Preface

Preface to the English Edition

Encouraged by the positive reception to our German textbook on mathematical modeling, we were motivated to create an English version of this book. Although there is a long tradition of English textbooks on mathematical modeling to the best of our knowledge, most of them differ from the one presented here, as our book is ordered according to a hierarchy of mathematical subjects with increasing complexity. We start from simple models given by (linear) equations and end up with complex mathematical models involving nonlinear partial differential equations with free boundaries. In this way, we offer a variety of mathematically more and more inclined subjects from which both elementary undergraduate and sophisticated graduate courses can be composed.[1] In this sense, we hope that our book fills a gap and will be of use also in the curricula for mathematical modeling in English-speaking universities.

We have taken the opportunity of the translation process to correct further misprints and small inconsistencies which have come to our knowledge.

This English version of this book would not have come into existence without support of the following people: Mrs. Eva Rütz, Mrs. Astrid Bigott, and Mrs. Cornelia Weber have created the TEX version on the basis of the first translation provided by the authors, as usual with enormous exactness, speediness, and dedication. This first draft has been considerably improved by Prof. Serge Kräutle and Dr. Kei Fong Lam, who worked through the whole text and removed various inconsistencies and mistakes not only in the usage of the English language but also concerning the mathematical contents. We thank Clemens Heine of Springer-Verlag for his continuous support and encouragement.

[1] See the preface to the German edition for more detailed suggestions for the use of this textbook.

It fills us with deep sadness that our scholar, colleague, and friend Christof Eck could not participate in this enterprise. Extremely untimely, just being 43 years of age, he passed away after a long illness. This volume is dedicated to his memory.

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July 2016

Preface to the First German Edition

With the notion *mathematical modeling*, we denote the description of phenomena from nature, technology, or economy by means of mathematical structures. The aim of modeling is the derivation of a meaningful mathematical formulation from which statements and solutions for the original problem can be derived. Here, in principle, every branch of mathematics is "applicable." In technological applications, quite often mathematical problems arise with such complexity that they have to be simplified by neglecting certain influences or they can only be solved with the help of numerical methods.

If one simplifies mathematical models, for example by neglecting certain "small" terms in an equation, then it is the task of mathematicians to check to which extent the behavior of solutions has been changed. Historically, a variety of mathematical concepts that have been developed were driven by the requirements of applications. Having this in mind, it is neither by chance nor astonishing that mathematics and real world "fit together." Mathematical concepts whose development has been triggered from a specific field of application often also proved to be usable in other fields of applications. Furthermore, for a specific problem from applications, there are in general a wide range of mathematical models feasible. The choice of the model certainly depends on the degree of detail in the (temporal or spatial) level of consideration.

Considering this, an education in mathematical modeling appears to be indispensable, if a study in mathematics is also supposed to contain professional qualifications. Often the classical courses of a mathematics curriculum can deliver such an education only insufficiently. Typical courses concentrate on the development of a mathematical theory, in which sample applications—if at all—only play the role of an "optional" motivation. Also, the study in a minor subject from an application field in general cannot close the gap because students in mathematics are often overburdened: On the one hand, they have to learn the results of an application field, and on the other hand, they have to extract mathematical structures from them to make mathematical knowledge useful. Against this background, specific courses in mathematical modeling have entered the curriculum of mathematical studies at various universities. In this context, the present textbook wants to be helpful.

On the one hand, this textbook presents knowledge from an area "between" mathematics and the sciences (e.g., from thermodynamics and from continuum mechanics) which students and lecturers of mathematics need in order to understand models for problems in the sciences and engineering and also to derive them. On the other hand, this book contains a variety of interesting, practically relevant examples for the mathematical theories often only experienced at an abstract level during the study of mathematics and thus answers the question often posed “what do I need this for?”. While it cannot substitute any of the textbooks dealing with the underlying mathematical structures such as linear systems of equations/linear algebra or ordinary or partial differential equations, it nevertheless contains essential aspects of the analysis of the models. One aim in particular is to illustrate the interactions between mathematics and applications, which unfortunately are often neglected in mathematics courses.

Furthermore, this textbook also addresses students from the sciences and from engineering and offers them an introduction into the methods of applied mathematics and mechanics.

The content of this book is restricted to deterministic models with continuous scales, as they are in the center of classical natural sciences and engineering. In particular, stochastic models are beyond the scope of this book, and the same applies to processes at very small scales for which particular models or models from quantum mechanics and its approximations are feasible. Also, models from economics are not in the focus of this book, as stochastic approaches play an important role there.

An essential concept of this book consists in using the mathematical structures (and the knowledge about them) as an ordering principle and not the different fields of applications. This reflects the strength of mathematics, lying in the fact that one concept can be used for totally different problem classes and fields of applications. It allows dealing with examples from different fields of applications efficiently without being forced to always repeat the same mathematical basic structures: This line will be followed in Chapter 1 and in Chapters 2, 4, 6, and 7. In this order, one finds embedded Chapters 3 (thermodynamics) and 5 (continuum mechanics). They provide the necessary links to the natural sciences and engineering. Of course, these chapters are also shaped by the application of mathematical tools.

The restriction of the subjects at the level of application corresponds to a restriction at a mathematical level: Throughout this book, we use knowledge from linear algebra and analysis intensively. Chapter 4 relies on the knowledge provided by courses in analysis or in ordinary differential equations, and Chapter 5 makes use in an essential way of the methods of the multidimensional differentiation and integration (integral theorems) and in this way from the more advanced aspects of analysis. In Chapter 7, the foundations of the geometry of curves and surfaces play an important role. It is impossible to define a clear delimitation to the analysis of partial differential equations. Knowledge from this field and also from linear functional analysis certainly is useful for Chapter 6, but not necessary.

A discussion of mathematical results about partial differential equations takes place in this chapter only insofar as there is a tight linkage to the model interpretation. Therefore, the presentation cannot be completely rigorous, but possible gaps and necessary consolidations are pointed out. In this way, this chapter does not necessarily require an intense study of the analysis of partial differential equations, but hopefully, it stimulates to do so. There is no chapter dedicated solely to optimization, but concepts of optimization are used in different chapters. This reflects that optimization problems in modeling often appear as an equivalent formulation of other mathematical problems when expressed in a variational formulation. We have totally excluded the treatment of numerical methods from this book, although they represent a central tool in the practical treatment of technical and scientific problems. We think that this can be justified as there is already a vast amount of excellent textbooks available.

The material of this book can be used in various ways for undergraduate and for graduate students, both at an elementary level and at a more advanced level. The simplest usages are two lecture courses of four hours each week, which cover the whole spectrum of this book. Here, the first part should be placed in the second half of undergraduate studies and the second part in the graduate studies. Alternatively, a course with two hours of lecture per week of introductory character is also possible at an early stage of graduate studies based on parts of Chapters 1, 2, and 4. This can be further complemented by another two-hour-per-week course consisting of parts of Chapters 5 and 6. If there is only room for one course, then it is also possible to build a course of four hours of lecture per week out of parts of Chapters 1, 2, 3, 4, 5, and some aspects of 6. Alternatively, Chapters 5, 6, and 7 can be used for a course about mathematical models and continuum mechanics or from the basic sections of Chapter 5 (derivation of the conservation equations) and Chapters 6 and 7, a course on "applied partial differential equation" can be extracted. Furthermore, this book can be used efficiently for self-study, at undergraduate, graduate, or postgraduate level. Postgraduates working with a mathematical model related to certain fields of application may find the necessary foundations here.

The aim of all the courses described can only be to close a certain gap to the application fields, and they cannot substitute the concrete realization of modeling projects. Intensive treatment of many exercises presented here is helpful, but finally, to our understanding, modeling can only be learned by modeling practice. A possible teaching concept as it has been used by the authors consists of problem seminars in which tasks from the applications are posed without any mathematical material, i.e., the development of feasible mathematical concepts is an essential part of the work. But we hope that courses based on this textbook provide an essential foundation for such a modeling practice.

Finally, this book seems to be also suitable for students of the natural sciences (physics, chemistry) and of engineering. For these students, some of the modeling aspects will be familiar from specific cases, but a rigorous inclusion in a mathematical methodology should lead to further insight.

Of course, we are not going to judge the existing textbooks on mathematical modeling. Certainly, there is a variety of excellent books, but in quantity much less than in other mathematical fields, and many textbooks restrict themselves to an elementary level, in particular also addressing high school students. A textbook which tries to cover the whole spectrum from elementary aspects to recent research is not known to us. Often, textbooks also follow different ordering principles. The present textbook originated from courses which were given by the second author at the University of Regensburg and by the first and third author at the University of Erlangen several times, and therefore, it is the result of a complex developing process. During this process, the authors received important support. The authors express their thanks to Bernd Ammann, Luise Blank, Wolfgang Dreyer, Michael Hinze, and Willi Merz for valuable suggestions. Sincere thanks are given to Barbara Niethammer, who together with the second author has lectured a course on mathematical modeling at the University of Bonn from which much material entered the present book. For careful proofreading, we thank Martin Butz, Daniel Depner, Günther Grün, Robert Haas, Simon Jörres, Fabian Klingbeil, David Kwak, Boris Nowak, Andre Oppitz, Alexander Prechtel, and Björn Stinner. In TEX writing, we were supported by Mrs. Silke Berghof and in particular by Mrs. Eva Rütz who typed a large part of the manuscript and worked on the numerous figures with strong dedication—to both our cordial thanks. We thank Serge Kräutle who provided the figure at the cover—a numerical simulation of the Kármán vortex street. We would also like to thank cordially Ulrich Weikard for providing Figure 6.14 and James D. Murray for providing Figure 6.10.

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