

Chapter 2

Theory

2.1 Definitions

The theory presented here comes mainly from Pedersen and Rasmussen (1990), Beiki and Pedersen (2010) and from our recent papers Kalvoda et al. (2013) and Klokočník et al. (2014, 2016). A short outline of the theory for a convenience of the reader (non-specialist in physical geodesy) follows.

The disturbing static gravitational potential outside the Earth masses in the spherical harmonic expansion is given by the well-known formula:

$$T(r, \phi, \lambda) = \frac{GM}{r} \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{R}{r}\right)^l \left(C'_{l,m} \cos m\lambda + S_{l,m} \sin m\lambda\right) P_{l,m}(\sin \phi) \quad (2.1)$$

where GM is a product of the universal gravitational constant and the mass of the Earth (also known from satellite analyses as the geocentric gravitational constant), r is the radial distance of an external point where T is computed, R is the radius of the Earth (which can be approximated by the semi-major axis of a reference ellipsoid), $P_{l,m}(\sin \phi)$ are the Legendre associated functions, l and m are the degree and order of the harmonic expansion, (ϕ, λ) are the geocentric latitude and longitude, $C'_{l,m}$ and $S_{l,m}$ are the **harmonic geopotential coefficients (Stokes parameters)**, fully normalized, $C'_{l,m} = C_{l,m} - C_{l,m}^{el}$, where $C_{l,m}^{el}$ belongs to the reference ellipsoid.

The spherical approximation of the **gravity anomaly** Δg (on free air) is computed

$$\Delta g = -\frac{\partial T}{\partial r} - 2\frac{T}{r} \quad (2.2)$$

or one can use the **gravity disturbance** (it is the same as Eq. 2.2 but without the second term, which is usually numerically small). The gravity anomalies/

disturbances are computed from measurements by gravimeters or derived from measurements performed by means of satellite altimetry.

Gravity gradient tensor $\mathbf{\Gamma}$ (the **Marussi tensor**) is a tensor of the second derivatives of the disturbing potential T of the particular gravitational field model. It is assumed that the model is known to a maximum degree L_{max} (see below some details about EGM 2008 and EIGEN 6C4). The second derivatives are expressed in the local north-oriented reference frame (x, y, z) , where z has the geocentric radial direction, x points to the north, and y is directed to the west (Pedersen and Rasmussen 1990):

$$\mathbf{\Gamma} = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 V}{\partial x^2} & \frac{\partial^2 V}{\partial x \partial y} & \frac{\partial^2 V}{\partial x \partial z} \\ \frac{\partial^2 V}{\partial y \partial x} & \frac{\partial^2 V}{\partial y^2} & \frac{\partial^2 V}{\partial y \partial z} \\ \frac{\partial^2 V}{\partial z \partial x} & \frac{\partial^2 V}{\partial z \partial y} & \frac{\partial^2 V}{\partial z^2} \end{bmatrix} \quad (2.3)$$

Outside of the source masses (in other words, in source-free region), $\mathbf{\Gamma}$ satisfies Laplace's differential equation; it means that the trace of the Marussi tensor is zero. Tensor $\mathbf{\Gamma}$ is symmetric and thus contains **just five** linearly independent components. These can conveniently be computed by means of the formulae in Hotine (1969).

In the local investigations, the tensor components are used to identify and to map the geological contact information, be it the edges of the source targets or the structural/stratigraphic contact information. The horizontal components help to identify the shape and the geological setting of a target body. The quantity T_{zz} is best suited for a target body detection; T_{zz} helps to define isopath/density relationships of a body mass with relation to its geological setting, see, e.g., Saad (2006).

Under any coordinate transformation, the Marussi tensor $\mathbf{\Gamma}$ preserves **three invariants**; here they are labelled I_0 , I_1 , and I_2 :

$$I_0 = \text{trace}(\mathbf{\Gamma}) = T_{xx} + T_{yy} + T_{zz}$$

and it is zero outside the masses of the studied body (known also as Laplace's equation). Then

$$I_1 = (T_{xx}T_{yy} + T_{yy}T_{zz} + T_{xx}T_{zz}) - (T_{xy}^2 + T_{yz}^2 + T_{xz}^2) = \sum_{\{i,j\} \in \{x,y,z\}} (T_{ii}T_{jj} - T_{ij}^2) \quad (2.4)$$

$$I_2 = \det(\mathbf{\Gamma}) = T_{xx}(T_{yy}T_{zz} - T_{yz}^2) + T_{xy}(T_{yz}T_{xz} - T_{xy}T_{zz}) + T_{xz}(T_{xy}T_{yz} - T_{xz}T_{yy}). \quad (2.5)$$

An invariant can be looked upon as a non-linear filter enhancing sources with big volumes (Pedersen and Rasmussen 1990). They discriminate major density anomalies into separate units (the reader will see many examples in Chap. 5).

Pedersen and Rasmussen (1990) showed that the ratio I of I_1 and I_2 defined as

$$0 \leq I = -\frac{(I_2/2)^2}{(I_1/3)^3} \leq 1 \quad (2.6)$$

lies always between zero and unity for any potential field. If the causative body is strictly 2D, then $I = 0$. Thus the ratio I can be an indicator of two-dimensionality. If $I = 0$ then we have the necessary but not a sufficient condition for two-dimensionality. When the causative body as seen from the observation point looks more and more 3D-like (say a volcano), then I increases and eventually approaches unity (Pedersen and Rasmussen 1990, p. 138).

The second order derivatives and the invariants provide evidence about the details of near-surface (not deep) structures. The Marussi tensor was already used locally (areas of a few kilometres) for petroleum, metal, diamond, groundwater etc. explorations [e.g., Saad (2006), Mataragio and Kieley (2009), Murphy and Dickinson (2009) and many others]. The full Marussi tensor is a rich source of information about the density anomalies providing useful details about the objects located closely to the Earth's surface. This extra information can be used by tensor imaging techniques to enhance target anomalies, as tested for local features (economic minerals, oil and gas deposits, fault location, etc.), see, e.g., Murphy and Dickinson (2009), Saad (2006).

The third derivatives has been studied by Šprlák et al. (2015), but the question is their practical use (noisy values) and also their physical meaning should be clarified. We tested T_{zzz} , but we do not work with it here.

The gradient tensor Γ contains information about strikes. Pedersen and Rasmussen (1990) defined the **strike angle** θ (also known as strike lineaments or strike direction) as follows:

$$\tan 2\theta_s = 2 \frac{T_{xy}(T_{xx} + T_{yy}) + T_{xz}T_{yz}}{T_{xx}^2 - T_{yy}^2 + T_{xz}^2 - T_{yz}^2} = 2 \frac{-T_{xy}T_{zz} + T_{xz}T_{yz}}{T_{xz}^2 - T_{yz}^2 + T_{zz}(T_{xx} - T_{yy})} \quad (2.7)$$

within a multiple of $\pi/2$. Provided that the ratio I (2.6) is small, the strike angle may indicate a dominant 2D structure. The strike angle indicates how the gradiometer measurements rotate within the main directions of the underground structures. In other words, if we were able to rotate with the structure in such a way that the elements of the first row and first column of Γ would be identically equal to zero, then we would reach the “correct” direction described by θ . For more details see Beiki and Pedersen (2010) or Murphy and Dickinson (2009).

To define the new quantity “**virtual deformation**” (Kalvoda et al. 2013; Klokočník and Kostecký 2014) (vd), an analogy with the tidal deformation was utilized; one can imagine directions of such a deformation due to “erosion” brought about solely by “gravity origin”. If there would be a tidal potential T , then the

horizontal shifts (deformations) would exist due to it and they could be expressed in the north-south direction (latitude direction) as

$$u_{\Phi} = l_S \frac{1}{g} \frac{\partial T}{\partial \phi} \quad (2.8)$$

and in the east-west direction (longitudinal direction) as

$$u_{\lambda} = l_S \frac{1}{g \cos \phi} \frac{\partial T}{\partial \lambda} \quad (2.9)$$

where g is the gravity acceleration 9.81 m s^{-2} , l_S is the elastic coefficient (Shida number) expressing the elastic properties of the Earth as a planet ($l_S = 0.08$), ϕ and λ are the geocentric coordinates (latitude and longitude) of a point P where we measure T ; the potential T is expressed in $[\text{m}^2 \text{ s}^{-2}]$. In our case, T is represented by Eqs. (2.1), (2.8) and (2.9). This mechanism is applied to a standard Earth gravitational model, but the real values of the Shida parameters l_S for the Earth's surface (for our purpose) are not known.

The apparatus of mechanics of continuum is applied to derive the main directions of the tension (Brdička et al. 2000). The tensor of (small) deformation $\mathbf{E}$ is defined as a gradient of the shift. It holds that

$$\mathbf{E} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{21} & \epsilon_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} \end{pmatrix}. \quad (2.10)$$

where x and y coordinates are a) equivalent to the spherical coordinates in a local coordinate system in the respective point or b) are planar coordinates in stereographic projection. The tensor $\mathbf{E}$ can be separated into two parts:

$$\mathbf{E} = \mathbf{e} + \mathbf{\Omega} = (\mathbf{e}_{ij}) + (\mathbf{\Omega}_{ij}) \quad (2.11)$$

where $\mathbf{e}$ is the symmetrical tensor and $\mathbf{\Omega}$ the anti-symmetrical tensor of deformation, respectively. The symmetrical tensor is:

$$\mathbf{e} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & (\epsilon_{12} + \epsilon_{21})/2 \\ (\epsilon_{12} + \epsilon_{21})/2 & \epsilon_{22} \end{pmatrix} \quad (2.12)$$

and the deformation parameters are:

$\Delta = e_{11} + e_{22}$ total dilatation

$\gamma_1 = e_{11} - e_{22}$ pure cut

$\gamma_2 = 2e_{12}$ technical cut

$\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$ total cut

$a = \frac{1}{2} (\Delta + \gamma)$ major semi-axis of ellipse of deformation

$b = \frac{1}{2} (\Delta - \gamma)$ minor semi-axis of ellipse of deformation

$\alpha = \frac{1}{2} \tan^{-1} (\gamma_2 / \gamma_1)$ direction of main axis of deformation.

Note that different branches use different terminology for the same or similar quantities.

To illustrate the virtual deformations vd , the semi-axes of deformation ellipse a and b are computed in their relative size. The values of l_s are not known, and, therefore, only the main directions of the vd (and not its amplitudes) can be shown.

The computations of all quantities (2.2)–(2.12) defined above, sometimes called “gravitational aspects”, shortly *aspects*, have been organized by software based on Holmes et al. (2006), or developed by Sebera et al. (2013), Bucha and Janák (2013), and by the authors of this atlas themselves. Various gravity field models and not only for the Earth were applied; here we present results only for Antarctica with the EIGEN 6C4 and RET 14 (see Sect. 3 about the data below). Our computations with EIGEN 6C4 cover the whole planet; the grid used over the whole world is 5 arcmin in latitude and longitude.

2.2 Comments on the Theory

To show how the various aspects (derivates, functions) of the disturbing geopotential, defined in the previous section, behave (decrease) with increasing distance from the source of the causative body (a density anomaly), it is assumed that the source body is a point mass, with a selected value of GM , located on the Earth surface; all masses of the Earth are concentrated in its centre. It is obvious from the definitions in Sect. 2.1 that in this simplified case, some aspects do not exist or are equal to zero. The remaining aspects are shown in Fig. 2.1; here the simplification leads to formulae for the invariants I_1 and I_2 , namely: $I_1 = -3 (GM)^2 r^{-6}$ and $I_2 = -2 (GM)^3 r^{-9}$. A different speed of decrease of the “gravitational signal” with increasing depth of the source mass body for various aspects can be seen. The slowest decrease is for the gravity anomaly (with r^2). The invariants decrease quickly (with r^6 or r^9). It tells us that aspects I_1 or I_2 best describe the density anomaly dominantly at the surface or in shallow depths under the surface while Δg and T_{zz} are related dominantly to deeper structures. Although all these aspects have the same “common

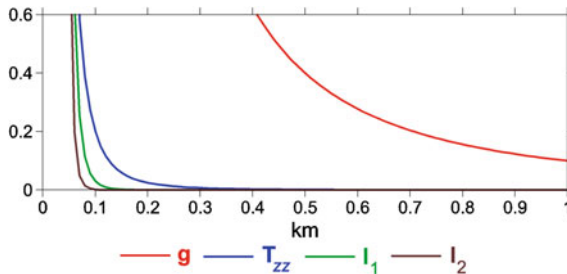


Fig. 2.1 An example of decrease in the values of the aspects of the disturbing gravitational potential with increasing distance (depth) from their source. On the x axis there are kilometres, on the y axis there is an arbitrary quantity (for a simple intercomparison). This is illustrative case for a point mass, representing a density anomaly, with a random value of GM

mother” (a recent gravity field model, Sect. 3.1), they have different properties (behaviour, sensitivity). Thus, the set of the gravitational aspects from Sect. 2.1 provides much more information about the density anomalies than only the traditional Δg , or Δg together with T_{zz} , can do.

A *systematic screening* of the whole planet Earth was performed. The results of the screening were presented by means of selected examples from regions with varying surface types, high mountain ranges, collision zones of oceanic and continental lithospheric plates, regional fault zones, volcanic chains and large impact craters in Kalvoda et al. (2013), Klokočník et al. (2010, 2014), or Klokočník and Kostecký (2014), www.asu.cas.cz/~jklokocn.

Geocentric and geodetic latitude. We need to know both types of the “latitude”, but all our maps contain geodetic latitude only. The difference between these two quantities is clear from Fig. 2.2; the geocentric latitude ϕ relates to a sphere, the geodetic latitude φ to a reference (rotational) ellipsoid. A relationship between the two latitudes is given by

$$\tan \phi = (1 - e^2) \tan \varphi,$$

where e is eccentricity of the used reference ellipsoid.

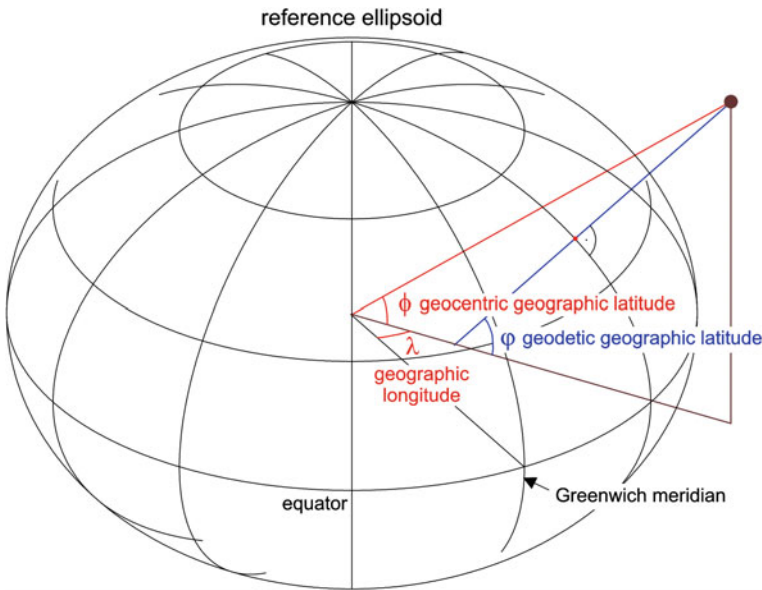


Fig. 2.2 Geocentric and geodetic latitudes

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