Chapter 2
Old and Novel Tools for the Calculus of the Hazard Rate

The major subjects covered in this chapter revolve around the following topics:

– Deductive reasoning needs a precise description of the intended objects of study and we begin with discussing the *physical models* of systems.
– The concept of entropy is very prolific in science [1] and this chapter will put forward the Boltzmann-like entropy as an innovative mathematical tool to calculate the systems' hazard rate.

We recall that Shannon's entropy is used in a broad variety of research areas but here we shall introduce a new form of entropy that has nothing to do with the concepts of information and noise. Secondly, we shall overlook software systems.

2.1 The Reliability of What?

A principle-based theory should calculate artificial and natural systems, man-made and biological entities and, in fact, Gnedenko et al. [2] adopts the *chain* as the *physical model* capable of describing this multitude.

2.1.1 We mean to particularize the hazard rate depending on different circumstances where the Markovian chain assumes a variety of forms, hence we take that *S* is a *continuous-time stochastic system with a set of discrete states* and append the following remarks:

1. A device or a biological entity normally has some alternative macro-states, i.e. a body is living or otherwise dead; a machine is running or otherwise is under reparation. The *m states* or *macro-states* are mutually exclusive, and we obtain the structure of *S*

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$$S = (A_1\, OR\, A_2\, OR \ldots OR\, A_m), \quad m > 0. \tag{2.1}$$

2. Each state is equipped with *sub-states* (sometimes called *components* or *parts*) which work together toward the same purpose. That is to say, the generic state A_i includes n *sub-states* that cooperate in order to fulfill the mission of A_i

$$A_i = (A_{i1}\, AND\, A_{i2}\, AND \ldots AND\, A_{in}), \quad i = 1, \ldots m;\ n > 1. \tag{2.2}$$

3. We integrate (2.1) and (2.2) and obtain the *structure of levels,* which has three grades of granularity at least. We give the name *macro-scale level, meso-scale level* and *micro-scale level* to them from the top to the bottom. This can be also called an *OR/AND structure* because of the relationships that govern it

$$\begin{aligned} S &= (A_1\, OR\, A_2\, OR \ldots OR\, A_r) \\ &= (A_{11}\, AND\, A_{12}\, AND \ldots) OR \ldots OR (A_{r1}\, AND\, A_{r2}\, AND \ldots). \end{aligned} \tag{2.3}$$

A stochastic system is basically a Markov chain however we shall not exploit the Markovian dependencies; rather we shall look into the various forms of the structure of level and the interrelationships amongst the components. It may be said that we shall examine the *topology of S*.

2.1.2 Structure (2.3) can be placed close to the engineering design schemes which represent a device at different scales of measurement. The levels allow the progressive zoom of S and offer a basic aid to exploring intricate systems. Factually, the levels prove to be flexible in analyzing failures that includes different mechanisms at different metric scales.

Example Biologists hold that the human body encompasses the immune system, the circulatory and respiratory systems, and many others. Each system includes organs e.g. the circulatory system is equipped with the heart, veins and arteries. Each organ is composed of tissues and the tissues are made of cells. The structure of the human body includes six principal levels

$$\begin{aligned} \textit{Human Body} &= \{\text{Alive}\}\, OR\, \{\text{Dead}\} \\ &= \{[\text{System}_\text{A}]\, AND\, [\text{System}_\text{B}]\, AND\, [\text{System}_\text{C}]\, AND \ldots\}\, OR\, \{\ldots\} \\ &= \{[(\text{Organ}_\text{A1})\, AND\, (\text{Organ}_\text{A2})\, AND \ldots]\, AND\, [\ldots] \ldots\}\, OR\, \{\ldots\} \\ &= \{[(\text{Tissue}_\text{A11}\, AND\, \text{Tissue}_\text{A12}\, AND \ldots)\, AND\, (\text{Tissue}_\text{A21}\, AND \ldots) \ldots]\, AND\, [\ldots] \ldots\}\, OR\, \{\ldots\} \\ &= \{[((\text{Cell}_\text{A111}\, AND\, \text{Cell}_\text{A112}\, AND \ldots)\, AND\, (\text{Cell}_\text{A121}\, AND\, \text{Cell}_\text{A122} AND \ldots) \ldots) \ldots]\, AND\, [\ldots] \ldots\}\, OR\, \{\ldots\} \end{aligned} \tag{2.4}$$

2.1.3 The hierarchical decomposition of systems is rooted in the pioneering works of Herbert Simon [3] who writes:

> Scientific knowledge is organized in levels, not because reduction in principle is impossible, but because nature is organized in levels, and the pattern at each level is most clearly discerned by abstracting from the detail of the levels far below.

The *hierarchy theory* offers efficient suggestions on how to surmount complexity in nature. The structure serves to subdivide a complex system into organizational levels and into discrete components within each level. The hierarchy theory clarifies other concepts such as surfaces, stability, nesting, filters and the integration of disturbances into systems [4].

2.1.4 States and sub-states are mapped into probabilities which indicate how likely it is that S stays in those states and sub-states

$$P_i = P(A_i).$$

One can calculate the probability in accordance with the relationships *OR* and *AND*. In particular, the *AND* amongst the sub-states in (2.2) provides the probability of A_i

$$P_i = (P_{i1} \cdot P_{i2} \cdot P_{i3} \cdot \ldots \cdot P_{in}), \quad n > 1. \tag{2.5}$$

2.1.5 In principle, [5] the essential states of S in the reliability domain are the following:

A_f = the *functioning state* during which the system runs,
A_r = the *recovery state* during which the system is repaired, or renewed and so forth,
A_l = the *idle state* during which the system is good and idles,
A_d = the *dereliction state* during which the broken system is not manipulated.

Thus (2.1) has this precise form

$$S = \left(A_f \, OR \, A_r \, OR \, A_l \, OR \, A_d\right). \tag{2.6}$$

The system evolves from one state to another according to the transition graph in Fig. 2.1.

2.1.6 We shall investigate various stochastic structures that vary depending on the goal pursued by each enquiry. We shall tailor the structure of levels according to the problem being tackled, specifically:

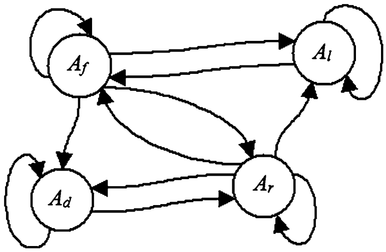

Fig. 2.1 System states and transactions

- The present chapter discusses the general properties of systems using this *physical model*

$$S = (A_1\, OR\, A_2\, OR \ldots OR\, A_m), \quad m > 0. \tag{2.1}$$

$$A_i = (A_{i1}\, AND\, A_{i2}\, AND \ldots AND\, A_{in}), \quad i = 1, 2, \ldots m;\; n > 1. \tag{2.2}$$

- Chapters from 3 to 8 examine exclusively the functioning state

$$S = A_f. \tag{2.7}$$

- Chapter 9 studies the dynamical system that is repaired or maintained

$$S = \left(A_f\, OR\, A_r\right). \tag{2.8}$$

2.1.7 Machines, devices and other artificial constructions are built as determined by specified standard. Manufacturers scrap the units that do not pass the quality control phase. This management method allows theorists to employ the stochastic model in engineering; but living beings diverge from man-made artifacts. Generically speaking, biological systems do not undergo any 'quality control' and are more intricate than the artificial. Individuals of the same species differ one another, and even present monstrous deformities at the birth.

Are mathematical models, specifically stochastic models, appropriate *physical models* for living beings?

All mathematical models simplify the reality, but they can offer some significant advantages: a relatively simple explanation can depict complex situations. A rigorous pattern can predict experimental data with precision. As second we recall how different studies require different viewpoints of the intended facts, and depending on the purpose scientists develop different models for the same phenomenon. The present work means to clarify the various aspects of the mortality rate function, and theorists in the reliability domain show how the stochastic model can add a great deal to our understanding of living beings' life span and longevity [6].

2.2 Entropic Systems

Generically speaking, the use of the entropy function is not new in the treatment of failures but the entropy function which we are going to present, the issues that we mean to tackle and the direction which we address are very different from the approaches taken in the current literature. Despite some trivial similarities we have followed a special pathway; that is why we discuss the argument since its early beginnings.

2.2.1 When we started to undertake the fundamental issues of reliability, we believed that new mathematical tools should be devised and thermodynamics proved to be a fertile ground for inspiration.

Experts in thermodynamics noted how a certain amount of energy released from combustion is always lost inside a thermal engine and thus is not transformed into useful work. Uncompensated transformations led theorists to define the *reversible process* which basically can be restored to the initial state from the final state without requiring a modification of the properties of the surrounding environment. By contrast, the *irreversible process* is a thermodynamic change which can be reversed thanks to an additional exchange of energy with the context (Fig. 2.2).

The entropy H_{T} qualifies the thermodynamic cycle T which starts from the state A—we tacitly refer to equilibrium states—and goes back to it

$$\mathsf{T} = (\mathsf{A} \to \mathsf{B} \to \mathsf{A})$$

In particular Clausius' inequality holds that for the reversible process T the change in entropy is zero. Whereas the change in the entropy H_{T} is always lower than zero for the irreversible cycle

$$\Delta H_{\mathsf{T}} = \oint \frac{\delta Q}{T} \leq 0. \tag{2.9}$$

Where δQ is the heat exchanged at any point of the cycle T and T the temperature of those exchange points. Note how the reversible system is an ideal model, whereas all the physical systems are irreversible in the reality.

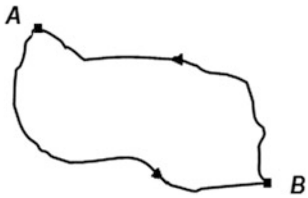

Fig. 2.2 The process T

2.2.2 Ludwig Boltzmann devised the thermodynamic entropy H_A that has two special properties:

- It calculates the property of the thermodynamic states A, B, C etc. and not the process T,
- It is related to the notion of probability and not to heat Q and temperature T, the parameters typical of H_T in (2.9).

Factually, the state A of the thermodynamic system (*TS*)—e.g. a sample of gas—is realized by different positions and momenta of the molecules contained in that sample. The energy of *TS* can be arranged in different ways and each *complexion of molecules* corresponding to a single A constitutes a *microstate* of A. The thermodynamic state A has several complexions or microstates: a, b, c, d….

$$A = (a\, OR\, b\, OR\, c\, OR \ldots).$$

Boltzmann defines his entropy this way

$$H_A(W) = k_B \log(W_A). \tag{2.10}$$

Where W_A is the number of alternative complexions which correspond to the macroscopic state A and k_B is Boltzmann's constant. The entropy $H_A(W)$ is the natural logarithm of the number of possible microscopic configurations of the individual molecules which give rise to the observed macroscopic state A of *TS*.

The thermodynamic system can assume only one microstate at a time, hence the number of favorable cases is 1 and the total number of possible cases is W_A. The various complexions are equally likely and the probability of the generic complexion X is

$$P_X = 1/W_A.$$

It seems useful to specify these details because there is a certain confusion between the so-called *thermodynamic probability* W_A—which Boltzmann introduces but is not the authentic mathematical probability—and the mathematical probability P_X [7].

2.2.3 Boltzmann notices that the higher is W_A and the higher the irreversibility of A. The thermodynamic system spontaneously reaches the *equilibrium state* which has the highest number of possible microstates W_A. On the other hand, the minimum irreversibility (and maximum reversibility) occurs when W_A is the unit; the molecules of A have only one microscopic configuration, and the entropy is null

$$H_A(W) = k_B \log(1) = 0. \tag{2.11}$$

This situation happens at the absolute temperature zero

$$T = T_0 = 0\,^{\circ}\mathrm{K}. \tag{2.12}$$

All the molecules are motionless and this entails that there is only one mechanical microstate which pertains to the macroscopic T_0-state. Various theorems demonstrate that it is impossible by any material procedure—no matter how idealized—to reduce the temperature of any system to T_0 in a finite number of operations. The T_0-state is so much reversible that a physical device can merely approximate it.

In Boltzmann's view, reversibility is the feature opposite to irreversibility, in the sense that when one increases, the other diminishes and vice versa. This double way relation could be written as follow

$$Reversibility/Irreversibility \tag{2.13}$$

We shall cite the coupled properties R and I with the symbol: R/I.

2.2.4 It is demonstrated that phenomena manageable in the real world are irreversible, including gas heating, liquid diffusion, friction between solid surfaces and chemical reactions. As a consequence, the *closed* thermodynamic system cannot finish a real process with as much useful energy as it had to start with. Some energy is always wasted. The second law of thermodynamics holds that the entropy of *TS* —either H_T or H_A—becomes greater in the world and is often enunciated in this form

$$\textit{The entropy of the universe tends to a maximum.} \tag{2.14}$$

Entropy can decrease with time in systems that are not closed. Factually, many *TS*s reduce local entropy at the expense of an environmental increase, resulting in a net increase in entropy.

From one point of view, statement (2.14) holds that the physical reality has an inherent tendency towards disorder and a general predisposition towards decay. A huge process of annihilation has been going on everywhere in the universe according to classic thermodynamics, and this suggested to us that there was an intriguing parallel with the reliability theory when we started to investigate the general trend of $\lambda(t)$. We observed how thermal processes and reliable systems alike are condemned to evolve toward demolition and these parallel behaviors intimated to us to introduce the entropy function for the study of reliable/reparable systems *S*. We had the intention to express some general properties of *S* in a manner similar to the properties of *TS* and introduced a function symmetric to (2.10) to which we gave the name *Boltzmann-like entropy*.

2.3 Boltzmann-like Entropy

We apply the notion of reversibility and irreversibility to the *physical model* (2.1).

2.3.1 For the sake of simplicity, suppose that there are two alternative and rather extreme situations for the stochastic system S. Once it has entered the generic state A_i ($i = 1, 2, \ldots m$), it may happen that S runs steady in A_i or otherwise S moves from A_i. In the former case S does not evolve from A_i, this state is rather stable and we say that A_i is *irreversible*. In the latter event, S abandons A_i, this state is somewhat unstable and we say A_i is *reversible*. E.g. a man/woman goes into an irreversible coma, this means that he/she does not leave this state and no longer recovers. E.g. the machine S was immediately repaired, this means that the failure was minor and the failure state was very reversible. In substance, the notion of reversibility is not simply a matter of 'playing the movie' backwards; the joined properties reversibility/irreversibility are a construct that allows one to judge the quality of the system behavior as it is being performed as well as evaluate its consequences.

2.3.2 We intend to calculate R/I of the state A_i in similarity with the Boltzmann entropy. Mathematicians prove the function $H_A(W)$ on the basis of three axioms [8] which we intend to examine:

(1) The number of complexions W expresses a certain idea of frequency; in parallel we demand that the Boltzmann-like entropy of the state A_i depends on the probability P_i.
(2) The entropy $H_A(W)$ is minimal when A is absolutely reversible [see (2.11)]; the Boltzmann-like entropy likewise reaches the minimum when A_i is *reversible*. The Boltzmann-like entropy is at its maximum when the stochastic state is *irreversible*.
(3) The microstates of a gas sample influence the overall state A and in turn the entropy H_A. Similarly we assume that the irreversibility of each component A_{i1}, A_{i2}, ... A_{in} affects the state A_i. By way of illustration, suppose that the part A_{ik} reaches an irreversible state because it breaks down, and affects the whole machine which stops. In practice, the entropy of A_{ik} changes the entropy of the overall operational state A_i and in turn the whole equipment. As a further example, let all the parts of a machine work steadily and in consequence the system fairly runs. The irreversible functioning micro-states A_{f1}, A_{f2}, ... result in the irreversible functioning macro-state A_f. The present assumption, which relates the R/I of the parts to the R/I of the whole, is the most meaningful one. It clarifies the system's complexity and the behavior of its parts.

We translate criteria (1), (2) and (3) into three assumptions as follows.

2.3.3 Let the generic state A_i has n cooperating parts

$$A_i = (A_{i1} AND\, A_{i2} AND \ldots AND\, A_{in}), \quad n > 1. \tag{2.2}$$

(1) The Boltzmann-like entropy H_i of A_i is continuous in the P_i

$$H_{\boldsymbol{i}} = H(P_{\boldsymbol{i}}). \tag{2.15}$$

(2) $H(P_{\boldsymbol{i}})$ is a monotonic increasing function of $P_{\boldsymbol{i}}$

(3) $H(P_{\boldsymbol{i}})$ is an additive function, namely it is the summation of the entropies of the sub-states A_{i1}, A_{i2},... A_{in}

$$H_i = H_{i1} + H_{i2} + H_{i3} + \ldots + H_{in}. \tag{2.16}$$

Theorem 2.1 *The H_i satisfying assumptions (**1**), (**2**) and (**3**) is of the form*

$$H_i = H(P_i) = a \log(P_{\boldsymbol{i}}) + b, \quad a > 0,\ b \geq 0. \tag{2.17}$$

Where a is the scale parameter, and b is the position parameter that depend on the system. We give the name of Boltzmann-like entropy to H_i.

Proof In order to simplify the formal expressions, let the state A_i consist of two sub-states (n = 2). We apply the multiplication law from (2.5)

$$P_i = P_{i1} \cdot P_{i2}. \tag{2.18}$$

From axiom (**3**) we obtain

$$H = H(P_{i1} \cdot P_{i2}) = H(P_{i1}) + H(P_{i2}). \tag{2.19}$$

We develop this expression by the Taylor series and preliminary we unify the variables in this way

$$\begin{aligned} P_{i1} &= P_i, \\ P_{i2} &= 1 - \varepsilon. \end{aligned} \tag{2.20}$$

Where ε is infinitesimal of the first order. Formula (2.19) becomes

$$H(P_i - \varepsilon P_i) = H(P_i) + H(1 - \varepsilon).$$

We develop this expression in series and neglect the terms of the highest order

$$H(P_i) - \varepsilon P_i \cdot H'(P_i) = H(P_i) + H(1) - \varepsilon \cdot H'(1). \tag{2.21}$$

The maximum of the entropy function is zero in accordance with (2.11)

$$H(1) = 0. \tag{2.22}$$

Hence

$$P_i \cdot H'(P_i) = H'(1). \tag{2.23}$$

Namely

$$P_i \cdot H'(P_i) = k, \quad k \geq 0. \tag{2.24}$$

This expression means

$$H'(P_i) = \frac{k}{P_i}. \tag{2.25}$$

The integration yields

$$H(P_i) = a \log(P_i) + b. \tag{2.26}$$

For the sake of simplicity we use $a = 1$ and $b = 0$ (Fig. 2.3), and obtain

$$H(P_i) = \log(P_i). \tag{2.27}$$

Secondly, the Boltzmann entropy depends on the number of the molecules complexions W_A that varies between 1 and $+\infty$. The domain of $H_A(W)$ is $(0, +\infty)$, instead $H(P_i)$ varies within

$$(-\infty, 0). \tag{2.28}$$

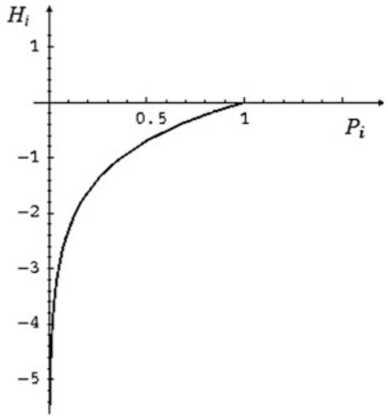

Fig. 2.3 The Boltzmann-like entropy

2.4 Physical Meaning of the Boltzmann-like Entropy

The Boltzmann-like entropy is intended to study the evolvability of this system

$$S = \left(A_f\, OR\, A_r\, OR\, A_l\, OR\, A_d\right). \tag{2.6}$$

The idle and the dereliction states do not carry out any transformation for S and do not contribute to any change. They turn out to be inessential for the entropic study, we confine attention to A_f and A_r, and in turn we inquiry:

(1) The *reliability or functional entropy* $H_f = H(A_f) = H(P_f)$
(2) The *recovery entropy* $H_r = H(A_r) = H(P_r)$

2.4.1 H_f and H_r specify the inherent characteristic of S in the functioning state and the recovery state in the order. The following intuitive remarks can help the reader understand 'how' the *functional entropy* H_f and the *recovery entropy* H_r depict S in the states A_f and A_r.

(1.a) When H_f is 'high', the functioning state is irreversible and the system works steadily. In particular, the more H_f is high, the more A_f is irreversible, and S is *capable of working*.
(1.b) On the other hand, when H_f is low, S often abandons A_f and switches to H_r, we say that *S is incapable of working*.
(2.a) When H_r is 'high', the recovery state is irreversible, and the workers operate on S with effort. In particular, the more H_r is high, the more A_r is stable, and in practice *S is hard to repair* and/or maintain in the world.
(2.b) On the other hand, when H_r is low, S leaves A_r and one says that *S can be easily restored or maintained.*

I/R are coupled opposite characters in accordance to (2.13); thus the function H_f illustrates the *capability to work* (point 1.a) and also the *incapability of working* (point 1.b). The function H_r qualifies the *capacity to be repaired* (point 2.b) and the *incapacity of being repaired* (point 2.a). The following two statements merge annotations 1.a with 1.b; and 2.a with 2.b as follows:

$$\begin{aligned}&\textit{The reliability entropy expresses the aptitude of S to work without failures;}\\&\textit{The recovery entropy illustrates the disposition of S toward reparation.}\end{aligned} \tag{2.29}$$

2.4.2 Practitioners adopt a very high number of variables to calculate the devices' capability of working including maximum speed, minimum noise, rapid answers etc. They assess several parameters even within a single engineering context. Doctors measure up the health state of a patient using many clinical tests. The vigor of a young body is qualified by means of fatigue resistance, indifference to low/high environmental temperature, low blood pressure etc. Some theorists qualify the performances of S using a set of states; for example, Mesbah et al. [9] introduce a four-state model which includes the *good, neutral, bad* and *dead states*. This scale supports longitudinal inquiries of the quality of life. In summary, many criteria are employed to qualify the effectiveness of each class of systems; it seems noteworthy that the entropy H_f expresses the aptitude of S to work which does not depend on the technology, on the form and dimension of S, on the nature of S—say artificial or natural—and on other special features.

Usually engineers qualify the labor required to repair S using an assortment of markers: the number of work-hours, the replaced components, the financial costs and so forth. Doctors give an account of the days necessary for a patient to recover from a surgery, an infection or another disease, and the various treatments required: drugs, hours of physiotherapy etc. The recovery entropy H_r expresses the aptitude of S to be repaired by reason of the reversibility and irreversibility criterion that crosses numerous fields of application.

2.4.3 Let us examine a pair of numeric examples.

Example Suppose a and b are two devices in series with $P_f(a) = 10^{-200}$, $P_f(b) = 10^{-150}$. We can calculate the probability of good functioning and then the stability of the overall system with the entropy

$$P_f(S) = [P_f(a) \cdot P_f(b)] = 10^{-350}$$

$$H_f(S) = \log\left[P_f(S)\right] = \log(10^{-350}) = -805.9$$

The R/I of the whole S depends on the R/I of the parts, the Boltzmann-like entropy is additive [assumption (2.16)] and one can follow this way with the same result

$$\begin{aligned} H_f(S) &= \left[H_f(a) + H_f(b)\right] = \log\left[P_f(a)\right] + \log\left[P_f(b)\right] \\ &= -345.3 + (-460.5) \approx -805.9 \end{aligned}$$

Example Suppose a device degrades during the interval (t_1, t_2); and the probability values are the following: $P_f(t_1) = 10^{-10}$, $P_f(t_2) = 10^{-200}$. The entropies qualify the irreversibility of the device

$$H_f(t_1) = \log\left(10^{-10}\right) = -23.0$$

$$H_f(t_2) = -460.5$$

One obtains how much the capability of good functioning has slowed down

$$\Delta H_f = H_f(t_2) - H_f(t_1) = -460.5 - (-23.0) = -437.5.$$

2.4.4 The calculation of the Boltzmann-like entropy does not oppose great difficulties in numerical terms; instead it raises somewhat serious arguments from the conceptual viewpoint as the significance of the free variable of $H(P)$—the probability P—is still arguable from many aspects.

The *frequency* and the *subjective theories* are currently under discussion and in a way these constructions underpin the classical and Bayesian statistics respectively. Some results of classical statistics have a similar Bayesian counterpart, and this has compounded the question even more. The correspondences between the two statistical schools have encouraged some working statisticians to minimize the differences and emphasize how parallel results can be obtained in both the statistical courses [10, 11].

Someone still objects that the mentioned differences and even other divergences do not seem dramatic, they appear smaller in magnitude compared to the distances between the parametric versus nonparametric approaches, the design versus model modes etc.

We mean to answer as follows.

The debate about the divergences of the classical and Bayesian statistics and even the discussion on the superiority of one approach over the other, remains on the surface of the questions until one assumes an operational viewpoint. As matter of facts, nobody has a meter in hand that is capable of measuring the virtues of a statistical method. There is no shared scale to quantify the distance between the right side and the left side of Table 2.1.

We note how the substantial contrast between the classical and Bayesian statistics does not emerge during the calculation phase but when the parallel use of the two statistical methods finishes. We mean to say that a statistician gets two numerical values at the end of the calculations, but the probability obtained through the classical method is *an objective quantity*, the probability obtained through Bayesian procedures expresses *a personal belief*. The two probabilities can even be equal in numerical terms but their meanings turn out to be irreconcilable (Table 2.2).

Table 2.1 Some opposite traits of the statistical methods

Classical Statistics	Bayesian Statistics
Unknown variables are treated as deterministic quantities that happen to be unknown	Unknown variables are treated as random variables with known distributions
Classical statisticians concentrate on the full set of data that might arise	Bayesians concentrate on the observed data only
The existence of true parameters is assumed	True parameters are not supposed to exist
Underlying parameters remain constant during repeated sampling: parameters are fixed	Parameters are unknown and described probabilistically: data are fixed

Table 2.2 Opposite conclusions of the two statistics

Classical Statistics	Bayesian Statistics
Probability is a physical and objective quantity	Probability is a means to quantify uncertainty

In consequence of the disparate significance of P, the Boltzmann-like entropy $H(P)$ assumes altogether contrasting significance and the purpose of this book, which means to provide a unified ground to the dependability domain, seems destined to be short lived.

The recent volume [12] addresses the problem of the probability interpretation and remarks that the debates about the double nature of probability has apparent philosophical origin. The book presents a pair of theorems to circumvent the philosophical approach that unfits mathematical logic. The theorems demonstrate that a statistician can select either the classic or the Bayesian approach without logical incongruence. As a result, a scholar becomes aware of what kind of entropies are involved whenever probability is defined in terms of chances, i.e. physical probabilities, or credence, i.e. degrees of belief.

2.4.5 A non-negligible group of scholar shares the idea that there is a certain parallel between thermodynamics and reliability. Feinberg and Widom [13] write this sentence which we fully subscribe to:

> The reliability science for physics-of-failure lacks a basic foundation. Thermodynamics is a natural candidate. Many engineers do not realize how closely thermodynamics is tied to reliability.

Some authors view degradation as an irreversible process that dissipates energies and can be qualified by the entropy function. They extend thermodynamics to the sector of dependability in a direct manner and calculate the variation of total thermodynamic entropy dH in order to assess the damage of components. Experimentalists apply this method especially in the mechanical context [14–16]. This vein of research, which could be labeled '*thermodynamic reliability*,' is summarized by the following cause-effect inference

$$Degradation \rightarrow Damage \rightarrow Dissipated\, energy \rightarrow Entropy\, production$$

In a way, this cause-effect analysis mirrors the initial motivation of the present work since both the approaches read the entropy function as an optimal tool that is capable of qualifying degradation and does not depend on the choice of observables. Although, there are great differences in the purposes and the mathematical methodologies between the present approach and the approach of the thermodynamic reliability, the state-of-the-art of which is illustrated in the work of Amiri and Modarres [17]. The latter deems degradation to be lost energy, whereas the former sees degradation as a somewhat reversible state of functioning. The latter calculates the thermodynamic entropy; the former the Boltzmann-like entropy. The present approach aims to tackle the definition of the hazard rate in general, researchers in thermodynamic reliability address specific empirical cases.

2.4.6 The *dynamical system theory* is a significant theoretical area that deals with the long-term behavior of systems and their evolution over time. The basic elements of the dynamical system theory are:

– The set X of all possible *states* x of the dynamical system.
– The *phase space* $\mathbb{R}^n$ of the system; $x \in \mathbb{R}^n$.
– The *family of transformations* T_t: $X \rightarrow X$ mapping the phase space to itself.

The transformations depend on time notably when t is a positive real number, the system's dynamics is continuous. If t is a natural number, S execute a discrete set of actions [18]. The Kolmogorov-Sinai (KS) entropy measures the difficulty to foresee S, in the sense that the higher the unpredictability is, the higher the KS entropy. This property matches nicely with the Shannon entropy, where the unpredictability of the next character coming from the source is equivalent to new information. It is also consistent with the concept of entropy in thermodynamics, where disorder increases the entropy; in fact, disorder and unpredictability are closely related. The KS entropy provides a significant aid to understanding the complexity of dynamical systems [19]. For instance, it contributes to the *chaos theory*, a mathematical frame of deterministic behaviors that are characterized by great disorder and confusion.

The dynamical system theory presents some traits that are not so distant from the present frame. For instance, Courbage and Prigogine [20] examine the concept of the irreversibility of dynamical systems. However, the objectives of that theory differ from the scope of the present work, and the pathways that the two constructions follow are diverging.

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