

Preface

This book is devoted to the design of a unified mathematical approach to the modeling and analysis of large systems constituted by several interacting living entities. It is a challenging objective that needs new ideas and mathematical tools based on a deep understanding of the interplay between mathematical and life sciences. The authors do not naively claim that this is fully achieved, but simply that a useful insight and some significant results are obtained toward the said objective.

The source of the contents of this book is the research activity developed in the last 20 years, which involved several young and experienced researchers. This story started with a book edited, at the beginning of this century, by N.B. with *Mario Pulvirenti* [51], where the chapters of the book presented a variety of models of life science systems which were derived by kinetic theory methods and theoretical tools of probability theory. The contents of [51] were motivated by the belief that an important new research frontier of applied mathematics had to be launched. The basic idea was that methods of the mathematical kinetic theory and statistical mechanics ought to be developed toward the modeling of large systems in life science differently from the traditional application to the fluid dynamics of large systems of classical particles.

Here, particles are living entities, from genes, cells, up to human beings. These entities are called, within the framework of mathematics, active particles. This term encompasses the idea that these particles have the ability to express special strategies generally addressing to their well-being and hence do not follow laws of classical mechanics as they can think, namely possess both an individual and a collective intelligence [84]. Due to this specific feature, interactions between particles are nonlinearly additive. In fact, the strategy developed by each particle depends on that expressed by the other particles, and in some cases develops a collective intelligence of the whole viewed as a swarm. Moreover, it often happens that all these events occur in a nonlinear manner.

An important conceptual contribution to describing interactions within an evolutive mathematical framework is offered by the theory of evolutive games [186, 189]. Once suitable models of the dynamics at the scale of individuals have been derived, methods of the kinetic theory suggest to describe the overall system

by a probability distribution over the microstate of the particles, while a balance of the number of particles within the elementary volume of the space of the microstates provides the time and space dynamics of the said distribution, viewed as a dependent variable. Quantities at the macroscale are useful in several applications, and these can be obtained from averaged moments of the dependent variable.

The hallmarks that have been presented above are somehow analogous to those proposed in the book [26], where the approach, however, was limited to linear interactions. Therefore, this present monograph provides, in the authors' belief, a far more advanced approach, definitely closer to physical reality. Moreover, an additional feature is the search of a link between mutations and selections from post-Darwinist theories [173, 174] to game theory and evolution. Indeed, applications, based on very recent papers proposed by several researchers, have been selected for physical systems, where nonlinearities appear to play an important role in the dynamics. Special attention is paid to the onset of a rare, not predictable event, called black swan according to the definition offered by Taleb [230].

The contents of the book are presented at the end of the first chapter after some general speculations on the complexity of living systems and on conceivable paths that mathematics can look for an effective interplay with their interpretation. Some statements can possibly contribute to understanding the conceptual approach and the personal style of presentation:

- The study of models, corresponding to a number of case studies developed in the research activity of the author and coworkers, motivated the derivation of mathematical structures, which have the ability to capture the most important complexity features. This formal framework can play the role of paradigms in the derivation of specific models, where the lack of a background field theory creates a huge conceptual difficulty very hard to tackle.
- Each chapter is concluded by a critical analysis, proposed with two goals: focusing on the developments needed for improving the efficacy of the proposed methods and envisaging further applications, possibly in fields different from those treated in this book. Applications cover a broad range of fields, including biology, social sciences, and applied sciences in general. The common feature of all these applications is a mathematical approach, where all of them are viewed as living, hence complex, systems.
- The authors of this book do not naively claim that the final objective of providing a mathematical theory of living systems has been fully achieved. It is simply claimed that a contribution to this challenging and fascinating research field is proposed and brought to the attention of future generations of applied mathematicians.

Finally, I wish to mention that this book represents what has been achieved until now. Hopefully, new results can be obtained in future activities. However, I decided to write a book, in collaboration with *Abdelghani Bellouquid*, *Livio Gibelli*, and *Nisrine Outada*, according to the feeling that defining the state of the art at this stage is a necessary step to look forward. *Abdelghani*, *Livio*, and *Nisrine* were kind

enough to allow me to write this Preface, as my experience in the field was developed in a longer (not deeper) lapse of time. Therefore, I have stories to tell, but mainly persons to thank.

I mentioned that many results have been achieved by various authors. Among them the coworkers are very many and I will not mention them explicitly, as they appear in the bibliography. However, I would like to acknowledge the contribution of some scientists who have motivated the activity developed in this book.

The first hint is from *Helmut Neunzert*, who is arguably the first to understand that a natural development of the mathematical kinetic theory needed to be addressed to systems far from that of molecular fluids. Namely, the pioneer ideas on vehicular traffic by Prigogine should have been applied, according to his hint, also to biology and applied sciences. He organized a fruitful, small workshop in Kaiserslautern, where discussions, critical analysis, and hints left a deep trace in my mind.

Subsequently I met *Wolfgang Alt*, who also had the feeling that methods of kinetic theory and statistical mechanics in general could find an interesting area of application in biology. He invited me to an Oberwolfach workshop devoted to mathematical biology, although I had never made, as a mathematician, a contribution to the specific field of the meeting. I was a sort of a guest scientist, who was lucky to have met, on that occasion, *Lee Segel*. His pragmatic way of developing research activity opened my eyes and convinced me to initiate a twenty-year activity, which is still going on and looks forward.

However, I still wish to mention three more lucky events. The first one is the collaboration with *Guido Forni*, an outstanding immunologist who helped me to understand the complex and multiscale essence of biology and of the immune competition in particular. Indeed, my first contributions are on the applications of mathematics to the immune competition. Recently, I met *Constantine Dafermos* in Rome, who strongly encouraged me to write this book to leave a trace on the interplay between mathematics and life. Finally, I had the pleasure to listen the opening lecture of *Giovanni Jona Lasinio* at the 2012 meeting of the Italian Mathematical Union. I am proud to state that I do share with him the idea that evolution is a key feature of all living systems and that mathematics should take into account this specific feature.

My special thanks go to Abdelghani Bellouquid. He has been a precious coworker for me and several colleagues. For my family, he has been one of us. I have to use the past tense, as he passed away when the book was reaching the end of the authors' efforts. My family and I, the two other authors of this book, and many others will never forget him.

The approach presented in this book was certainly challenging and certainly on the border of my knowledge and ability. In many cases, I have been alone with my thoughts and speculations. However, my scientific friends know that I have never been really alone, as my wife *Fiorella* was always close to me. Without her, this book would have simply been a wish.

Turin, Italy
January 2017

Nicola Bellomo

A Quest Towards a Mathematical Theory of Living
Systems

Bellomo, N.; Bellouquid, A.; Gibelli, L.; Outada, N.

2017, XIII, 181 p. 30 illus., 27 illus. in color., Hardcover

ISBN: 978-3-319-57435-6

A product of Birkhäuser Basel