

Contents

Preface	ix
Chapter 1. Introduction to the theory of topological vector spaces	1
1.1. Linear spaces and topology	1
1.2. Basic definitions	11
1.3. Examples	17
1.4. Convex sets	29
1.5. Finite-dimensional and normable spaces	35
1.6. Metrizability	41
1.7. Completeness and completions	45
1.8. Compact and precompact sets	53
1.9. Linear operators	59
1.10. The Hahn–Banach theorem: geometric form	63
1.11. The Hahn–Banach theorem: analytic form	71
1.12. Complements and exercises	81
Uniform spaces (81). Convex compact sets (84). Fixed point theorems (86). Sequence spaces (89). Duals to Banach spaces (90). Separability properties (91). Continuous selections and extensions (93). Exercises (94).	
Chapter 2. Methods of constructing topological vector spaces	101
2.1. Projective topologies	101
2.2. Examples of projective limits	104
2.3. Inductive topologies	109
2.4. Examples of inductive limits	113
2.5. Grothendieck’s construction	119
2.6. Strict inductive limits	125
2.7. Inductive limits with compact embeddings	127
2.8. Tensor products	130
2.9. Nuclear spaces	134
2.10. Complements and exercises	139
Properties of the spaces \mathcal{D} and \mathcal{D}' (139). Absolutely summing operators (143). Local completeness (145). Exercises (147).	

Chapter 3. Duality	153
3.1. Polars	153
3.2. Topologies compatible with duality	158
3.3. Adjoint operators	162
3.4. Weak compactness	164
3.5. Barrelled spaces	170
3.6. Bornological spaces	175
3.7. The strong topology and reflexivity	180
3.8. Criteria for completeness	186
3.9. The closed graph theorem	193
3.10. Compact operators	199
3.11. The Fredholm alternative	205
3.12. Complements and exercises	208
Baire spaces (208). The Borel graph theorem (211). Bounding sets (212). The James theorem (213). Topological properties of locally convex spaces (214). Eberlein–Šmulian properties (218). Schauder bases (219). Minimal spaces and powers of the real line (221). Exercises (224).	
Chapter 4. Differential calculus	243
4.1. Differentiability with respect to systems of sets	244
4.2. Examples	251
4.3. Differentiability and continuity	257
4.4. Differentiability and continuity along a subspace	261
4.5. The derivative of a composition	263
4.6. The mean value theorem	273
4.7. Taylor's formula	275
4.8. Partial derivatives	278
4.9. The inversion of Taylor's formula and the chain rule	279
4.10. Complements and exercises	289
The inverse function theorem (289). Polynomials (291). Ordinary differential equations in locally convex spaces (294). Passage to the limit in derivatives (297). Completeness of spaces of smooth mappings (300). Differentiability via pseudotopologies (305). Smooth functions on Banach spaces (307). Exercises (308).	
Chapter 5. Measures on linear spaces	311
5.1. Cylindrical sets	311
5.2. Measures on topological spaces	313
5.3. Transformations and convergence of measures	321
5.4. Cylindrical measures	327
5.5. The Fourier transform	333
5.6. Covariance operators and means of measures	337
5.7. Gaussian measures	345
5.8. Quasi-measures	354
5.9. Sufficient topologies	357
5.10. The Sazonov and Gross–Sazonov topologies	359

5.11.	Conditions for countable additivity	366
5.12.	Complements and exercises	372
	Convolution (372). 0–1 laws (376). Convex measures (378). The central limit theorem (381). Infinitely divisible and stable measures (383). Banach supports of measures (391). Infinite-dimensional Wiener processes (393). Prohorov locally convex spaces (394). Measurable linear and polylinear functions (399). Relations between different σ -algebras (409). Radonifying operators (412). Measurable norms (412). Exercises (413).	
	Comments	419
	Bibliography	425
	Author index	447
	Subject index	451

Topological Vector Spaces and Their Applications

Bogachev, V.; Smolyanov, O.G.

2017, X, 456 p., Hardcover

ISBN: 978-3-319-57116-4