

Chapter 2

Simulation of Laser Cutting

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Abstract Laser cutting is a thermal separation process widely used in shaping and contour cutting applications. There are, however, gaps in understanding the dynamics of the process, especially issues related to cut quality. This work describes the advances in fundamental physical modelling and process monitoring of laser cutting, as well as time varying processes such as contour cutting. Diagnosis of ripple and dross formation is advanced to observe the melt flow and its separation simultaneously as well as the spatial shape of the cut kerf. The cutting process is described with a spatial three-dimensional Free Boundary Problem for the motion of one phase boundary. In such dissipative dynamical systems a finite dimensional *inertial manifold* exists which contains the attractor of the system. The existence of a finite dimensional inertial manifold means that the motion of a finite set of degrees of freedom can give a good approximation to the complete solution. Asymptotic methods are used to identify the degrees of freedom, and integral methods are applied to derive their equations of motion. Experimental findings about the morphology of ripple formation guide the modelling approach and motivate the investigation of what is known as the one phase problem. Solving inverse problems and the properties of the thermal boundary layers are discussed. The model reproduces details of the U-shaped ripples evolving at the cut surface. In discussion of what is known as the two phase problem the properties of the melt flow are included. The additional degrees of freedom are the melt film thickness, the mass flow and the temperature at the melt

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film surface. The onset of evaporation and the increase in capillary forces are the two physical phenomena relevant to the build-up of adherent dross. The dynamic model predicts a modulation frequency for the laser power that leads to almost complete suppression of adherent dross in contour cutting. Heat transport in thin film flow is investigated demonstrating how to control the error of reduced models by spectral methods. To find the properties of the gas flow leading to melt ejection is a fundamental task in cutting. The interaction of the gas flow with the condensed phase is mediated by two quantities, namely the pressure gradient and the shear stress along the liquid surface. Results of a detailed analysis of the momentum boundary layer of the gas flow is compared with numerical calculations using the Euler equations as well as the viscous effects described by the compressible Navier-Stokes equations. Deflection and separation of a supersonic gas jet emanating from a nozzle and propagating into the cut kerf is investigated using Schlieren photography and theoretical analysis. Looking for the different situations present in cutting and trepanning, the formation of horizontal structures in the ripple pattern in the cut is discussed. The effect of design and alignment parameters on nozzle performance in cutting are investigated and two dominant effects are discussed, namely the feedback of the gas flow into the nozzle and deflection of the gas flow away from the cutting front. Discussion of the onset of dross formation is extended to include compressible gas flow in the simulation such that the nozzle pressure enters the calculation of the processing domain for a dross-free cut.

2.1 Introduction

Laser processing dominates the market for industrial laser applications, with metal sheet cutting more important than metal sheet welding and marking, as well as other applications that are not yet widespread. Scientific research and development are related to monitoring and quality assurance. Today the development of the technology high-performance cutting machines, which is already well established, is moving towards a so-called autonomous machine, which is thought to incorporate cognition of the underlying physical mechanisms. Related fundamental research activities are focussed on the influence of fluctuations of the parameters and the resulting dynamical features of the process as well as scatter in cutting results. The dominant dynamical features of the process are ripple formation [1] and adherent dross [2, 3]—in contour cutting [4], processing at high cutting speeds, precision machining of microstructures [5, 6]. The applicability to high-quality cutting of diode pumped solid state lasers [7] and fiber lasers in the multi kW power range is also under investigation and researchers are exploiting their potential. The fundamental physical processes—also present in carving, welding and drilling—are related to the movement of free boundaries separated by the melt flow establishing the dynamical shape of the cutting front. Fundamental results are available for the physics of thin film flow [8–13], wetting [14, 15] and flow separation [16–19], as well as evaporation and condensation [20–24]. Specific investigations deal with the development and application of

mathematical methods for the analysis of partial differential equations (asymptotic methods [25–27], integral (variational) [28, 29] and spectral methods [30]) and Free Boundary Problems [29], which yield the formulation of approximate (asymptotically exact) models [25, 26, 31–33]. Numerical methods for tracking of free boundaries (Level Set method [34, 35], and adaptive sparse grids [36, 37]) are applied. The comparison of approximate models with advanced numerical simulations and experimental identification [4, 38] by diagnostic methods with temporal and spatial resolution is used to improve understanding and to determine the physical limits of cutting-machine performance.

Historically, investigations of fundamental experiments [39–42] and modelling are carried out to discover the different physical mechanisms contributing to the technical process. The physical foundations and their possible application to thermal processing are given by van Allmen [43]. In the monograph of Steen [44] the dominant physical processes and their technical relevance for laser applications are discussed. The main modelling activities up to 1994 are reviewed by Steen [45]. Comprehensive surveys [46] of the practical tasks for laser cutting are available.

The physical mechanisms dominant in cutting—such as the balance of the thermal energy heating up the material to be cut, the two phase transitions of melting and evaporation, the melt flow and its hydrodynamical stability [47] as well as the flow of the assist gas in cutting related to momentum and heat transfer [48] from the inert gas jet to the melt—are explored through experimental evidence and have been discussed in connection with the early models. The thin melt film is driven by a supersonic gas jet which is in contact with the melt at the free moving liquid surface. Depending on the position and type of the gas nozzle, as well as the shape of the cutting front, the gas pressure and the shear stress on the melt film can change, and the compressible flow can separate from the cut metal sheet. Performing numerical simulations of supersonic gas flow and using the Schlieren method [42, 49–52], characteristics of the complex gas jet such as supersonic domains, shocks, expansion waves and flow separation have become more well understood. Turbulent effects of the gas flow have been discussed [53]. The main interest then changed from the discussion of specific problems to the question of how to give a closed formulation of the process, and what are the additional effects introduced by the coupling of the physical processes. It became essential to distinguish between the independent parameters of the process such as cutting speed, laser power, intensity distribution etc., and quantities, which evolve during the process and have to be calculated as part of the solution, such as the slope of the cutting front, the melt film thickness, the surface temperature, the width of the cut kerf [54–58] etc. Additional effects of the reactive gas cutting process have been investigated [53, 59–61]. The oxygen and iron concentrations within the liquid oxide layer were investigated [62–64] and yield the reaction rate and hence the energy released by the exothermal reaction. By comparing the action of the oxygen gas jet in laser cutting and in autogenous gas cutting the high performance laser gas cutting process, referred to as burning stabilised gas cutting, was invented [65] and later applied in industry.

2.1.1 Physical Phenomena and Experimental Observation

Direct observation of the melt flow as well as the formation of dross and striations is observable during a special type of laser cutting referred to as trimming or trim-cutting. Temporally and spatially resolved diagnosis of ripple and dross formation during trim-cutting (Fig. 2.1a) was introduced by Zefferer [66]. Zefferer combined the observation of thermal radiation emitted from the cutting front and light from an external illumination source reflected by the cut surface. This technique was subsequently applied by other scientists to cutting [67] as well as welding [38]. Trimming of sheet metal means cutting along the edge of the work piece, such that the corresponding trim-cut evolves with a one-sided cutting front producing only one cut surface. To allow visual inspection during cutting and to avoid expansion of the gas jet the open side of the trim-cut is covered by a plate made out of transparent material. Two drawbacks remain present in the diagnosis of trim-cutting introduced by the transparent plate, namely the high temperature at the cutting front, which limits the proximity of the plate and the sensitivity of the gas flow to position and bending of the plate. There are plausible and interesting ideas how to improve the trim-cut technique, such as using materials with low melting temperature ($T_m = 96\text{ °C}$ Rose's alloy) to reduce the overall thermal load of the transparent plate [68].

The ideas of Zefferer [66] and Beersiek [38] and the first demonstrations were improved by Moalem [69] and further refinements today open the possibility of simultaneously observing thermal emission from the liquid surface, separating molten material and vapour flow as well as the shape of the moving front and thermal expansion of the solid on top of the sheet in the environment of the melting front during real processing (Fig. 2.1c). Observation in the coaxial direction with respect to the laser beam axis can be carried out by recording thermal emission from the cutting front (Fig. 2.1b), which gives detailed information about the dynamics of the surface temperature. As the main result, the signal amplitude of fluctuations depends on the cutting speed being more pronounced for low cutting speed. However, while recording the thermal emission only, it is hard to see features of the spatial shape of the cutting front such as the width of the cut at the top and bottom surface. More

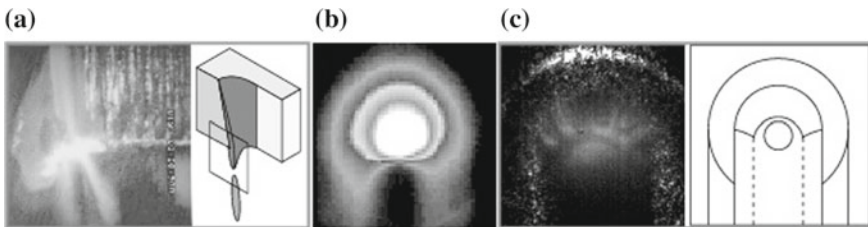


Fig. 2.1 Observation of ripple and dross formation. **a** Trim-cutting using in addition external illumination by a flash lamp [66]. Separation of the melt flow, droplet and dross formation as well as ripple formation. Observation of a real cut with an optical set-up coaxial with the laser beam axis. Without **(b)** and with **(c)** external illumination

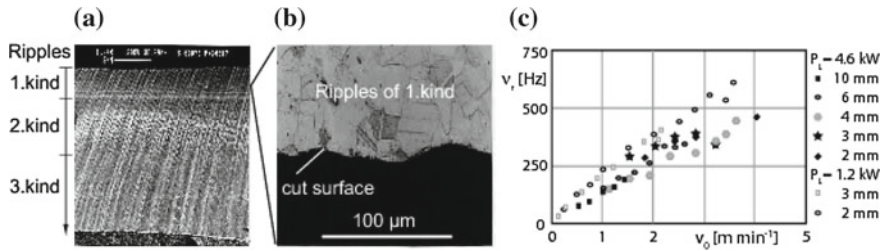


Fig. 2.2 Ripple morphology: **a** At least three different kinds of ripples can be distinguished. **b** For ripples of the first kind no resolidified melt can be observed and **c** the averaged ripple frequency increases linearly with cutting speed

detailed results are demonstrated by Moalem [69] combining the attenuated thermal signal with the spectral filtered reflection of an external light source (Fig. 2.1c). As shown in the sketch in Fig. 2.1c of the cutting front the different areas can be clearly distinguished in the camera recordings.

At least three different ripple patterns are observable, referred to as ripples of first, second, and third kind (Fig. 2.2a). They evolve subsequently with increasing sheet thickness and cutting speed. Ripples of the first kind appear at the top of the cut edge and up to feed rates of 4 m/min especially. Almost no resolidified material can be observed on metallurgical inspection of the cross section near the top of the cut edge (Fig. 2.2b). The averaged ripple frequency increases nearly linearly with respect to the cutting speed up to a value of about 500 Hz at 4 m/min (Fig. 2.2c). Ripples of the second kind are characterised by double the ripple frequency of first kind, and with increasing cutting speed their onset is shifted towards the top surface of the sheet metal. Ripples of the third kind are characterised by the dominance of resolidified molten material. To probe more deeply into the mechanisms leading to ripple formation requires the identification of the different types of ripple morphology and the relating of these findings to the processing parameters. Zefferer [66] compared the ripple frequency ν_r with the modulation frequency ν_{mod} and pointed out, that the visibility of ripples strongly depends on their shape. For this reason he introduced the notion of natural ripples and natural ripple frequency, which have been firmly established to be a consequence of natural fluctuations of the processing parameters or the temporally varying gas flow. U-shaped ripples with a sharp tip are clearly visible to the naked eye down to extremely small ripple amplitudes of about $<10 \mu\text{m}$. As result, the ripple frequency depends linearly on modulation frequency (Fig. 2.3a). However, directly observable ripples show this behaviour only in a narrow interval of modulation frequencies around the natural ripple frequency of a few hundred Hz observed for unmodulated laser radiation. Indeed the laser radiation fluctuates and induces ripple formation even for unmodulated power. In spite of that, any modulation frequency induces a movement of the melting front resulting in ripple formation at the cut surfaces. The observability of ripples at low modulation frequency suffers from their large wavelength as well as their smooth and wavy shape. At large modulation frequency the ripple amplitude goes down to a μm scale and a microscope is

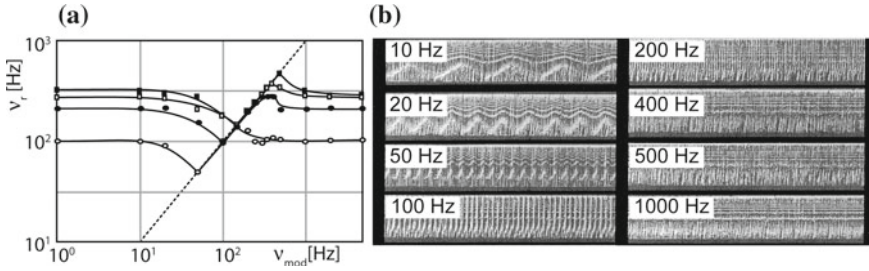


Fig. 2.3 Ripples of first kind: **a** The averaged ripple frequency ν_r changes with modulation frequency ν_{mod} of the laser power and the cutting speed v_0 ($v_0 = 1.5$ (filled square), 1.25 (square), 1.0 (filled circle), 0.5 (circle) m min⁻¹). **b** The ripple pattern for different values of the modulation frequency ν_{mod} ($v_0 = 1.25$ m min⁻¹). Laser power $P_L = 0.9 + 0.15 \cdot \sin(\nu_{mod}t)$ kW, sheet thickness $d = 2$ mm, stainless steel, gas pressure $P_k = 8 \cdot 10^5$ Pa.

necessary for observation. It is therefore misleading to ignore the strongly reduced visibility of ripples at high and low frequencies. The waviness of the cut surface as a consequence of low values for the modulation frequency is clearly visible at low modulation frequency (Fig. 2.3b). For high modulating frequencies the corresponding small ripple amplitude is found to be superimposed on natural ripples. These experimental findings are related especially to the movement of the melting front only, and therefore significantly guide the modelling approach. They motivate a closer look at the so-called one-phase problem (Fig. 2.4) and the detailed structure of the solution describing the thermal boundary layers. Subsequent iterative extension of the model by increasing the spatial dimension and investigating the meaningful properties of liquid and gaseous phases turns out to be more favourable than trying to solve an apparently comprehensive model. The most challenging task in the diagnostics of cutting is observation of the gas and vapour flow. The Schlieren method [70, 71] is a powerful technique for investigating the properties of the supersonic gas jet as it flows along the surfaces of a cold prepared kerf (model kerf). This model kerf is made out of three layers, substituting the shape of the cut front as well as the two cut surfaces. Interpreting the results from the Schlieren method requires transferring the data from a model flow along a cold, simplified (sharp edges, flat surfaces) set-up to the real cutting situation. Indeed the Schlieren results suffer from the effects of surface temperature, which strongly increase the temperature and momentum boundary layer thicknesses. The shape of the real cutting front and the interaction of gas and liquid flow introduce additional open questions.

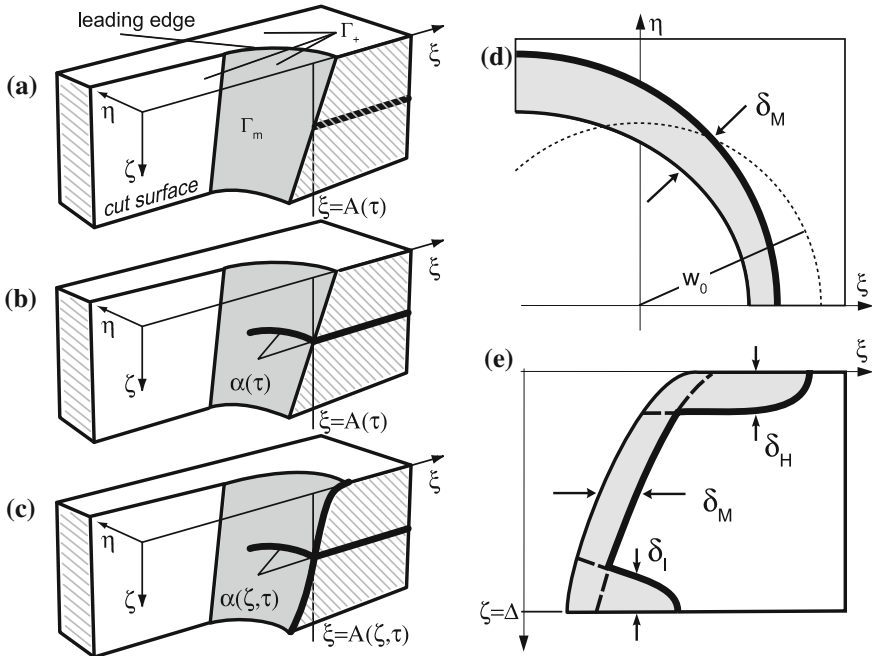


Fig. 2.4 Formulation of the one-phase Free Boundary Problem. The different length scales introduce a hierarchical structure of the solution: **a** spatially one-dimensional model where the front position $\xi = A(\tau)$ and heat content $Q(\tau)$ are the free parameters, **b** spatially two-dimensional model with curvature $\alpha(\tau)$ as additional free parameter, and **c** the axially resolved two-dimensional model, which reproduce the axial shape of the melting front. **d** Top view, lateral cross section with thermal boundary layer $\delta_M = \kappa/v_0$ and beam radius w_0 . **e** Side view, longitudinal section with thermal boundary layers δ_H due to p re-heating and δ_I due to thermal isolation. $x = \delta_M \xi$, $y = w_0 \eta$, $z = d \zeta$

2.2 Mathematical Formulation and Analysis

A typical property of dissipative infinite dimensional dynamical systems is the existence of a finite dimensional attractor [25], which is a subset of the phase space attracting all trajectories. From the theory of finite dimensional dynamical systems it is well known that, in systems with different time scales after the decay of the fast degrees of freedom, the dimension in phase space is reduced. By analogy, the analysis of some examples [26, 27, 31] of dissipative partial differential equations yields the result that the long time dynamic of such systems can also be concentrated in the neighbourhood of a manifold having a *finite* dimension. Such manifolds are called *inertial manifolds*. Obviously, the attractor is a part of the inertial manifold. In principle, the existence of a finite dimensional inertial manifold means that the motion of a finite set of degrees of freedom can give a good approximation to the complete solution. The contraction of phase space is also well known from systems of ordinary differential equations and the corresponding attractor is called the *central manifold*.

One example of such a behaviour is well known from the Boltzmann equation of kinetic theory. The solutions of this differential equation are known as distribution functions, describing the probability amplitudes for a particle in phase space. In the short time limit, which corresponds to non-equilibrium, these solutions have a rather complex structure. In the long time limit, which corresponds to quasi-equilibrium (equilibrium of flows), the solution is completely described by the equilibrium distribution. The equilibrium distribution is parameterised by a small number of averaged quantities like the thermodynamical temperature and the hydrodynamical drift velocity. The dynamics of these parameters are governed by the simpler hydrodynamical equations of motion. For this example the approximate dynamical system is given by the hydrodynamical approximation of the kinetic equations, where the approximating step is the assumption of the local thermodynamical equilibrium. The solution of the hydrodynamical equations of motion plays the role of the inertial manifold of the Boltzmann equation.

For many partial differential equations of continuum physics the existence of an inertial manifold has been proved and their dimension estimated. From these estimates we can expect a decrease of the dimension in phase space with the dissipation in the system.

In some cases the inertial manifold is explicitly constructed; see for example [72] for the case of the Chaffee-Infante reaction-diffusion equation. Historically, essential advances in the understanding of physical processes and analysis of the partial differential equations are established by the application of integral methods. Famous examples are the quantum mechanical treatment of the Helium atom by Hylleraas [73] applying variational methods and the formulation of the Bénard Problem by Saltzman [74] and Lorenz [75] using integral methods. It is shown that the motion of the complete system contracts on short time scales into a subspace of lower dimension. Hylleraas identified the three crucial characteristic dynamical variables for the description of the Helium atom, namely the orbital radii of the two electrons and their separation. The Lorenz system consists of three ordinary differential equations of first order with respect to time. The dynamics of these three characteristic dynamical variables or degrees of freedom only, are still able to reproduce the most important properties of the Bénard Problem.

Characteristic dynamical variables are encountered in long-time dynamics near the inertial manifold. The construction of equations of motion for the characteristic dynamical variables is also the crucial step for the testing and final identification of reduced models for Free Boundary Problems in thermal processing. The fundamental idea for deriving models with a reduced dimension in phase space is to find integral representations of the partial differential equations. The variational method introduced by Biot and Daughaday [28] allows the formulation of an integral equation which is equivalent to the Free Boundary Problem ((2.11)–(2.13), (2.34)–(2.35), (2.52)–(2.54)). The application of the variational method to the spatial one-dimensional Free Boundary Problem is given in [33]. The approximating step is the assumption of fast relaxation for the spatial distribution of the temperature, which is parameterised by slowly varying characteristic dynamical variables.

2.2.1 The One-Phase Problem

In cutting, the interaction of the gaseous, liquid and solid domains takes place only across spatially two-dimensional surfaces. The solid volume $\Omega(\tau)$ (Fig. 2.4) is bounded by the absorption front $\Gamma^+(\tau)$ and the bottom surface $\Gamma^-(\tau)$. A part of the absorption front is a free boundary called the melting front $\Gamma_m(\tau)$. Hence, adding the various effects of the gas- and melt flow only change the boundary values, while the structure of the model for the solid, called the one-phase model, remains unchanged. To find the properties of a comprehensive cutting model relies on the detailed analysis of the one-phase model, which was first formulated in 1997 by Enss et al. [32].

The phase boundary of the one-phase Free Boundary Problem can move into the material and erosion takes place or remains unchanged. Resolidification is not allowed. Mathematically the problem is formulated as follows (Fig. 2.4a). The material volume $\Omega(t) \subseteq R^2 \times (0, d)$ is bounded by the absorption front $\Gamma_+(t) = \partial\Omega(t) \cap \{z < d\}$ and the bottom surface of the solid material $\Gamma_-(t) = \partial\Omega(t) \cap \{z = d\}$. The intensity $I = I_0(t)f(|\mathbf{x} - \mathbf{v}_0 t|/w_0)$ of the laser beam is characterised by its maximum value $I_0(t)$, the spatial distribution f ($0 \leq f \leq 1$) and the laser beam radius w_0 . The laser beam is directed top to bottom with local direction $\mathbf{s} = \mathbf{s}(\mathbf{x}, t)$ ($|\mathbf{s}| = 1$) of the Poynting vector $\mathbf{S} = I_0 f \mathbf{s}$ and moves with the velocity v_0 in the positive x -direction, $\mathbf{v}_0 = v_0 \mathbf{e}_x$. The absorbed heat flux $q_a = -A_p \mathbf{n} \cdot \mathbf{S}$ depends on the degree of absorption A_p , where $\mathbf{n} = \mathbf{n}(\mathbf{x}, t)$ is normal with respect to the absorption front $\Gamma_+(t)$. The velocity $-v_p \mathbf{n}$ of the free boundary is therefore related to the degree of absorption $A_p = A_p(\mu)$, which depends on polarisation p and wavelength of the laser beam as well as the angle of incidence ϑ ($\mu = \cos(\vartheta)$). The normal component $v_p \geq 0$ of the velocity is restricted to non-negative values, since resolidification is not allowed. The absorption front $\Gamma_+(t)$ can be subdivided into two regions: the melting front $\Gamma_m(t)$, where the temperature equals the melting point $T = T_m$ and erosion takes place ($v_p \geq 0$), and the rest $\Gamma_+(t) \setminus \Gamma_m(t)$ ($T < T_m$ and $v_p = 0$). The melting front $\Gamma_m(t)$ is the *free boundary* of the one-phase problem. The resulting shape of the cut surfaces—where ripples are formed—is a result of the motion of the border line $\partial\Gamma_m(t)$, which is located at the leading edge and is a part of the cut surface. With these definitions the one-phase Free Boundary Problem has the following general form: Find the solution of the heat conduction equation

$$\frac{\partial T}{\partial t} = \kappa \Delta T, \quad T = T(\mathbf{x}, t), \quad \mathbf{x} \in \Omega(t), \quad (2.1)$$

subject to the free boundary conditions

$$q_a - \lambda \nabla T \cdot \mathbf{n} = \rho \mathbf{H}_m v_p, \quad \mathbf{x} \in \Gamma_m(t) \quad (2.2)$$

$$T = T_m, \quad \mathbf{x} \in \Gamma_m(t)$$

$$q_a - \lambda \nabla T \cdot \mathbf{n} = 0, \quad \mathbf{x} \in \Gamma_+(t) \setminus \Gamma_m(t) \quad (2.3)$$

$$\nabla T \cdot \mathbf{n} = 0, \quad \mathbf{x} \in \Gamma_-(t)$$

$$T|_{|\mathbf{x}| \rightarrow \infty} = T_a, \quad \mathbf{x} \in \Omega(t).$$

Here T_a and T_m are the ambient- and the melting temperatures. H_m is the melting enthalpy, λ is the heat conductivity, ρ the density, $\kappa = \lambda/(\rho c)$ the thermal diffusivity and c the specific heat capacity. In comparison with the well known Stefan-problem, as represented by Elliot et al. [29] or Fasano et al. [76] there is a different and more complicated situation involved in thermal erosion by a moving heat source,

- the energy transfer takes place directly at the free boundary,
- the heat flux absorbed at the free boundary depends on the angle of incidence and via the intensity distribution $f(\mathbf{x}, t)$ on the position and
- the boundary conditions are changing discontinuously at the boundary $\partial\Gamma_m(t)$ of the melting front.

There are typically three clearly different spatial scales involved in cutting.

$$\delta_M \ll w_0 \ll d \quad (2.4)$$

namely, the thickness $\delta_M = \kappa/v_0$ of a thermal boundary layer at the melting front Γ_m and the spatial scales of the laser beam. The shape of the isophote of the laser beam also introduces entirely different length scales. Typically, the beam radius $w_0 \ll z_R$ is small compared with the Rayleigh length z_R and these scales give the characteristic measures for the shape of the capillary in welding or the melting front in cutting. The scales

$$t = \frac{\delta_M^2}{\kappa} \tau, \quad x = \delta_M \xi_\delta, \quad y = w_0 \eta, \quad z = d \zeta, \quad \theta = \frac{T - T_a}{T_m - T_a} \quad (2.5)$$

are therefore characteristic for cutting. The non-dimensional form of the heat conduction equation (2.1) in a frame of reference moving with the laser beam axis at velocity v_0

$$\frac{\partial \theta}{\partial \tau} = \left[\frac{\partial \theta}{\partial \xi_\delta} + \frac{\partial^2 \theta}{\partial \xi_\delta^2} \right] + \left(\frac{\delta_M}{w_0} \right)^2 \frac{\partial^2 \theta}{\partial \eta^2} + \left(\frac{\delta_M}{d} \right)^2 \frac{\partial^2 \theta}{\partial \zeta^2}, \quad (2.6)$$

clearly demonstrates the importance of the different length scales. In particular, the spatially one-dimensional properties in the feed direction are dominant for fast movement of the beam axis (the square brackets in (2.6)) and the lateral term $\delta_M/w_0 \ll 1$ as well as the axial terms $\delta_M/d \ll 1$ only contribute small corrections. The smallness relation $\delta_M \ll w_0 \ll d$ of the length scales induce a set of three one-phase models hierarchically ordered with respect to the corrections introduced by higher spatial dimensions. As indicated in Fig. 2.4a–c the different magnitude of the contributions in (2.6) at the leading order gives the position $A(\tau)$ of the melting front with respect to the feeding direction (spatially one dimensional model), the width or radius $\alpha(\tau)$ of the melting front in the η -direction (spatially two dimensional model) and its axial shape as part of the solution of the spatially three-dimensional model. The ratio δ_M/w_0 of the boundary layer thickness and the beam radius is called the inverse Péclet-number Pe^{-1} ($\text{Pe} = w_0/\delta_M = w_0 v_0/\kappa$).

2.2.1.1 One-Dimensional One-Phase Model

The one-dimensional one-phase model is introduced in [33]. The idea by which the equations of motion for the melting front are derived and the fundamental properties of the solution are outlined. Consider the leading order of (2.6) with respect to the smallness relation $\delta/d \ll \delta/w_0 \ll 1$ the most fundamental limiting case of the boundary value problem can be extracted. The spatial one-dimensional problem

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial \theta}{\partial \xi_\delta} + \frac{\partial^2 \theta}{\partial \xi_\delta^2} \theta \Big|_{\xi_\delta^2=1} = 1, \quad \theta \Big|_{\xi_\delta^2 \rightarrow \infty} = 0 \quad (2.7)$$

has the general quasi-steady solution

$$\theta(\xi_\delta) = \exp[-\xi_\delta], \quad \xi_\delta = \frac{x}{\delta} = \text{Pe} \frac{x}{w_0} = \text{Pe} \xi. \quad (2.8)$$

As a result, there are three parameters well suited to describing the structure of the exact quasi-steady solution (2.8) for the temperature in advance of the melting front thus motivating the ansatz

$$\theta_{app}(\xi, \tau) = \theta_s(\tau) \exp \left[-\frac{\xi - A(\tau)}{Q(\tau)} \right], \quad \xi \geq A(\tau), \quad (2.9)$$

which is characterised by three time-dependent characteristic variables, the surface temperature $\theta_s(\tau)$, the position of the melting front $A(\tau)$ and the thermal penetration depth $Q(\tau)$. In the spatial one-dimensional problem the change of boundary conditions can take place with respect to time only, and spatial coexistence is not covered. During the melting phase the surface temperature $\theta_s(\tau) = 1$ equals the melting temperature. $Q(\tau)$ is the penetration depth of the temperature. For $Q(\tau) = 1/\text{Pe}$ the quasi-steady solution (2.8) is reproduced by (2.9) and the position $A(\tau) = \text{Pe} \tau$ of the melting front moves together with the beam axis at the cutting speed $\text{Pe} = (w_0/\kappa)v_0$. Of course, the spatially integrated temperature has the physical meaning of the energy content E ,

$$E(\tau) = \int_{A(\tau)}^{\infty} \theta_{app}(\xi, \tau) d\xi = \theta_s(\tau) Q(\tau); \quad (2.10)$$

the quantity $Q(\tau)$ during the melting phase ($\theta_s(\tau) = 1$) can also be interpreted as energy content. In the beginning of the material treatment or while the intensity is modulated, the dynamical system can remain in the heating phase. The surface temperature $\theta_s(\tau) < 1$ is smaller than the melting temperature and the position of the front $A(\tau) = \text{const.}$ remains unchanged.

A detailed derivation of the equations of motion for the characteristic dynamical variables $\theta_s(\tau)$, $A(\tau)$ and $Q(\tau)$ using a variational approach is given in [33]. The equations of motion reduce to the system of ordinary differential equations (ODE-system),

$$\begin{array}{ll} \text{heating phase } (\theta_s < 1) & \text{melting phase } (\theta_s = 1) \\ \dot{\theta}_s = \frac{1}{(1-b_1)Q}(\gamma f - b_1 \frac{\theta_s}{Q}) & \dot{\theta}_s = 0 \end{array} \quad (2.11)$$

$$\dot{A} = 0 \quad \dot{A} = \frac{1}{1+h_m-b_1}(\gamma f - b_1 \frac{1}{Q}) \quad (2.12)$$

$$\dot{Q} = \frac{1}{\theta_s}(\gamma f - \dot{\theta}_s Q) \quad \dot{Q} = \gamma f - (1+h_m)\dot{A}. \quad (2.13)$$

Here $\gamma f = \gamma(\tau)f(A(\tau) - \text{Pe } \tau)$ is the absorbed intensity and $b_1 = 3/5$.

The independent parameters of the problem are the Péclet-number Pe , the time-dependent maximum value $\gamma(\tau)$ of the intensity with distribution $f(x)$, and the inverse of the Stefan number h_m :

$$\text{Pe} = \frac{w_0 v_0}{\kappa}, \quad \gamma = \frac{A_p I_0}{\lambda(T_m - T_a)/w_0}, \quad h_m = \frac{H_m}{c(T_m - T_a)}. \quad (2.14)$$

The system (2.11)–(2.13) is an example of ordinary differential equations with a discontinuous right hand side [77]. The solution can switch between the heating phase and the melting phase (Fig. 2.5a) and is used to check the performance of numerical codes solving the corresponding partial differential equations

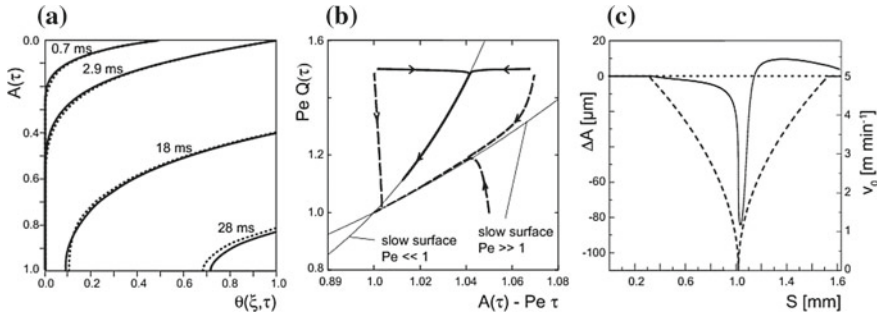


Fig. 2.5 **a** The solution of the ODE-system (2.11)–(2.13) (dotted) is reproduced well by the numerical solution (solid) of the corresponding PDE-system. A spatially one-dimensional domain is heated and ablated by the action of the absorbed laser radiation with intensity $\gamma(\tau)$. **b** Time scale separation depending on Péclet-number Pe changes the relaxation path. The solid lines correspond to the value $\text{Pe} = 1.5 \cdot 10^{-2}$; the dashed lines correspond to $\text{Pe} = 15$. **c** To compensate for a dynamical variation of the cutting speed $v_0(\tau)$ (dashed) the intensity $\gamma(\tau)$ can be controlled. Quasi-steady control (solid) leads to strong deviation ΔA of the front position, while the deviation is suppressed using the inverse solution of the ODE-system for the control (dotted)

(PDE-system). During the heating phase the surface temperature $\theta_s < 1$ is below the melting point and the front does not move $\dot{A} = 0$. In the melting phase the surface temperature $\theta_s = 1$ is equal to the melting point and the front moves $\dot{A} > 0$. As shown in Fig. 2.5a the initial heating phase terminates at $\tau_1 = 2.9$ ms while the surface at $A(\tau_1) = 0$ reaches the melting point $\theta(\xi = 0, \tau_1) = 1$. The system switches to the melting phase and part of the material is ablated $\dot{A} > 0$. At $\tau_l = 18$ ms thermal isolation at the bottom is already present.

2.2.1.2 Time Scale Separation

Consider the melting phase, where the front position $A(\tau)$ and the thickness $Q(\tau)$ of the thermal boundary layer relax towards the quasi-steady state $\{PeQ(\tau), A(\tau) - Pe\tau\} = \{1, 1\}$ (Fig. 2.5b). Time scale separation depending on Péclet-number Pe changes the relaxation path of the ODE system (2.11)–(2.13) towards the quasi-steady state $\{PeQ(\tau), A(\tau) - Pe\tau\} = \{1, 1\}$ of the melting phase. Relaxation of the fast variable towards the slow surface and subsequent relaxation of both fast and slow variables towards the quasi-steady state is characteristic for the melting phase. The corresponding slow surfaces are indicated in Fig. 2.5b. The time scales $t_\kappa = \kappa/v_0^2$ and $t_{v_0} = w_0/v_0$ are characteristic for heat conduction and movement of the melting front position, respectively. The ratio $t_{v_0}/t_\kappa = Pe$ of these time scales provides the meaning of the Péclet-number Pe for the dynamical properties of the solution. Analysis using the theorem of Vasil'eva, Gradshtejn and Tichonov [78] yields the following results:

- For $Pe \gg 1$ first the energy content relaxes fast towards the quasi-steady value, which corresponds to the actual value of the distance $A_1 = A(\tau) - Pe\tau$ between the melting front and the laser beam axis. Further development of the system takes place along the slow surface

$$(1 + h_m)\dot{A} = \gamma f, \quad Q = \frac{(1 + h_m)}{\gamma f}, \quad \gamma f = \gamma f(A(\tau) - Pe\tau) \quad (2.15)$$

in phase space.

- For $Pe \ll 1$ first the position $A(\tau)$ of the melting front relaxes fast to the quasi-steady value, which corresponds to the actual value of the energy content Q . Further development of the system takes place along the slow surface

$$\dot{Q} = b_1\left(\frac{1}{Q} - Pe\right), \quad \gamma f(A(\tau) - Pe\tau) = (1 + h_m - b_1)Pe + \frac{b_1}{Q}. \quad (2.16)$$

in phase space.

The slow surfaces intersect at the stable quasi-steady point.

Let the initial value $Q(\tau = 0) > Q_{stat}$ for the energy content be larger than the quasi steady value $Q_{stat} = 1/Pe$. The distance $A(\tau) - Pe\tau$ of the front with respect

to the laser beam axis then changes only slightly when $Pe \gg 1$. For small values $Pe \ll 1$ the front at first moves very rapidly out of the laser beam and further on approaches the quasi steady point together with its energy content, while moving slowly. For small values of the Péclet-number Pe , the position of the melting front responds more strongly to changes of the processing parameters than is the case for large values.

As result of quantitative analysis of the ODE-system (2.11)–(2.13) the Péclet-number has to be compared with the scaled gradient $G := 1/f(\partial f/\partial \xi) = O(1)$ of the intensity distribution $f(\xi)$. For a Gaussian distribution this value is of the order of unity. Moreover, the fast and slow time scales can be given explicitly:

$$Pe \gg \frac{|G|}{b_1} : \quad t_{fast} = \frac{1 + h_m - b_1}{1 + h_m} \frac{\kappa}{b_1 v_0^2}, \quad t_{slow} = \frac{w_0}{|G| v_0}, \quad (2.17)$$

$$Pe \ll \frac{|G|}{b_1} : \quad t_{fast} = \frac{1 + h_m - b_1}{1 + h_m} \frac{w_0}{|G| v_0}, \quad t_{slow} = \frac{\kappa}{b_1 v_0^2}. \quad (2.18)$$

These characteristic times are comparable with experimentally observed times for the changes of the thermal emission of the melting front. Inserting typical values for the distance $A(\tau) - Pe \tau = 1$ between the front position and the laser beam axis gives the corresponding value for the gradient $|G| = 4$. The value $w_0 = 150 \mu\text{m}$ for the laser beam radius, $v_0 = 5 \text{ cm s}^{-1}$ for the cutting speed and $\kappa = 6 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$ for the thermal diffusivity gives the values $Pe = 1.25$, $b_1 Pe/|G| = 0.19 \ll 1$. As result, the characteristic times $t_{fast} = 0.4 \text{ ms}$ for relaxation of the melting front and $t_{slow} = 4.0 \text{ ms}$ for the relaxation of the penetration depth are found.

2.2.1.3 An Inverse Problem

A control problem relevant to contour cutting or during the start of a cut can be posed. Using the reduced equations of motion (2.11)–(2.13) allows the solution of the inverse problem for the control of the processing parameters. Let $R_L(\tau)$ be the position of the laser beam axis, which is accelerated $\ddot{R}_L(\tau) \neq 0$ with respect to the material. An inverse problem can be formulated as follows. Find the power $\gamma(\tau)$ as a function of time, for which the distance $A_1 = A(\tau) - R_L(\tau)$ between the position of the front $A(\tau)$ and the laser beam axis $R_L(\tau)$ remains constant. Let $c(\tau)$

$$c(\tau) = \frac{\gamma(\tau)f(A_1)}{1 + h_m} - \dot{R}_L(\tau) \quad (2.19)$$

be the dynamical correction to the quasi steady control. Then the expression for the unknown power $\gamma(\tau)$ has the form

$$\gamma(\tau) = \frac{1 + h_m}{f(A_1)} (\dot{R}_L(\tau) + c(\tau)) \quad (2.20)$$

and the dynamical correction $c(\tau)$ is the solution of the initial value problem

$$\dot{c}(\tau) + b_1 c(\tau) [\dot{R}_L(\tau) + \frac{1 + h_m}{b_1} c(\tau)]^2 = \frac{b_1}{1 + h_m} \ddot{R}_L(\tau), \quad c(\tau = 0) = 0. \quad (2.21)$$

For uniform motion $\ddot{R}_L(\tau) = 0$, $\dot{R}_L(\tau) = \text{Pe} = \text{const}$, the correction $c(\tau) = 0$ is the unique solution and the power γ equals the quasi steady value $\gamma(\tau)f(A_1) = (1 + h_m) \text{Pe}$. Consider the deviation $\Delta A(\tau) = A_1 - [A(\tau) - R_L(\tau)]$ of the position of the front from its quasi steady value. The result for this deviation is shown in Fig. 2.5c (solid line) as a function of the cut length S for the quasi steady control $\gamma(\tau) \propto \dot{R}_L(\tau)$. With values of the acceleration $\ddot{R}_L \kappa / w_0 = 0.5 \text{ g}$ of cutting machines attainable today, the deviations reach values which are of the order of the laser beam radius $w_0 = 100 \mu\text{m}$.

2.2.1.4 Two-Dimensional One-Phase Model

The dynamics of the lateral extent of the cutting front (Fig. 2.4b, e) are described by equations of motion for the radius r_m of the melting front which may deviate from the beam radius w_0 . To derive the equations for the radius $\alpha = r_m / w_0$, the boundary problem for a moving cylindrical source, frequently cited in the literature related to welding and cutting applications, is considered:

$$0 = \text{Pe} \frac{\partial \theta}{\partial \xi} + \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2}, \quad \rho^2 = \xi^2 + \eta^2 \geq \alpha^2 \quad (2.22)$$

$$\theta|_{\xi^2 + \eta^2 = \alpha^2} = 1, \quad \theta|_{\xi^2 + \eta^2 \rightarrow \infty} = 0 \quad (2.23)$$

where the lengths are scaled with the beam radius w_0 . The solution can be given in explicit form and in plane polar coordinates $\{\rho, \varphi\}$ gives:

$$\theta(\rho, \varphi) = \exp[-\text{Pe} \rho \cos(\varphi)/2] S(\rho, \varphi), \quad \rho > \alpha \quad (2.24)$$

$$S(\rho, \varphi) = \theta_0 K_0(\text{Pe} \rho/2) + 2 \sum_{n=1}^{\infty} \theta_n K_n(\text{Pe} \rho/2) \cos(n\varphi),$$

$$\theta_n = \left[\frac{I_n(x)}{K_n(x)} \right]_{x=\text{Pe} \alpha/2}, \quad n = 0, 1, 2, \dots$$

Asymptotic analysis of the quasi-steady solution (2.24) with respect to the small parameter $\epsilon = 1/k$ gives an expression for temperature at the vertex line $\{\rho, \varphi = 0\}$

$$\theta(\rho, \varphi = 0) \sim \frac{\exp[-2k(\rho/\alpha - 1)]}{\sqrt{2}\sqrt{\rho/\alpha - \beta_0}}, \quad \beta_0 = \frac{1}{2}, \quad k = \frac{\text{Pe} \alpha}{2} \rightarrow \infty. \quad (2.25)$$

which indicates a lateral correction of square root type to the exponential dependence (2.28) which is well known from the spatial one-dimensional solution. The correction takes into account the additional heat flux necessary to reach the melting point on a boundary curve with radius α . Moreover, there is a shift β_0 with respect to the centre of the cylindrical axis with an asymptotic value of $\beta_0 = 1/2$. The asymptotic analysis suggest an approximate ansatz $\theta_{app}(\rho, \varphi, \tau)$ for the description of the time-dependent, two-dimensional melting front:

$$\theta_{app}(\rho, \varphi, \tau) = \left(\sqrt{\frac{\rho/\alpha - \beta}{1 - \beta}} \right)^{-1} \exp\left[-\frac{1 + \cos(\varphi)}{2} \frac{\rho/\alpha - 1}{Q/\alpha}\right] \quad (2.26)$$

where the shift $\beta = \beta(Q/\alpha)$ depends on the ratio $Q/\alpha = \delta_M/r_m$ of the thickness δ_M of the thermal boundary layer and radius r_m of the cutting front at the vertex line $\varphi = 0$

$$\beta = \beta(x) = x \exp[x] E_1[x], \quad x = Q/\alpha, \quad Q = Q(\tau), \quad \alpha = \alpha(\tau); \quad (2.27)$$

the parameters Q, α of the quasi-steady solution are identified as characteristic dynamical variables $Q(\tau)$ and $\alpha(\tau)$. The spatially one-dimensional solution is reproduced by the limiting case for large values of the radius α of the melting front:

$$\lim_{\alpha \rightarrow \infty} \theta_{app}(\nu + \alpha, \varphi = 0, \tau) = \lim_{\alpha \rightarrow \infty} \frac{\exp[-\frac{\nu}{Q}]}{\sqrt{1 + \frac{\nu/\alpha}{1 - \beta[Q/\alpha]}}} = \exp[-\frac{\nu}{Q}] \quad (2.28)$$

where the abbreviation

$$\nu = \rho - \alpha, \quad \rho/\alpha = 1 + \nu/\alpha. \quad (2.29)$$

for the distance ν from the boundary is introduced. Since the displacement β is non-negative $0 \leq \beta \leq 1/2$ and remains bounded, the temperature θ_{1D} is an upper bound $\theta_{app} \leq \theta_{1D}$ for the temperature in advance of the curved melting front.

The ansatz (2.26) is used to derive the equations of motion for the two-dimensional one-phase model of the melting front by using a variational approach. To apply the variational approach as discussed in [33] means to find the so called thermal potential $\mathbf{h}$ by solving the equations

$$\theta = -\nabla \cdot \mathbf{h}, \quad \nabla \times \mathbf{h} = \mathbf{0} \quad (2.30)$$

The expansion with respect to φ at the vertex line $\varphi = 0$ of the melting front

$$\theta(\rho, \varphi, \tau) = \theta_0(\rho, \tau) + \theta_2(\rho, \tau) \frac{\varphi^2}{2} + \dots$$

$$h_\rho(\rho, \varphi, \tau) = h_{\rho 0}(\rho, \tau) + h_{\rho 2}(\rho, \tau) \frac{\varphi^2}{2} + \dots \quad h_\varphi(\rho, \varphi, \tau) = h_{\varphi 1}(\rho, \tau) \varphi + \dots \quad (2.31)$$

gives a system of equations with the radius ρ as an independent variable

$$\begin{aligned} 0 &= \frac{\partial}{\rho \partial \rho} h_{\varphi 1} - \frac{h_{\rho 2}}{\rho} \\ -\theta_0 &= \frac{\partial}{\rho \partial \rho} (\rho h_{\rho 0}) + \frac{h_{\varphi 1}}{\rho} \\ -\theta_2 &= \frac{\partial}{\rho \partial \rho} (\rho h_{\rho 2}) \end{aligned}$$

for the calculation of the Taylor coefficients in the expansion (2.31). Inserting the ansatz (2.26) into the formal solution

$$h_{\rho 0}(\rho, \tau) = \frac{1}{\rho} \int_\rho^\infty \theta_0(\rho', \tau) \rho' d\rho' + \frac{1}{\rho} \int_\rho^\infty h_{\varphi 1}(\rho', \tau) d\rho' \quad (2.32)$$

$$h_{\varphi 1}(\rho, \tau) = -\frac{1}{\rho} \int_\rho^\infty h_{\rho 2}(\rho', \tau) d\rho', \quad h_{\rho 2}(\rho, \tau) = \frac{1}{\rho} \int_\rho^\infty \theta_2(\rho', \tau) \rho' d\rho' \quad (2.33)$$

and direct integration yields an explicit expression for the thermal potential $\mathbf{h}$. Applying the variational calculus discussed in [33] the final result for the equations of motion reads:

$$\dot{\alpha} = \frac{1}{1 + h_m} \left[(\gamma f_2 - \frac{b_2}{Q}) + \frac{1 + h_m + b_2}{1 + h_m - b_1} (\gamma f_0 - \frac{b_1}{Q}) \right], \quad (2.34)$$

$$\dot{A} = \frac{1}{1 + h_m - b_1} (\gamma f_0 - \frac{b_1}{Q}), \quad \dot{Q} = b_1 \left(\frac{1}{Q} - \dot{A} \right). \quad (2.35)$$

where $b_1 = 3/5$ and $b_2 = 1/10$. As a result, the dynamics of the spatially one-dimensional dynamical variables $\{A, Q\}$ is decoupled from the motion of the additional two-dimensional variable α . It is worth discussing the limiting case of large values of the Péclet-number. Owing to time scale separation as shown in Fig. 2.5b, the dynamical variables move on the slow surface: $(1 + h_m)\dot{A} = \gamma f$, $Q = (1 + h_m)/\gamma f$. Additionally, inserting a Gaussian intensity distribution $f(\mathbf{x}) = \exp(-2\mathbf{x}^2)$ in the intensity γf the Taylor coefficients f_0, f_2 are

$$f = f_0 + f_2 \frac{\varphi^2}{2}, \quad f_0 = \exp(-2\bar{A}^2), \quad f_2 = -4\alpha(\alpha - \bar{A})f_0. \quad (2.36)$$

The distance $\bar{A} = A - R_L$ between the position of the front $\xi = A(\tau)$ and the moving laser beam axis $\xi = R_L(\tau)$ has been introduced. Finally, in the limiting case $\text{Pe} \rightarrow \infty$ the lateral equations of motion

$$\dot{\alpha} = \frac{\gamma f_0}{1 + h_m} [-4\alpha(\alpha - \bar{A}) + 1], \quad \bar{A} = A - R_L \quad (2.37)$$

$$(1 + h_m)\dot{A} = \gamma f_0, \quad Q = \frac{1}{\dot{A}}, \quad (2.38)$$

are solved explicitly. The stable ($\alpha \geq 0$) quasi-steady ($\dot{A} = \text{Pe}$, $\dot{\alpha} = 0$) solution reads

$$\alpha = \frac{1}{2}\bar{A} + \frac{1}{2}\sqrt{\bar{A}^2 + 1}, \quad \gamma f_0(\bar{A}) = (1 + h_m)\text{Pe}, \quad Q = \frac{1}{\text{Pe}} \quad (2.39)$$

For the quasi steady state the radius $\alpha > \bar{A}$ is larger than the distance $\bar{A}$ between the melting front and the laser beam axis. Since the lines of constant values for the intensity are circles, the radius of curvature of the isophote at the vertex line equals the distance $\bar{A}$. Comparison with numerical results shows that the radius α of curvature at the vertex line is already closely connected to the width of the cutting kerf.

The laser beam axis and the feeding direction span the xz -plane ($y = 0$ or $\eta = 0$). The intersection of the xz -plane and the cutting front gives a curve referred to as the ceiling line, or centre line of the cutting front. The dynamics of $\alpha(\tau)$ is closely related to the formation of ripples at the cut edge, if the radius of curvature $\alpha(\tau)$ at the ceiling line $\phi = 0$ is accepted as an approximation to the shape of the cutting front. The boundary $\partial\Gamma_m(t)$ is defined by the spatial coexistence of the boundary conditions (2.34) and (2.35) for the melting phase, where the melting front moves $v_p > 0$, and the heating phase, where the cut edge evolves ($v_p = 0$). For the dynamical system, the non-dimensional velocity v_p of the boundary is $v_p = (\dot{A} - \dot{\alpha}) \cos(\phi) + \dot{\alpha}$ which determines the azimuth angle $\phi = \bar{\phi}(\tau)$ where the boundary conditions are changing, namely $v_p(\bar{\phi}) = 0$,

$$\cos(\bar{\phi}) = \frac{-\dot{\alpha}}{\dot{A} - \dot{\alpha}}. \quad (2.40)$$

The stationary solution ($\dot{A} = \text{Pe}$, $\dot{\alpha} = 0$) shows that $\bar{\phi}_{stat} = \pm\pi/2$ and any deviation from the stationary state leads to a dynamical response of the arc length $\alpha(\tau)2\bar{\phi}(\tau)$ of the melting front. In particular, the arc length can change faster than the cutting speed resulting in U-shaped ripples evolving at the cut surface.

2.2.1.5 Three-Dimensional One-Phase Model

The Free Boundary Problem for a single phase describing the axial motion of the melting front (Fig. 2.4) in the non-dimensional variables $\xi = x/w_0$, $\zeta = z/d$ reads

$$\begin{aligned}
\frac{\partial \theta}{\partial \tau} &= \frac{\partial^2 \theta}{\partial \xi^2} + \left(\frac{w_0}{d}\right)^2 \frac{\partial^2 \theta}{\partial \zeta'^2}, \\
\theta|_{\xi=a(\zeta')} &= 1, \\
\gamma(\tau)\mu A(\mu)f(a(\zeta) - \text{Pe}\tau) + \left[\frac{\partial \theta}{\partial \xi} - \left(\frac{w_0}{d}\right)^2 \frac{\partial \theta}{\partial \zeta'} \frac{\partial a}{\partial \zeta'}\right] &= -h_m \frac{\partial a}{\partial \tau}, \\
\frac{\partial \theta}{\partial \zeta'} \Big|_{\substack{\zeta'=0 \\ \xi \geq a(0)}} &= -\left(\frac{d}{w_0}\right)^2 \gamma A(1)f(\xi)
\end{aligned}$$

The asymptotic analysis ($w_0 \ll d$) of this problem shows that outside the boundary layers at the top δ_H and bottom δ_I of the metal sheet the terms containing $(w_0/d)^2 \ll 1$ can be neglected. Therefore the axial dynamics of the melting front are determined dominantly by spatial one-dimensional heat conduction in the feed-direction [32, 33]. Within the upper boundary layer of thickness δ_H the temperature field is determined by preheating of the laser beam (Fig. 2.4e).

An approximate solution for $(w_0/d)^2 \ll 1$ can be derived by solving the preheating problem separately, which gives the initial value for the position $\xi = A(\zeta, \tau)$ of the axially distributed melting front. The interaction of the boundary layers δ_H and δ_M can be calculated by matching the thermal boundary layers, in fact.

2.2.1.6 Thermal Boundary Layers

There are three different thermal boundary layers involved in cutting (Fig. 2.4e). Ahead of the laser beam, heating (Neumann type) of the material surface takes place resulting in a thermal boundary layer of thickness $\delta_H = \text{Pe}^{-1/2}$. The position where the surface temperature reaches the melting point is called the leading edge of the cutting front. The cut evolves at the leading edge. The cutting front is moving and so there is a second thermal boundary layer (Dirichlet type) with thickness $\delta_M = \text{Pe}^{-1}$. At the lower surface $\zeta = \Delta$ the heat transition to ambient conditions is fairly small compared to the flux inside the metal; this state of near thermal isolation is the reason for the third thermal boundary layer whose thickness is δ_I . It is worth mentioning that the cutting model includes different types of boundary conditions at the top surface (Neumann type) and the melting front (Dirichlet type). Moreover, at the leading edge $\{\xi, \zeta\} = \{A(\zeta = 0), 0\}$ the different boundary conditions coexisting spatially and interact. At the lower edge $\zeta = \Delta$ of the cutting front there are also two coexisting Neumann conditions with very different boundary values, namely the heat flux, which moves the melting front, coexists with the relatively small-valued thermal radiation condition; the latter may even be best represented by thermal isolation giving a homogeneous Neumann condition. In consequence a third boundary layer δ_I exists. The different thickness $\delta_H, \delta_M, \delta_I$ of the thermal boundary layers tend to support three-dimensional effects for the heat transport. In contrast to heat transport with fixed boundaries, the solution of free boundary problems is sensitive to the effects of coexisting boundary conditions. In this case, the position $A(\zeta = 0)$ of the leading

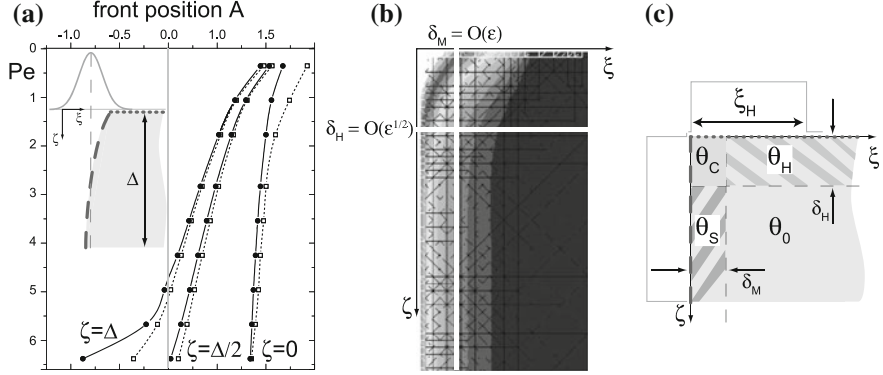


Fig. 2.6 Interaction of different thermal boundary layers with thickness δ_M, δ_H at the leading edge $\{\xi, \zeta\} = \{A(\zeta), 0\}$ and the lower edge $\zeta = \Delta$ of the cutting front depending on position $\zeta \in \{0, \Delta\}$ and Péclet-number Pe . **a** The front position $\xi = A(\zeta)$ calculated without (dotted, reduced model) and with (solid, numerical model) interaction of the boundary layers. **b** The numerical solution for the temperature (gray scale) reproduces the different scales $\delta_M = O(\epsilon)$ and $\delta_H = O(\epsilon^{1/2})$ which are derived from singular perturbation analysis for moderate values of the Péclet-number ($Pe = 1$). **c** Singular perturbation analysis gives the temperature as the sum of three contributions $\theta_H, \theta_M, \theta_C$ related to pre-heating at the surface $\zeta = 0$, the heat flux at the melting front $\xi = A(\zeta)$ and a correction taking into account the interaction of the thermal boundary layers δ_H and δ_M

edge is an initial value for the quasi-steady shape of the melting front $A(\zeta)$, and its shape at the lower surface $\zeta = \Delta$ influences stability of the cut as well as wetting and separation of the melt flow.

The effects of three-dimensional heat transport at the leading edge $\{\xi, \zeta\} = \{A(\zeta = 0), 0\}$ and the lower edge $\zeta = \Delta$ of the cutting front depend on position $\zeta \in \{0, \Delta\}$ and on the Péclet-number Pe (Fig. 2.6a). In general, thermal boundary layers are singular perturbations of the heat conduction equation and are therefore hard to analyse by numerical methods. To find the structure of the solution near the leading edge, to be able to evaluate the accuracy of numerical results and to identify an ansatz for the reduced model, the asymptotic solution has to be analysed. Introducing the spatial scales $x = w_0 \xi, z = w_0 \zeta$ and the smallness parameter ϵ

$$\epsilon = \frac{1}{Pe}, \quad Pe = \frac{w_0 v_0}{\kappa} \quad (2.41)$$

the quasi-steady model problem (Fig. 2.6c) can be restated as follows. Find the solution of the heat conduction equation

$$\frac{\partial \theta}{\partial \xi} + \epsilon \frac{\partial^2 \theta}{\partial \xi^2} + \epsilon \frac{\partial^2 \theta}{\partial \eta^2} = 0 \quad (2.42)$$

subject to the boundary conditions (Fig. 2.6c)

$$\frac{\partial \theta}{\partial \eta} \Big|_{\eta=0} = \begin{cases} -Q_H & \text{falls } 0 < \xi < \xi_H, \quad Q_H > 0 \\ 0 & \text{const} \end{cases} \quad (2.43)$$

$$\frac{\partial \theta}{\partial \xi} \Big|_{\xi=0} = -Q_M, \quad Q_M > 0 \quad (2.44)$$

$$\frac{\partial \theta}{\partial \eta} \Big|_{\eta=\Delta} = 0 \quad (2.45)$$

$$\theta(\xi, \eta) \Big|_{\xi \rightarrow \infty} = \theta_0. \quad (2.46)$$

In the singular limiting case $\epsilon \rightarrow 0$ the appropriate new scales δ_M and δ_H given by the thickness of the corresponding boundary layers

$$\hat{\xi} = \xi / \delta_M, \quad \hat{\eta} = \eta / \delta_H, \quad \delta_M = \epsilon, \quad \delta_H = \sqrt{\epsilon} \quad (2.47)$$

are found. The solution emerges as the superposition

$$\theta(\xi, \eta) = \theta_0 + \theta_M \left(\hat{\xi} = \frac{\xi}{\delta_M} \right) + \theta_H \left(\xi, \hat{\eta} = \frac{\eta}{\delta_H} \right) + \theta_C \left(\hat{\xi}, \hat{\eta} \right) + O(\epsilon) \quad (2.48)$$

of the outer solution θ_0 , which is determined by the boundary condition (2.46) and three contributions $\theta_M, \theta_H, \theta_C$ related to pre-heating at the surface $\zeta = 0$, the heat flux at the moving melting front $\xi = A(\zeta)$ and a correction θ_C which describes the interaction of the thermal boundary layers δ_H and δ_M near the leading edge $\{\xi, \zeta\} = \{A(\zeta = 0), 0\}$. As a result, the explicit solution reads:

$$\theta_M \left(\hat{\xi} \right) = \epsilon Q_M \exp(-\hat{\xi}) \quad (2.49)$$

$$\theta_H \left(\xi, \hat{\eta} \right) = \epsilon^{1/2} Q_H \left[\frac{2\sqrt{\xi_H - \xi}}{\sqrt{\pi} \exp\left(\frac{\hat{\eta}^2}{4\sqrt{\xi_H - \xi}}\right)} - \hat{\eta} \operatorname{erfc}\left(\frac{\hat{\eta}^2}{2\sqrt{\xi_H - \xi}}\right) \right] \quad (2.50)$$

$$\theta_K \left(\hat{\xi}, \hat{\eta} \right) = -\epsilon^{3/2} \pi^{-1/2} Q_H \exp\left(-\frac{\hat{\eta}^2}{4}\right) \exp(-\hat{\xi}) \quad (2.51)$$

The numerical solution for the temperature (Fig. 2.6b, gray scale) reproduces the different scales $\delta_M = O(\epsilon)$ and $\delta_H = O(\epsilon^{1/2})$ which are derived from singular perturbation analysis. It is worth mentioning that the asymptotic values remain significant and give a suitable approximation even for moderate values of the Péclet number ($Pe = 1$). As result of the singular perturbation analysis, numerical values for the spatial resolution in numerical calculations can be given, which are needed to resolve the thermal boundary layers.

The effects of the thermal boundary layer in the lateral direction are pronounced in the case of contour cutting. The lateral shape of the evolving cut kerf is relevant to the cut quality in connection with the evenness of the cut surface, and for melt flow

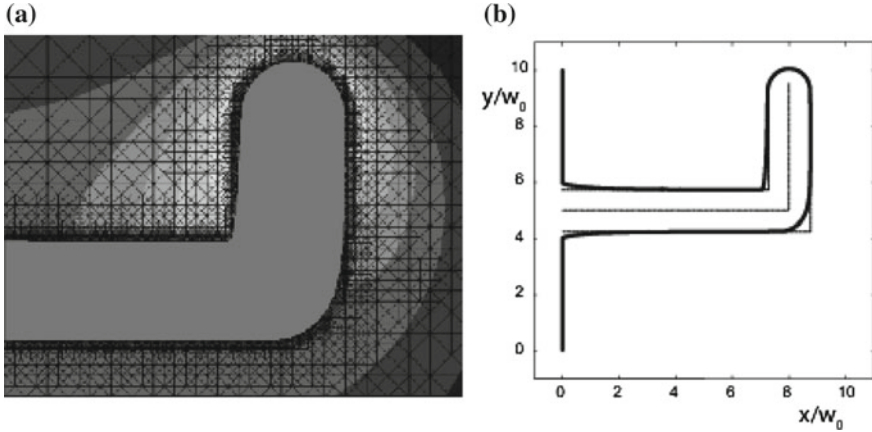


Fig. 2.7 **a** Numerical calculation of temperature, motion of the cutting front and development of the cut surface. Detail at the corner of the cut. The intensity is considered to be constant and no ripple formation occurs. **b** At the start of the cut and near the corner, the temperature and thermal boundary layers and also the cut shape (*solid*) deviate from the ideal shape (*dotted*). The trace of the laser beam axis is indicated as a *dashed line*

which changes with the formation of the gas flow pattern. Numerical simulation (Fig. 2.7) of the Free Boundary Problem is carried out using a modified Level-Set method [34, 35] for the motion of the melting front and adaptive methods on hierarchically ordered and sparse grids [37] for accurate approximation of the temperature distribution within the boundary layers. Deviations from the ideal shape of the cut kerf are limited by the extent of the thermal boundary layers. The results indicate that at the start of the cut and near the corner, deviations of the boundary layer thickness from the ideal values are present, which can be calculated in detail by taking into account the local curvature.

2.2.2 The Two-Phase Problem

Approximate solutions of the variational equation describing long-time behaviour are found. The corresponding finite dimensional dynamical system describes the movement of time-dependent characteristic parameters, such as the front position A , front radius α , surface temperature θ_s and energy content Q . The extent of the model for the melt flow—one example of a sequel-process—can be treated in a similar way, by describing the dynamics of integrated quantities like the melt film thickness h and mass flow m . The mass flow m is the melt velocity integrated with respect to the melt film thickness. The onset of evaporation at the irradiated metal surface is a further example of a sequel-process. Quality degradation such as scorings in oxygen cutting,

unevenness in contoured fusion cutting and ripple formation from resolidifying melt are a direct consequence of melt flow.

The suitable dynamical variables are the melt film thickness $h(\tau)$, the mass flow $m(\tau)$ and the temperature $\theta_s(\tau)$ at the melt film surface. These variables are time-dependent parameters of the spatial distributions of the velocity and the temperature in the melt. The dynamics on shorter time scales governing relaxation to the equilibrium distributions can be described by spectral methods. Using the integral representation of the balance equations for mass, momentum, and energy the equations of motion for the dynamical variables $h(\tau)$, $m(\tau)$, $\theta_s(\tau)$ read

$$\text{Re} \left(\frac{\partial m}{\partial \tau_l} + \frac{6}{5} \frac{\partial}{\partial \zeta} \left[\frac{m^2}{h} \right] \right) = -\frac{\partial \Pi_g}{\partial \zeta} h + 3 \left(\frac{\Sigma_g}{2} - \frac{m}{h^2} \right), \quad \text{momentum} \quad (2.52)$$

$$\frac{\partial h}{\partial \tau_l} + \frac{\partial m}{\partial \zeta} = v_p, \quad \text{mass} \quad (2.53)$$

$$\frac{1}{2} \frac{\partial}{\partial \tau_l} \left[\theta_s h g_1 \right] + \frac{5}{8} \frac{\partial}{\partial \zeta} \left[\theta_s m g_2 \right] = \frac{1}{Pe_l} \left[q_a - \frac{\theta_s}{h} g_3 \right] \quad \text{energy} \quad (2.54)$$

$$m|_{\zeta=0} = h|_{\zeta=0} = \theta_s|_{\zeta=0} = 0. \quad \text{initial values} \quad (2.55)$$

The dynamical system (2.52)–(2.55) for the melt flow is coupled to the gaseous and the solid phases by the pressure gradient $\partial \Pi_g / \partial \zeta$ as well as by the shear stress Σ_g exerted by the gas flow and by friction at the melting front, respectively. The cutting problem is extended by the dynamical system for the motion of the melting front (2.34)–(2.35). Inserting the time dependent characteristic parameters into the ansatz for the spatial distribution allows the reconstruction of the three-dimensional representation of the movement in phase space. Two examples, namely the formation of ripples and adherent dross, will be used to demonstrate that the analysis in phase space—spanned by integral quantities—is more transparent than the interpretation of spatially three dimensional results from direct numerical integration.

The formation of ripples is a characteristic dynamical feature of the cutting process. Taking the melt flow into account (2.52)–(2.55) an additional property of the cutting process—the formation of ripples of the second kind—becomes part of the solution [4]. Ripples of the first kind, namely U-shaped ripples evolving at the cut surface (Fig. 2.8a), are reproduced by the dynamical system as a result of the

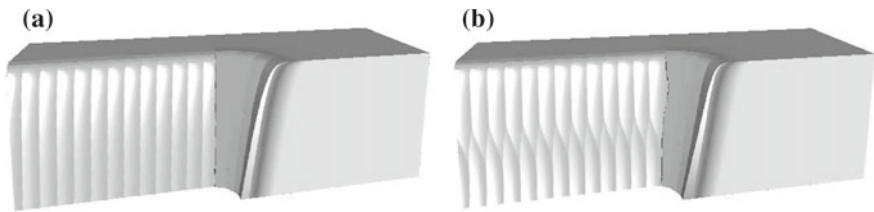


Fig. 2.8 Ripples of the first kind (a) and second kind (b) are reproduced by the dynamical system

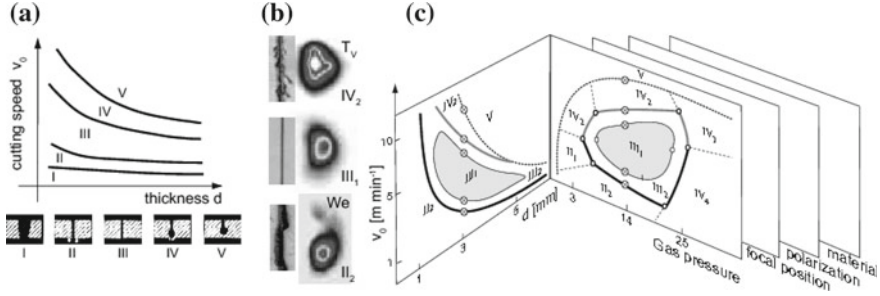


Fig. 2.9 Processing domains related to cut quality. Quality classes introduced by Arata [39] for oxygen cutting (a) are applicable to fusion cutting (b, c). There are at least two different kinds of dross observable, which are related to droplet formation by capillary forces (We , solid black) or by the onset of evaporation (T_v , solid gray). The maximum cutting speeds (dotted) are limited by material thickness and laser power

moving melting front (2.40). In the upper part of the sheet the influence of the melt flow on the movement of the melting front is negligible, since the heat contained in the melt film, its thickness and the mass flow of molten material, remains fairly small. Heat transport by convection in the melt film is involved when ripples of the second kind are formed (Fig. 2.8b). Any ripple of the first kind causes a melt wave propagating in the axial direction. The subsequent heat transport—convection in the axial direction—leads to an additional and delayed motion of the melting front by a heat wave (2.74) moving the melting front. As a result, the ripple frequency is doubled. Ripples of the third kind are a consequence of resolidified melt at the cut surfaces. Introduction of quality classes by Arata [39] is applicable to fusion cutting (Fig. 2.9a, class II, III, IV). The quality classes are related to the dynamical processing domains and can be identified with the underlying physical mechanisms. Three main processing domains in fusion cutting (Fig. 2.9b) are characterised by different morphology of the adherent dross and become observable by online measurement of the thermal emission with a CCD-camera [4]. The ratio of the stagnation pressure ρu_m^2 of the melt flow at the bottom of the sheet metal and the counteracting capillary pressure σ/d_m is called the Weber number We

$$We = \frac{\rho u_m^2}{\sigma/d_m} \quad (2.56)$$

where ρ , u_m , σ and d_m are the liquid density, velocity of melt, surface tension and thickness of the melt film, respectively. If the capillary forces become comparable with the inertia of the melt $We \approx 1$ then the separation of melt flow becomes unstable. The tendency to the formation of dross is estimated from the solution (Fig. 2.9c solid black curve, We) and occurs for low cutting speeds. The dross shows a circular shape and adheres on one side of the cutting kerf. The thermal emission recorded with the camera images shows low signal levels, strongly fluctuating and asymmetric

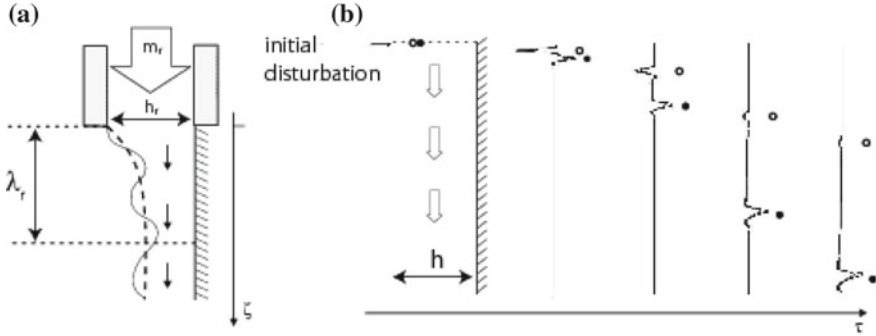


Fig. 2.10 **a** Water tap model. **b** Solution for the propagation of an initial perturbation from the linear stability analysis (*dots*) compared with the numerical solution (*solid*). The hollow λ_- and solid circles λ_+ give the time dependent position of the travelling melt wave from linear stability analysis

with respect to the feeding direction. The interpretation for low values of the Weber number is that the CCD-image shows an asymmetric position of the single melt thread (Fig. 2.9b, We) and there is dross at one side of the kerf, called dross of kind 1. For increased cutting speeds the melt film starts to evaporate depending on cutting speed v_0 and thickness d of the sheet metal as the model predicts (Fig. 2.9c solid gray curve, T_V). With the onset of the evaporation pressure the CCD-image shows a symmetric splitting of the melt thread and dross at both sides of the kerf appears, referred to as dross of kind 2. Finally the maximum cutting speed achievable (Fig. 2.9c dotted curve) is given by energy balance. The correlation between the processing domains II (We) and IV (T_V), and the characteristics of the CCD-images and the formation of the different kinds of dross is striking. The equations of motion (2.52) and (2.53) for mass flow $m = m(\zeta, \tau)$ and melt film thickness $h = h(\zeta, \tau)$ are analysed with respect to stability of the travelling melt waves (Fig. 2.10). A model problem—we call it the water tap model—is considered; it describes the outflow from an orifice with diameter h_r and mass flow m_r down a rigid wall in the ζ -direction, $\zeta \in [0, \infty)$. The flow is accelerated by a shear stress Σ at the free liquid surface and a no-slip condition is imposed at the rigid wall. The equations of motion for the water tap read:

$$\text{Re} \left(\frac{\partial m}{\partial \tau} + \frac{6}{5} \frac{\partial}{\partial \zeta} \left[\frac{m^2}{h} \right] \right) = \frac{3}{2} \Sigma - 3 \frac{m}{h^2} \quad (2.57)$$

$$\frac{\partial h}{\partial \tau} + \frac{\partial m}{\partial \zeta} = 0 \quad (2.58)$$

$$h(\zeta, \tau) \Big|_{\zeta=0} = h_r, \quad m(\zeta, \tau) \Big|_{\zeta=0} = m_r(\tau) \quad (2.59)$$

$$h(\zeta, \tau) \Big|_{\tau=0} = h_r, \quad m(\zeta, \tau) \Big|_{\tau=0} = m_r(\tau) \Big|_{\tau=0} \quad (2.60)$$

In the long time limit the thickness $h(\zeta, \tau)$ of the liquid undergoes relaxation towards the value h_∞ on the relaxation length λ_r . Linear stability analysis shows that there are two solutions with different velocities of propagation $\lambda^{(\pm)}$ which are related to fanned and damped disturbances of the liquid waves. The (2.57) and (2.58)

$$u(\zeta, \tau) = \begin{pmatrix} m(\zeta, \tau) \\ h(\zeta, \tau) \end{pmatrix} = u_0(\zeta) + u_1(\zeta, \tau) \quad (2.61)$$

are linearised with respect to the deviation u_1 from the quasi-steady solution u_0

$$\frac{\partial u_1}{\partial \tau} + A(u_0) \frac{\partial u_1}{\partial \zeta} = B(u_0) u_1. \quad (2.62)$$

The eigenvalues $\lambda^{(\pm)}$ and eigenvectors $v_{(\pm)}$ of the matrix $A(u_0)$ are given by

$$\lambda^{(\pm)} = \frac{1}{5} \frac{m_0}{h_0} \left(6 \pm \sqrt{6} \right) > 0, \quad v_{(\pm)} = \begin{pmatrix} \lambda^{(\pm)} \\ 1 \end{pmatrix}, \quad (2.63)$$

the velocities and the direction of propagation for the perturbation u_1 , respectively. The dispersion relation for harmonic perturbations with respect to a constant melt film thickness with frequency ω and complex valued wave-number $k \in \mathbb{C}$ is

$$k^2 - \frac{k}{\lambda^{(+)}} \left[\omega \left(1 + \frac{1}{c} \right) - i \left(\tilde{b}_1 - \frac{\tilde{b}_2}{c} \right) \right] = -\frac{1}{\lambda^{(+)}} \left[\frac{\omega^2}{\lambda^{(-)}} - i \frac{\omega}{\lambda^{(-)}} (\tilde{b}_1 - \tilde{b}_2) \right], \quad (2.64)$$

$$c = \frac{6 - \sqrt{6}}{6 + \sqrt{6}}, \quad \tilde{b}_{1,2} = \frac{1}{\text{Re}} \frac{3}{5} \frac{m_0}{h_0^3} \left(4 \mp \sqrt{6} \right).$$

The solution shows growing $\text{Im}(k_1) > 0$ and damped $\text{Im}(k_2) < 0$ contributions. With increasing Reynolds number Re the absolute value $|k|$ of the solutions decays and the length scale for amplitude growth as well as for damping increases. As a result, the modulation frequency of the laser power producing one single wave crest of melt moving down the cutting front can be estimated by the ratio of velocity of propagation for the growing wave amplitude and the sheet thickness. This kind of power modulation is used to avoid the formation of adherent dross in contour cutting.

The capillary forces become dominant for low cutting speeds and for cw operation. The melt flow does not separate completely from the sheet metal. Low cutting speeds are encountered in contour cutting and therefore a reliable control strategy is of interest. For this task it is crucial to differentiate between the cutting speed v_0 and the velocity $v_p(t)$ of the cutting front. From the results about the capillary forces (Fig. 2.9) the formation of adherent dross is related to small values of the Weber number $\text{We} \propto u_m^2 d_m$, which has to be kept at values larger than unity. Modulation of the laser power results in a correspondingly modulated velocity $v_p(t)$ of the melting front

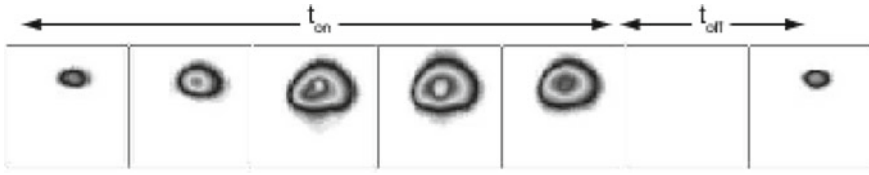


Fig. 2.11 The cutting process can evolve periodically using modulated laser power. The recorded CCD-images in this case show the typical properties of high quality cuts of class III known from CW operation (cutting speed 1.2 m/min, thickness 3 mm, N_2 14 bar, nozzle diameter 1.4 mm, maximum CO2 laser power 2.6 kW/pulsed, duty cycle 70%, pulse frequency 450 Hz, beam radius 150 μ m, Rayleigh length 2.2 mm, focal position -2.4 mm, stainless steel (1.4301), CCD camera frame rate 2.9 kHz)

and hence the time dependent mass flow $u_m d_m = v_p(t)d \gg v_0 d$ can be significantly larger than its value $v_0 d$ averaged with respect to the time of the modulation period. The modulation parameters, namely pulse duration and the duty cycle, can be chosen such that only one wave of ejected melt is produced during each laser pulse. The laser pulse duration t_{on} has to be matched to the typical time d/u_m for the propagation of a melt wave propagating down the melting front with sheet thickness d . Additionally, the cycle time $T = t_{on} + t_{off}$ has to meet the time averaged energy balance, such that the cut length $\int v_p(t')dt' = v_0 T$ matches the movement of the laser beam axis with velocity v_0 . Using these constraints, the parameters for the pulse mode operation, namely pulse duration and duty cycle, are determined. Figure 2.11 gives an example for a cut at a low cutting speed $v_0 = 1.2$ m/min performed with modulated laser power which remains dross-free. The CCD-images show that the melt thread remains at the centre of the cutting front as for cw-cutting and parameters corresponding to quality class III in Fig. 2.9.

2.2.2.1 Heat Transport in Thin Film Flow

Taking into account the liquid flow in cutting requires extending the one-phase problem by introducing the heat transport through the melt film. To investigate the dominant effects of heat transport in thin film flow the heat transport through a liquid layer with an established Couette-flow profile is a suitable model problem. The convective heat transport in thin film flow and the accuracy of equations for the boundary layer dynamics reduced by spatial integration, are two fundamental aspects which can be discussed in detail. The corrections to the spatially integrated equations of motion are found by applying a spectral method, which can be applied in a similar way to more detailed formulations of the problem. Consider a viscous, incompressible fluid flowing on an infinitely extended horizontal plane under the influence of a constant shear force, created by a homogeneous gas flow along the surface. We assume that the gas pressure along the surface is constant. The momentum balance of the Navier-Stokes equations

$$\text{Re}(u_t + uu_z + vu_x) = -p_z + u_{zz} + u_{xx}, \quad (2.65)$$

$$\text{Re}(v_t + uv_z + vv_x) = -p_x + v_{zz} + v_{xx} \quad (2.66)$$

with no-slip boundary conditions at the rigid wall $x = 0$ and continuity boundary condition at the free surface $x = h(z, t)$

$$u = v = 0, \quad x = 0, \quad h_t + uh_z = v, \quad x = h(z, t) \quad (2.67)$$

has Couette flow as the well known stationary solution [79, 80]:

$$v(x, z) = 0, \quad u(x, z) = x, \quad h(z) = 1. \quad (2.68)$$

The heat conduction in the melt film [81] is described by the partial differential equation

$$\frac{\partial \theta}{\partial t} + \text{Pe } u \frac{\partial \theta}{\partial z} + \text{Pe } v \frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial x^2} \quad (2.69)$$

with boundary conditions

$$\theta|_{x=0} = 0, \quad \frac{\partial \theta}{\partial x} \Big|_{x=h} = \gamma(t)f(z). \quad (2.70)$$

Here Re , Pe denote the Reynolds and Péclet-number respectively. We consider the instantaneous solution $\theta_0(x, z, t) = \gamma(t)f(z)x$ of the (2.69). The function θ_0 satisfies the (2.69) for $\text{Pe} = 0$ in the stationary case and the boundary conditions (2.70). Looking for a solution of the form $\theta(x, z, t) = \theta_0(x, z, t) + \theta_1(x, z, t)$, the function θ_1 satisfies the inhomogeneous equation with homogeneous boundary conditions:

$$\frac{\partial \theta_1}{\partial t} + \text{Pe } u \frac{\partial \theta_1}{\partial z} - \frac{\partial^2 \theta_1}{\partial x^2} = -\frac{\partial \theta_0}{\partial t} - \text{Pe } u \frac{\partial \theta_0}{\partial z} \quad (2.71)$$

$$\theta_1|_{x=0} = 0, \quad \frac{\partial \theta_1}{\partial x} \Big|_{x=h} = 0 \quad (2.72)$$

We look for a spectral decomposition $\theta_1(x, z, t) = \sum a_k(z, t)w_k(x)$ of θ_1 , where $k \in \{0, 1, \dots, \infty\}$ and $w_k(x)$ are the eigenfunctions of the eigenvalue problem

$$w_k''(x) = -\lambda_k u(x)w_k(x), \quad w_k(0) = 0, \quad w_k'(h) = 0, \quad (2.73)$$

including the convective transport proportional to the flow velocity $u(x)$. Normalising, and introducing the quantity e_k so that

$$\int_0^h w_j(x)w_k(x)u(x)dx = \delta_{jk}, \quad e_k = \sqrt{3}\text{Ai}'(-h\lambda_k^{1/3}) - \text{Bi}'(-h\lambda_k^{1/3}),$$

the eigenvalues $\lambda_k(h)$ are determined by the transcendental equation $e_k = 0$ having an infinite number of solutions. The two lowest solutions $k = 0, 1$ of this equation for $h = 1$ are $\lambda_0 = 3.47662 \dots$ and $\lambda_1 = 44.1385 \dots$ and are separated by an order of magnitude. The corresponding eigenfunctions have the form

$$w_k(x; h) = N_k(h)^{-1/2} \left[\sqrt{3} \text{Ai}(-\lambda_k(h)^{1/3} x) - \text{Bi}(-\lambda_k(h)^{1/3} x) \right],$$

where $N_k(h)$ represents the normalisation constant. As a result, the equations of motion for the spectral components $a_k(z, t)$

$$c_k \frac{\partial a_k(z, t)}{\partial t} + \text{Pe} \frac{\partial a_k(z, t)}{\partial z} + \lambda_k a_k(z, t) = q_k(z, t; \text{Pe}) \quad (2.74)$$

$$c_k = \int_0^h w_k^2(x; h) dx = N_k(h)^{-1} \left\{ h e_k^2 - \left(\frac{43}{\lambda_k(h)} \right)^{1/3} \frac{1}{\Gamma(1/3)^2} \right\}$$

$$q_k(z, t, \text{Pe}) = -\gamma'(t) f(z) \int_0^h x w_k(x; h) dx - \text{Pe} \gamma(t) f'(z) \int_0^h x u(x) w_k(x; h) dx$$

have the form of a wave equation and the wave velocity c_k of the k -th mode depends on the thickness h of the melt film. Temporal $\gamma'(t)$ and spatial $\text{Pe} f'(z)$ changes of the absorbed intensity together with the flow velocity Pe enter the source term $q_k(z, t, \text{Pe})$ for the spectral components of the heat wave. The quasi-steady solution $\gamma'(t) = 0$ of (2.74) can be integrated directly thus giving

$$a_k(z) = \gamma \left[\frac{\sqrt{2\pi} \lambda_k}{4\text{Pe}} \exp \left\{ \frac{\lambda_k^2 - 8z\lambda_k \text{Pe}}{8\text{Pe}^2} \right\} \left(1 + \text{erf} \left(\frac{4\text{Pe} z - \lambda_k}{2\sqrt{2}\text{Pe}} \right) \right) - e^{-2z^2} \right] \quad (2.75)$$

$$\times \int_0^h x u(x) w_k(x; h) dx. \quad (2.76)$$

To evaluate the accuracy of the equations of motion for boundary layer dynamics reduced by spatial integration we apply the method of integrated balance [81] to the heat conduction equation (2.69) as an approximate approach. The heat conduction equation is integrated with respect to the melt film thickness $x \in \{0, h\}$ obtaining

$$\frac{\partial}{\partial t} \int_0^h \theta dx + \text{Pe} \frac{\partial}{\partial z} \int_0^h x \theta dx = \gamma(t) f(z) - \frac{\partial \theta}{\partial x} \Big|_{x=0}.$$

Assuming time scale separation such that the surface temperature $\hat{\theta}$ is the slow variable and the spatial distribution is the fast variable, the temperature is given by a linear function $\theta(x, z, t) = \hat{\theta}(z, t)x$ of the spatial coordinate x . The integrated balance becomes an equation for the surface temperature $\hat{\theta}$:

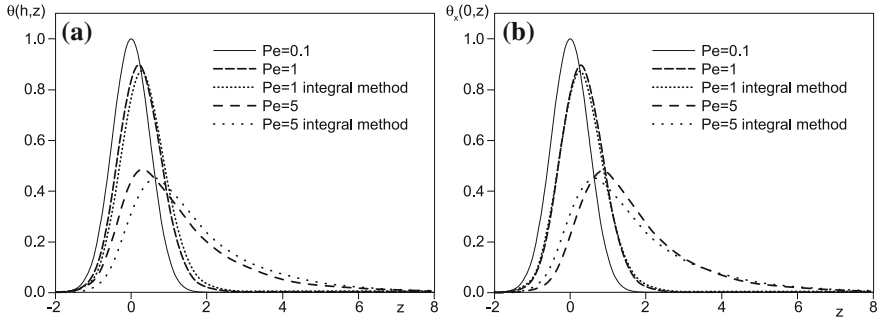


Fig. 12.12 Comparison of the solutions from the integral method (*dotted*) with the spectral method (*dashed*) for $Pe = 1.5$. As reference, the solution from the spectral method is given for a low value of the Péclet-number $Pe = 0.1$. **a** The temperature $\theta(h, z)$ at the liquid surface $x = h$ and **b** the heat flux $\theta_x(0, z)$ at the solid boundary $x = 0$

$$\frac{h^2}{2} \frac{\partial \hat{\theta}}{\partial t} + Pe \frac{h^3}{3} \frac{\partial \hat{\theta}}{\partial z} = \gamma(t) f(z) - \theta_0(z, t). \quad (2.77)$$

In particular, for the Gaussian distribution $f(z)$ and constant maximum intensity $\gamma(t) = \gamma = \text{const.}$ the result is given by

$$\hat{\theta}(z) = \frac{3\sqrt{2\pi}\gamma}{4h^3Pe} \exp \left\{ \frac{9}{8h^6Pe^2} - \frac{3z}{h^3Pe} \right\} \text{erfc} \left(\frac{3\sqrt{2}}{4h^3Pe} - \sqrt{2}z \right). \quad (2.78)$$

The resulting surface temperature $\theta(x = h, z)$ and the heat flux $\theta_x(0, z)$ at the solid boundary $x = 0$ reconstructed from the spectral and integral method (Fig. 12.12) show displacement by convection and widening by diffusion of heat compared with the absorbed intensity. The results agree very well and the error introduced by the integral approximation can be determined using spectral methods. Both methods are suitable to evaluate numerical calculations. The equation of motion (2.77) from the integrated balance shows a pronounced dependence on the melt film thickness h . The spectral approach already showing the effects of temporal variations of the intensity can be directly extended to include spatial variations of the melt film thickness $h(z, t)$ as well. Spatial variations of the melt film thickness $h(z, t)$ in fact change the structure of the mathematical problem qualitatively by introducing coupling of the equations for the spectral components $a_k(z, t)$.

2.2.3 Three-Phase Problem

To find the properties of the gas flow leading to melt ejection is a fundamental task in cutting. The interaction of the gas flow with the condensed phase is mediated by two quantities, namely the pressure gradient and the shear stress along the liquid surface.

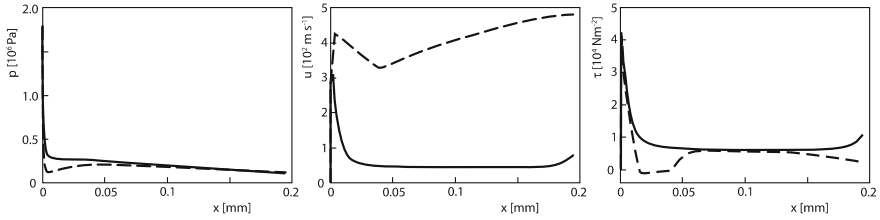


Fig. 2.13 Pressure p , velocity u and shear stress τ at the condensed surface in fine cutting. Numerical solution for the compressible flow calculated with the Navier-Stokes equations (*solid*) and the non-viscous Euler equations (*dashed*). The shear stress τ from the solution of the Euler equations is calculated using the Prandtl boundary layer expression given in (2.91)

There remain a couple of intriguing questions about the effects of the gas flow on the melt flow and their relation to quality criteria in cutting such as ripples of the third kind and dross formation. These quantities are related to separation and resolidification of the melt. In particular, separation of the supersonic gas flow from the cutting front are observed using Schlieren photography [42, 66, 68]. Other effects like the feedback of a wavy melt surface on gas flow are not considered in detail. However, discussion of the properties of supersonic gas flow in cutting suffers from the numerical difficulties involved [82]. Compared to well established simulations of compressible flow in aerofoil construction the situation in cutting is much more difficult due to the widely different temperature scales and spatial scales involved. The smallest temporal and spatial scales involved in cutting are related to gas flow phenomena and the most detailed properties of the shape of the condensed boundary, namely the liquid surface at the melting front, have to be addressed. In particular, to probe deeper into medical or photovoltaic applications like fine cutting of metallic stents or silicon wafers requires the inclusion of evaporation and flow of gas and vapour at a tube wall, or wafer thickness in the range of a few hundreds of micrometers (Fig. 2.13). Therefore, for the analysis of the gas flow phenomenon it is crucial to support the development and evaluation of numerical schemes by rigorous mathematical analysis. To find the details of the analysis of compressible non-viscous flow described by the Euler equations, boundary layer theory and improved experimental methods are used to guide efforts to extend numerical methods for the solution of the compressible Navier-Stokes equations.

2.2.3.1 Momentum Boundary Layer of the Gas Flow

Using asymptotic analysis of the compressible Navier-Stokes equations the viscous effects near the condensed boundary can be described by a set of coupled ordinary differential equations. The structure of the solution for the momentum boundary layer can be found and moreover, explicit expressions for the shear stress $\tau_g(\xi)$ at the condensed boundary $\eta = 0$ and the thickness $\delta_g(\xi)$ of the viscous boundary layer in the flow direction ξ can be derived. Near the condensed surface viscous forces

slow down the velocity of the cutting gas and a boundary layer for the momentum evolves. The thickness δ_G of the momentum boundary layer is characterised by the distance from the condensed surface where the flow changes from viscosity to inertia dominated. For laminar flows the ratio $\epsilon = \delta_G/L$ of the thickness δ_G and the length L along the flow becomes a smallness parameter for the Navier-Stokes equations. In the limiting case $\epsilon \rightarrow 0$ the compressible Navier-Stokes equations take the form of the well known Prandtl boundary layer equations for compressible flows:

$$\partial_t \rho + \nabla(\rho \mathbf{v}) = 0, \quad (2.79)$$

$$\rho(\partial_t + \langle \mathbf{v}, \nabla \rangle)u + \partial_x p = \partial_y \mu \partial_y u, \quad (2.80)$$

$$\partial_y p = 0, \quad (2.81)$$

$$\rho(\partial_t + \langle \mathbf{v}, \nabla \rangle)h - u \partial_x p = \mu(\partial_y u)^2 + \frac{1}{\text{Pr}} \partial_y \lambda \partial_y h, \quad (2.82)$$

for the four dependent quantities density ρ , velocity $\mathbf{v} = \mathbf{v}(x, y)$ and gas pressure p , subject to the no-slip boundary conditions

$$\mathbf{v}(x, y) \big|_{y=0} = 0, \quad T(x, y) \big|_{y=0} = T_s(x) \quad (2.83)$$

at the condensed surface $y = 0$, where $T_s(x)$ is the temperature of the condensed surface. The boundary conditions for large values of $y \rightarrow \infty$ are given by the properties of the outer flow.

$$\mathbf{v}(x, y) \big|_{y \rightarrow \infty} = \mathbf{v}_\infty(x), \quad T(x, y) \big|_{y \rightarrow \infty} = T_\infty(x), \quad (2.84)$$

$$\rho(x, y) \big|_{y \rightarrow \infty} = \rho_\infty(x) \quad (2.85)$$

The Prandtl number $\text{Pr} = \mu c_p / \lambda$ is the ratio of dynamical viscosity μ times specific heat capacity c_p and heat conductivity λ . Here u, v are the components of the velocity $\mathbf{v} = (u, v)$ parallel and perpendicular to the condensed boundary $y = 0$, respectively. The specific enthalpy $h = e + p/\rho$ of the gas depends on the internal energy $e = e(T) = c_v p / \rho$ ($c_v = (\gamma - 1)^{-1}$), where the temperature T is related to density and pressure via the equation of state ($p = \rho \tilde{R} T$, $\tilde{R}$ specific gas constant). As a consequence of the boundary layer approximation the pressure does not change $\partial_y p = 0$ within the boundary layer. The mass balance (2.79) can be satisfied by introducing a potential $\Psi(x, y)$, such that $\partial_y \Psi = \rho u$ and $\partial_x \Psi = -\rho v$. Introducing the transformation $\{\xi, \eta\} = \{\xi(x), \eta(x, y)\}$ for the spatial variables

$$\xi(x) = \int_0^\infty \mu_\infty \rho_\infty u_\infty dx', \quad \eta(x, y) \sqrt{2\xi(x)} = u_\infty \int_0^y \rho dy' \quad (2.86)$$

the Prandtl boundary layer equations (2.79)–(2.82) are reduced to an equivalent form for two dependent quantities $g(\xi, \eta), f(\xi, \eta)$ as discussed by Rogers [83]. The most interesting result is found by considering the limiting case for slowly varying properties of the outer non-viscous flow characterised by the values $\rho_\infty, u_\infty, \mu_\infty, h_\infty$

and H_∞ far outside the viscous boundary layer. The total enthalpy $H_\infty = h_\infty + v^2/2$ of the outer flow includes the contribution $v^2/2$ from the stagnation pressure. More precisely, the boundary values for the viscous flow within the boundary layer change with the spatial coordinate ξ in the flow direction on a length scale large compared to the boundary layer thickness itself. Consequently, the solution for $f = f(\eta)$ and $g = g(\eta)$ describing the gas flow within the boundary layer can be seen to depend on ξ only as a parameter. The boundary layer flow can be found by solving a set of coupled ordinary differential equations

$$f_{\eta\eta\eta} + f f_{\eta\eta} + \beta(g - f_\eta^2) = 0, \quad \beta = \beta(\text{Ma}_\infty, \gamma) \quad (2.87)$$

$$g_{\eta\eta} + \text{Pr} f g_\eta + (\text{Pr} - 1) \bar{\sigma} (f_\eta f_{\eta\eta})_\eta = 0, \quad \bar{\sigma} = \bar{\sigma}(\text{Ma}_\infty, \gamma) \quad (2.88)$$

where Ma_∞ and γ are the Mach-number and the adiabatic coefficient, subject to the boundary conditions

$$f(\eta = 0) = 0, \quad f_\eta(\eta = 0) = 0, \quad g(\eta = 0) = \frac{h(T(\xi, 0))}{H_\infty(\xi)} \quad (2.89)$$

$$f_\eta(\eta \rightarrow \infty) = 1, \quad g(\eta \rightarrow \infty) = 1 \quad (2.90)$$

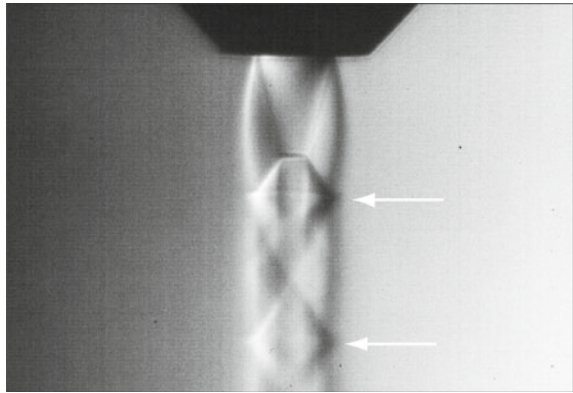
at the condensed surface (2.89) describing the no-slip condition $f_\eta = u$ and prescribing the surface temperature $T(\xi, 0)$ as well as the properties of the outer flow (2.90). As a result, the explicit expressions for the shear stress $\tau_g(\xi)$ and the thickness $\delta_g(\xi)$ of the viscous boundary layer read

$$\tau_g(\xi) = c_\tau \frac{\mu(\xi, 0) \rho(\xi, 0) u_\infty(\xi)}{\sqrt{2\xi}}, \quad c_\tau = f_{\eta\eta}(0) \quad (2.91)$$

$$\delta_g(\xi) = c_\delta \frac{\sqrt{2\xi}}{\rho_\infty(\xi) u_\infty(\xi)}, \quad c_\delta = \left[\int_0^\infty \frac{H_\infty}{h_\infty} (g - f_\eta^2) - f_\eta (1 - f_\eta) d\eta \right] \quad (2.92)$$

where the factors c_τ and c_δ reflect the details within the boundary, are of the order of unity, and give only a small correction to the scales depending on ξ . Comparing the numerical solution for the compressible flow along the free boundary calculated from the Navier-Stokes equations and the inviscid Euler equations (Fig. 2.13) shows deviations for pressure p and shear stress τ only in the regions where the flow undergoes large spatial variations due to expansion from the stagnation point near the upper edge $x = 0$ and separation at the lower edge $x = 0.2$ mm. The velocity u of the Euler flow corresponds to the outer flow velocity u_∞ and from the Navier-Stokes flow it does not reach the no-slip boundary value $u = 0$, depending on the spatial extent of the boundary cell element. The Prandtl boundary layer approximation gives a useful scale for the driving force exerted by the shear stress τ_g given in (2.91) that guides the development of reliable numerical schemes. As result, the numerical scheme can be used to reproduce the limiting case of a viscous boundary layer.

Fig. 2.14 Density gradient of a supersonic gas jet. The positions of two Mach discs are indicated (white arrows)



2.2.3.2 Deflection and Separation

A supersonic gas jet emerging from a nozzle into the ambient gas forms an interacting wave and flow pattern. The wave pattern within the jet consists of shock and expansion waves as well as Mach discs (Fig. 2.14, white arrows). The outer boundary of the jet forming the flow pattern is characterised by a curved contact discontinuity between the gas jet and the ambient gas. For free expansion of the jet, the shock and expansion waves have an almost linear spatial shape and the contact discontinuity is curved. Across the contact discontinuity the gas density as well as the velocity component in the tangential direction changes discontinuously while pressure and the normal component of the velocity are continuous. The shape of the contact discontinuity depends on the nozzle parameters and thus is a free surface within the ambient gas. The resulting flow pattern undergoes cyclic changes. At the edge of the orifice, where the contact discontinuity evolves, the nozzle pressure changes to the ambient pressure discontinuously and therefore the contact discontinuity as well as a converging shock spatially coexist. Expansion waves arise during the transition from the converging shock wave to the contact discontinuity. With increasing distance from the orifice the expansion waves are reflected at the contact discontinuity, leading to local minima of pressure at positions where the gas jet reaches its maximum diameter. The reflected expansion waves together with the converging shocks build a disc-like shock called the Mach disc. A cyclic change from reflected shocks and expansion waves forms the diamond-like characteristic structure observed by Schlieren photography. The jet is accelerated and decelerated between sonic and supersonic flow regions, where sonic flow is established at the Mach discs.

Schlieren diagnostics and simulation are performed for the gas flow impinging on an edge and for a gas flow in a prepared kerf (model kerf) made out of three layers substituting the shape of the cut front. Agreement between Schlieren photography and simulation is striking (Fig. 2.15).

An additional oblique shock is formed between body and nozzle when the gas jet impinges on a rigid body. The oblique shock deflects and accelerates the gas jet

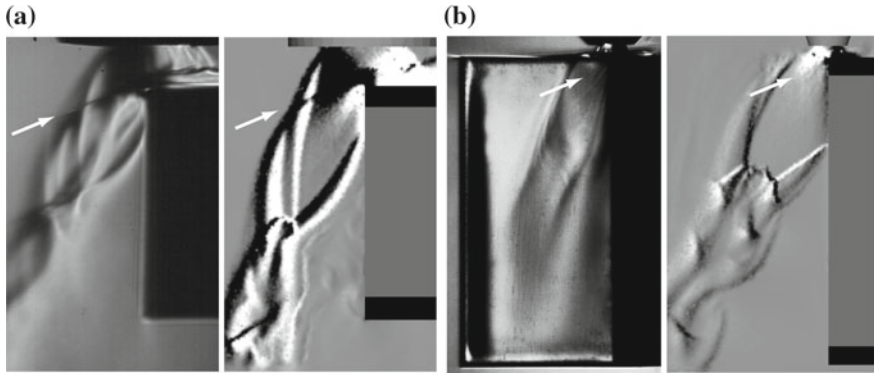
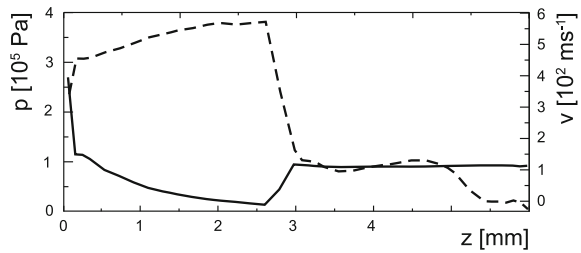


Fig. 2.15 Density gradient of the gas flow through a model kerf. Comparison of Schlieren photography and simulation both showing the oblique shock (*white arrows*) above the rigid body deflecting the gas jet **a** without glass plates and free expansion to the sides (sheet thickness $d = 6$ mm) and **b** with glass plates confining the kerf on both sides (sheet thickness $d = 20$ mm). Nozzle diameter $D_3 = 1.5$ mm, displacement $V_x = V_y = 0$ mm, stand-off distance $A = 0.75$ mm, nozzle pressure $p_K = 15 \cdot 10^5$ Pa, kerf width $500 \mu\text{m}$

Fig. 2.16 Pressure and tangential velocity component along the edge related to Fig. 2.15a. A jump in the gas flow properties occurs at the point of separation ($z \approx 2, 6$ mm)



into the kerf. The jet is partially aligned to the body and deflected by the oblique shock. Although the oblique shock leads to a deflection of the jet, the wave pattern of the deflected jet remains qualitatively unchanged. As a consequence the aligned part of the jet is related to the driving forces for melt acceleration along the rigid body, i.e. the pressure gradient and shear stress. Strength and shape of the oblique shock depend on the nozzle position and parameters as well as on the body's shape. The range of jet alignment depends on the nozzle position and parameters as well as on the body shape; the driving forces can therefore be adjusted by a proper choice of nozzle position and parameters. The main difference between the flows along the rigid body without glass plates and free expansion to the sides (Fig. 2.15a) and the flow through the model kerf with two glass plates bounding the kerf on both sides (Fig. 2.15b) is the magnitude of the gas pressure behind the oblique shock. For the model kerf the pressure p is larger ($p \sim 10 \cdot 10^5$ Pa) than the pressure ($p \sim 3 \cdot 10^5$ Pa) without lateral kerf walls. Consequently, the flow within the kerf separates far down the front of the model kerf at a separation depth $d_s \sim 5.7$ mm (Fig. 2.19) compared to $d_s \sim 2.6$ mm (Fig. 2.16).

The spatial extent of shock and expansion wave patterns is well reproduced in the upper part of the kerf, while in the lower part the contrast of the experimental results suffers additionally from larger amplitudes of the non-stationary oscillating flow. The simulation gives the instantaneous flow while the experimental observation is averaged with respect to time. Simulation therefore shows that the instantaneous density changes even in the lower part of the kerf are much stronger than expected from Schlieren photography. Improved recording of the flow using high-speed videography reproduce the oscillatory behaviour of the gas flow. While strong reflections occur in channel flow, these reflections perpendicular to the kerf walls are suppressed in the vicinity of the cutting front. Simulations and experimental observations show that the shape of the deflecting oblique shock changes depending on the shape of the kerf. The shape of the oblique shock and the subsequent expansion taking place in the direction of the open side of the cut kerf tends to suppress the appearance of reflections perpendicular to the kerf walls. Induced by a shock, the gas flow separates from the wall and the depth d_s of separation can be determined from the wave pattern. Numerical results reproduce the jump-like change of the gas flow where the flow separates from the rigid body (Fig. 2.16). The pressure gradient changes its sign and the velocity component tangential to the wall is slowed down considerably. The driving forces on a melt film induced by the gas flow change suddenly in a drastic way, leading to a deceleration of the melt flow as well as to a growing thickness of the melt film. The measured depth d_s of separation changes significantly when a flow around an edge is compared with a flow through a model kerf. The depth d_s of separation is significantly larger in the case of the model kerf, showing that an analysis of a flow around an edge will not give valid results when flow situations in a cutting kerf are considered.

The analysis of gas flows through the model kerf is detailed with respect to the nozzle displacement V_x (Fig. 2.17). The agreement of experiment and simulation concerning the depth where separation of the gas flow from the condensed surface sets in (Fig. 2.19) and the related wave pattern (Fig. 2.18) is extremely good. A larger nozzle pressure p_K leads to an increase of the separation depth d_s . Reducing the stand-off distance A can increase the depth of separation as well, until feedback into

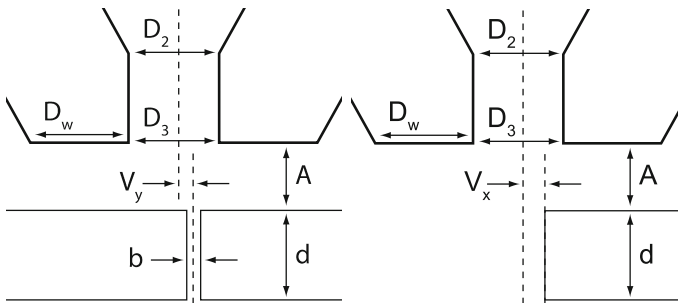


Fig. 2.17 The parameters for design and alignment of a conical-cylindrical nozzle with respect to the cut kerf

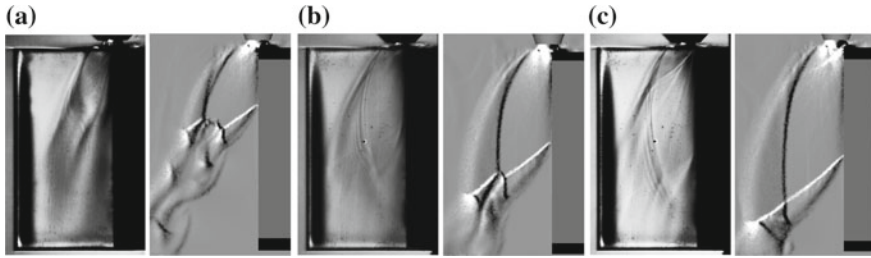


Fig. 2.18 Density gradient of the gas flow within a kerf. Comparison of Schlieren photography and simulation for different values V_x of displacement. **a** $V_x = 0$ (a), 0.35(b), 0.75(c) mm. Nozzle diameter $D_3 = 1.5$ mm, displacement $V_y = 0$ mm, stand-off distance $A = 0.75$ mm, nozzle pressure $p_K = 15 \cdot 10^5$ Pa, kerf width $500 \mu\text{m}$, sheet thickness of $d = 20$ mm

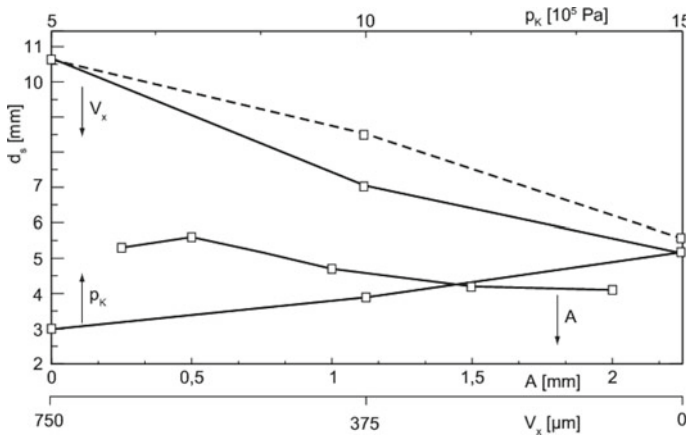


Fig. 2.19 The depth d_s where the gas flow separates from the condensed surface depends on nozzle pressure p_K , the stand-off distance A and the nozzle displacement V_x . Experimental results (*solid*) and the simulation (*dashed*) indicate that the depth d_s of separation is dominantly influenced by the nozzle displacement V_x

the nozzle takes place due to stagnation of the gas flow at the sheet metal (Fig. 2.20). For the chosen experimental setup the onset of feedback can be observed for stand-off distances smaller than $A < 0.5$ mm. Experimental results and the simulation indicate that the depth of separation is dominantly influenced by the nozzle displacement V_x . The effect of jet deflection is minimised by reducing the overlap between nozzle and material.

Obviously, the displacement V_x and the width b of the cut kerf cannot be prescribed directly, are a part of the solution, and depend on different processing parameters such as cutting speed, spatial distribution of the laser radiation etc. Moreover, with increasing depth of separation the flow tends to lower oscillatory amplitudes. From these findings it is quite clear that high-quality cutting relies on a gas flow where it does not separate from the front and the flow relaxes to a quasi-steady state

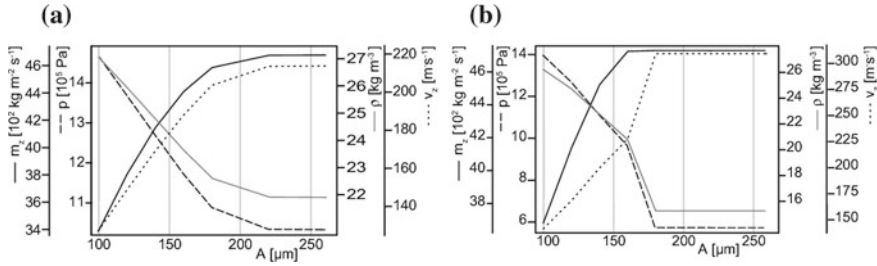


Fig. 2.20 Feedback into the nozzle due to stagnation of the gas flow at the sheet metal surface. The mass flow m_z out of the nozzle, the pressure p , the density ρ and the gas velocity v_z at the nozzle exit are integrated with respect to the diameter D_3 of the orifice, and their dependence on the stand-off distance A is given. **a** Conical nozzle ($D_2 > D_3$) and **b** conical-cylindrical nozzle ($D_2 = D_3$). $D_2 = D_3 = 0.24$, $A = 0.175$, $d = 0.26 \text{ mm}$, $p_K = 19 \cdot 10^5 \text{ Pa}$ (Argon)

on time scales much shorter than those prescribed by other parameters such as cutting speed and laser power. It remains an intriguing problem to find the time scales of relaxation and the dynamical properties of the gas flow depending on the parameters of nozzle design and alignment.

2.2.3.3 Gas Flow in Cutting and Trepanning

Periodically repeated reflection of shocks and compression waves along the side walls are observed in Schlieren photography and are related to the formation of horizontal structures of the ripple pattern in the real cutting situation. Looking for the different situations present in cutting and trepanning the main difference for the gas flow is related to the open kerf in cutting compared with the circular hole in trepanning. Simulation shows that the shape of the deflecting oblique shock is related to the shape of the kerf. Subsequent expansion taking place in the direction of the open side of the cut kerf tends to suppress the appearance of strong reflections perpendicular to the kerf walls. However, in trepanning the extent of the circular cut can be small enough to enhance the strength of shocks reflected from the condensed wall behind the actual position of the cutting front (Fig. 2.21). This behaviour can be observed in trepanning, where a drill hole is widened by performing a trim cut along the circular wall. In particular, performing trepanning and having the laser beam axis inclined with respect to the sheet metal surface the effect of reflected compression waves is further enhanced leading to periodic changes of the driving forces for the melt flow and corresponding changes in the thickness residual recast layers. If the distance for expansion of the gas flow by widening the hole diameter or by proper alignment of the nozzle by shifting the nozzle wall towards the right-hand upper edge of the circular drilled wall (Fig. 2.21a) is increased, the reflections in the gas flow are absent and resolidification can be avoided.

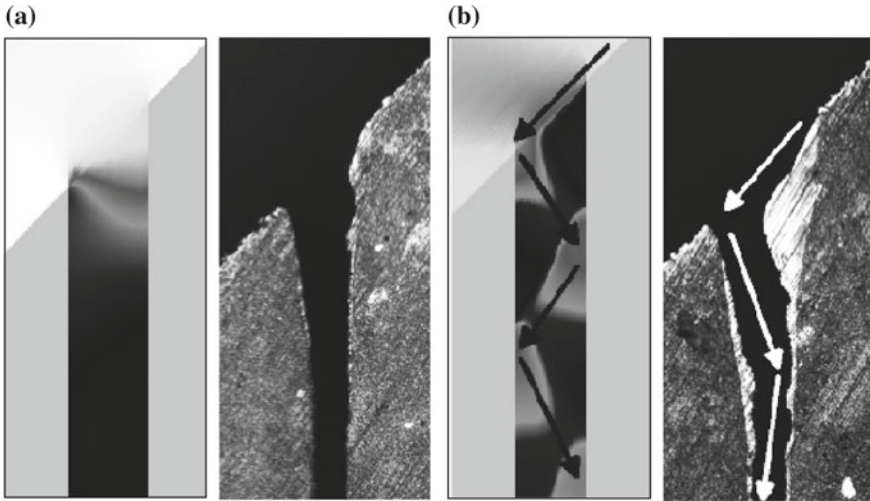


Fig. 2.21 Reflection of shocks and compression waves takes place in trepanning of inclined sheet metal leading to separation of the gas flow and pronounced periodic axial variation of the pressure (simulation) and periodic formation of recast layers (cross section). **a** Nozzle alignment towards the right-hand upper edge of the circular cut (wall position on the right side of the cut) and **b** nozzle axis aligned to the laser beam axis

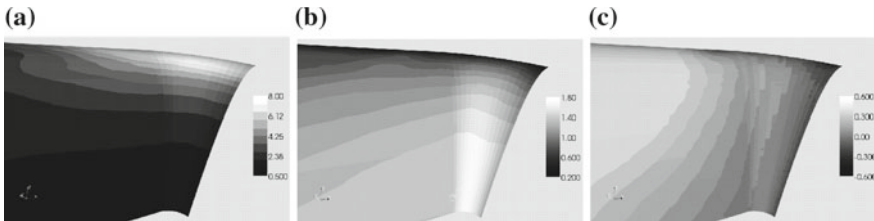


Fig. 2.22 Simulation of the three dimensional problem including the gas flow from the compressible Navier-Stokes equations. **a** Pressure p [10^5 Pa] and tangential components **b** v_{axial} ($3.3 \cdot 10^2$ m s $^{-1}$) and **c** $v_{lateral}$ ($3.3 \cdot 10^2$ m s $^{-1}$) of the gas velocity in axial and lateral direction, respectively. ($D_3 = 1.5$, $A = 0.75$, $d = 4$ mm, $p_K = 15 \cdot 10^5$ Pa, N_2 , $w_0 = 300$ μ m, $P_L = 4$ kW, $v_0 = 1.5$ m min $^{-1}$, $d = 4$ mm)

Simulation of the three dimensional problem including the moving cutting front and the gas flow calculated with the compressible Navier-Stokes equations are carried out to compare the effects of the driving forces in the axial and lateral directions along the cutting front (Fig. 2.22). As a consequence of the processing parameters the gas flow does not separate from the cutting front. The pressure gradient as well as the gas velocity are directed dominantly in the axial direction. There are no shocks at the cut surfaces and even the pressure and velocity vary smoothly along the cutting front and the cut surfaces.

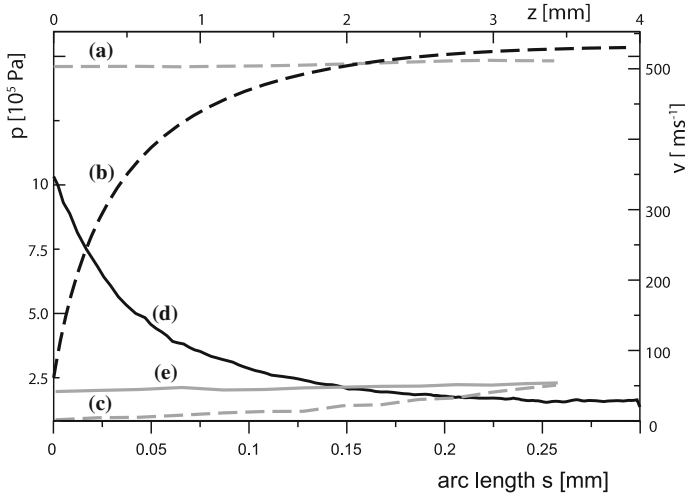


Fig. 2.23 Pressure p (solid, (d), (e)) and tangential components $v_{axial}, v_{lateral}$ of the velocity v (dashed, (a), (b), (c)) are given depending on the lateral (gray) direction measured by the arc length s at a depth of $z_0 = 2$ mm (a) $v_{axial}(s, z_0)$, (c) $v_{lateral}(s, z_0)$, (e) $p(s, z_0)$ and the axial direction (black) at the ceiling line $s = 0$ as a function of the depth z (b) $v_{axial}(s = 0, z)$, (d) $p(s = 0, z)$. ($D_3 = 1.5, A = 0.75, d = 4$ mm, $p_K = 15 \cdot 10^5$ Pa, $N_2, w_0 = 300$ μ m, $P_L = 4$ kW, $v_0 = 1.5$ m min $^{-1}$, $d = 4$ mm)

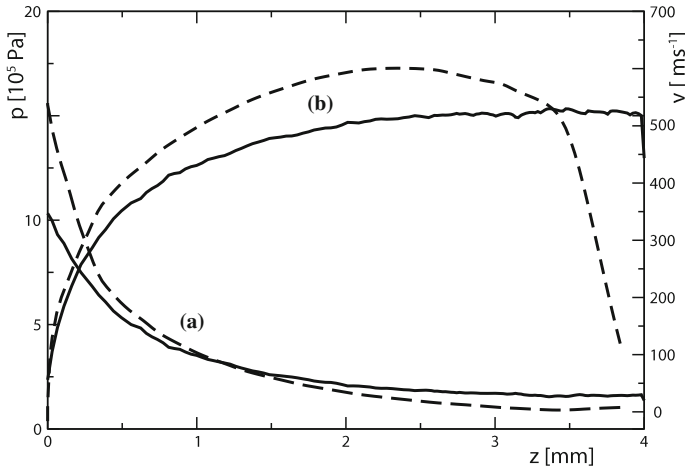


Fig. 2.24 Simulation of the three dimensional problem including the compressible Navier-Stokes equations. Comparison of gas flow taking place at a spatially two- (dashed) and three-dimensional (solid) domain. The values for the three-dimensional flow are taken from the ceiling line. a Pressure p (10^5 Pa) and b axial component v ($m s^{-1}$) of the gas velocity

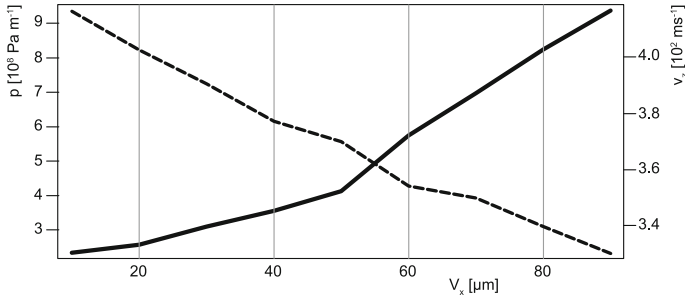


Fig. 2.25 Pressure gradient p_z (solid) and gas velocity v_z (dashed) depending on the displacement V_x between nozzle axis and front. $D_2 = D_3 = 0.24$, $A = 0.175$, $d = 0.26$ mm, $p_K = 19 \cdot 10^5$ Pa (Argon)

Spatial variations of pressure and velocities of the gas flow taken from the results in Fig. 2.22 are compared (Fig. 2.23). The ratio of lateral (gray curves) and axial quantities (black curves) are of the same order of magnitude as the ratio $2\pi w_0 d^{-1} \sim 0.18$ of the lateral $2\pi w_0$ and axial d length of the cutting front. Due to the presence of the cut surfaces the oblique shock is enhanced leading to a large pressure $p \sim 10 \cdot 10^5$ Pa at the top of the cut compared with the maximum pressure $p \sim 3 \cdot 10^5$ Pa for the case where free expansion perpendicular to the cutting direction takes place (Fig. 2.16).

There is an extended range of processing parameters, where the two-dimensional properties of the gas flow already reproduce the qualitative behaviour of the three dimensional flow (Fig. 2.24). In general, the spatial changes of gas pressure p along the cutting front change by an order of magnitude. As a rule of thumb, for a typical situation in cutting the pressure p along the cutting front changes at an average rate of about $\sim 10^8$ Pa m $^{-1}$ which equals 1 bar mm $^{-1}$. The value of 10^8 Pa m $^{-1}$ corresponds to a cutting depth $z > 0.5$ mm in Fig. 2.24 (a, solid line). The maximum value of the pressure gradient located near the upper edge of the front is up to ten times larger, but the extent of this region of high pressure gradient is limited to a few hundred microns, as shown in Fig. 2.25.

2.2.3.4 Alignment of Nozzle Position and Cutting Front

The effect of design and alignment parameters on nozzle performance in cutting are investigated in more detail. Two dominant effects are discussed, namely the feedback of the gas flow into the nozzle and deflection of the gas flow away from the cutting front. Feedback into the nozzle sets in due to stagnation of the gas flow at the sheet metal. Feedback diminishes both the mass flow out of the nozzle orifice and the outflow velocity. With increasing deflection, first the driving forces along the cutting front (pressure gradient and shear stress) are reduced and finally the gas flow separates before reaching the full cut depth.

As is well known from experimental evidence, the simulation also shows that it is advantageous to use a conical nozzle design in cutting instead of the Laval or Laval-Venturi design. The Laval nozzle generates larger gas velocities and lower gas pressure at the orifice, which turns out to be of minor importance due to the stagnation-type flow evolving at the sheet metal surface and the subsequent transition of the flow through an oblique shock into the cutting kerf; moreover, the pressure gradient at the cutting front is smaller for the Laval design. In particular, the alignment parameters: stand-off distance A of the nozzle with respect to the surface of the sheet metal, and the displacement V_x of the nozzle axis and the cutting front, are more sensitive for the Laval nozzle.

The parameters of the conical nozzle are given by the thickness of the nozzle wall D_w , diameters D_2 , D_3 of the orifice and stand-off distance A as well as the kerf parameters width and depth b , d of the cut and displacements V_x , V_y with respect to feed and lateral direction as sketched in Fig. 2.17. Additional parameters such as length of the cylindrical part between D_2 and D_3 of the orifice and the length of the conical part above D_2 are of less importance.

The simulation results are related to feedback into the nozzle caused by stagnation of the gas flow at the sheet metal, and the resulting pressure, velocity, density and mass flow out of the nozzle and along the cutting front.

The gas velocity at the orifice reaches the velocity of sound $c = \sqrt{\gamma p / \rho}$ for a conical nozzle. However, feedback due to stagnation is present due to the compressibility of the gas and a sufficiently large stand-off distance is necessary to avoid the unwanted counteracting effect.

The displacement V_x of the nozzle with respect to the feed direction dominantly changes the driving forces for the melt flow (Fig. 2.25). Variations along the front are pronounced for the pressure gradient p_z , while the axial component v_z of the gas velocity changes only slightly.

2.2.3.5 Dross Formation Depending on Gas Pressure

Discussion of the onset of dross formation is carried out with respect to cutting speed v_0 and sheet thickness d (Fig. 2.9), where the driving forces exerted on the melt flow were estimated by their scales only. The analysis can be extended to include compressible gas flow in the simulation so that the nozzle pressure p_K enters the calculation of the processing domain (Fig. 2.26b). The transitions to the adjacent domains II and IV related to droplet formation and evaporation are characterised by low values for the Weber number ($We \leq 1$, domain II) and the onset of evaporation ($\Theta_s \geq 1$, domain IV). Both quantities are determined by the gas flow via the free quantities' outflow velocity $u = u_m / u_0$ and thickness $h = d_m / d_{m0}$ of the melt film (Fig. 2.26c). The scale u_0 for the outflow velocity results from global mass balance in the melt film. The scale d_{m0} for the melt film thickness is obtained from the momentum in the outflow direction at the ceiling line (centre line) of the cutting front and depends on dynamical viscosity η , cutting speed v_0 , sheet thickness d and a typical value for

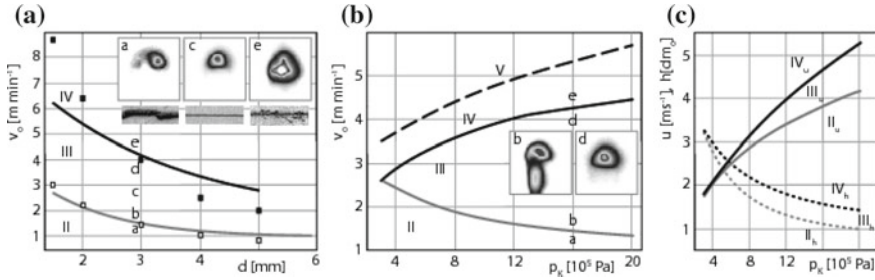


Fig. 2.26 The processing domain III for determination for a cut free of dross. Dross of kind 1. (domain II: droplet formation, *gray*, insert (a) and kind 2. (domain IV, onset of evaporation, *black*, insert (e). CCD-images of the thermal emission from the cutting front. **a** Comparison of experiment (*squares*) and simulation (*solid lines*). Nozzle pressure $p_K = 14 \cdot 10^5$ Pa. **b** The calculated boundaries of the processing domains depending on nozzle pressure at sheet thickness $d = 3$ mm. **c** Outflow velocity u (*solid*) and thickness h (*dotted*) of the melt film depending on nozzle pressure at $d = 3$ mm. (CO_2 laser, $P_L = 2.6$ kW, $w_0 = 150$ μm , $z_R = 2.2$ mm, focal position $z_0 = d/2$, stainless steel)

the driving forces. Here, the dominant driving force, shear stress τ_g or the pressure gradient dp_g/dz exerted to the melt film surface by the cutting gas, has to be inserted. The scales for the Weber number We (droplet formation) and the surface temperature T_s (evaporation) are thus

$$We = \frac{\rho}{\sigma} u_0^2 d_{m0}, \quad u_0 = v_0 \frac{d}{d_{m0}}, \quad (2.93)$$

$$\Theta_s = \frac{T_s - T_m}{T_v - T_m} = \exp\left(\frac{v_0 d_{m0}}{\kappa}\right), \quad d_{m0,\tau}^2 = 2 \frac{\eta v_0 d}{\tau_g}, \quad d_{m0,p}^3 = \frac{\eta v_0 d}{dp_g/dz} \quad (2.94)$$

where σ is the surface tension of the liquid and T_v , T_m denote the evaporation and melting temperature, respectively. As a result, the simulation reproduces the experimental results for the onset of evaporation and droplet formation as functions of the cutting speed v_0 and sheet thickness d (Fig. 2.26a). Additionally, the simulation shows that the processing domain III has the tendency to be closed for decreasing nozzle pressure at $p_K \sim 3 \cdot 10^5$ Pa. The most interesting result is related to the sensitivity of the outflow velocity u and thickness h of the melt film on the nozzle pressure p_K . Scatter of the processing parameters shows different sensitivity with respect to their actions on the outflow velocity and the melt film thickness. The values for the processing parameters within the dross free processing domain III, 1 indicated in Fig. 2.9c and the allowable fluctuations leading to adherent dross can be derived. Consequently, the confidence interval (the distance between region III, 1 and III, 2) can be determined.

To probe more deeply into the understanding of dross formation requires identification of additional features of the dross morphology, and the detailing and identification of the dominance of either the fluctuations or the temporal averaged parameter settings of the contributing mechanisms.

2.3 Outlook

Laser cutting machines account for the highest percentage of the laser market for industrial applications. In spite of the technical success already achieved, laser cutting is expected to exploit additional future potential offered by new laser systems such as multi-kW fibre laser and cutting machines with improved performance. To simplify handling and trouble-shooting the machine suppliers aim to introduce cognition in relation to the process into the machine, and development efforts are directed towards an “autonomous cutting machine”. One crucial step towards a cutting machine having more autonomous properties consists of extending knowledge about the processing domains and including a larger set of the processing parameters. For example, models for the propagation of laser radiation could take into consideration more details than just free propagation and the geometric optics of reflections. Extending the number of parameters spanning the phase space of the processing domains also means including not only a sufficient number of parameters but also understanding the influence of their fluctuations. To know more details about the accuracy necessary for a save parameter-setting requires the ability to distinguish between fluctuations of processing parameters, the scatter of residual cut quality, and the sensitivity of the dependence of the cut quality on the processing parameters.

While some properties of macro cutting can be applied to processing of small sheet thickness, simulation of fine cutting remains a challenge [84, 85]. To probe deeper into medical or photovoltaic applications like fine cutting of metallic stents or silicon wafers requires the inclusion of evaporation and flow of gas and vapour at a tube wall, or wafer thickness in the range of a few hundreds of micrometres. Processes of this kind are at the border line between drilling and cutting, since vapour flow from evaporation and gas flow from the nozzle are involved simultaneously.

The need for more detailed analysis of numerical schemes, their performance as well as their reliability in solving coupled dynamical equations with very different length scales along the free boundaries, becomes apparent. The smallest length scales of a few tenth of μm for the momentum boundary layer up to scales of about ten mm corresponding to the sheet thickness favour advanced developments including cartesian grid methods, discontinuous Galerkin methods and the adaptive grid refinement. Such methods are not directly applicable at present and appropriate modification of them requires detailed mathematical analysis.

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