

Contents

1	Introduction	1
2	Setting, Hypotheses and Main Results	7
2.1	Description of the Abstract Setting	7
2.2	The Perturbed Dynamics: Bridge to Analysis	12
2.3	Linear Response and the Kubo Formula	18
2.4	The Adiabatic Limit and the Kubo–Strěda Formula	21
2.5	Zero Temperature Limit and Topological Interpretation	24
2.6	The Tight-Binding Type Simplification	25
3	Mathematical Framework	27
3.1	Algebra of Observables	27
3.1.1	The von Neumann Algebra of Observables	27
3.1.2	The Algebra of Affiliated Operators	29
3.1.3	Finite Versus Semi-finite von Neumann Algebras	30
3.2	Non-commutative \mathfrak{L}^p -Spaces	31
3.2.1	F.n.s Trace State	31
3.2.2	Convergence in Measure and Measurable Operators	32
3.2.3	Integration and \mathfrak{L}^p -Spaces	35
3.2.4	Isospectral Transformations and Induced Isometries	39
3.3	Generalized Commutators	41
3.3.1	Commutators Between \mathcal{T} -Measurable Operators	41
3.3.2	Commutators Between \mathcal{T} -Measurable and Affiliated Operators	42
3.3.3	Commutators Between Unbounded Operators	47
3.4	Non-commutative Sobolev Spaces	47
3.4.1	\mathcal{T} -Compatible Spatial Derivations	48
3.4.2	Non-commutative Gradient and Sobolev Spaces	51

4	A Unified Framework for Common Physical Systems.	53
4.1	Von Neumann Algebra Associated to Ergodic Topological Dynamical Systems	53
4.1.1	Projective Representations of \mathbb{G}	54
4.1.2	Randomly Weighted Hilbert Spaces	55
4.1.3	Direct Integral of Hilbert Spaces	56
4.1.4	The Algebra of Covariant Random Operators	58
4.2	The Trace per Unit Volume.	61
4.3	Generators Compatible with the Trace per Unit Volume	65
4.4	Reduction to the Non-random Case	66
5	Studying the Dynamics	69
5.1	Unperturbed Dynamics	69
5.1.1	The Generator of the Unperturbed Dynamics.	70
5.1.2	A Formula for the Projection in Theorem 2.4.1	72
5.2	Perturbed Dynamics	75
5.2.1	Adiabatic Isospectral Perturbations	76
5.2.2	Additive Versus Multiplicative Perturbations	77
5.2.3	Existence of the Unitary Propagator	80
5.2.4	Evolution of Observables	85
5.2.5	Interaction Evolution of Observables	90
5.3	Comparison of Perturbed and Unperturbed Dynamics	92
6	The Kubo Formula and Its Adiabatic Limit	97
6.1	Comparing the Evolutions of Equilibrium States	97
6.1.1	Initial Equilibrium States	98
6.1.2	Existence of $\rho_{\text{full}}(t)$ and Its Expansion in Φ	103
6.2	The Kubo Formula for the Conductivity	107
6.2.1	The Macroscopic Net Current and the Conductivity Tensor	107
6.2.2	Proof of the Kubo Formula	113
6.3	The Adiabatic Limit of the Conductivity Tensor	115
7	Applications	121
7.1	Linear Response Theory for Periodic and Random Light Conductors	121
7.1.1	Schrödinger Formalism of Electromagnetism	122
7.1.2	Random Media	124
7.1.3	Open Questions	126
7.2	Quantum Hall Effect in Solid State Physics	127
7.2.1	Continuum Models	128
7.2.2	Discrete Models	130
	References	133

Linear Response Theory
An Analytic-Algebraic Approach
De Nittis, G.; Lein, M.
2017, X, 138 p., Softcover
ISBN: 978-3-319-56731-0