

Substituting given component values,

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\Delta = 3 \cdot 3 - 1 = 8, \quad \Delta_1 = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5, \quad i_1 = \frac{\Delta_1}{\Delta} = \frac{5}{8}\,\text{A}$$

$$V_{R1} = i_1 R_1 = \frac{5}{8} \cdot 2 = \frac{10}{8} = 1.25\,\text{V}$$

$$V_x = V_1 - V_{R1} = 2 - 1.25 = 0.75\,\text{V}$$

$$V_{R2} = V_x - V_2 = 0.75 - 1 = 0.25\,\text{V}.$$

**Problem 2.2.6** Find the values of mesh currents and the node voltage in the circuit shown in Fig. 2.40. $R1 = 10\ \Omega$, $R2 = 2\ \Omega$, $R3 = 1\ \Omega$, $V1 = 4$ V, $V2 = 2$ V, $V3 = 1$ V (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
KVL in mesh1:

$$-4 + 10i_1 + 2(i_1 - i_2) + 2 = 0$$

$$12i_1 - 2i_2 = 2. \tag{2.57}$$

KVL in mesh2:

$$-2 + 2(i_2 - i_1) + i_2 + 1 = 0$$

$$-2i_1 + 3i_2 = 1. \tag{2.58}$$

Using Eqs. (2.57)–(2.58), one obtains the following matrix equation:

$$\begin{bmatrix} 12 & -2 \\ -2 & 3 \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

The solution of this matrix equation gives

$$i_1 = 0.25\,\text{A}$$

$$i_2 = 0.5\,\text{A}$$

$$v_x = V_2 + (i_1 - i_2) \times R_2 = 2 - 0.25 \times 2 = 1.5\,\text{V}.$$

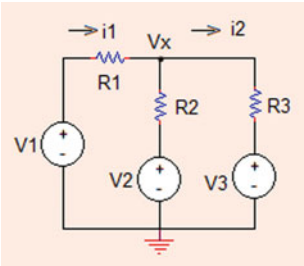



**Fig. 2.40** The circuit for Problem 2.2.6

Alternatively,

$$v_x = V_3 + i_2 R_3 = 1 + 0.5 \times 1 = 1.5\,\mathrm{V}.$$

**Problem 2.2.7** In the circuit shown in Fig. 2.41, determine the voltage drop across $R2$ using mesh analysis.

$R1 = 3\ \Omega$, $R2 = 5\ \Omega$, $R3 = 4\ \Omega$, $v1 = 2\ \mathrm{V}$, $I = 2\ \mathrm{A}$.

**Solution**
Since a current source exists in the second mesh,

$$i_2 = -I = -2\,\mathrm{A}.$$

The mesh equation for the other mesh,

$$-2 + 3i_1 + (i_1 - i_2) \times 5 = 0.$$

Solving this equation for the unknown current,

$$3i_1 + 5i_1 + 10 - 2 = 0 \quad \rightarrow \quad 8i_1 + 8 = 0 \quad \rightarrow \quad i_1 = -1\,\mathrm{A}.$$

The voltage drop across $R_2$ is

$$V_{R_3} = 5(i_1 - i_2) = 5(-1 + 2) = 5\,\mathrm{V}.$$

**Problem 2.2.8** Find the values of numbered (clockwise flowing) mesh currents in the circuit shown in Fig. 2.42. Use Cramer's rule when necessary $U = 4\,\mathrm{V}, I = 2\,\mathrm{A}, R = R_1 = R_2 = R_3 = R_4 = 1\,\Omega$ (mesh1.cir).

**Solution**
Assume clockwise rotation for mesh currents.

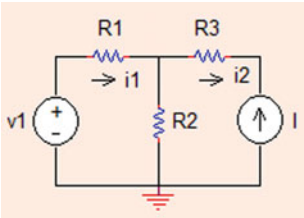
Since $i_2 = I$, it is not necessary to write down KVL equation associated with second mesh.

KVL in mesh 1:

$$-U = (i - i_3)R_1 + (i_1 - i_2)R_3 = 0$$

$$i_1(R_1 + R_3) - i_3 R_1 = U + IR_3. \tag{2.59}$$

Fig. 2.41 The circuit for Problem 2.2.7



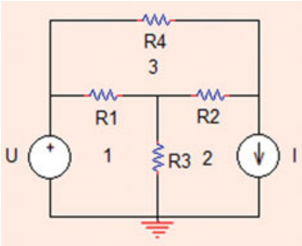



**Fig. 2.42** The circuit for Problem 2.2.8

KVL in mesh 3:

$$i_3R_4 + (i_3 - i_1)R_1 + (i_3 - i_2)R_2 = 0$$

$$i_3(R_1 + R_3 + R_4) - i_1R_1 = U + IR_2. \tag{2.60}$$

From (2.59) and (2.60),

$$\begin{bmatrix} R_1 + R_3 & -R_1 \\ -R_1 & R_1 + R_2 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} U + IR_3 \\ IR_2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \quad \Delta = 6 - 1 = 5, \quad \Delta_1 = \begin{vmatrix} 6 & -1 \\ 2 & 3 \end{vmatrix} = 20,$$
$$\Delta_2 = \begin{vmatrix} 2 & 6 \\ -1 & 2 \end{vmatrix} = 10$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{20}{5} = 4\,\text{A}, i_2 = \frac{\Delta_2}{\Delta} = \frac{10}{5} = 2\,\text{A}.$$

**Problem 2.2.9** For the circuit shown in Fig. 2.43, write down the circuit equation in matrix form and solve for mesh currents. $R = 2\ \Omega$, $V1 = V2 = V3 = 1$ V (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
Analysis by inspection yields

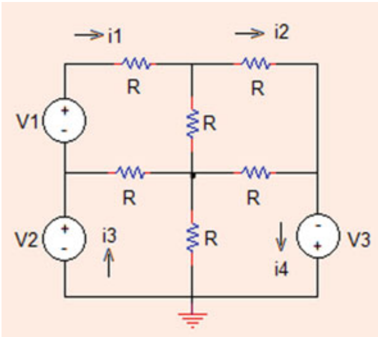



**Fig. 2.43** The circuit for Problem 2.2.9

$$\begin{bmatrix} 6 & -2 & -2 & 0 \\ -2 & 6 & 0 & -2 \\ -2 & 0 & 4 & -2 \\ 0 & -2 & -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} (\mathrm{V}).$$

Solution of this equation (using given EXCEL or MATLAB tools) gives

$$I_1 = 0.818\,\mathrm{A}, \quad I_2 = 0.682\,\mathrm{A}, \quad I_3 = 1.273\,\mathrm{A}, \quad I_4 = 1.227\,\mathrm{A}.$$

**Problem 2.2.10** In the circuit shown in Fig. 2.44, find the value of current $i_{AB}$ through 3 Ω resistor.($i_{AB} = -i_{BA}$). Use Cramer's rule, when necessary.

$V1 = 7$ V, $V2 = 6$ V, $R1 = R5 = 1$ Ω, $R2 = R4 = 2$ Ω, $R3 = 3$ Ω.

**Solution**

$$i_{AB} = i_3 - i_2.$$

The mesh current equations yield the following matrix equation:

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7-6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 & \vdots & 3 & -1 \\ -1 & 6 & -3 & \vdots & -1 & 6 \\ -2 & -3 & 6 & & -2 & -3 \end{vmatrix} = 3 \times 36 + (-6) + (-6) - [24 + 27 + 6] = 39$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 & \vdots & 3 & 1 \\ -1 & 0 & -3 & \vdots & -1 & 0 \\ -2 & 6 & 6 & & -2 & 6 \end{vmatrix} = 0 + 6 + 12 - (0 - 54 - 6) = 18 - (-60) = 78$$

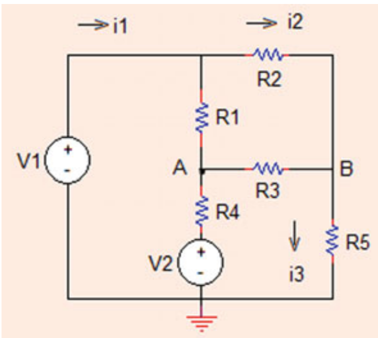



**Fig. 2.44** The circuit for Problem 2.2.10

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 1 & \vdots & 3 & -1 \\ -1 & 6 & 0 & \vdots & -1 & 6 \\ -2 & -3 & 6 & & -2 & -3 \end{vmatrix} = 108 + 0 + 3 - (-12 + 0 + 6)$$
$$= 111 - (-6) = 117$$

$$i_2 = \frac{\Delta_2}{\Delta} = \frac{78}{39} = 2\,\text{A}, \quad i_3 = \frac{\Delta_3}{\Delta} = \frac{117}{39} = 3\,\text{A}, \quad i_{AB} = i_3 - i_2 = 3 - 2 = 1\,\text{A}.$$

**Problem 2.2.11** For the circuit shown in Fig. 2.45, determine the ratio of currents, $r = \dfrac{i_{R1}}{i_{R3}}$, for $k = -1, 0, 1, \infty$.

**Solution**

Assume mesh currents$(i_1, i_2)$ flow clockwise in the left and right meshes, respectively,

$$v = (i_2 - i_1)R_2, \quad i_1 = i_{R1}, \quad i_2 = i_{R3}.$$

KVL at mesh 1:

$$kv + (R_1 + R_2)i_1 - i_2R_2 = 0 \quad \rightarrow \quad k(i_2 - i_1)R_2 + (R_1 + R_2)i_1 - i_2R_2 = 0$$

$$i_1(R_1 + R_2 - kR_2) + i_2(kR_2 - R_2) = 0. \tag{2.61}$$

KVL at mesh 2:

$$-R_2i_1 + (R_2 + R_3)i_2 = U. \tag{2.62}$$

From Eqs. (2.61) and (2.62),

$$\begin{bmatrix} R_1 + R_2(1-k) & -R_2(1-k) \\ -R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} U \\ 0 \end{bmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & -R_2(1-k) \\ U & R_2 + R_3 \end{vmatrix} = UR_2(1-k)$$

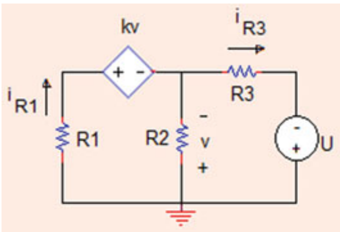



**Fig. 2.45** The circuit for Problem 2.2.11

$$\Delta_2 = \begin{vmatrix} R_1 + R_2(1-k) & 0 \\ -R_2 & U \end{vmatrix} = U[R_1 + R_2(1-k)]$$

$$i_1 = \frac{\Delta_1}{\Delta}, i_2 = \frac{\Delta_2}{\Delta}, r = \frac{i_1}{i_2} = \frac{i_{R1}}{i_{R3}} = \frac{\Delta_1}{\Delta_2} = \frac{R_2(1-k)}{R_1 + R_2(1-k)} = \frac{R_2}{\frac{R_1}{1-k} + R_2}.$$

Values of r for different $k$ parameters are shown in Table 2.3.

**Problem 2.2.12** In the circuit shown in Fig. 2.46, use mesh analysis and calculate the value of current through 10 Ω internal resistance of the 24 V voltage source, currents through $R_2$ = 12 Ω and $R_3$ = 4 Ω. Find the node voltage.

**Solution**
Applying KVL in the left mesh, taking clockwise current directions, ($i = i_1$)

$$10i_1 + 12i_1 - 12i_2 = 24$$

$$11i_1 - 6i_2 = 12. \tag{2.63}$$

KVL 2:

$$\begin{aligned} 12i_2 - 12i_1 + i_2 &= -4V_x \\ &= -4(12(i_1 - i_2)) \\ &= 48i_1 + 48i_2 \\ &\quad - 12i_1 + 48i_1 + 16i_2 - 48i_2 = 0 \end{aligned}$$

$$36i_1 - 32i_2 = 0$$

$$9i_1 - 8i_2 = 0 \tag{2.64}$$

$$9i_1 = 8i_2 \quad \rightarrow \quad i_1 = \frac{8}{9} i_2 \tag{2.65}$$

Table 2.3 Values of r for different k parameters

| $k$ | $-1$ | $0$ | $1$ | $\infty$ |
|---|---|---|---|---|
| $r$ | $\frac{R_2}{\frac{R_1}{2} + R_2}$ | $\frac{R_2}{R_1 + R_2}$ | 0 | 1 |

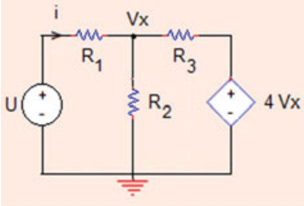



Fig. 2.46 The circuit for Problem 2.2.12

$$11\left(\frac{8}{9}i_2\right) - 6i_2 = 12 \quad \rightarrow \quad i_2\left(\frac{8}{9} - 6\right) = 12 \quad \rightarrow \quad i_2 = \frac{12}{\frac{34}{9}} = \frac{108}{34} = 3.176\,\mathrm{A},$$

$$i = i_1 = \frac{8}{9} \times (3.176) = 2.824\,\mathrm{A}$$

$$i_{R2} = i_1 - i_2 = 2.824 - 3.176 = -0.352\,\mathrm{A}$$

$$V_x = 12 \times i_{R2} = -4.224\,\mathrm{V}.$$

**Problem 2.2.13** Find the values of currents $i_1$, $i_2$, $i_3$, $i_{AB}$ in the circuit shown in Fig. 2.47. Use Cramer's rule, when necessary. $V1 = 4$ V, $R1 = 10\ \Omega$, $R2 = R3 = 4\ \Omega$, $R4 = 6\ \Omega$ (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**

$$i_0 = i_1 - i_2$$

$$\text{KVL at mesh1:} \; -4 + 10(i_1 - i_2) + 6(i_1 - i_3) = 0 \quad \rightarrow \quad 16i_1 - 10i_2 - 6i_3 = 4$$

$$\text{KVL at mesh2:} \; 4i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \quad \rightarrow \quad -10i_1 + 18i_2 - 4i_3 = 0$$

$$\text{KVL at mesh3:} \; 2(i_1 - i_2) + 6(i_3 - i_1) + 4(i_3 - i_2) = 0 \quad \rightarrow \quad -4i_1 - 6i_2 + 10i_3 = 0.$$

Collecting three equations in a matrix form,

$$\begin{bmatrix} 16 & -10 & -6 \\ -10 & 18 & -4 \\ -4 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

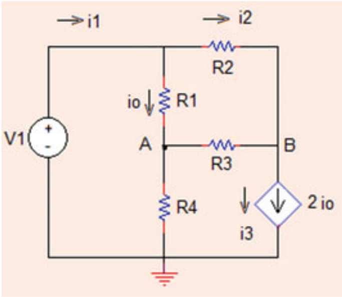



**Fig. 2.47** The circuit for Problem 2.2.13

$$\Delta = \begin{vmatrix} 16 & -10 & -6 \\ -10 & 18 & -4 \\ -4 & -6 & 10 \end{vmatrix} = 544, \quad \Delta_1 = \begin{vmatrix} 4 & -10 & -6 \\ 0 & 18 & -4 \\ 0 & -6 & 10 \end{vmatrix} = 624$$

$$\Delta_2 = \begin{vmatrix} 16 & 4 & -6 \\ -10 & 0 & -4 \\ -4 & 0 & 10 \end{vmatrix} = 464, \quad \Delta_3 = \begin{vmatrix} 16 & -10 & 4 \\ -10 & 18 & 0 \\ -4 & -6 & 0 \end{vmatrix} = 528$$

$$i_1 = \frac{\Delta_1}{\Delta} \cong 1.15\,\text{A}, \quad i_2 = \frac{\Delta_2}{\Delta} \cong 0.85\,\text{A}, \quad i_3 = \frac{\Delta_3}{\Delta} \cong 0.97\,\text{A}, \quad i_{ab} = i_3 - i_2 \cong 0.12\,\text{A}.$$

**Problem 2.2.14** In the circuit shown in Fig. 2.48, $R = 1\ \Omega$, $V1 = 2$ V, $V2 = 1$ V. Find the value of current $i_X$ (Sim_Lin_Eq_Solve.m, matrix_solve.xlsx).

**Solution**
KVL at mesh 1:

$$-2 + (i_1 - i_2) + (i_1 + i_3) + (i_1 - i_5) + i_1 = 0.$$

Simplifying,

$$2 = 4i_1 - i_2 - i_3 - i_5$$

Similarly,

$$\text{KVL at mesh2}: i_2 + (i_2 - i_3) + (i_2 - i_1) = 0 \quad \rightarrow \quad 0 = -i_1 + 3i_2 - i_3$$

$$\text{KVL at mesh3}: 1i_3 + (i_3 - i_1) + (i_3 - i_2) = 0 \quad \rightarrow \quad 0 = -i_1 - i_2 + 4i_3 - i_4$$

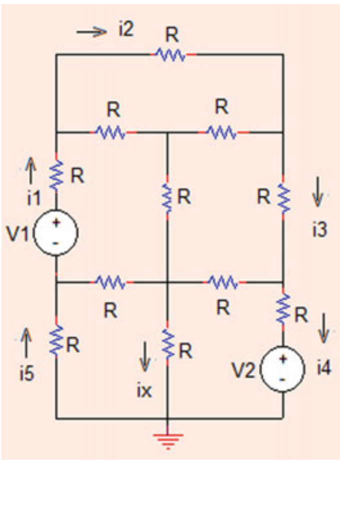



**Fig. 2.48** The circuit for Problem 2.2.14

$$\text{KVL at mesh4}: (i_4 - i_3) + i_4 + 1 + (i_4 - i_5) = 0 \quad \rightarrow \quad 0 = -i_3 + 3i_4 - i_5$$

$$\text{KVL at mesh5}: i_5 + (i_5 - i_1) + (i_5 - i_4) = 0 \quad \rightarrow \quad 0 = -i_1 - i_4 + 3i_5.$$

Collecting these equations in a matrix form,

$$\begin{bmatrix} 4 & -1 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 & 0 \\ -1 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}.$$

Solution of this matrix equation by employing available software (see, MATLAB m file or EXCEL file) yields

$$i_1 = 0.646\,\text{A}, \quad i_2 = 0.273\,\text{A}, \quad i_3 = 0.172\,\text{A}, \quad i_4 = -0.23\,\text{A}, \quad i_5 = 0.139\,\text{A}$$

$$i_y = i_2 = 0.273\,\text{A}$$

$$i_x = i_5 - i_4 = 0.139\,\text{A} + 0.23\,\text{A} = 0.369\,\text{A}.$$

**Problem 2.2.15** Find the values of mesh currents and the node voltage $V_X$ in the circuit shown in Fig. 2.49.

$R$ = 1 kΩ (matrix_solve.xlsx).

**Solution**

$$-10 + i_1 + i_1 - i_2 + i_1 - i_4 = 0$$

$$3i_1 - i_2 - i_4 = 10 \tag{2.66}$$

$$i_2 + i_2 - i_3 + i_2 - i_5 + i_2 - i_1 = 0$$

$$-i_1 + 4i_2 - i_3 - i_5 = 0 \tag{2.67}$$

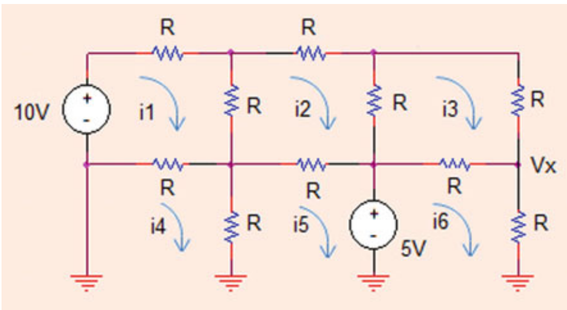



**Fig. 2.49** The circuit for Problem 2.2.15

$$i_3 + i_3 - i_6 + i_3 - i_2 = 0$$

$$-i_2 + 3i_3 - i_6 = 0 \tag{2.68}$$

$$i_4 - i_1 + i_4 - i_5 = 0$$

$$-i_1 + 2i_4 - i_5 = 0 \tag{2.69}$$

$$i_5 - i_4 + i_5 - i_2 + 5 = 0$$

$$-i_2 - i_4 + 2i_5 = -5 \tag{2.70}$$

$$-5 + i_6 - i_3 + i_6 = 0$$

$$-i_3 + 2i_6 = 5. \tag{2.71}$$

Using Eqs. (2.66)–(2.71), one obtains the following matrix equation:

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ -5 \\ 5 \end{bmatrix}.$$

The solution of this matrix equation gives

$$i_1 = 4.211\,\text{mA}, \quad i_2 = 1.118\,\text{mA}, \quad i_3 = 1.447\,\text{mA}$$

$$i_4 = 1.513\,\text{mA}, \quad i_5 = -1.184\,\text{mA}, \quad i_6 = 3.224\,\text{mA}$$

$$v_x = 5 - (i_6 - i_3) \times R = 5 - 1.777 = 3.223\,\text{V}.$$

**Problem 2.2.16** In the circuit shown in Fig. 2.50, use mesh currents method to determine currents flowing through each resistor (mesh6.xlsx) $R_1 = R_2 = R_3 = 2\,\Omega$, $R_4 = R_5 = R_6 = 4\,\Omega, R_7 = R_{10} = 8\,\Omega, R_8 = R_9 = 1\,\Omega, V_1 = V_2 = 10\,\text{V}$.

**Solution**

Analysis by inspection, $[R][I] = [V]$,

$$\begin{bmatrix} 14 & -8 & 0 & -4 & 0 & 0 \\ -8 & 15 & -1 & 0 & -4 & 0 \\ 0 & -1 & 7 & 0 & 0 & -4 \\ -4 & 0 & 0 & 5 & -1 & 0 \\ 0 & -4 & 0 & -1 & 13 & -8 \\ 0 & 0 & -4 & 0 & -8 & 12 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \\ 0 \\ -10 \end{bmatrix}.$$

Solution of this matrix equation (via Excel) gives mesh currents as

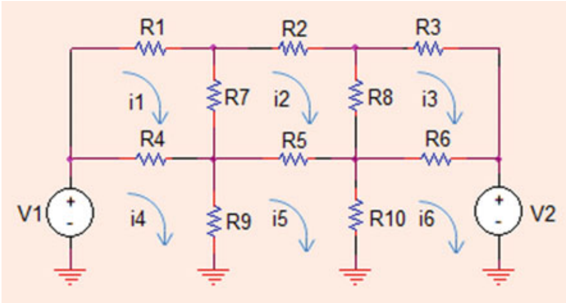



**Fig. 2.50** The circuit for Problem 2.2.16

$$I_1 = 0.790\,\text{A}, \quad I_2 = 0.147\,\text{A}, \quad I_3 = -0.937\text{A}$$

$$I_4 = 2.473\,\text{A}, \quad I_5 = -0.796\,\text{A}, \quad I_6 = -1.677\,\text{A}.$$

Individual currents flowing through each resistor are calculated as follows:

$$I_{R1} = I_1 = 0.790\,\text{A}, \quad I_{R2} = I_2 = 0.147\,\text{A}, \quad I_{R3} = I_3 = -0.937$$

$$I_{R4} = I_1 - I_4 = -1.683\,\text{A}, \quad I_{R5} = I_2 - I_5 = 0.943\,\text{A}, \quad I_{R6} = I_3 - I_6 = 0.740\,\text{A}$$

$$I_{R7} = I_1 - I_2 = 0.644\,\text{A}, \quad I_{R8} = I_2 - I_3 = 3.269\,\text{A}, \quad I_{R9} = I_4 - I_5 = 3.269\,\text{A}$$

$$I_{R10} = I_5 - I_6 = 0.880\,\text{A}.$$

**Problem 2.2.17** Use mesh analysis and determine the node voltages at nodes 2 and 4 in the circuit shown in Fig. 2.51. $V_a = 12\,\text{V}, V_b = 6\,\text{V}, I = 8\,\text{A}, R_1 = 2R_2 = 3R_3 = 12\,\Omega$ (Supermesh1.cir).

**Solution**
Since there is a current source between two meshes, a supermesh results by excluding the current source and resistor connected in series with it. Thus KVL around supermesh (Fig. 2.52):

$$-V_a + i_1R_1 + i_2R_2 + V_b = 0 \rightarrow V_b - V_a = -i_1R_1 - i_2R_2$$

$$V_a - V_b = i_1R_1 + i_2R_2 \rightarrow 6 = 12i_1 + 6i_2$$

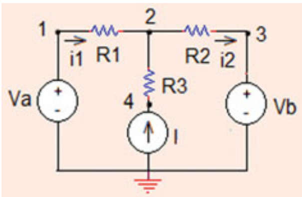



**Fig. 2.51** The circuit for Problem 2.2.17

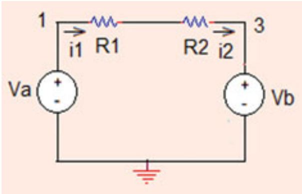



**Fig. 2.52** The supermesh for the circuit of Problem 2.2.17

$$2i_1 + i_2 = 1. \tag{2.72}$$

Constraint equation is obtained by applying KCL at node,

$$i_1 - i_2 + 8 = 0 \rightarrow i_1 - i_2 = -8. \tag{2.73}$$

From (2.72) and (2.73),

$$-i_2 = -8 - i_1 \rightarrow i_2 = 8i_1 \tag{2.74}$$

$$2i_1 + 8 + i_1 = 1 \rightarrow 3i_1 = -7 \rightarrow i_1 = -\frac{7}{3}A = -2.333\,\mathrm{A}$$

$$i_2 = 8 - \frac{7}{3} = \frac{24-7}{3} = \frac{17}{3}A = 5.667\,\mathrm{A}$$

$$v_2 = V_b + i_2 \times R_2 = 6 + (5.667)(6) = 40\,\mathrm{V}, v_4 = 40 - 4 \times 8 = 8\,\mathrm{V}.$$

Following is the SPICE netlist for the operating point analysis of this circuit:

```
supermesh1
*OP Analysis
R1 1 2 12
R2 2 3 6
R3 2 4 4
Va 1 0 12
Vb 3 0 6
I1 0 4 8
```

**Problem 2.2.18** In the circuit shown in Fig. 2.53, determine voltages at nodes 2, 3, and 4 using mesh current analysis (Supermesh2.cir).

$$V_a = 10\,\mathrm{V}, \quad g = 0.1(\mathrm{A/V}), \quad h = 10(\mathrm{V/A}), \quad R_1 = 5\,\Omega, \quad R_2 = 4\,\Omega,$$
$$R_3 = 3\,\Omega.$$

**Solution**

There is a current source between two meshes. For the supermesh circuit, (KVL) (Fig. 2.54),

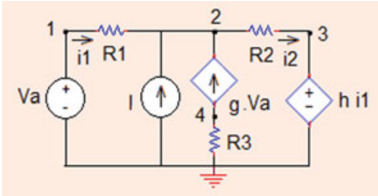



**Fig. 2.53** The circuit for Problem 2.2.18

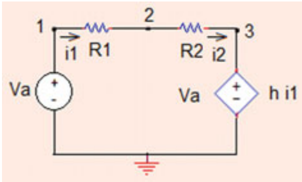



**Fig. 2.54** The supermesh for the circuit of Problem 2.2.18

$$-V_a = R_1 i_1 + R_2 i_2 + h i_1 = 0$$

$$-10 = 5i_1 + 4i_2 + 10i_1 = 0 \rightarrow 15i_1 + 4i_2 = 10. \tag{2.75}$$

KCL at node 2 (constraint equation):

$$1 + i_1 - i_2 + \frac{V_a}{10} = 0 \rightarrow 1 + i_1 - i_2 + \frac{10}{10} = 0$$

$$i_1 - i_2 = -2\,\mathrm{A}. \tag{2.76}$$

From (2.71) and (2.72),

$$i_2 = \frac{40}{19} = 2.1053\,\mathrm{A}, \quad i_1 = \frac{2}{19} = 0.1053\,\mathrm{A}$$

$$V_2 = i_2 R_2 + 10 i_1 = \frac{40}{19} 4 + 10 \frac{2}{19} = \frac{180}{19} = 9.474\,\mathrm{V}.$$

Alternatively,

$$V_2 = V_a - i_1 R_1 = 10 - \frac{2}{19} \times 5 = 9.474\,\mathrm{V}$$

$$V_4 = 9V_a \times R_3 = 0.1 \times 10 \times 3 = -3\,\mathrm{V}$$

$$V_3 = h i_1 = 10 \times 0.1053 = 1.053\,\mathrm{V}.$$

Following is the SPICE netlist for the operating point analysis of this circuit:

```
supermesh2
*OP Analysis
Va 1 0 10
I1 0 2 1
R1 5 2 5
R2 2 3 4
R3 4 0 3
*CCVS: hxx N+ N- VNAME VALUE
*Controlling current is through a zero volt voltage source
VREF 1 5 0
h1 3 0 VREF 10
*VCCS: gxx N+ N- NC+ NC- VALUE
g1 4 2 1 0 0.1
```

## 2.3 Linearity and Superposition

**Problem 2.3.1** In the circuit shown in Fig. 2.55, find the value of the current flowing through $R_2 = 9\,\Omega$ resistor using superposition. $R_1 = 6\Omega$, $U = 3\,\text{V}$, $I = 2\,\text{A}$.

**Solution**
Current due to voltage source alone is

$$i' = \frac{3}{9+6} = \frac{1}{5}\ \text{A}.$$

Current due to current source alone is

$$i'' = \frac{6}{9+6} \times 2 = \frac{4}{5}\ \text{A}.$$

The sum:

$$i = i' + i'' = \frac{1}{5} + \frac{4}{5} = 1\ \text{A}.$$

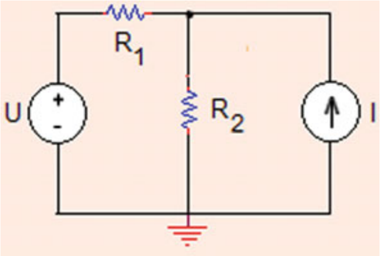



**Fig. 2.55** The circuit for Problem 2.3.1

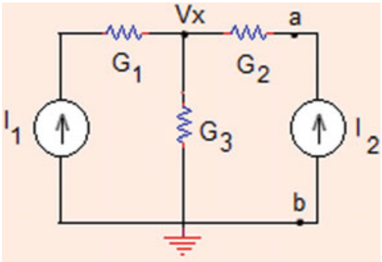



**Fig. 2.56** The circuit for Problem 2.3.2

**Problem 2.3.2**

(a) Determine the node voltage in the circuit shown in Fig. 2.56 (use superposition).
(b) Calculate the node voltage if all conductances are 2 S, and current source values are both 1 A.

**Solution**

(a) By superposition,
$I_1$ off, $I_2$ on; $V'_x = I_2/G_3$; $I_1$ on, $I_2$ off; $V''_x = I_1/G_3$;

$$V_x = V'_x + V''_x = \frac{I_2}{G_3} + \frac{I_1}{G_3} = \frac{1}{G_3}(I_1 + I_2).$$

(b)
$$G_1 = G_2 = G_3 = 2\,\mathrm{S}, \quad I_1 = I_2 = 1\,\mathrm{A}$$

$$V_x = \frac{1}{2}(1+1) = 1\,\mathrm{V}.$$

**Problem 2.3.3**

(a) In the circuit shown in Fig. 2.57, determine the voltage at node a, if $V1 = 1$ V, $V2 = 2$ V, $R = 1$ kΩ.
(b) $i = ?$ (Use superposition theorem).

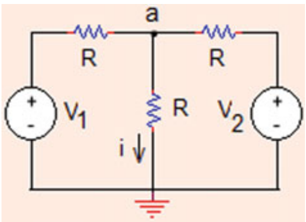



**Fig. 2.57** The circuit for Problem 2.3.3

**Solution**

(a)
$$V'_a = \frac{V_1}{3}\,\mathrm{V}, \quad V''_a = \frac{V_2}{3}\,\mathrm{V},$$

$$V_a = V'_a + V''_a = \frac{V_1}{3} + \frac{V_2}{3} = \frac{1}{3} + \frac{2}{3} = 1\,\mathrm{V}$$

(b)
$$i = \frac{V_a}{R} = \frac{1\,\mathrm{V}}{1\,\mathrm{k}\Omega} = 1\,\mathrm{mA}.$$

**Problem 2.3.4**

(a) Assuming a single-input and single-output (SISO) system, state criteria to determine the linearity of such a system.
(b) If y is the output and $x$ is the input of a system of the form $y = mx + n$, what can be said about its linearity?

**Solution**

(a) Assuming $y$ is the output and $x$ is the input of a system, three criteria to determine the linearity of such a system are as follows:

1. Homogeneity: if $y = f(x)$ then $k.y = f(k.x)$ where $k$ is a constant factor (more generally stated, $k$ is any real number for real systems and it is any complex number for complex-valued signals and systems).
2. Additivity: If $y1 = f(x1)$ and $y2 = f(x2)$, then $y1 + y2 = f(x1 + x2)$.
3. For $x = 0$, then $y = f(0) = 0$.

If a system satisfies all of these criteria stated above, it is a linear system.

(b) Let $m = 2$, $n = 1$, then $y = 2x + 1$, then

| | | | | |
|---|---|---|---|---|
| $x$ | $x1 = 0$ | $x2 = 1$ | $x3 = 2$ | $x4 = 3$ |
| $y$ | $y1 = 1$ | $y2 = 3$ | $y3 = 5$ | $y4 = 7$ |

All criteria are violated. For example, $y(x2 + x3) \neq y(x2) + y(x3)$.

Therefore, this system is “incrementally linear” so that the output is a scaled reproduction of the input except for a fixed offset in the output.

**Problem 2.3.5** What can be said about the linearity of the modified voltage divider circuit shown in Fig. 2.58?

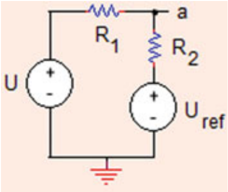



**Fig. 2.58** The circuit for Problem 2.3.5

**Solution**

$$V_a(U) = \frac{R_2}{R_1 + R_2} U + \frac{R_1}{R_1 + R_2} U_{\text{ref}} \quad \rightarrow \quad y = mx + n, \quad \text{where } x = U.$$

This circuit is "incrementally linear" so that the output voltage is a scaled reproduction of the input voltage except for a fixed offset in the output voltage.

**Problem 2.3.6** In the circuit shown in Fig. 2.59, use linearity principle to find the values for the voltage at node C (=$V_C$) and the current $i$ through the resistor $R_6$ ($R_1$ = $R_2$ = $R_3$ = 1 Ω, $R_4$ = $R_5$ = $R_6$ = 4 Ω, $i_S$ = 2 A).

**Solution**
Let $i$ = 1 A

$$V_{\text{C}} = 4 \cdot i = 4 \cdot 1 = 4\,\text{V}$$

$$i_{R3} = \frac{V_{\text{C}}}{R_3} = \frac{4}{1} = 4\,\text{A}$$

$$i_{\text{BC}} = i_{R3} + i = 4 + 1 = 5\,\text{A}$$

$$V_{\text{B}} = i_{\text{BC}} \cdot 4 + V_c = 5 \cdot 4 + 4 = 24\,\text{V}$$

$$i_{R2} = \frac{V_{\text{B}}}{R_2} = \frac{24}{1} = 24\,\text{A}$$

$$i_{\text{AB}} = i_{R2} + i_{\text{BC}} = 24 + 5 = 29\,\text{A}$$

$$V_{\text{A}} = 4 \cdot i_{\text{AB}} + V_{\text{B}} = 4 \cdot 29 + 24 = 116 + 24 = 140\,\text{V}$$

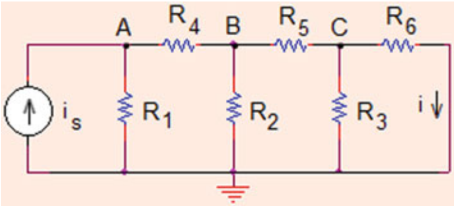
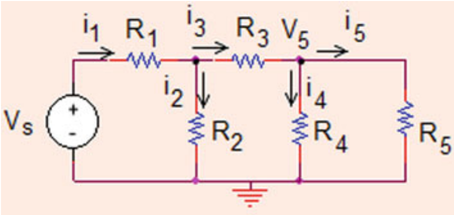



**Fig. 2.59** The circuit for Problem 2.3.6

**Fig. 2.60** The circuit for Problem 2.3.7




$$i_{R1} = \frac{V_A}{R_1} = 140\,\text{A}$$

$$i_s' = i_{R1} + i_{AB} = 140 + 29 = 169\,\text{A} \quad \rightarrow \quad \frac{i_s'}{i_s} = \frac{1}{i} \quad \rightarrow \quad i = \frac{2}{169} = 0.011834\,\text{A}$$

$$V_C = i \cdot 4 = 0.04734\,\text{V}.$$

**Problem 2.3.7** Find the current through resistor $R_5$ in the circuit shown in Fig. 2.60 (use linearity principle). $V_s = 10\,\text{V}, R_1 = 0.5\,\Omega, R_2 = 8\,\Omega, R_3 = 2\,\Omega,$ $R_4 = 2\,\Omega, R_5 = 1\,\Omega.$

**Solution**

$$\text{Let }\; i_5 = 1\,\text{A}, v_5 = i_5 \cdot R_5 = 1\,\text{V}$$

$$i_4 = \frac{v_5}{R_4} = 0.5\,\text{A}$$

$$i_3 = i_4 + i_5 = 1.5\,\text{A}$$

$$v_3 = i_3 R_3 + v_5 = (1.5)(2) + 1 = 4\,\text{V}$$

$$i_2 = \frac{v_3}{R_2} = \frac{4}{8} = 0.5\,\text{V}$$

$$i_1 = i_2 + i_3 = 0.5 + 1.5 = 2\,\text{A}$$

$$V_{sx} = i_1 R_1 + v_3 = (2)(0.5) + 4 = 5\,\text{V}$$

$$\text{When } V_s = 5\,\text{V}, \quad i_5 = 1\,\text{A}$$

$$\text{But } v_s = 10\,\text{V}, \quad \text{then } i_5 = 2\,\text{A}.$$

**Problem 2.3.8** Determine the current ($I_X$) in the circuit shown in Fig. 2.61. Use linearity principle.

$R$ = 10 Ω, $U$ = 20 V.

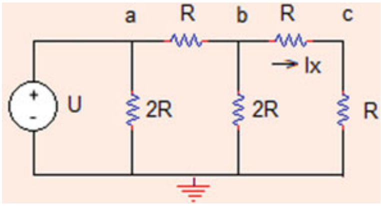



**Fig. 2.61** The circuit for Problem 2.3.8

**Solution**
Let $I_x = 1$ A,

$$V_b = 2R \times 1 = 20\,\text{V}$$

$$V_a = U = i_{\text{ab}}R + V_b = (1+1)R + 20 = 2 \times 10 + 20 = 40\,\text{V}.$$

Since given value of U = 20 V (which is half the calculated value), $I_x = 0.5$ A.

**Problem 2.3.9** Calculate the value of currents through $R_3$ and $R_1$ in the circuit shown in Fig. 2.62. Use superposition. $R_1 = R_2 = 1\,\text{k}\Omega, R_3 = 2\,\text{k}\Omega, \text{I}_1 = 9I_2 = 9\,\text{mA}$.

**Solution**
By current division rule due to $I_1$

$$i_{R31} = I_1 \cdot \frac{R_1}{R_1 + R_2 + R_3}.$$

By current division rule due to $I_2$,

$$i_{R32} = -I_2 \cdot \frac{R_2}{R_1 + R_2 + R_3}.$$

The sum of the currents:

$$i_{R3} = i_{R31} + i_{R32} = \frac{1}{R_1 + R_2 + R_3}(I_1R_1 - I_2R_2) = \frac{1}{1+1+2}(9 \times 1 - 1 \times 1) = 2\,\text{mA}.$$

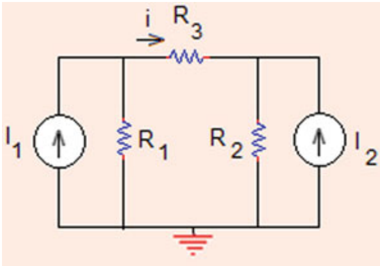



**Fig. 2.62** The circuit for Problem 2.3.9

By Kirchhoff's current law, $9 = i_{R1} + i_{R3} = i_{R1} + 2$ or,

$$i_{R3} = 7\,\text{mA}.$$

Note that application of superposition principle is somewhat lengthy even though it is straightforward.

**Problem 2.3.10** Using superposition theorem in the circuit shown in Fig. 2.63, find the value of

(a) $Vx$.
(b) $Vx$, if $R_1 = 0\,\Omega$.
(c) $Vx$, if $R_2 = 0\,\Omega$.

**Solution**

Step 1. (Fig. 2.64), $i_0 = 0\,\text{A} \rightarrow -\text{V}_1 + i\text{R}_1 + i\text{R}_2 + \text{kV}_1 = 0$

$$i(R_1 + R_2) = V_1 - kV_1 = V_1(1-k), \quad \rightarrow \quad i = \frac{V_1(1-k)}{R_1 + R_2}$$

$$V_{x1} = i \cdot R_2 + kV_1 = \frac{V_1(1-k)}{R_1 + R_2} \cdot R_2 + kV_1 = V_1\left[\frac{(1-k)R_2}{R_1 + R_2} + k\right].$$

Step 2. $V_1 = 0\,\text{V}$,

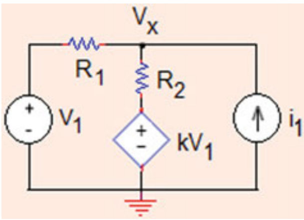



**Fig. 2.63** The circuit for Problem 2.3.10

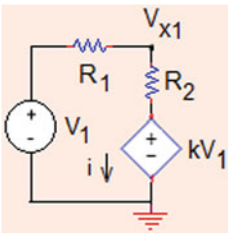



**Fig. 2.64** The circuit after "killing" the current source

$$V_{x2} = i_1 \cdot \frac{R_1 \cdot R_2}{R_2 + R_2}$$

a. $V_x = V_{x1} + V_{x2} = V_1 \left[ \frac{(1-k)R_2}{R_1 + R_2} + k \right] + i_1 \cdot \frac{R_1 R_2}{R_1 + R_2} \quad [\mathrm{V}]$
b. $R_1 = 0\,\Omega, \quad V_x = V_1 \quad [\mathrm{V}]$
c. $R_2 = 0\,\Omega, \quad V_x = kV_1 \quad [\mathrm{V}].$

**Problem 2.3.11** In the circuit shown in Fig. 2.65, $R_1 = R_2 = R_3 = 2\,\Omega$. Find the values of $V_a, V_b, I_1 = I_{R1}, I_2 = I_{R2}, I_3 = I_{R3}$.

**Solution**
According to superposition theorem,

$$V_a = V_{a1} + V_{a2} + V_{a3} \text{ and } V_b = V_{b1} + V_{b2} + V_{b3}.$$

When 3 A current source is closed, $I_2$ = 1 A, $I_1$ = $I_3$ = 2 A.

$$V_{a1} = 2 \cdot I_2 = 2 \cdot 1 = 2\,\mathrm{V}$$

$$V_{b1} = 2 \cdot I_3 = 2 \cdot 2 = 4\,\mathrm{V}.$$

When 1 A current source is closed, $I_2$ = 5 A, $I_1$ = 1 A and $I_3$ = 5 A.

$$V_{a2} = 2 \cdot I_2 = 2 \cdot 5 = 10\,\mathrm{V}$$

$$V_{b2} = 2 \cdot I_3 = 2 \cdot 5 = 10\,\mathrm{V}$$

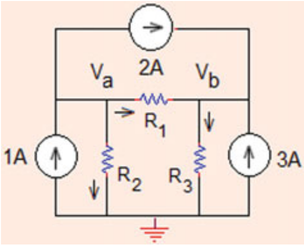
When 2 A current source is closed, $I_2$ = 4 A, $I_1$ = 2 A and $I_3$ = 4 A.

$$V_{a3} = 2 \cdot I_2 = 2 \cdot 4 = 8\,\mathrm{V}, V_{b3} = 2 \cdot I_3 = 2 \cdot 4 = 8\,\mathrm{V}$$

$$V_a = V_{a1} + V_{a2} + V_{a3} = 2 + 10 + 8 = 20\,\mathrm{V}$$

$$V_b = V_{b1} + V_{b2} + V_{b3} = 4 + 10 + 8 = 22\,\mathrm{V}.$$

**Fig. 2.65** The circuit for Problem 2.3.11



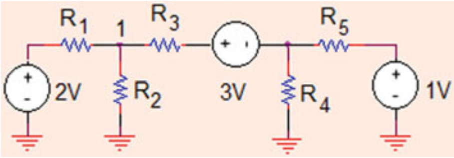



**Fig. 2.66** The circuit for Problem 2.3.12

### Problem 2.3.12

(a) Use superposition theorem and find the value of voltage at node 1 of the circuit shown in Fig. 2.66 ($R_1 = R_2 = R_4 = R_5 = 2\ \Omega$, $R_3 = 1\ \Omega$).
(b) Check your result using SPICE analysis. Print netlist (superposition check1.cir).

### Solution

(a)

(i) See Fig. 2.67.

$$V_{11} = 2 \cdot \frac{(R_4//R_5 + R_3)//R_2}{(R_4//R_5 + R_3)//R_2 + R_1} = 2 \cdot \frac{1}{1+2} = \frac{2}{3}\ \mathrm{V}.$$

(ii) See Fig. 2.68.

$$i = \frac{3}{1+1+1} = 1\ \mathrm{A}, \quad V_{12} = 1 \cdot 1 = 1\ \mathrm{V}.$$

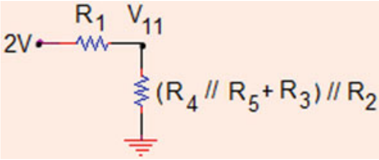



**Fig. 2.67** The circuit for the calculation of $V_{11}$ for Problem 2.3.12

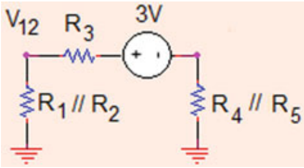



**Fig. 2.68** The circuit for the calculation of $V_{12}$ for Problem 2.3.12

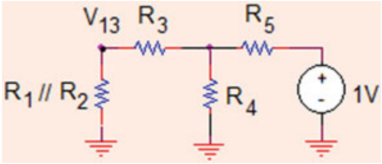



**Fig. 2.69** The circuit after "killing" the current source

(iii) See Fig. 2.69.

$$V_{13} = \frac{(R_3 + R_1//R_2)//R_4}{(R_3 + R_1//R_2) + R_4} = \frac{(1+1)//2}{[(1+1)//2] + 2} = \frac{2//2}{2//2 + 2} = \frac{1}{1+2} = \frac{1}{3}\ \text{V}$$

(iv) $$V_{11} + V_{12} + V_{13} = \frac{2}{3} + 1 + \frac{1}{6} = \frac{4+6+1}{6} = \frac{11}{6} = 1833\ \text{V}.$$

(b) SPICE netlist,

```
*OP analysis,superposition check1
  v1 1 0 2
  v2 3 4 3
  v3 5 0 1
  R1 1 2 2
  R2 2 0 2
  R3 2 3 1
  R4 4 0 2
  R5 4 5 2
```

**Problem 2.3.13** Use superposition theorem and find the value of voltage *Vx* in the circuit shown in Fig. 2.70.

**Solution**

First, voltage source is short circuited (Fig. 2.71).

The current flow in 4 Ω branch is calculated by current division,

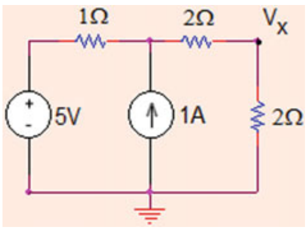



**Fig. 2.70** The circuit for Problem 2.3.13

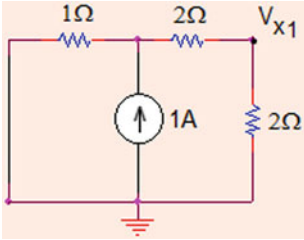



**Fig. 2.71** The circuit after voltage source is "killed"

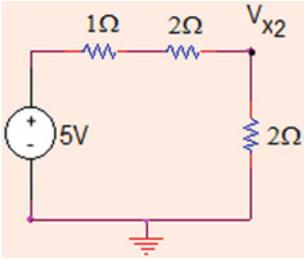



**Fig. 2.72** The circuit after current source is "killed"

$$i_1 = 1 \times \frac{1}{1+4} = 0.2\,\mathrm{A} \quad \rightarrow \quad V_{x1} = 0.2 \times 2 = 0.4\,\mathrm{V}.$$

Second, current source is open circuited (Fig. 2.72).

In this circuit, one may use the voltage division rule and obtain the unknown voltage as

$$V_2 = 5 \times \frac{2}{5} = 2\,\mathrm{V}.$$

Finally, superposition results are collected together, $V_x = V_{x1} + V_{x2} = 2.4\,\mathrm{V}$.

**Problem 2.3.14** Using superposition theorem, find the values of currents and voltages in the circuit shown in Fig. 2.73 ($i_1$ = 1 A, $V$ = 10 V, $R_1$ = 1 kΩ, $R_2$ = 2 kΩ).

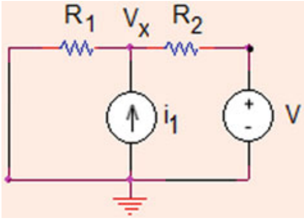



**Fig. 2.73** The circuit for Problem 2.3.14

**Solution**
First, the voltage source is ignored (short circuited, Fig. 2.74).

$$R_{\mathrm{eq}} = (R_1 // R_2) = 0.667\,\mathrm{kV}$$

$$V_{x1} = I \cdot R_{\mathrm{eq}} = 1 \cdot 0.667 = 0.667\,\mathrm{kV}.$$

Then, the current source is ignored (open circuited, Fig. 2.75). By voltage division,

$$V_{x2} = \frac{R_1}{R_1 + R_2} \cdot V = \frac{1}{1+2} \cdot 10 = 3.333\,\mathrm{V}$$

$$V_x = V_{x1} + V_{x2} = 667\,\mathrm{V} + 3.33\,\mathrm{V} = 670.33\,\mathrm{V}$$

$$I_{R_1} = \frac{V_x}{R_1} = \frac{670.33\,\mathrm{V}}{1\,\mathrm{k}\Omega} = 0.67033\,\mathrm{A}$$

$$I_{R_2} = I - I_{R_1} = 1 - 0.67033 = 0.32967\,\mathrm{A}.$$

**Problem 2.3.15** In the circuit shown in Fig. 2.76, find the value of $i$ (in mA) by using superposition theorem.

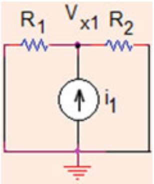



**Fig. 2.74** The circuit after voltage source is "killed"

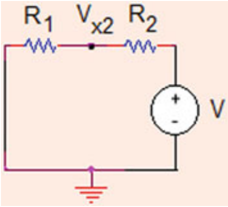



**Fig. 2.75** The circuit after current source is "killed" (open circuited)

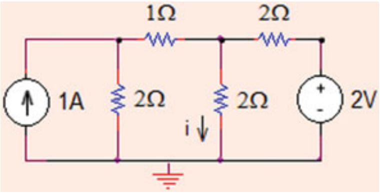



**Fig. 2.76** The circuit for Problem 2.3.15

**Solution**

Deactivated voltage source: (Fig. 2.77). Applying current division,

$$i' = \frac{1}{2}\,\mathrm{A}$$

$$V' = \frac{1}{2}\,\mathrm{A} \cdot 1\,\mathrm{V} = \frac{1}{2}\,\mathrm{V}.$$

Deactivated current source (Fig. 2.78): By voltage division,

$$V'' = 2 \cdot \frac{3//2}{3//2 + 2} = 2 \cdot \frac{\frac{3 \cdot 2}{3+2}}{\frac{3 \cdot 2}{3+2} + 2} = 2 \cdot \frac{\frac{6}{5}}{\frac{6}{5} + 2} = 2 \cdot \frac{6}{16} = \frac{12}{16} = \frac{3}{4}\,\mathrm{V}$$

$$V = V' + V' = \frac{1}{2} + \frac{3}{4} = 1.25\,\mathrm{V}$$

$$i = \frac{V}{2} = \frac{1.25}{2} = 0.625\,\mathrm{A} = 625\,\mathrm{mA}.$$

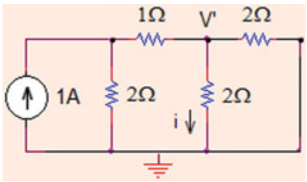



**Fig. 2.77** Deactivated voltage source

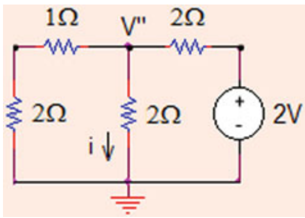



**Fig. 2.78** Deactivated current source

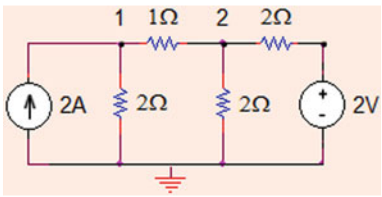



**Fig. 2.79** The circuit for Problem 2.3.16

**Problem 2.3.16** Use superposition theorem to find the values of voltages at nodes 1 and 2 in the circuit shown in Fig. 2.79.

**Solution**

(a) Kill the voltage source as shown in Fig. 2.80:

$$i_{1\Omega} = 1\,\mathrm{A}, \quad V_2' = 1\,\Omega \times 1\,\mathrm{A} = 1\,\mathrm{V}$$

$$V_1' = 2[2//(1+1)] = (2//2) \times 2 = 1\,\Omega \cdot 2\,\mathrm{A} = 2\,\mathrm{V}.$$

(b) Kill the current source as shown in Fig. 2.81.

By voltage division,

$$R_p = \frac{(2+1)2}{(2+1)+2} = \frac{6}{5}\,\Omega$$

$$V_2'' = 2 \cdot \frac{R_p}{R_p + 2} = 2 \cdot \frac{\frac{6}{5}}{\frac{6}{5}+2} = 2 \cdot \frac{\frac{6}{5}}{\frac{16}{5}} = \frac{12}{16} V = \frac{3}{4}\,\mathrm{V}.$$

Voltage division:

$$V_1'' = V_2'' \cdot \frac{2}{2+1} = \frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}\,\mathrm{V}$$

$$V_1 = V_1' + V_1'' = 2 + 0.5 = 2.5\,\mathrm{V}$$

$$V_2 = V_2' + V_2'' = 1 + \frac{3}{4} = 1.75\,\mathrm{V}.$$

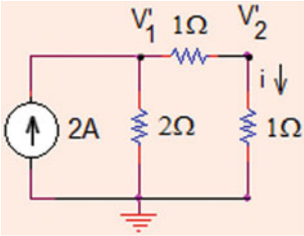



**Fig. 2.80** Deactivated voltage source (short circuited)

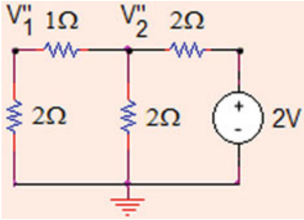



**Fig. 2.81** Deactivated current source (open circuited)

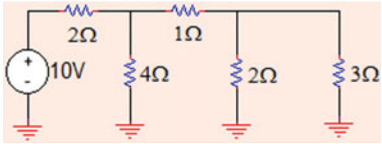



**Fig. 2.82** The circuit for Problem 2.3.17

**Problem 2.3.17** The supply voltage *v* and output current *i* are mutually transferable in a linear passive circuit. A circuit composed of linear bilateral elements (e.g., *R*, *L*, *C*) is reciprocal.

The ratio of *v* and *i* is called the transfer resistance (trans-resistance).

This means that if the positions of a voltage source and an ammeter are interchanged, the reading of ammeter remains the same, assuming ideal situation (i.e., internal resistance of both the voltage source and ammeter are null).

Alternatively, interchanging a current source and a voltmeter in a linear bilateral circuit does not change the voltmeter reading.

Reciprocity is based on the symmetry property of nodal conductance (mesh resistance) matrix. Thus, even a circuit containing dependent sources can be reciprocal for some specific dependent source coefficients, provided that its conductance or resistance matrix is symmetric.

Application of reciprocity theorem is limited only to circuits containing a single independent source.

(a) Use SPICE and determine the current flowing through 3 Ω resistor in the circuit shown in Fig. 2.82, assuming that an ammeter is placed in that branch. What is trans-resistance value?
(b) Interchange the ammeter and the voltage source and determine the new ammeter reading, again. What is new transresistance value?
(c) If the voltage is 50 V in part (b), determine the new ammeter reading.

**Solution**

(a) The current flowing through 3 Ω resistor (ammeter reading) is 0.754717 A. Trans-resistance is 13.24999966 Ω.
(b) Interchanging the ammeter and the voltage source, the ammeter reading is 0.754717 A, again.
    Therefore, trans-resistance is 13.24999966 Ω, as well.
(c) If the voltage is 100 V in part (b), (due to linearity) the new ammeter reading is 7.54717 A. This is also verified by SPICE analysis.

SPICE netlist (Reciprocity1.cir) is given below.

```
Reciprocity
*v1 1 0 10
V1 4 0 0
R1 1 2 2
R2 2 0 4
R3 2 3 1
R4 3 0 2
R5 3 4 3
*VX 4 0 0
VX 1 0 10
```

## 2.4 Source Transformation

**Problem 2.4.1** In the circuit shown in Fig. 2.83, find the value of node voltage $Vx$, if $V_1$ = 3 V, $I$ = 9 A, $R$ = 1 Ω using source transformation.

**Solution**
Applying source transformation to the given circuit gives the circuit shown in Fig. 2.84; then,

$$\frac{V_1}{R} - I + \frac{V_1}{R} + I = 3\frac{V_x}{R} = 3V_x$$

$$2V_1 = 3V_x$$

$$V_x = \frac{2}{3}V_1 = \frac{2}{3} \times 3 = 2\,\mathrm{V}.$$

**Problem 2.4.2** In the circuit shown in Fig. 2.85, use source transformation method and determine the current through resistor $R_2$.

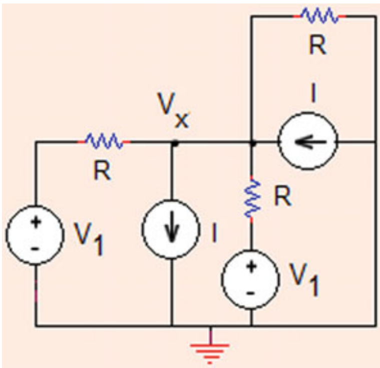



**Fig. 2.83** The circuit for Problem 2.4.1

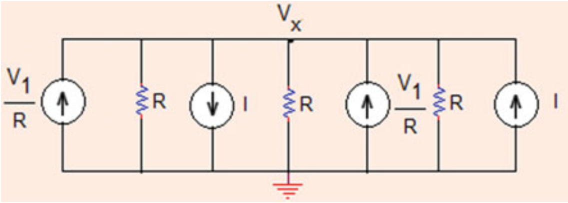


**Fig. 2.84** Source transformed circuit of Problem 2.4.1

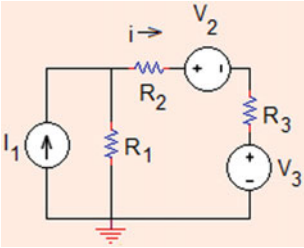


**Fig. 2.85** The circuit for Problem 2.4.2

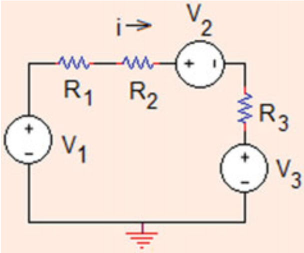


**Fig. 2.86** Source transformation applied to circuit of Problem 2.4.2

**Solution**

By source transformation and KVL, (see Fig. 2.86),

$$V_1 - V_2 - V_3 = i(R_1 + R_2 + R_3)$$

$$I_1R_1 - V_2 - V_3 = i(R_1 + R_2 + R_3)$$

$$i = \frac{I_1R_1 - V_2 - V_3}{R_1 + R_2 + R_3}.$$

**Problem 2.4.3** Use source transform to calculate the value of node voltage $V_a$ in the circuit shown in Fig. 2.87. $R_1 = R_2 = 2R_3 = 8\ \Omega$, $I_1 = I_2 = 1$ A.

**Solution**

$$i = \frac{I_1R_1 - I_2R_3 - 4V_a}{R_1 + R_2 + R_3} = \frac{8 - 4 - 4V_a}{8 + 4 + 4} = \frac{4}{20}(1 - V_a) = \frac{1 - V_a}{5} \tag{2.77}$$

$$V_a = iR_3 + I_2R_3. \tag{2.78}$$

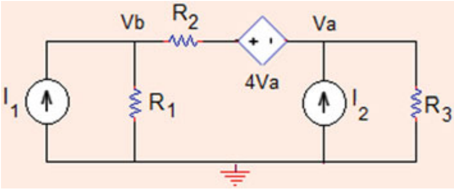



**Fig. 2.87** The circuit for Problem 2.4.3

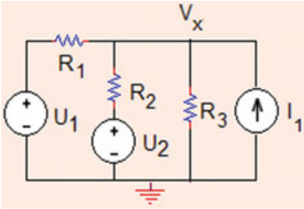



**Fig. 2.88** The circuit for Problem 2.4.4

Substitute (2.77) in (2.78), use given data,

$$V_a = \frac{1 - V_a}{5} \times 4 + 1 \times 4 = \frac{4}{5} - \frac{4V_a}{5} + 4$$

$$\frac{9}{5} V_a = \frac{24}{5} \quad \rightarrow \quad V_a = \frac{24}{9} = \frac{8}{3} = 2.667\,\text{V}.$$

**Problem 2.4.4** In the circuit shown in Fig. 2.88, find the value of voltage $V_x$ using source transformation.

$U_1 = 3$ V, $U_2 = 5$ V, $I_1 = 2$ A, $R_1 = R_3 = 2R_2 = 4\ \Omega$.

**Solution**

Application of source transformation to voltage sources results in with the following circuit equation:

$$\frac{3}{4} + \frac{5}{2} + 2 = \frac{V_x}{4}.$$

Solving for the unknown voltage yields $V_x = 21\,\text{V}$.

**Problem 2.4.5** Using source transformation, find the node voltage $V_X$ in the circuit shown in Fig. 2.89.

$R_1 = R_2 = 4\ \Omega$.

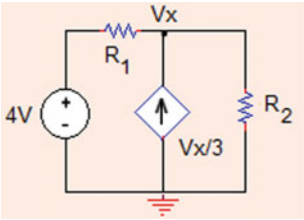



**Fig. 2.89** The circuit for Problem 2.4.5

**Solution**

By source transformation, (Fig. 2.90), $R_1 \| R_2 = 2\Omega$.

KCL at node $x$:

$$1 + \frac{V_x}{3} - \frac{V_x}{2} = 0$$

$$V_x\left(\frac{1}{3} - \frac{1}{2}\right) = -1$$

$$V_x = -\frac{1}{\left(-\frac{1}{6}\right)} = 6\,\text{V}.$$

**Problem 2.4.6** In the circuit shown in Fig. 2.91, find the value of node voltage using superposition theorem, and source transformation ($E_1$ = 20 V, $R_2$ = 10 Ω, $R_1$ = 10 Ω, $I_1$ = 2 A).

**Solution**

First, the current source is deactivated, and voltage source is transformed to current source. By KCL,

$$\frac{E_1}{R_2} + \frac{V_{x1}}{10} - \frac{V_{x1}}{R_p} = 0, \quad R_p = R_1//R_2 = 10//10 = 5\,\Omega$$

$$\frac{E_1}{R_2} + V_{x1}\left(\frac{1}{10} - \frac{1}{R_p}\right) = 0 \quad \rightarrow \quad V_{x1} = -\frac{\frac{E_1}{R_2}}{\frac{1}{10} - \frac{1}{R_p}} = \frac{-\frac{20}{10}}{\frac{1}{10} - \frac{1}{5}} = 20\,\text{V}.$$

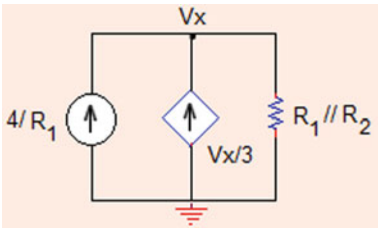


**Fig. 2.90** Source transformation applied to the circuit of Problem 2.4.5

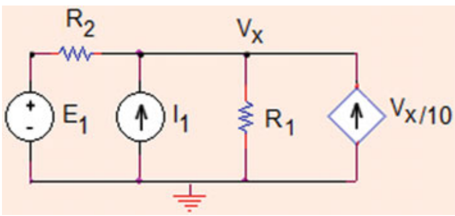


**Fig. 2.91** The circuit for Problem 2.4.6

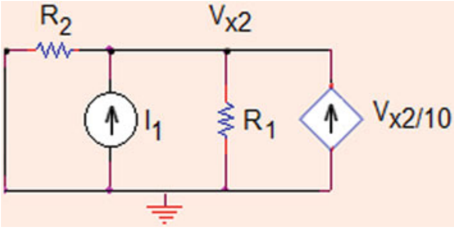



**Fig. 2.92** Deactivated voltage source (short circuited)

Then, the voltage source is deactivated by short circuiting it; and using KCL (Fig. 2.92),

$$I_1 + \frac{V_{x2}}{10} - \frac{V_{x2}}{R_p} = 0 \quad \rightarrow \quad I_1 + V_{x2}\left(\frac{1}{10} - \frac{1}{R_p}\right) = 0 \quad \rightarrow \quad V_{x2} = -\frac{I_1}{\frac{1}{10} - \frac{1}{R_p}} = 20\,\text{V}.$$

Finally, adding these superposition results, $V_x = V_{x_1} + V_{x_2} = 20 + 20 = 40\,\text{V}$.

**Problem 2.4.7** In the circuit shown in Fig. 2.93, find the values of $V_X$ by using source transformation and Kirchhoff's current law. Use Cramer's rule when necessary.

$$R_1 = R_2 = 2R_3 = R_4 = R_5 = 2R_2 = 2\,\Omega, \quad U_1 = U_2 = U_3 = 2\,\text{V}, \quad I_1 = 1\,\text{A}.$$

**Solution**
The voltage sources are transformed into 1 A current sources (Fig. 2.94), and simplified as shown in Fig. 2.95.

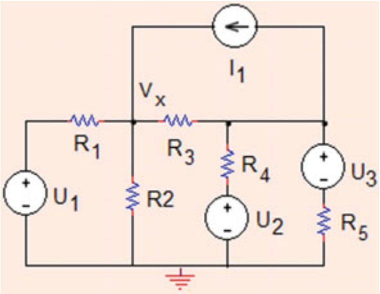



**Fig. 2.93** The circuit for Problem 2.4.7

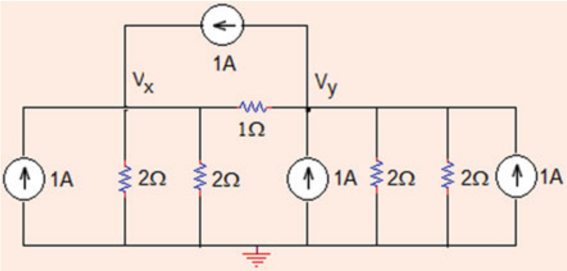
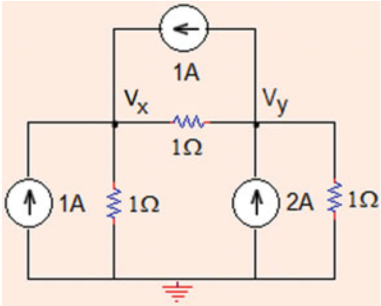



**Fig. 2.94** The voltage sources are transformed into current sources

**Fig. 2.95** Simplified circuit for Problem 2.4.7




Equivalent resistance of 2 Ω parallel resistors is calculated and the current sources are added:

$$[I] = [G][V]$$

$$\begin{bmatrix} 1+1 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 1+1 & -1 \\ -1 & 1+1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

$$\Delta = 2 \cdot 2 - 1 \cdot 1 = 4 - 1 = 3$$

$$\Delta_x = 2 \cdot 2 - (-1)(1) = 5$$

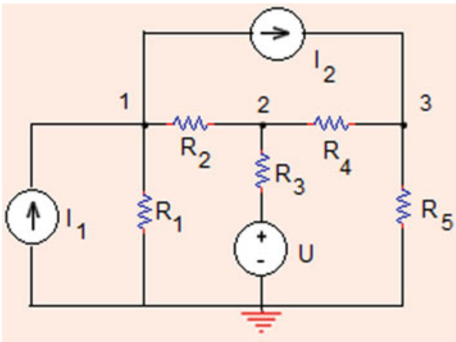
$$V_x = \frac{\Delta_x}{\Delta} = \frac{5}{3}\,\mathrm{V} = 1.667\,\mathrm{V}.$$

**Problem 2.4.8** Determine the value of voltage at node 2 in the circuit shown in Fig. 2.96.

Use source transformation and Cramer's rule, when necessary.

$$R_1 = R_2 = R_3 = 2\,\mathrm{k}\Omega, \quad R_4 = R_5 = 4\,\mathrm{k}\Omega, \quad I_1 = 4I_2 = 4\,\mathrm{mA}, \quad U = 2\,\mathrm{V}.$$

**Fig. 2.96** The circuit for Problem 2.4.8



**Solution**

$$GV = I$$

$$\begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 & 0 \\ -1/R_2 & 1/R_2 + 1/R_3 + 1/R_4 & -1/R_4 \\ 0 & -1/R_4 & 1/R_4 + 1/R_5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ \dfrac{U}{R_3} \\ I_2 \end{bmatrix}$$

$$10^{-3} \times \begin{bmatrix} 1/2+1/2 & -1/2 & 0 \\ -1/2 & 1/2+1/2+1/4 & -1/4 \\ 0 & -1/4 & 1/4+1/4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} (4-1) \times 10^{-3} \\ 2 \times 10^{-3} \\ 10^{-3} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \times 10^{-3}$$

or

$$10^{-3} \times \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{5}{4} & -\frac{1}{4} \\ 0 & -\frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \times 10^{-3}$$

$10^{-3}$ terms cancel out;

$$V_2 = \frac{\Delta_2}{\Delta},$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 0 \\ -\frac{1}{2} & 2 & -\frac{1}{4} \\ 0 & 1 & \frac{1}{2} \end{vmatrix} = 2, \quad \Delta = \begin{vmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{5}{4} & -\frac{1}{4} \\ 0 & -\frac{1}{4} & \frac{1}{2} \end{vmatrix} = \frac{5}{8} - \left(\frac{1}{16} + \frac{1}{8}\right) = \frac{4}{8} - \frac{1}{16} = \frac{7}{16}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2}{\frac{7}{16}} = \frac{32}{7}\,\text{V} = 4.571\,\text{V}.$$

**Problem 2.4.9** In the circuit shown in Fig. 2.97, find the value of current $I$ using source transformation method $2R_1 = 2R_2 = R_3 = R_4 = 2\,\Omega, I_1 = 1\,\text{A}, U = 2\,\text{V}$ (matrix_solve.xlsx).

**Solution**
2 V voltage source is transformed to 1 A independent current source, VCVS is transformed and circuit is simplified by taking only equivalent resistance of parallel resistors into consideration, see Fig. 2.98.

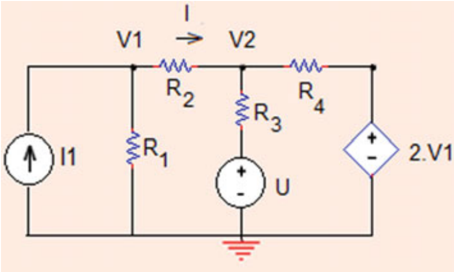



**Fig. 2.97** The circuit for Problem 2.4.9

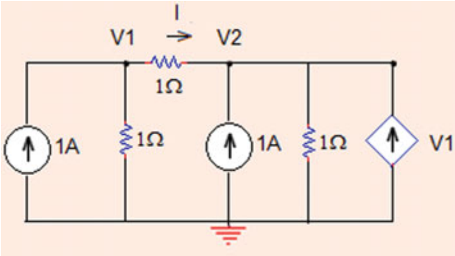



**Fig. 2.98** Simplified circuit for Problem 2.4.9

Apply Kirchhoff's Current Law:
At node 1: $1 = V_1 + V_1 - V_2$

$$2V_1 - V_2 = 1. \tag{2.79}$$

At node 2: $V_1 + 1 + V_1 - V_2 - V_2 = 0$

$$V_1 - V_2 = -\frac{1}{2}. \tag{2.80}$$

Put (2.80) and (2.81) in matrix form:

$$\begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}.$$

Solving the matrix equation yields

$$V_1 = 1.5\,\mathrm{V}, \quad V_2 = 2\,\mathrm{V},$$

$$I = (V_1 - V_2)/R = 1.5 - 2 = -0.5\,\mathrm{A}.$$

**Problem 2.4.10** In the circuit shown in Fig. 2.99, use source transformation to find the value of node voltage $V_x$, if $V_1 = V_2 = 10$ V, $I_1 = 1$ A, $R_1 = R_2 = R_3 = 1$ kΩ.

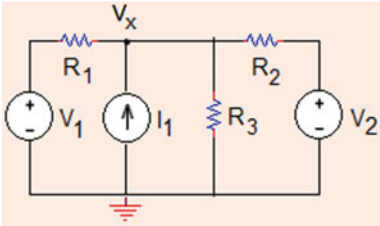



**Fig. 2.99** The circuit for Problem 2.4.10

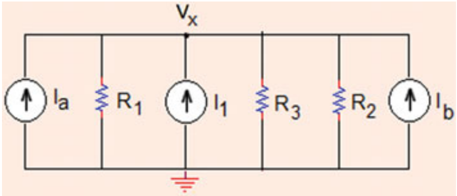



**Fig. 2.100** Source transformation applied to the circuit of Problem 2.4.10

**Solution**

If source transformation is used for the circuit (Fig. 2.100),

$V_1$ is transformed into $I_a$ and $V_2$ transformed into $I_b$

$$I_a = \frac{10\,\mathrm{V}}{10^3\,\Omega} = 10^{-2}\,\mathrm{A}$$

$$I_b = \frac{10\,\mathrm{V}}{10^3\,\Omega} = 10^{-2}\mathrm{A}, \quad I_1 + I_a + I_b = 1.02\,\mathrm{A}$$

$$\frac{1}{R_t} = \frac{1}{10^3} + \frac{1}{10^3} + \frac{1}{10^3} = \frac{3}{10^3} \quad \rightarrow \quad R_t = 333.\bar{3}\,\Omega,$$

$$V_x = I \cdot R_t = 1.02 \times 333.\bar{3} = 340\,\mathrm{V}.$$

**Problem 2.4.11** In the circuit shown in Fig. 2.101, find the value of current through resistor $R_1$ using source transformation method ($E_1$ = 20 V, $R_1$ = 2 Ω, $R_2$ = 3 Ω, $R_3$ = 5 Ω, $I_1$ = 8 A).

**Solution**

Source transformation is applied on $E_1$ and $R_3$ (see, Fig. 2.102).

$$I_2 = \frac{E_1}{R_3} = \frac{20\,\mathrm{V}}{5\,\Omega} = 4\,\mathrm{A}$$

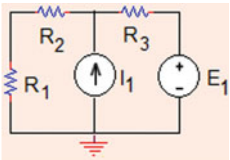



**Fig. 2.101** The circuit for Problem 2.4.11

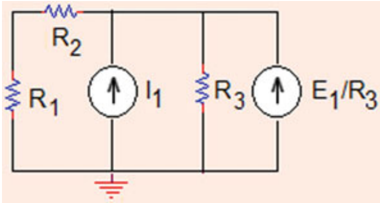



**Fig. 2.102** Source transformation applied to the circuit of Problem 2.4.11

$I_1$ and $I_2$ added together, $I = 8 + 4 = 12\,\mathrm{A}$

$$R_{\mathrm{eq}} = (R_1 + R_2)//R_3 = (2 + 3)//5 = 2.5\,\Omega$$

$$V_{R_{\mathrm{eq}}} = I \cdot R_{\mathrm{eq}} = 12 \cdot 2.5 = 30\,\mathrm{V}$$

$$I_{R1} = \frac{V_{R_{\mathrm{eq}}}}{R_1 + R_2} = \frac{30\,V}{5\,\Omega} = 6\,\mathrm{A}.$$

**Problem 2.4.12** In the circuit shown in Fig. 2.103, $V_S$ = 1 V, $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$, $f$ = 4 A/A.

$V_1$ = ?, $V_a$ = ?, $i$ = ? Use source transformation and node voltage method.

**Solution**

Use source transformation and note that $V_a = V_2$ (see, Fig. 2.104),

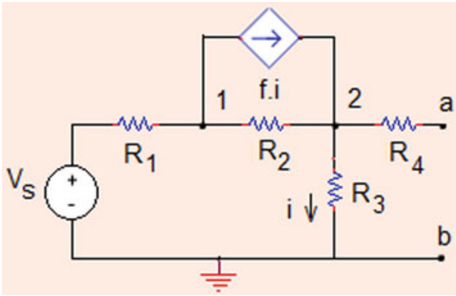



**Fig. 2.103** The circuit for Problem 2.4.12

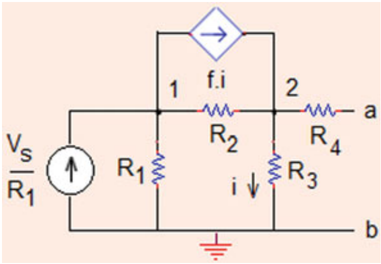



**Fig. 2.104** Source transformation applied to the circuit of Problem 2.4.12

$$\frac{V_s}{R_1} - f \cdot \frac{V_2}{R_3} - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_2} = 0 \tag{2.81}$$

$$f \cdot \frac{V_2}{R_3} + \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = 0. \tag{2.82}$$

Substituting numerical values and rearranging above equations,

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solution of this matrix equation gives

$$V_1 = 2\,\text{V}, \quad V_2 = V_a = -1\,\text{V}, \quad i = \frac{V_2}{R_3} = -\frac{1}{1} = 1\,\text{A}.$$

## 2.5 Thévenin–Norton Equivalent Circuits and Maximum Power Transfer

**Problem 2.5.1** A signal source has an open-circuit voltage of 1 mV and a short-circuit current of 100 nA. What is the source resistance?

**Solution**
Open-circuit voltage = Thévenin voltage, Short-circuit current = Norton current

$$R_s = \frac{V_\text{T}}{I_\text{N}} = \frac{10^{-3}}{100 \cdot 10^{-9}} = \frac{10^{-3}}{10^{-7}} = 10^4\,\Omega = 10\,\text{k}\Omega.$$

**Problem 2.5.2** For which one of the following circuits in Fig. 2.105, Thévenin's theorem cannot be applied?

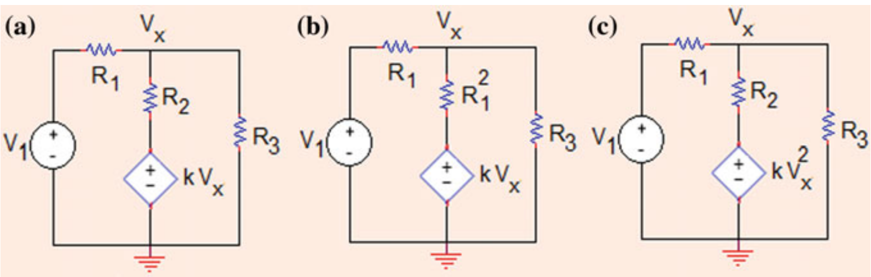



**Fig. 2.105** The circuit for Problem 2.5.2

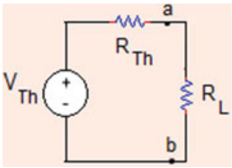



**Fig. 2.106** The circuit for Problem 2.5.3

**Solution**
Thévenin's theorem helps to reduce any one-port linear electrical network to a single-voltage source and a single impedance. The circuit in Fig. 2.105c contains a nonlinear-dependent source. Therefore, it does not suit for the application of this theorem.

**Problem 2.5.3** A carbon–zinc battery can be thought of its Thévenin's equivalent circuit with Thévenin resistance being the internal battery resistance, see Fig. 2.106. In an experiment, the open-circuit voltage of a battery is measured as 1.596 V. When a resistor of $R = 33.0\ \Omega$ is connected across its terminals, the load voltage is measured as 1.580 V. What is the internal battery resistance?

**Solution**
Assuming that measuring equipment probes and battery terminals have no contact resistances, internal resistance of the battery is serially connected to the load. The load current is

$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{V_L}{R_L}$$

$$I_L = \frac{1.596}{R_{\text{Th}} + 33} = \frac{1.580}{33} = 0.47878\ \text{A}$$

$$R_{\text{Th}} = (28.886)(1.596) - 33$$

$$R_{\text{Th}} = 0.334\ \Omega.$$

**Problem 2.5.4** In the circuit shown in Fig. 2.107, determine the current through $R_L$ at maximum power transfer condition if $I = 4$ A.

**Solution**
Turning off the current source and calculating Thévenin resistance gives

$$R_{\text{Th}} = 2R.$$

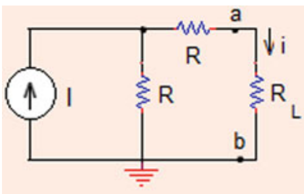



**Fig. 2.107** The circuit for Problem 2.5.4

Thévenin voltage is the voltage drop across the grounded resistor,

$$V_{\mathrm{Th}} = V_{\mathrm{oc}} = IR$$

$$i_{\mathrm{RLpmax}} = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} = \frac{IR}{2R + 2R} = \frac{I}{4} = 1\,\mathrm{A}.$$

**Problem 2.5.5**

(a) Determine the Thévenin and Norton equivalents for the circuit shown in Fig. 2.108, between a and b terminals.
(b) Find the power delivered to a load resistance, if $R_L = 5\,\Omega$.
(c) Determine the value of load resistor for maximum power transfer. $R_1 = R_2 = 6\,\Omega$, $R_3 = 12\,\Omega$, $U = 10\,\mathrm{V}$.

**Solution**

(a) Thévenin equivalent circuit values are

$$R_{\mathrm{Th}} = 12 + 6\|6 = 12 + 3 = 15\,\Omega$$

$$V_{\mathrm{Th}} = V_{\mathrm{ab}} = V_{\mathrm{oc}} = \frac{6}{6+6} \cdot 10 = 5\,\mathrm{V}.$$

Note that 12Ω resistance has no effect here.
Norton equivalent circuit values are

$$R_{\mathrm{N}} = R_{\mathrm{Th}} = 15\,\Omega$$

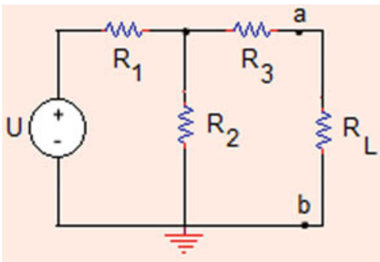
$$I_{\mathrm{N}} = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}}} = \frac{5}{15} = \frac{1}{3} = 0.333\,\mathrm{A}$$

(b) $I = \dfrac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} = \dfrac{5}{15+5} = \dfrac{1}{4}\,\mathrm{A}$

$$P = I^2 R_L = \left(\frac{1}{4}\right)^2 \times 5 = \frac{5}{16} = 0.3125\,\mathrm{W}$$

(c) For maximum power transfer; $R_L = R_{\mathrm{Th}} = 5\,\Omega$.

**Fig. 2.108** The circuit for Problem 2.5.5



**Problem 2.5.6** Use Thévenin's theorem to find the value of current, $i_{R6}$ (Fig. 2.109).

$$(R_1 = 50\,\Omega, \quad R_2 = 5\,\Omega, \quad R_3 = R_4 = 10\,\Omega, \quad R_5 = 4\,\Omega, \quad R_6 = 2\,\Omega, \quad V_1 = 20\,V).$$

**Solution**

Remove $R_6$ from circuit: $R_3//R_4 = 5\,\Omega, \quad V_{\mathrm{ab}} = V_{\mathrm{Th}}$

Voltage division:

$$V_{\mathrm{ab}} = \frac{R_3//R_4}{R_2 + R_3//R_4} \cdot 20$$

$$V_{\mathrm{Th}} = V_{\mathrm{ab}} = \frac{5}{5+5} \cdot 20 = 10\,\mathrm{V}.$$

Thévenin resistance: When the voltage source is short circuited, $V_1 = 0$ V, $R_1$ is shorted (see, Fig. 2.110):

$$R_{\mathrm{Th}} = R_5 + (R_2 \| R_3 \| R_4) = 4 + (5\|10\|10)$$

$$R_{\mathrm{Th}} = 4 + (5\|5) = 4 + 2.5 = 6.5\,\Omega.$$

The value of the current flowing through resistor $R_6$

$$i_{R_6} = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_6} = \frac{10}{6.5+2} = \frac{10}{8.5} = 1.177\,\mathrm{A}.$$

**Problem 2.5.7** In the circuit shown in Fig. 2.111, use Thévenin's Theorem and source transformation method to determine the current through the resistor $R_L$.

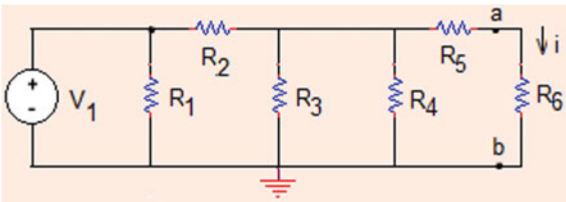



**Fig. 2.109** The circuit for Problem 2.5.6

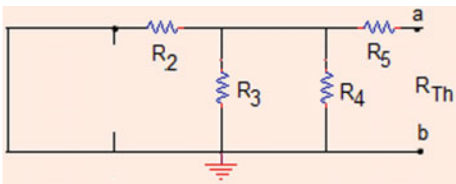
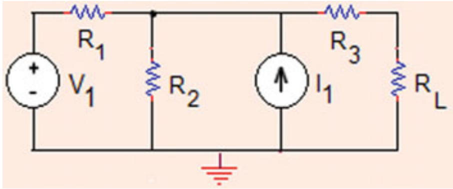



**Fig. 2.110** The circuit for the calculation of Thévenin resistance in Problem 2.5.6

**Fig. 2.111** The circuit for Problem 2.5.7




$$R_1 = R_2 = 2\,\Omega, \quad R_3 = R_L = 1\,\Omega, \quad V_1 = 1\,\text{V}, \quad I_1 = 0.5\,\text{A}.$$

**Solution**

First calculate Thévenin resistance for the circuit to the left of $R_L$ (Fig. 2.112):

$$R_{\text{Th}} = (R_1 \| R_2) + R_3.$$

Then, determine Thévenin voltage, noting that $R_3$ has no current flow at the output terminals (a-ground), as shown in Fig. 2.113.

The node voltage $V_x$ becomes the open-circuit voltage.

Using KCL at this node,

$$\frac{V_1}{R_1} + I_1 = \frac{V_x}{R_1} + \frac{V_x}{R_2} \quad \rightarrow \quad V_x = V_{\text{Th}} = \frac{\dfrac{V_1}{R_1} + I_1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}.$$

The load current through the resistor $R_L$ is found using Fig. 2.114:

$$i_{RL} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{\dfrac{\dfrac{V_1}{R_1} + I_1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}}{(R_1 \| R_2) + R_3 + R_L} = \frac{\dfrac{\dfrac{1}{2} + \dfrac{1}{2}}{\dfrac{1}{2} + \dfrac{1}{2}}}{(2\|2) + 1 + 1} = \frac{1}{3}\ \text{A}.$$

**Fig. 2.112** The circuit for the calculation of Thévenin resistance in Problem 2.5.7

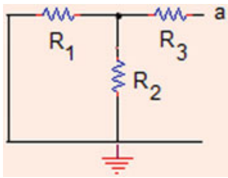
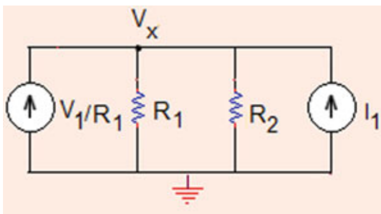



**Fig. 2.113** The circuit for the calculation of Thévenin voltage in Problem 2.5.7



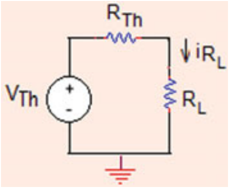



**Fig. 2.114** The circuit for the calculation of load current in Problem 2.5.7

**Problem 2.5.8** Determine Thévenin equivalent parameters between a and b terminals of the circuit shown in Fig. 2.115. $R = 2\ \Omega$. (Hint: Apply source transformation for the current source in calculating Thévenin equivalent voltage.)

**Solution**

Thévenin equivalent resistance is found by eliminating independent sources (i.e., short-circuit voltage source and open-circuit current source) in the circuit and finding the resistance between a and b terminals.

$$R_{\mathrm{Th}} = R_{\mathrm{ab}} = [(R + R) \| 2R] + R = 2R = 4\,\Omega.$$

Thévenin equivalent voltage between (a) and (b) terminals of the circuit can be found by applying source transformation to the current source, and then determining the voltage at node (c). This is due to the fact that the voltage at terminal (a) equals to voltage at node (c), $V_{\mathrm{cb}} = V_{\mathrm{ab}}$ (Fig. 2.116).

By KVL, $i = (2 - 1)/(R + R + 2R) = 1/8 = 0.125\,\mathrm{A}$

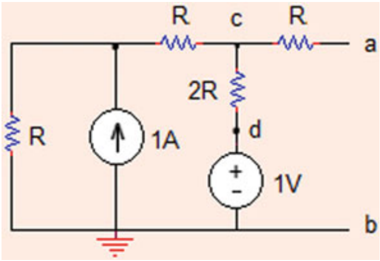



**Fig. 2.115** The circuit for Problem 2.5.8

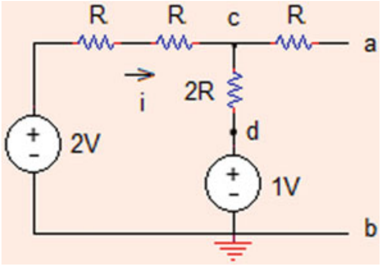



**Fig. 2.116** The circuit for the calculation of Thévenin voltage in Problem 2.5.8

$$V_{\mathrm{Th}} = V_{\mathrm{ab}} = V_{\mathrm{cb}} = 2R \cdot i + 1 = 4 \times 0.125 + 1 = 1.5\,\mathrm{V}.$$

In summary, $V_{\mathrm{Th}} = 1.5\,\mathrm{V}, R_{\mathrm{Th}} = 4\,\Omega$.

**Problem 2.5.9** For the circuit shown in Fig. 2.117,

(a) Find Thévenin's equivalent to the left of terminals a and b.
(b) If $R_1 = 1$ kΩ, $R_2 = 1$ kΩ, $R_3 = 100$ Ω, $k = 0.1$, $R_L = 1$ kΩ, what is $V_{\mathrm{ab}}$?

**Solution**
For Thévenin's equivalent circuit to the left of terminals a and b, the voltage source is removed, the first circuit becomes a short circuit, so the $R_{\mathrm{Th}}$ only depends on $R_3$ (Fig. 2.118).

$$R_{\mathrm{Th}} = R_3 = 100\,\Omega.$$

The current generated with current-controlled current source, CCCS becomes

$$i_{\mathrm{Th}} = k \times i_{R_2} = k \times \frac{V_1}{R_1 + R_2} = 0.1 \times \frac{V_1}{2000} = \frac{V_1}{2} \times 10^{-4}\,\mathrm{A}$$

$$V_{\mathrm{Th}} = R_{\mathrm{Th}} \times i_{\mathrm{Th}}$$

$$V_{\mathrm{Th}} = 100 \times \frac{V_1}{2} \times 10^{-4} = 50 \times 10^{-4} \times V_1$$

$$i_L = \frac{V_{\mathrm{Th}}}{R_3 + R_L} = \frac{50 \times 10^{-4} \times V_1}{100 + 1000} = 4.6 \times 10^{-6} \times V_1\,\mathrm{A}$$

$$V_{R_L} = R_L \times i_L = 1000 \times 4.6 \times 10^{-6} \times V_1 = 4.6 \times 10^{-3} \times V_1.$$

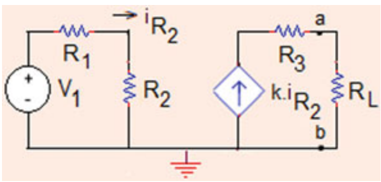


**Fig. 2.117** The circuit for Problem 2.5.9

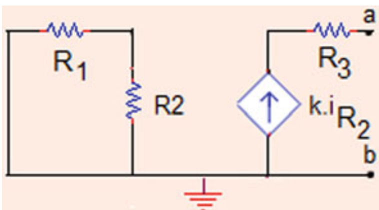
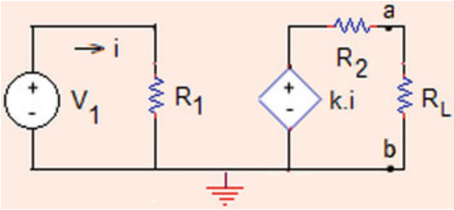


**Fig. 2.118** $R_{\mathrm{Th}}$ only depends on $R_3$

Fig. 2.119 The circuit for Problem 2.5.10




**Problem 2.5.10** For the circuit shown in Fig. 2.119,

a. Find the current through $R_L$, and voltage across $R_L$, using Thévenin's method (as function of $k$, $V_1$, $R_1$, $R_2$, $R_L$)
b. What is the Norton's equivalent circuit to the left of a–b if $V_1 = 2$ V, $k = 2$ V/V, $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$.

**Solution**

(a) $i = \frac{V_1}{R_1}, \quad V_{\text{oc}} = V_{\text{T}} = k \cdot i = k \cdot \frac{V_1}{R_1}$

$$R_{\text{T}} = R_2,$$

$$I_{R_L} = \frac{V_{\text{T}}}{R_2 + R_L} = \frac{kV_1}{R_1(R_2 + R_L)}, \quad V_{R_L} = I \cdot R_L = \frac{k \cdot V_1 R_L}{R_1(R_2 + R_L)}.$$

b. Thévenin equivalent circuit is shown in Fig. 2.120.

$$V_{\text{T}} = k \cdot \frac{V_1}{R_1} = 2 \cdot \frac{2}{10} = 0.4\,\text{V}, \quad R_{\text{T}} = R_{\text{N}} = R_2 = 5\,\Omega.$$

Norton equivalent circuit is shown in Fig. 2.121.

Fig. 2.120 Thévenin equivalent circuit

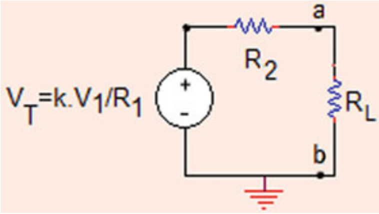
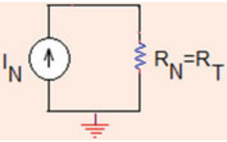



Fig. 2.121 Norton equivalent circuit



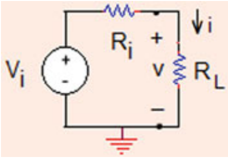



**Fig. 2.122** The circuit described in Problem 2.5.11

**Problem 2.5.11** A DC voltage source with internal resistance of 10 Ω and with an open-circuit voltage of 12 V feeds a resistive load, $R_L$. Determine the range of load resistance values so that the circuit operates for $V_L \geq 5\,\text{V}$ and $i_L \geq 0.5\,\text{A}$.

**Solution**

Figure 2.122 shows the equivalent circuit.

Using voltage constraint, $R_{\text{Th}} = R_i = 10\,\Omega, V_{\text{oc}} = V_{\text{Th}} = 12\,\text{V}$

$$\frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}} \geq 5 \quad \rightarrow \quad \frac{R_L}{10 + R_L} \times 12 \geq 5 \quad \rightarrow \quad 7R_L \geq 50 \quad \rightarrow \quad R_L \geq 7.14\,\Omega.$$

Using current constraint,

$$i = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} \geq 0.5\,\text{A} \quad \rightarrow \quad \frac{12}{10 + R_L} \geq 0.5 \quad \rightarrow \quad 12 \geq 5 + 0.5R_L \quad \rightarrow \quad R_L \leq 14\,\Omega.$$

Therefore, $7.14\,\Omega \leq R_L \leq 14\,\Omega$.

A proper value of load resistance is to choose the arithmetic mean of limiting values. This allows for component variations in the circuit. In that case, $R_L = 10\,\Omega$ can be a suitable value of the load resistance.

**Problem 2.5.12** Maximum power delivered by a DC circuit to a resistor of $R$ = 20.25 Ω is 1 W. Find the open-circuit voltage at the output of the circuit. Draw its Thévenin's equivalent circuit.

**Solution**

$$P_{\max} = \frac{V_{\text{th}}^2}{4R_L}, \quad V_{\text{Th}} = V_{\text{oc}}, \quad R = R_{\text{Th}}$$

$$1 = \frac{V_{\text{oc}}^2}{4 \cdot (20.25)} = \frac{V_{\text{oc}}^2}{81} \quad \rightarrow \quad V_{\text{oc}}^2 = 81 \quad \rightarrow \quad V_{\text{oc}} = 9\,\text{V}.$$

Figure 2.123 describes the Thévenin equivalent circuit. $V_{\text{Th}} = V_{\text{oc}} = 9\,\text{V}, R_{\text{Th}} = R = 20.25\,\Omega$.

**Problem 2.5.13** Determine Thévenin and Norton equivalent circuit parameters for the circuit between terminal b and ground, as shown in Fig. 2.124.

**Fig. 2.123** Thévenin equivalent circuit

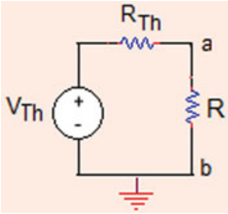
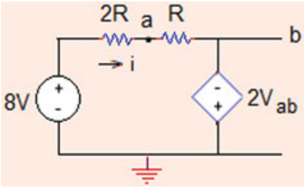



**Fig. 2.124** The circuit described in Problem 2.5.13




**Solution**

First, determine the loop current under open-circuit conditions (i.e., no load is connected between terminal b and ground). By KVL and assuming clockwise current flow in the loop,

$$8 - (4 + 2)i + V_{ab} = 8 - 6i + 2(2i) = 8 - 2i = 0 \quad \rightarrow \quad i = 4\,\text{A}$$

$$V_{oc} = V_{th} = 2V_{ab} = 2(2i) = 4i = 4 \times 4 = 16\,\text{V}.$$

When dependent source is shorted to ground, one can determine the short-circuit current at the output port, as

$$i_{sc} = \frac{8}{4+2} = \frac{8}{6}\,\text{A}.$$

Then,

$$R_{th} = R_N = \frac{V_{oc}}{i_{sc}} = \frac{16}{\frac{8}{6}} = 12\,\Omega, \quad I_N = \frac{V_{th}}{R_{th}} = \frac{16}{12} = 1.333\,\text{A}.$$

**Problem 2.5.14** For the circuit shown in Fig. 2.125, find

**Fig. 2.125** The circuit described in Problem 2.5.14

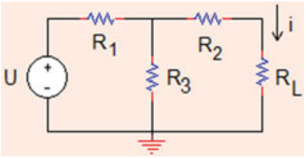
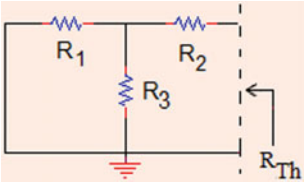


**Fig. 2.126** The circuit for the calculation of Thévenin resistance in Problem 2.5.14




(a) the value of load at maximum power transfer.
(b) the value of current through load resistor at maximum power.
(c) the value of maximum power transferred to the load $(U = 2\,\text{V}, R1 = 6\,\Omega, R2 = 8\,\Omega, R3 = 4\,\Omega)$.

**Solution**

(a) Thévenin resistance (see Fig. 2.126)

$$R_{\text{Th}} = (6//4) + 8 = \frac{24}{10} + 8 = \frac{104}{10}$$

$$R_L = R_{\text{Th}} = 10.4\,\Omega.$$

(b) The value of current at maximum power is calculated from Thévenin's equivalent circuit, see Figs. 2.127 and 2.128.

By voltage division,

**Fig. 2.127** The circuit for the calculation of Thévenin voltage in Problem 2.5.14

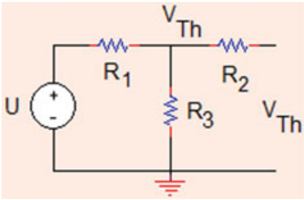
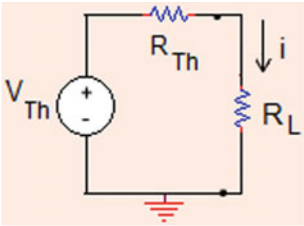



**Fig. 2.128** Thévenin equivalent circuit



$$V_{\mathrm{Th}} = V_{\mathrm{oc}} = 2 \cdot \frac{4}{4+6} = \frac{8}{10} = 0.8\,\mathrm{V}$$

$$i = \frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L} = \frac{0.8}{10.4+10.4} = \frac{0.8}{20.8} = 0.038\,\mathrm{A}.$$

(c) $P_{\max} = i^2 \cdot R_L = (0.038)^2 \cdot (10.4) = 0.015\,\mathrm{W}.$

**Problem 2.5.15** In the circuit shown in Fig. 2.129, all resistors (except $R_x$) have the same resistance of $R = 10\ \Omega$, and $V_1 = 10$ V. Find the value of $R_x$ for maximum power transfer (matrix_solve.xlsx) (Fig. 2.130).

**Solution**
Thévenin's equivalent circuit

$$R_x = R_{\mathrm{Th}} = \frac{v_{\mathrm{oc}}}{i_{\mathrm{sc}}}.$$

Open-circuit voltage (Fig. 2.131),

$$\begin{bmatrix} 3R & -R & 0 & -R & 0 \\ -R & 4R & -R & 0 & -R \\ 0 & -R & 4R & 0 & 0 \\ -R & 0 & 0 & 2R & -R \\ 0 & -R & 0 & -R & 3R \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

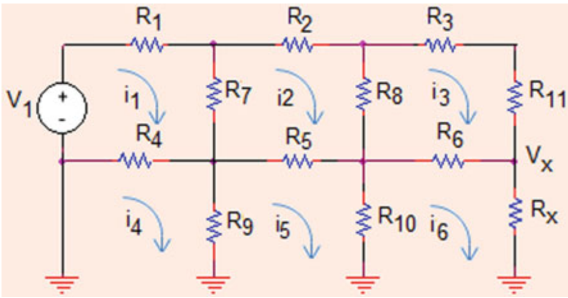



**Fig. 2.129** The circuit of Problem 2.5.15

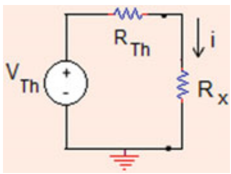



**Fig. 2.130** Thévenin's equivalent circuit in Problem 2.5.15

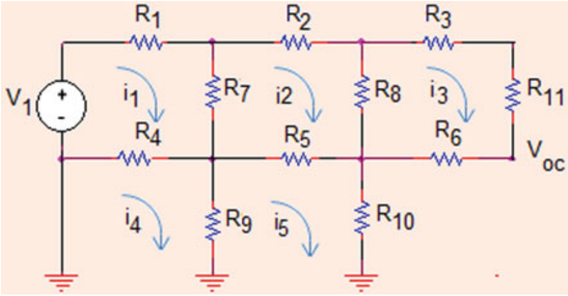



**Fig. 2.131** Circuit for Thévenin voltage calculation in Problem 2.5.15

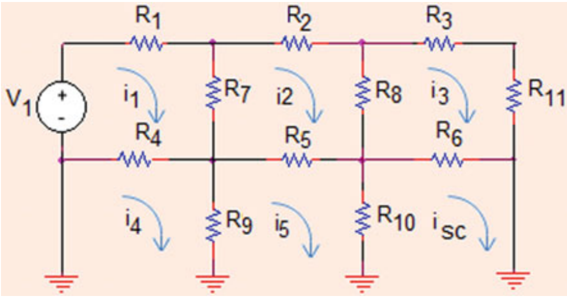



**Fig. 2.132** Circuit for short circuit current calculation in Problem 2.5.15

Mesh currents can be found here using an EXCEL spreadsheet as

$$i_1 = 0.509\,\text{A}, \quad i_2 = 0.185\,\text{A}, \quad i_3 = 0.046\,\text{A}, \quad i_4 = 0.343\,\text{A}, \quad i_5 = 0.176\,\text{A}$$

$$V_{\text{oc}} = V_{\text{Th}} = i_5 \cdot R + i_3 \cdot R = 10(0.046 + 0.176) = 10 \times 0.222 = 2.22\,\text{V}.$$

Short-circuit current (Fig. 2.132):

$$\begin{bmatrix} 3R & -R & 0 & -R & 0 & 0 \\ -R & 4R & -R & 0 & -R & 0 \\ 0 & -R & 4R & 0 & 0 & -R \\ -R & 0 & 0 & 2R & -R & 0 \\ 0 & -R & 0 & -R & 3R & -R \\ 0 & 0 & -R & 0 & -R & 2R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_{\text{sc}} \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$i_{\text{sc}} = 0.195\,\text{A}$$

$$R_x = R_{\text{Th}} = \frac{V_{\text{oc}}}{i_{\text{sc}}} = \frac{2.2}{0.195} = 11.282\,\Omega.$$

**Problem 2.5.16** In the circuit shown in Fig. 2.133,

(a) Determine Thévenin's equivalent circuit parameters.
(b) If $R_1 = R_2 = R_3 = 1\ \Omega$, $V_1 = 2$ V, $I_1 = 1$ A, $R_{\text{Th}} = ?$, $V_{\text{Th}} = ?$

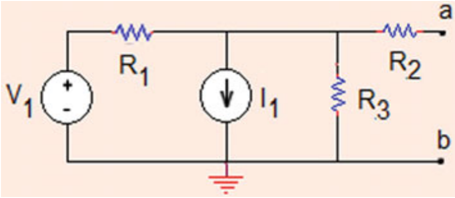



**Fig. 2.133** The circuit of Problem 2.5.16

(c) Determine the condition for $V_{ab} < 0$.
(d) Norton equivalent circuit parameters in (b) ?

## Solution

(a) De-activate all independent sources (see, Fig. 2.134):

$$R_{\mathrm{Th}} = R_2 + (R_1 // R_3) = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_3}.$$

Because node a is open, $R_2$ has no effect. By source transformation of the voltage source, following circuit is obtained. Then, applying KCL at node a (Fig. 2.135),

$$\frac{V_1}{R_1} - \frac{V_a}{R_1} - I_1 - \frac{V_a}{R_3} = 0 \quad \rightarrow \quad \frac{V_1}{R_1} - I_1 = V_a\left(\frac{1}{R_1} + \frac{1}{R_3}\right) \quad \rightarrow$$
$$V_a = \frac{\left(\frac{V_1}{R_1} - I_1\right)}{\left(\frac{1}{R_1} + \frac{1}{R_3}\right)} = V_{\mathrm{Th}}.$$

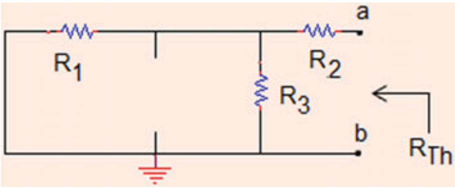



**Fig. 2.134** The circuit for the calculation of Thévenin resistance in Problem 2.5.16

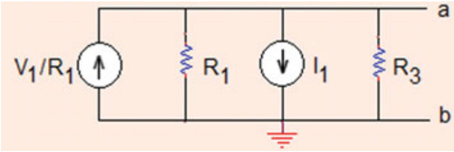



**Fig. 2.135** The circuit for the calculation of Thévenin voltage in Problem 2.5.16

(b) $R_1 = R_2 = R_3 = 1\ \Omega$, $V_1 = 2$ V, $I_1 = 1$ A, substituting the values,

$$R_{\text{Th}} = \frac{1+1+1}{1+1} = 1.5\,\Omega$$

$$V_{\text{Th}} = V_a = \frac{\frac{2}{1} - 1}{1+1} = 0.5\,\text{V}$$

(c) If $\frac{V_1}{R_1} < I_1, \quad V_{\text{Th}} = V_a < 0.$

(d) Norton equivalent circuit parameters are

$$I_{\text{Th}} = \frac{V_{\text{Th}}}{R_{\text{Th}}} = \frac{0.5}{1.5} = 0.333\,\text{A}$$

$$R_{\text{N}} = R_{\text{Th}} = 1.5\,\Omega.$$

**Problem 2.5.17**

(a) Find Thévenin equivalent circuit for the circuit shown in Fig. 2.136.
(b) Find the limiting value of $k$ if $R_1 = R_2 = 1\ \Omega$.
(c) Norton equivalent circuit?

**Solution**

(a) The circuit has no independent sources. Apply source transform to dependent source and 1 A current at terminals a, b (Fig. 2.137).

KCL at terminal (a), with $i = V_a/R_2$

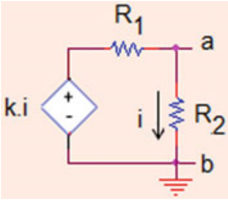



**Fig. 2.136** The circuit of Problem 2.5.17

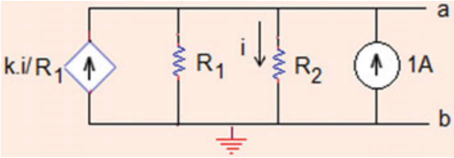



**Fig. 2.137** Source transformation of dependent source and application of 1 A current at terminals a, b of the circuit of Problem 2.5.17

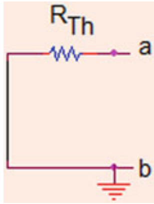



**Fig. 2.138** Thévenin's (=Norton's) equivalent of the circuit in Problem 2.5.17

$$\frac{kV_a}{R_1 \cdot R_2} + 1 - \frac{V_a}{R_1} - \frac{V_a}{R_2} = 0 \quad \rightarrow \quad V_a\left(\frac{k}{R_1R_2} - \frac{1}{R_1} - \frac{1}{R_2}\right) = -1$$

$$V_{\mathrm{Th}} = V_a = \frac{-1}{\dfrac{k}{R_1R_2} - \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)}$$

$$R_{\mathrm{Th}} = \frac{V_a}{1\,\mathrm{A}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} - \dfrac{k}{R_1R_2}}.$$

(b) Denominator of Thévenin resistance must be different than zero,

$$\frac{k}{R_1R_2} \neq \frac{1}{R_1} + \frac{1}{R_2}.$$

If $R_1 = R_2 = 1\ \Omega$, then $k \neq 2$

(c) Norton equivalent circuit = Thévenin equivalent circuit (Fig. 2.138).

**Problem 2.5.18** In the circuit shown in Fig. 2.139, determine the inequality condition on parameter $C$ in terms of known quantities of the circuit so that, $R_{ab} < 0\ \Omega$.

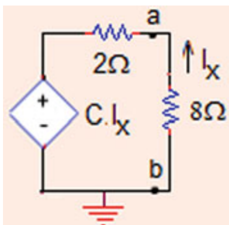
**Solution**

The circuit does not contain an independent source, therefore its Thévenin equivalent circuit has only a Thévenin resistance. We assign a current source at the output, and source transform-dependent source (Fig. 2.140a, b),

Nodal equation:

$$I_0 + I_x + \frac{C}{2}I_x - \frac{V_a}{2} = 0, \quad (V_b = 0\,\mathrm{V}).$$

**Fig. 2.139** The circuit of Problem 2.5.18



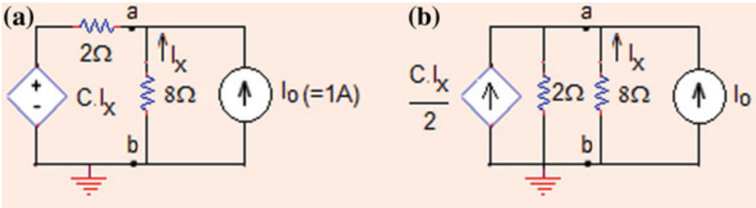



**Fig. 2.140** Assign a current source at the output, and source transform-dependent source

But,

$$V_a = -8I_x \quad \rightarrow \quad I_x = -\frac{V_a}{8}$$

$$I_0 - \frac{V_a}{8} + \frac{C}{2}\left(-\frac{V_a}{8}\right) - \frac{V_a}{2} = 0,$$

$$I_0 = \frac{CV_a}{16} + \frac{V_a}{8} + \frac{V_a}{2} = V_a\left(\frac{C}{16} + \frac{1}{8} + \frac{1}{2}\right) = V_a\left(\frac{C+10}{16}\right)$$

$$V_a = \frac{16I_0}{10+C}.$$

Thévenin equivalent of the circuit,

$$R_{\mathrm{Th}} = \frac{V_a}{I_0} = \frac{\left(\frac{16I_0}{10+C}\right)}{I_0} = \frac{16}{10+C}.$$

Therefore, for $R_{\mathrm{ab}} = R_{\mathrm{Th}} < 0$, $C < -10$.

**Problem 2.5.19** Find Thévenin equivalent circuit for the circuit shown in Fig. 2.141 ($Vs$ = 1 V, $f$ = 4, $R_1 = R_2 = R_3 = R_4 = 1\ \Omega$) (matrix_solve.xlsx).

**Solution**

$R_4$ has no influence since node a is open. $V_{\mathrm{Th}} = V_{\mathrm{oc}} = V_2$. Apply source transform to voltage source and write nodal equations at 1–2, with $i = V_2/R_3$ (see, Fig. 2.142)

$$\frac{V_s}{R_1} - f \cdot \frac{V_2}{R_3} - \frac{V_1}{R_1} - \frac{V_1 - V_2}{R_2} = 0 \tag{2.83}$$

$$f \cdot \frac{V_2}{R_3} + \frac{V_1 - V_2}{R_2} - \frac{V_2}{R_3} = 0. \tag{2.84}$$

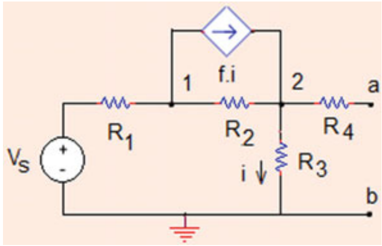



**Fig. 2.141** The circuit of Problem 2.5.19

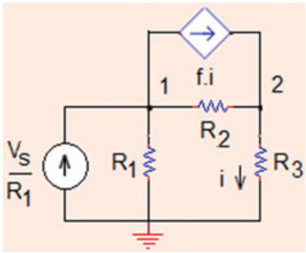



**Fig. 2.142** The circuit of Problem 2.5.19 after independent source transformation

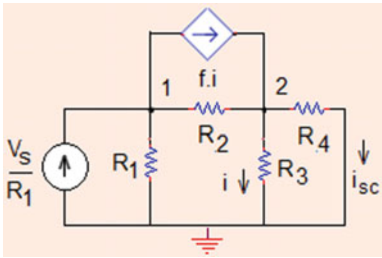



**Fig. 2.143** Determining short-circuit current

Substituting numerical values and rearranging equations,

$$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2.85}$$

Solving for $V_2$,

$$V_2 = V_{\mathrm{Th}} = -1\ \mathrm{V} = V_{\mathrm{oc}}$$

$$R_{\mathrm{Th}} = \frac{V_{\mathrm{oc}}}{I_{\mathrm{sc}}}.$$

KCL at nodes 1, 2; with $i = V_2/R_3$, (Fig. 2.143)

$$1 - 4V_2 - V_1 - (V_1 - V_2) = 0 \tag{2.86}$$

$$4V_2 + (V_1 - V_2) - V_2 - V_2 = 0 \tag{2.87}$$

Simplify (2.86) and (2.87),

$$2V_1 + 3V_2 = 1 \tag{2.89}$$

$$V_1 + V_2 = 0. \tag{2.89}$$

Solving these two simultaneous equations for $V_2$ yields

$$V_2 = 1\,\mathrm{V}, \quad I_{sc} = \frac{V_2}{R_4} = \frac{1}{1} = 1\,\mathrm{A}.$$

Therefore, Thévenin equivalent circuit consists of a negative resistor (Fig. 2.144), with

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = -\frac{1}{1} = -1\,\Omega.$$

**Problem 2.5.20** In the circuit shown in Fig. 2.145, determine the maximum power (in Watts) that can be transferred to load resistance $R_L$. Given that, when $R_L$ is removed from the circuit, the voltage at node 4 is measured as 4.25 V:

$$R_1 = R_5 = R_6 = R_7 = 1\,\Omega, \quad R_2 = R_3 = R_4 = 2\,\Omega, \quad I_1 = I_2/2 = 2\,\mathrm{A}.$$

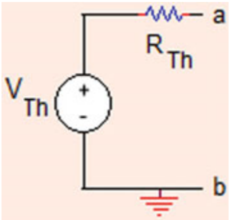



**Fig. 2.144** Thévenin equivalent circuit consists of a negative resistor

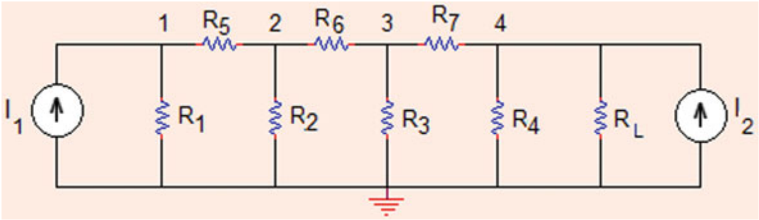



**Fig. 2.145** The circuit of Problem 2.5.21

**Solution**

$$P_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}} = \frac{4.25^2}{4 \times R_{\mathrm{Th}}}$$

$$R_{\mathrm{th}} = \{\{[(R_1 + R_5)\|R_2] + R_6\}\|R_3 + R_7\}\|R_4 = 1\,\Omega$$

$$P_{\max} = \frac{4.25^2}{4 \times 1} = \frac{18.0625}{4} = 4.516\,\mathrm{W}.$$

**Problem 2.5.21** In the circuit shown in Fig. 2.145, determine the maximum power transferred to $R_L$ :

$R_1 = R_5 = R_6 = R_7 = 1\,\Omega, R_2 = R_3 = R_4 = 2\Omega, i_1 = 2\,\mathrm{A}, i_2 = 4\,\mathrm{A}$ (matrix_-solve.xlsx).

**Solution**

$$R_{\mathrm{th}} = \{\{[(R_1 + R_5)\|R_2] + R_6\}\|R_3 + R_7\}\|R_4$$

$$R_{\mathrm{th}} = 1\,\Omega$$

$$V_{\mathrm{oc}} = V_4, \quad (R_L = \infty).$$

Using node voltages method, finding $V_4$ yields $\mathrm{Th} = V_{\mathrm{oc}} = V_4$.

$$GV = I$$

$$G = \begin{bmatrix} 1/R_1 + 1/R_5 & -1/R_5 & 0 & 0 \\ -1/R_5 & -1/R_5 + 1/R_2 + 1/R_6 & -1/R_6 & 0 \\ 0 & -1/R_6 & 1/R_3 + 1/R_6 + 1/R_7 & -1/R_7 \\ 0 & 0 & -1/R_7 & 1/R_4 + 1/R_7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2.5 & -1 & 0 \\ 0 & -1 & 2.5 & -1 \\ 0 & 0 & -1 & 1.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 4 \end{bmatrix}.$$

Solution of this matrix equation for $V_4$ yields, $V_4 = 4.25\,\mathrm{V}$

$$(V_1 = 1.844\,\mathrm{V}, V_2 = 1.588\,\mathrm{V}, V_3 = 2.375\,\mathrm{V}).$$

Condition for maximum power transfer to load resistor is $R_L = R_{\mathrm{Th}} = 1\,\Omega$. Maximum power transferred to load resistor,

$$P_{\max} = \frac{V_{\mathrm{Th}}^2}{4R_{\mathrm{Th}}} = \frac{4.25^2}{4 \times 1} = \frac{18.0625}{4} = 4.516\,\mathrm{W}.$$

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