2 Heat transfer

2.1 Introduction

In this chapter, we will look more closely at mechanisms that facilitate heat transfer. Heat can be transferred by three basic mechanisms, which are

1. *conduction*,
2. *convection* and
3. *radiation*.

In short time periods, the amount of transferred heat is practically always proportional to time. Heat transferred in 1 min will be 60 times larger than heat transferred in 1 s. It is therefore convenient to define the *heat flow rate* with the unit *watt* Φ (W) as a ratio of transferred heat to time:

$$\Phi = \frac{\mathrm{d}Q}{\mathrm{d}t}. \tag{2.1}$$

The watt therefore equals J/s. For a stationary situation, that is, a time-independent heat flow, a nondifferential form can be used:

$$\boxed{\Phi = \frac{Q}{t}.} \tag{2.2}$$

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The transfer of heat is best described by the density of heat flow rate.

In most practical situations, the heat flow rate is proportional to the area, so it is also convenient to define the *density of heat flow rate* q (W/m^2) as

$$\boxed{\Phi = A\,q.} \tag{2.3}$$

2.2 Conduction

Heat *conduction* is a mechanism of heat transfer facilitated by microscopic particles without bulk movement of particles. This mechanism is typical for solids (see Section 1.1) because most particles are moving only around a fixed equilibrium position (oscillation), although some (electrons) may also move randomly through the material (*diffusion*). Heat transfer is a consequence of *collisions between particles* and *particle diffusion*. Note that in terms of energy, the process is diffusive, so conduction can be also called *heat diffusion*.

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Conduction is facilitated by stationary particles, so it primarily occurs in the solid state.

M. Pinterić, *Building Physics*,
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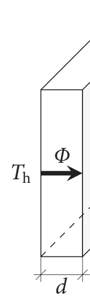

Figure 2.1: Heat transfer (conduction) through a solid slab whose opposite faces are at different temperatures, $T_h > T_l$. The slab's cross-sectional area is A, and the thickness is d.

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Equilibrium stands for temperatures independent of space and time. Steady heat transfer stands for the space-dependent and time-independent temperatures. Steady heat transfer is studied due to its simplicity.

2.2.1 Fourier's law

We start our study of conduction with *steady heat transfer*. Steady implies that we allow for temperature variations in space but no variations in time.

Let's observe heat transfer through a solid slab whose opposite faces are at different temperatures; that is, the hotter face is at higher temperature T_h, and the colder face is at lower temperature T_l (Fig. 2.1). The slab's cross-sectional area is A, and the thickness is d.

It can be shown experimentally that the heat flow rate is always

- proportional to cross-sectional area A,
- inversely proportional to thickness d and
- proportional to *temperature difference* $\Delta T = T_h - T_l$.

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If there is a spatial difference in temperature, the conduction—heat diffusion—from the region with the higher temperature to the region with the lower temperature sets in.

We can join these statements into the expression

$$\Phi = \lambda \frac{A \Delta T}{d}. \tag{2.4}$$

This statement is called *Fourier's law* or the *law of thermal conduction*. The coefficient of proportionality λ $(\mathrm{W/(m\,K)})$ is called *thermal conductivity*.

Thermal conduction also depends on the slab substance. Thermal conductivity is thus material dependent and has to be determined experimentally. Values for a few typical building materials are presented in Table A.3.

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Thermal insulators, that is, materials that hinder heat transfer, have low thermal conductivity.

Thermal insulators are materials that hinder heat transfer (Fig. 2.2). Note that good thermal insulators have *low* thermal conductivity.

Because the temperature difference is the same for both temperature scales (1.2), and degrees Celsius is used more commonly in engineering, we will rewrite

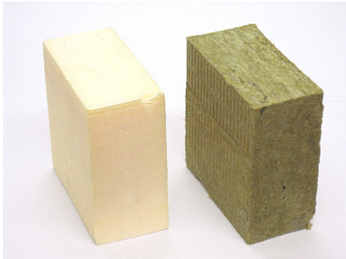

Figure 2.2: Thermal insulators: extruded polystyrene (left) and mineral wool (right). The primary ingredient of insulators is air (see Section 2.3).

Fourier's law as

$$\Phi = \lambda \frac{A\Delta\theta}{d}. \tag{2.5}$$

In terms of density of heat flow rate (2.3), Fourier's law is transformed to

$$q = \lambda \frac{\Delta\theta}{d}. \tag{2.6}$$

Usually, the layer in the building component is characterised by *thermal resistance* R $(\mathrm{m^2\,K/W})$, that is, the ratio of thickness to thermal conductivity

$$R = \frac{d}{\lambda}, \tag{2.7}$$

or, alternatively, though rarely, by the inverse of thermal resistance, that is, the *coefficient of heat transfer* k $(\mathrm{W/(m^2\,K)})$ as

$$k = \frac{1}{R} = \frac{\lambda}{d}. \tag{2.8}$$

In terms of thermal resistance, Fourier's law is therefore

$$\Phi = \frac{A\Delta\theta}{R}, \tag{2.9}$$

$$q = \frac{\Delta\theta}{R}. \tag{2.10}$$

2.2.2 Thermal resistance of multiple layers

We will now attend to the problem of heat transfer through multiple layers in building components when they are in parallel or serial positions (Fig. 2.3).

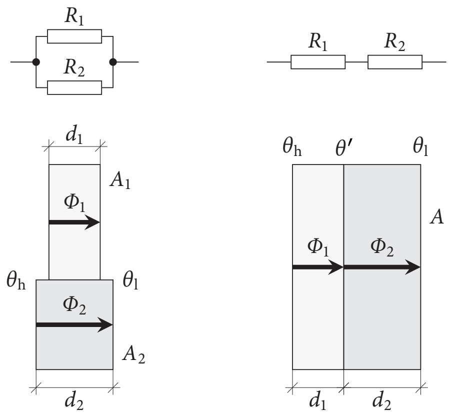

Figure 2.3: Conduction through multiple layers in building components when they are positioned in parallel (left) or serial (right). Equivalent electrical circuits are drawn above.

For *parallel layers* (Fig. 2.3, left), both layers have temperature θ_h at the left face and temperature θ_l at the right face. Taking this into consideration, we can write

$$\Phi_1 = \frac{A_1(\theta_h - \theta_l)}{R_1}, \quad \Phi_2 = \frac{A_2(\theta_h - \theta_l)}{R_2}.$$

The effective heat flow rate from the left side to the right side is simply a sum of the heat flow rates through the individual layers:

$$\Phi = \Phi_1 + \Phi_2 = \left(\frac{A_1}{R_1} + \frac{A_2}{R_2}\right)(\theta_h - \theta_l) \equiv \frac{A_{eq}}{R_{eq}}(\theta_h - \theta_l)$$
$$\Longrightarrow \frac{A_{eq}}{R_{eq}} = \frac{A_1}{R_1} + \frac{A_2}{R_2}.$$

We can generalise this result for an arbitrary number n of layers as

$$\frac{A_{eq}}{R_{eq}} = \sum_{i=1}^{n} \frac{A_i}{R_i}. \tag{2.11}$$

For *serial layers* (Fig. 2.3, right), both layers have the same cross-sectional area $A_1 = A_2 = A$. There is also another temperature involved, that is, the temperature at the boundary of two layers θ', which is usually called the *interface temperature*. Taking this into account, we can write

$$\Phi_1 = \frac{A(\theta_h - \theta')}{R_1}, \qquad \Phi_2 = \frac{A(\theta' - \theta_l)}{R_2}.$$

What is the relationship between the heat flow rates in both layers? If the first heat flow rate is larger than the second, $\Phi_1 > \Phi_2$, then more heat will enter the boundary of the two layers than will leave it; therefore, temperature θ' will increase. On the other hand, if the first heat flow rate is smaller than the second, $\Phi_1 < \Phi_2$, then less heat will enter the boundary of the two layers than will leave it; therefore, temperature θ' will decrease. We conclude that in order to have steady heat transfer, that is, constant temperature θ', both flow rates must be equal as in

$$\Phi_1 = \Phi_2.$$

Using that, we can readily calculate

$$\theta' = \frac{\theta_l R_1 + \theta_h R_2}{R_1 + R_2}.$$

Putting θ' in the equation for Φ_1 or Φ_2, we obtain

$$\Phi = \Phi_1 = \Phi_2 = \frac{A}{R_1 + R_2}(\theta_h - \theta_l) \equiv \frac{A_{eq}}{R_{eq}}(\theta_h - \theta_l)$$
$$\Longrightarrow R_{eq} = R_1 + R_2.$$

Here we took into account that $A_{eq} = A$.

For an arbitrary number n of layers

$$\Phi = A\frac{\theta_h - \theta_1'}{R_1} = A\frac{\theta_1' - \theta_2'}{R_2} = A\frac{\theta_2' - \theta_3'}{R_3} = \cdots = A\frac{\theta_{n-1}' - \theta_l}{R_n}, \qquad (2.12)$$

we similarly get

$$\Phi = A\frac{\theta_h - \theta_l}{R_1 + R_2 + R_3 + \cdots + R_n},$$

which leads to the generalised form of

$$\boxed{R_{eq} = \sum_{i=1}^{n} R_i.} \qquad (2.13)$$

It is always instructive to point to the similarity between heat conduction and electrical current conduction for electrical circuits, which was shown in Fig. 2.3 above. In the parallel case, heat splits and flows partially through the upper layer and partially through the lower layer. In the serial case, heat flows through both layers. Similarly, in the parallel case, electrical current splits and flows partially through the upper and partially through the lower resistor; in the serial case, electrical current flows through both resistors.

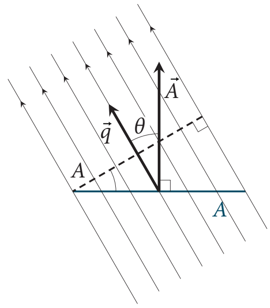

Figure 2.4: Nonperpendicular heat flow through a surface of area A. The density of heat flow rate is defined in terms of the surface perpendicular to heat flow of area A', so angle of flow incidence θ has to be taken into account.

2.2.3 Multidimensional conduction

If heat does not flow perpendicularly to the surface, angle of incidence θ has to be taken into account (Fig. 2.4). Equation (2.3) is defined in terms of the surface perpendicular to heat flow of area A', so heat flow in the slanted case amounts to

$$\Phi = A'q = A\cos\theta\, q. \tag{2.14}$$

We see that the heat flow rate can differ for the same density of heat flow rate and the same surface area but a different angle of flow incidence. In Section 2.4, we will use that fact to explain different temperatures on the Earth's surface.

We can define heat flow rate as a vector

$$\vec{q} = q_x\vec{i} + q_y\vec{j} + q_z\vec{k} \tag{2.15}$$

and mathematically describe the surface in terms of a vector $\vec{A}$ perpendicular to the surface with the length corresponding to the surface area. In this case, the heat flow rate can be rewritten in terms of a scalar product between those two vectors

$$\Phi = \vec{A}\cdot\vec{q}. \tag{2.16}$$

In one dimension, for thin layers, the thickness and temperature difference in (2.6) tend to zero, $d \to \mathrm{d}x$, $\Delta\theta \to \mathrm{d}\theta$, leading to the differential version

$$\boxed{q = -\lambda\frac{\mathrm{d}\theta}{\mathrm{d}x},} \tag{2.17}$$

where we have taken into account that when temperature increases in the $+x$ direction, heat flows in the opposite $-x$ direction and vice versa.

In more complex situations, however, temperature is a function of all three coordinates, $\theta(x, y, z)$, and heat transfers in all three directions. We must therefore write Fourier's law for each of the dimensions

$$q_x = -\lambda\frac{\partial\theta}{\partial x},\ q_y = -\lambda\frac{\partial\theta}{\partial y},\ q_z = -\lambda\frac{\partial\theta}{\partial z},$$

where the symbol ∂ denotes a partial derivative. Using the density of heat flow rate vector (2.15) and *nabla operator*

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}, \tag{2.18}$$

we can transform the Fourier's law to a three-dimensional form of

$$\vec{q} = -\lambda\left(\frac{\partial\theta}{\partial x}\vec{i} + \frac{\partial\theta}{\partial y}\vec{j} + \frac{\partial\theta}{\partial z}\vec{k}\right),$$

$$\boxed{\vec{q} = -\lambda\vec{\nabla}\theta.} \tag{2.19}$$

The density of heat flow rate is therefore a product of the thermal conductivity and negative *gradient* of temperature.

The gradient operator is the generalisation of the derivative operator in multiple dimensions. If the value of a certain scalar quantity is specified by three-dimensional function $f(x, y, z)$, the gradient at an arbitrary position returns the vector with two properties:

1. The vector is directed along the increase of the function.
2. The vector length is proportional to the slope of the increase.

Figure 2.5 presents an example of a gradient for a two-dimensional function.

The scalar quantity of interest in our case is temperature, whereas the vector is the density of the heat flow rate. Because the heat flow rate is directed along the decrease of temperature, whereas the gradient is directed along the increase of the temperature, the minus sign in (2.19) is required in order to reverse the direction of the gradient vector.

For any multidimensional function, it is also possible to define contour lines, which are curves that connect points in space with the same value. The gradient vector has two properties:

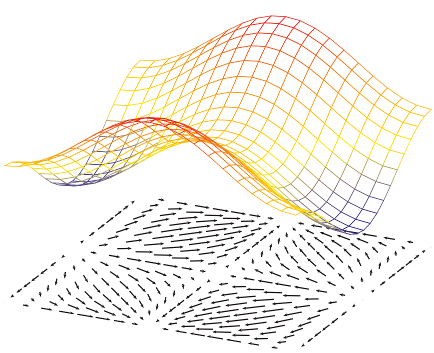

Figure 2.5: An illustrative two-dimensional function $f(x, y) = \cos x - \cos y$. The gradient (vector) is directed towards the increase of the function, and its length is proportional to the slope of the function.

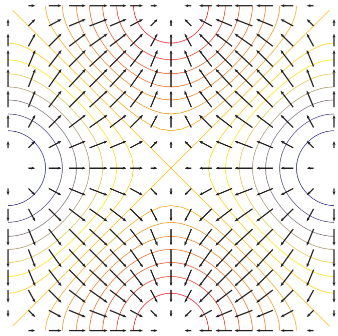

Figure 2.6: Contour lines of an illustrative two-dimensional function $f(x, y) = \cos x - \cos y$. The gradient vector is perpendicular to the contour lines, and its length is proportional to the density of the contour lines.

1. The vector is perpendicular to the contour lines.
2. The vector length is proportional to the density of the contour lines.

Figure 2.6 presents an example of contour lines and gradient for a two-dimensional function. Here we are interested in the isothermal lines, which are curves that connect points with the same temperature. The density of heat flow rate vector is therefore always perpendicular to the isothermal lines.

A practical example of two-dimensional conduction, temperatures and contours for a simple geometric thermal bridge is shown later in Fig. 3.13 on page 75.

2.2.4 Dynamic conduction

Until now, we have studied *steady heat transfer* (conduction), that is, situations with time-independent temperatures that disregard the accumulation of energy in building components. Because the temperature and internal energy of the building component are constants, the heat flow rate entering the building component on one side equals the heat flow rate leaving the component on the other side (Fig. 2.7, left).

Now we will take a closer look at *dynamic heat transfer* (conduction). Because the heat flow rate entering the building component on one side differs from the

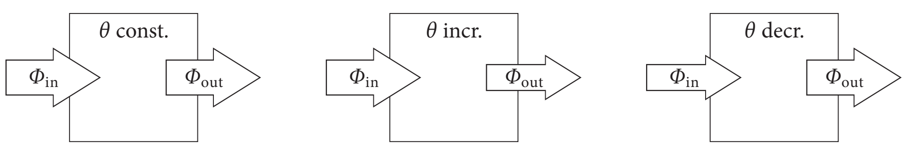

Figure 2.7: Difference between steady conduction (left) and dynamic conduction (middle, right). When heat flow rate entering building component differs from the one leaving, temperature of building component changes.

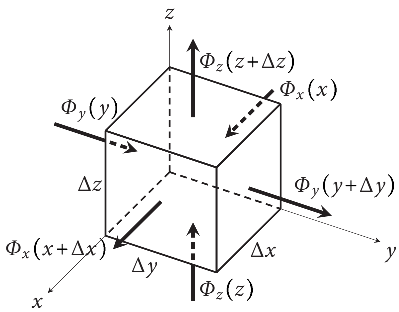

Figure 2.8: A small fragment of the building component with dimensions $\Delta x \times \Delta y \times \Delta z$. In a dynamic situation, the heat flow rate that enters the fragment is different from the heat flow rate that leaves it, which leads to the temperature change.

heat flow rate leaving the component on the other side, the building component's internal energy and temperature must change (Fig. 2.7, middle and right).

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Dynamic heat transfer conditions occur for time-dependent temperatures and are used for studying more complex processes.

In order to describe temperature change, we observe a small fragment of the building component with dimensions $\Delta x \times \Delta y \times \Delta z$ (Fig. 2.8). In a dynamic situation, the heat flow rate that enters fragment, Φ_{in}, is different from the heat flow rate that leaves it, Φ_{out}. The difference of heat flow rates corresponds to the net heat transferred to/from the fragment in a given time period (2.1):

$$\Phi_{\mathrm{net}} = \Phi_{\mathrm{in}} - \Phi_{\mathrm{out}} = \frac{\mathrm{d}Q}{\mathrm{d}t}.$$

Because the heat flow is not necessarily directed along one of the coordinate axes, we have to decompose it into components

$$\vec{\Phi} = \Phi_x \vec{i} + \Phi_y \vec{j} + \Phi_z \vec{k}.$$

As shown in Fig. 2.8, x, y and z components of heat flow rate Φ_x, Φ_y and Φ_z enter and leave the fragment along x, y and z axes, respectively. For simplicity, we will assume that $\Phi_{\mathrm{in}} > \Phi_{\mathrm{out}}$ and consequently conclude that the net heat flow rate to the fragment increases its temperature (1.20):

$$[\Phi_x(x) + \Phi_y(y) + \Phi_z(z)] - [\Phi_x(x+\Delta x) + \Phi_y(y+\Delta y) + \Phi_z(z+\Delta z)] = mc\frac{\partial \theta}{\partial t}.$$

The mass of the fragment can be written in terms of its density and volume as

$$m = \rho V = \rho\, \Delta x \Delta y \Delta z.$$

Because the heat flow rate is a smooth function, the exiting heat flow rate components can be related to the entering heat flow rate components using Taylor series

$$\Phi_x(x+\Delta x) = \Phi_x(x) + \frac{\Delta x}{1!}\frac{\partial \Phi_x}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 \Phi_x}{\partial x^2} + \cdots \approx \Phi_x(x) + \Delta x\frac{\partial \Phi_x}{\partial x},$$
$$\Phi_y(y+\Delta y) = \Phi_y(y) + \frac{\Delta y}{1!}\frac{\partial \Phi_y}{\partial y} + \frac{\Delta y^2}{2!}\frac{\partial^2 \Phi_y}{\partial y^2} + \cdots \approx \Phi_y(y) + \Delta y\frac{\partial \Phi_y}{\partial y},$$
$$\Phi_z(z+\Delta z) = \Phi_z(z) + \frac{\Delta z}{1!}\frac{\partial \Phi_z}{\partial z} + \frac{\Delta z^2}{2!}\frac{\partial^2 \Phi_z}{\partial z^2} + \cdots \approx \Phi_z(z) + \Delta z\frac{\partial \Phi_z}{\partial z},$$

where we have neglected the higher order contributions. By using the preceding equations, we obtain

$$-\Delta x\frac{\partial \Phi_x}{\partial x} - \Delta y\frac{\partial \Phi_y}{\partial y} - \Delta z\frac{\partial \Phi_z}{\partial z} = \rho c\,\Delta x\Delta y\Delta z\,\frac{\partial \theta}{\partial t}.$$

The density of heat flow rate is the heat flow rate divided by the appropriate cross-sectional area

$$q_x = \frac{\Phi_x}{\Delta y\Delta z},\ q_y = \frac{\Phi_y}{\Delta x\Delta z},\ q_z = \frac{\Phi_z}{\Delta x\Delta y},$$

leading to

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = -\rho c\frac{\partial \theta}{\partial t},$$

$$\boxed{\vec{\nabla}\cdot\vec{q} = -\rho c\frac{\partial \theta}{\partial t}.} \tag{2.20}$$

Here we used the definition of the heat flow rate as a vector (2.15) and nabla operator (2.18). This is the *heat continuity equation* in absence of heat sources, which states that the *divergence* of the density of heat flow rate is the product of the volumetric heat capacity (1.21) and temperature change. Note that materials with larger volumetric heat capacities will have smaller temperature changes for the same heat flow rate; therefore, they will damp the temporal temperature variations more intensely.

Info box

Materials that damp temporal temperature variations more intensely have larger volumetric heat capacities.

The divergence operator is somewhat similar to a gradient. It probes the values of vectors in the vicinity of the arbitrary position. If vectors pointing out of the infinitesimal environment are 'larger' than vectors pointing towards the infinitesimal environment, divergence is positive, and vice versa.

Joining the heat continuity equation (2.20) and the three-dimensional Fourier's law (2.19), we obtain the following *heat diffusion equation*:

$$\vec{\nabla}\cdot(\lambda\vec{\nabla}\theta) = \rho c\frac{\partial \theta}{\partial t}. \tag{2.21}$$

The heat diffusion equation is a partial differential equation of the second order that describes the spatial and temporal variations of temperature θ. It is also used to study heat conduction, where the equation is first solved to determine temperatures, after which the density of heat flow rate is obtained using (2.19). Finally, the heat flow rate can be obtained by integrating the density of heat flow rate over surface A using the differential version of the expression (2.16):

$$\Phi = \int_A \vec{q} \cdot \mathrm{d}\vec{A}.$$

Note that, in general, heat conductivity depends on spatial coordinates, as well as temperature and mass concentration of water w (see Section 4.4), $\lambda = f(x, y, z, \theta, w)$; therefore, solving differential equations for real problems can be extremely complicated. Usually, the solution is found by the splitting building component into parts with constant heat conductivity, in which case, (2.21) is simplified into the form

$$\frac{\partial \theta}{\partial t} = \alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right),$$

$$\boxed{\frac{\partial \theta}{\partial t} = \alpha \nabla^2 \theta,} \qquad (2.22)$$

where *thermal diffusivity* $\alpha\,(\mathrm{m}^2/\mathrm{s})$ is the ratio of heat conductivity and volumetric heat capacity (1.21):

$$\boxed{\alpha = \frac{\lambda}{\rho c}.} \qquad (2.23)$$

Values for a few typical building materials are presented in Table 2.1 on page 35. Operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \qquad (2.24)$$

is called the *Laplace operator*. For a proximity of an arbitrary position, the Laplace operator of the function presents the rate at which the average value of the function deviates from the central value as the distance increases.

Because differential equations (2.21) and (2.22) include a time derivative, we also need the following to get the solution:

- *Boundary conditions*, value of the temperature or density of heat flow rate at the boundary of the observed system.
- *Initial conditions*, value of the temperature within the observed system at the initial moment.

Practically all real problems are so complex that the solutions to (2.21) and (2.22) must be found numerically. However, there are a few idealised theoretical models that have analytical solutions and are instructive enough to be presented here. Because this is beyond the scope of this book, the mathematical procedure for solving differential equations will be skipped, and only the solution to the model will be revealed. All models of interest are one-dimensional, so we will drop the x and y dependence and look for $\theta(z, t)$ as the solution of

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial z^2}. \qquad (2.25)$$

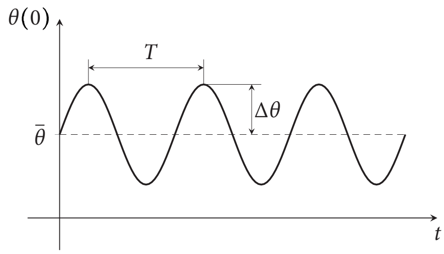

Figure 2.9: Boundary condition: harmonic variation of the temperature at the surface of the material.

2.2.5 Half-space material with the harmonic change of temperature

In the first theoretical model, we assume that the half-space ($z \geq 0$) is occupied by a material of known heat conductivity λ, density ρ and specific heat capacity c. We also assume that the boundary condition is a harmonic variation of the temperature at the surface, $z = 0$, as shown in Fig. 2.9,

$$\theta(0, t) = \bar{\theta} + \Delta\theta \sin\left(\frac{2\pi}{T} t\right),$$

where $\bar{\theta}$ is the average temperature, $\Delta\theta$ is the temperature amplitude and T is the period of temperature change (time required for the temperature to make one cycle).

The solution of equation (2.25) is

$$\theta(z, t) = \bar{\theta} + \Delta\theta\, e^{-z/d} \sin\left(\frac{2\pi}{T}(t - t_0)\right), \tag{2.26}$$

where

$$d = \sqrt{\frac{T\alpha}{\pi}}$$

is the *damping depth*, and

$$t_0 = \sqrt{\frac{T}{4\pi\alpha}}\, z$$

is the *time delay*. The validity of the solution can be checked by inserting (2.26) into (2.25).

The temporal component of the solution is displayed in Fig. 2.10. With increasing depth, the temperature oscillation amplitude decreases, and the time delay increases.

The spatial component of the solution is displayed in Fig. 2.11. On the left side, we can see how the temperature oscillation 'travels' into the interior of the material with decreasing amplitude and increasing time delay. Note that the second dot is always delayed for $T/4$ against the first dot, and the third point for $T/2$ against the first dot. This means that for a material that is deep enough, there exists a depth at which the temperature will be lowest for the highest temperature on the surface, and vice versa.

On the right side of Fig. 2.11, we can observe how smaller thermal diffusivity (or period) corresponds to a stronger reduction of amplitude and a larger time delay at the same depth z.

There are two practical applications of this theoretical model:

1. *Heat transfer dynamics within the building component*. The most important temperature cycle of interest is the daily temperature cycle, whose period is $T = 24\,\mathrm{h}$ (Fig. 2.12). The building component absorbs part of the entering heat during the day when the outside temperature is high, and then releases that heat during the night when the outside temperature is low. This damps the effect of the external temperature oscillation in the internal environment.

   The largest effect is obtained for small thermal diffusivity, that is, for small heat conductivity λ and large volumetric heat capacity ρc (2.23). Namely, as pointed out before, materials with small heat conductivity hinder heat transfer, whereas materials with large volumetric heat capacity damp temperature change. However, materials possess either high heat conductivity and volumetric heat capacity or small heat conductivity and volumetric heat capacity, so heat diffusivity variation is rather small (Table 2.1).

   The thermal diffusivity is smallest for timber, so timber is a perfect material for building components that are composed of a single material. This is demonstrated in Table 2.1 where damping depths and time delays for the most common building materials are presented. Usually, the effect of 'small' heat diffusivity for nontimber building components is achieved by combining two different materials, for example, an insulating part that provides small heat conductivity and a massive part (concrete, brick) that provides large volumetric heat capacity.

2. *Temperature of the ground*. The most important temperature cycle of interest is the yearly temperature cycle, whose period is $T = 365.24$ days

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Materials with small thermal diffusivity damp spatial and temporal temperature variations more intensely.

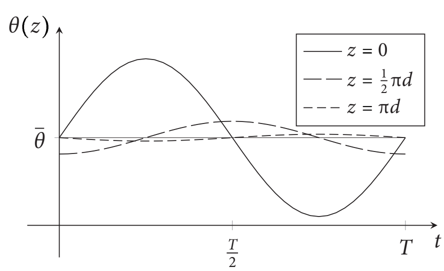


Figure 2.10: Temporal variation of the temperature for the half-space material with a harmonic change of temperature at the surface. Three depths, corresponding to the three dots shown in Fig. 2.11, are presented.

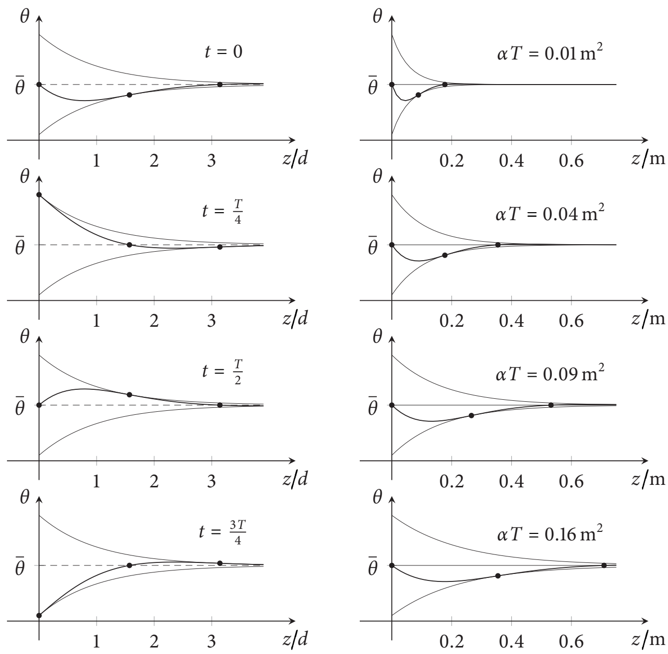


Figure 2.11: Spatial variation of the temperature for the half-space material with a harmonic change of temperature. On the left side, the temperature time evolution is shown. On the right side, the αT dependence of the spatial distribution at the initial moment is displayed.

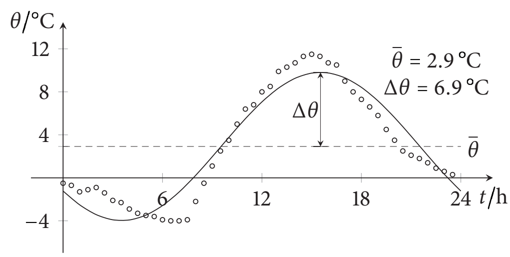


Figure 2.12: Actual temperature profile for a cloudy winter day in continental climate (open dots) and approximated harmonic function.

(Fig. 2.13). This situation is important for studying heat losses in the building due to contact with the ground.

Furthermore, (2.26) and Fig. 2.11 indicate that for large depths, the temperature equals the average yearly temperature. If the average yearly temperature exceeds 0 °C, there is a certain depth below which the ground never freezes.

This theoretical model has limited value for building components because it does not take into account that they are not infinitely deep, they consist of several different layers and that on the other side, there are also other sources of heat that

Table 2.1: Diffusive properties of most common building-related materials. The period of temperature variation is taken to be 24 h for building materials and 365.24 d for soils.

$T = 24\,\mathrm{h}$				
Material	$\alpha \Big/ 10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{s}}$	$b \Big/ \frac{\mathrm{W\,s}^{1/2}}{\mathrm{m}^2\,\mathrm{K}}$	d /cm	$\frac{t_0}{x} \Big/ \frac{\mathrm{h}}{\mathrm{cm}}$
expanded polystyrene	0.80	39	14.9	0.26
mineral wool	0.34	60	9.7	0.40
timber	0.16	320	6.7	0.57
brick, solid †	0.44	1200	11.1	0.35
brick, perforated †	0.32	390	9.4	0.40
gypsum plasterboard	0.30	380	9.1	0.42
concrete, medium density	0.75	1910	14.4	0.27
soda lime glass	0.53	1370	12.1	0.32
ceramic, porcelain	0.67	1580	13.6	0.28
steel	14.25	13 250	62.6	0.06
stone, crystalline	1.25	3130	18.5	0.21
stone, sedimentary	0.88	2450	15.6	0.24
$T = 365.24\,\mathrm{d}$				
Material	$\alpha \Big/ 10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{s}}$	$b \Big/ \frac{\mathrm{W\,s}^{1/2}}{\mathrm{m}^2\,\mathrm{K}}$	d /m	$\frac{t_0}{x} \Big/ \frac{\mathrm{d}}{\mathrm{m}}$
soil, sand/gravel	0.98	2020	3.1	17
soil, clay/silt	0.48	2170	2.2	24

influence dynamic heat transfer. To get exact results for real problems, it is necessary to solve the heat diffusion equation (2.21) numerically.

2.2.6 Contact of two materials at different temperatures

In the second theoretical model, we assume that the half-space ($z \leq 0$) is occupied by one material of known heat conductivity λ_1, density ρ_1 and specific heat capacity c_1 at initial temperature θ_1', and the other half-space ($z \geq 0$) is occupied by another material of known heat conductivity λ_2, density ρ_2 and specific heat capacity c_2 at initial temperature θ_2'. At the initial moment, we put those materials into contact, which leads to a heat flow from the material at the higher tempera-

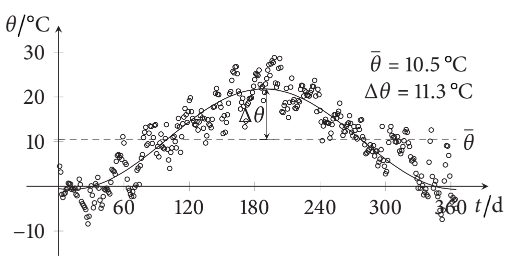


Figure 2.13: The actual temperature profile for a year in a continental climate (open dots) and the approximated harmonic function.

ture to the material at the lower temperature. The obvious boundary conditions are that the temperatures of the both materials on the common border must be equal as

$$\theta_1(z=0) = \theta_2(z=0),$$

and the density of heat flow rate leaving one material must equal the density of heat flow rate entering the other (2.17):

$$\lambda_1 \left.\frac{\mathrm{d}\theta_1}{\mathrm{d}z}\right|_{z=0} = \lambda_2 \left.\frac{\mathrm{d}\theta_2}{\mathrm{d}z}\right|_{z=0}.$$

The solution of equation (2.25) is in the form

$$\theta_1(z,t) = \theta_0 + (\theta_1' - \theta_0)\,\mathrm{erf}\left(\frac{z}{2\sqrt{\alpha_1 t}}\right),$$
$$\theta_2(z,t) = \theta_0 + (\theta_2' - \theta_0)\,\mathrm{erf}\left(\frac{z}{2\sqrt{\alpha_2 t}}\right),$$

where

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\,\mathrm{d}t$$

is the error function. The contact temperature (temperature at the border of two materials) equals

$$\theta_0 = \frac{b_1\theta_1' + b_2\theta_2'}{b_1 + b_2}, \tag{2.27}$$

where

$$b = \sqrt{\lambda\rho c} \tag{2.28}$$

is *thermal effusivity* b $(\mathrm{W\,s^{1/2}}/(\mathrm{m^2\,K}))$. Note that the contact temperature is closer to the temperature of the material with the larger effusivity.

Info box

The contact temperature is closer to the temperature of the material with the larger effusivity.

In Fig. 2.14, spatial distribution of temperature is presented after two materials with different temperatures come into contact. Constant contact temperature θ_0 is immediately established, whereas the temperature of the bulk of the material gradually drifts towards the contact temperature.

A practical application of this theoretical model is experienced when a bare foot comes into contact with a room floor. The temperature of the foot is about 36 °C, whereas, in general, the temperature of the floor equals the temperature of the room, regardless of the material. However, ceramic tiles have a much larger thermal effusivity than timber parquet (Table 2.1), which means that the contact temperature in the former case is much higher than in the latter case (2.27). This is the reason why tiles are perceived as 'colder' compared to timber parquet, despite the fact they both have the same temperature.

2.3 Convection

Heat *convection* is a mechanism of heat transfer facilitated by microscopic particles, including bulk movement of particles (advection). This mechanism appears only in fluids (see Section 1.1), that is, liquids and gases, because in order to have bulk movement of particles, they must be able to move freely. Besides *particle advection*, heat transfer is also caused by *collisions between particles* and *particle diffusion*; therefore, in principle, convection includes conduction.

Info box
Convection is facilitated by travelling particles; therefore, it occurs in liquid and gaseous states.

Convection can be classified into two groups:

1. *Natural convection* appears when the bulk movement of particles is facilitated by some natural process, for example, buoyancy.

2. *Forced convection* appears when the bulk movement of particles is facilitated by a device, for example, a fan for gases or a pump for liquids.

A practical example of convection is the central heating system in a building (Fig. 2.15). The circulation of water in the pipes system corresponds to forced convection. Water is heated in the boiler and then forced by pump to the radiators, where it is cooled down and than returned to the boiler. The circulation of the air in the room corresponds to natural convection. First, air is warmed near the radiator. Because warmer air has lower density, it rises due to buoyancy. Moving through the room, the air eventually cools down, drops to the floor and returns back to the radiator. The circulation of air is entirely due to natural processes—no device is required.

An illustration of the importance of the convection regards thermal insulators. The primary ingredient of the most common insulating materials, expanded polystyrene (EPS), extruded polystyrene (XPS), stone and glass wool, is air (Fig. 2.2 on page 23). This is not surprising because still air is a good thermal insulator (see Table A.3). However, as shown in the case of air in the room, circulating air can transfer considerably larger quantities of heat, so it is essential to prevent its movement. This is done either by closing the air off within small pockets of polystyrene bubbles (EPS and XPS) or by hindering the movement of air by

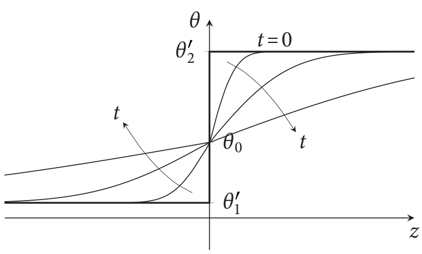


Figure 2.14: Spatial distribution of temperature for the contact of two materials at different temperatures. Initially, both materials have constant temperatures θ_1' and θ_2'. Subsequently, the temperatures of the materials near the contact gradually change towards the contact temperature θ_0.

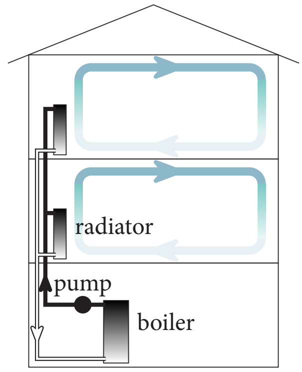


Figure 2.15: Central heating system. Forced convection is represented by the movement of water (shades of black) and natural convection by movement of air (shades of blue). (color online)

dense fibres (stone and glass wool). Note that due to the polystyrene/fibre presence, the thermal conductivity of insulating materials is somewhat larger than the thermal conductivity of the air itself.

2.3.1 Newton's law of cooling

When a radiator is opened in a cold room, air starts circulating between the warm radiator and the cold objects in the room, as shown in Fig. 2.15. This corresponds to dynamic heat transfer because the temperature of the objects and the air in the room is increasing. Eventually, the heat entering the room by the radiator equals the heat leaving the room through external building elements. This corresponds to steady heat transfer as the temperature of the objects in the room becomes time independent. In this case we can assume that all objects in the room, including the air, except the radiator and external building elements, have the same temperature. Therefore, the temperature gradients, that is, the temperature spatial variations, appear only in the thin air layer next to surfaces of the radiator and next to the external building elements.

Info box

In the steady heat transfer situation, convection is the heat transfer between the solid surface and the fluid bulk.

Generally, in the steady heat transfer situation, the temperature gradients appear only in the thin layer of the fluid next to the solid surface. Because heat transfer is possible only in the presence of temperature gradients, *convection is then the heat transfer between the solid surface and the fluid bulk.*

We will take a closer look at the microscopic picture of the convection. For simplicity, however, we will consider only the case in which the temperature of the surface is higher than the temperature of the fluid, as shown in Fig. 2.16.

The left side of Fig. 2.16 presents the microscopic picture of the forced convection. The fluid bulk velocity is v_0, and the fluid bulk temperature is θ_0. On the other hand, the solid surface temperature, as well as the fluid temperature next to the surface, is $\theta_s > \theta_0$. Due to the friction, the velocity of the fluid next to the

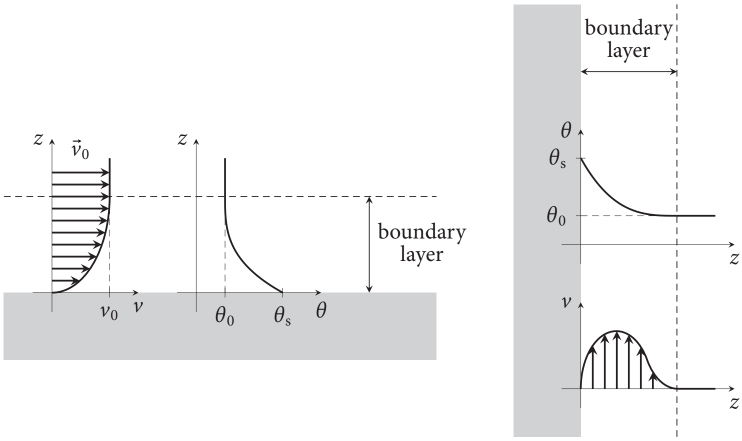


Figure 2.16: Microscopic picture of forced convection (left) and natural convection (right). In the boundary layer, the temperature and velocity of particles are gradually changing.

surface equals zero. Furthermore, because of the viscosity (friction between particles of fluid), velocity of fluid gradually changes from 0 to v_0, creating a boundary layer. In an approximately equally thick boundary layer, the temperature gradually changes from θ_s to θ_0.

The right side of Fig. 2.16 presents the microscopic picture of the natural convection. Because there is no external coercion, the fluid bulk velocity is zero, and the fluid bulk temperature is θ_0. On the other hand, the solid surface temperature, as well as the fluid temperature next to the surface, is $\theta_s > \theta_0$. Due to the higher temperature, the fluid next to the surface is thinner, and buoyancy forces it up. Nevertheless, due to the friction, the velocity of fluid next to the surface also equals zero. Due to the viscosity (friction between particles of fluid), the velocity of the fluid gradually changes from 0 to v_0 and back to 0, creating a boundary layer. In an approximately equally thick boundary layer, the temperature gradually changes from θ_s to θ_0.

In the case of central heating, forced convection corresponds to the heat transfer from the pipe to the water in the boiler (surface is hotter than fluid) and from the water to the radiator (surface is colder than fluid). On the other hand, natural convection corresponds to the heat transfer from the radiator to the air (surface is hotter than fluid) and from the air to the external wall during the winter (surface is colder than fluid).

It is also possible to have a combination of forced and natural convection. When water is forced through a vertical pipe, forced convection due to the water pump is combined with natural convection due to buoyancy. On the other hand, when a fan is put near the radiator in the room, natural convection due to buoyancy is combined with forced convection due to the fan. In either of these cases, the mechanism of heat transfer is extremely complex and beyond the scope of this book.

Info box

Convection heat flow is proportional to the temperature difference between the solid surface and the fluid bulk.

Regardless of the convection type, the heat transfer rate is proportional to the area of the surface A and the temperature difference between the surface and fluid bulk $\theta_s - \theta_0$. This statement is called *Newton's law of cooling* and can be written as

$$\Phi = A\, h_c(\theta_s - \theta_0), \tag{2.29}$$

where constant h_c $(\mathrm{W/(m^2\,K)})$ is called the *convective surface coefficient*. This statement also can be written in terms of the density of heat flow rate (2.3):

$$q = h_c(\theta_s - \theta_0). \tag{2.30}$$

Typical values of the convective surface coefficient are listed in Table 2.2. Note that the coefficient for gases is considerably smaller than the one for liquids, and the same is true for natural convection as compared to forced convection.

Convection is the principal physical mechanism behind *heat exchangers*, that is, devices that transfer heat between two or more fluids. In civil engineering it is common to encounter two types of heat exchangers:

- *Radiators* transfers heat from liquid water to air through two convective processes: first from water to an internal metal surface and then from an external metal surface to the air. The most obvious way to increase its efficiency, that is, transferred heat, is by increasing the contact area (2.29). This is especially important for metal-to-air heat transfer because the metal-to-air convective surface coefficient is much smaller than the metal-to-water coefficient (Table 2.2). The contact area is increased by a large flat design, by introducing sections and columns, and by adding convector fins, that is, the zigzagging metal strips welded to the pipes that transport liquid water (Fig. 2.17). Heat transfer can be further increased by changing from natural to forced air convection, so fans are often used to bolster the efficiency of building convector heaters and vehicle radiators.
- *Recuperators* recover energy by transferring heat between the exhaust air leaving the building and the fresh air coming into it. This way the temperature of the fresh air is brought closer to the internal temperature and less energy is being lost. We will elaborate on recuperator application in Section 3.4.

The convective surface coefficient for civil engineering cases are specified in ISO 6946 [20]. Its values for internal surfaces, or external surfaces adjacent to

Table 2.2: Typical values of the convective surface coefficient depending on the fluid type [1].

$h_c \big/ \frac{\mathrm{W}}{\mathrm{m^2\,K}}$	Type of Convection	
	Natural	Forced
gas	2–25	25–250
liquid	50–1000	100–20 000

Figure 2.17: Details of a domestic radiator. In order to increase the heat flow rate, the metal-to-air contact area is increased by the flat design, introducing sections and columns, and adding convector fins.

Table 2.3: Convective surface coefficients in civil engineering [20].

$h_c \,/\, \frac{\mathrm{W}}{\mathrm{m^2\,K}}$	Direction of Heat Flow		
	Upward	Horizontal	Downward
internal, h_{ci}	5.0	2.5	0.7
external, h_{ce}	$4 + 4\,v$	$4 + 4\,v$	$4 + 4\,v$

a well-ventilated air layer, are shown in Table 2.3. At external surfaces, the coefficient can be calculated using the expression

$$h_{ce} = 4 + 4\,v,$$

where v (m/s) is the wind speed adjacent to the surface.

Note that in the winter, perception of coldness is increased with wind speed. This is due to the fact that wind increases convection heat flow from human skin to the air and increases the cooling of exposed body parts.

2.4 Radiation

Radiation is a mechanism of heat transfer that does not require matter and can occur even in a vacuum (absence of matter). Radiation in general implies many forms of energy transmission, but with heat transfer, it applies exclusively to *electromagnetic waves*. Because electromagnetic waves are emitted by all bodies (whose temperature is above absolute zero), they represent an important contribution to heat transfer.

Info box

Radiation is facilitated by electromagnetic waves; therefore it primarily occurs in gases and vacuums.

A typical example of heat transfer by radiation occurs between the Sun and the Earth. Because these two objects are separated by a vacuum, no other mechanism of heat transfer can occur.

Bodies emit electromagnetic waves in a broad spectrum (wide range of wavelengths), and the density of heat flow rate is dependent on the body temperature. We will first address the radiation of *black bodies*, which are idealised physical objects that absorb all incident electromagnetic radiation. For a black body at

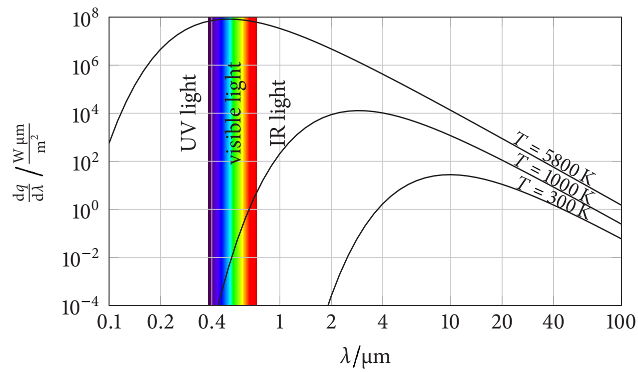


Figure 2.18: The spectral density of heat flow rate for three prominent temperatures, most notably Sun temperature and room temperature. Only very hot objects can radiate in the visual spectrum.

temperature T, the emitted density of heat flow rate per wavelength λ or *spectral density of heat flow rate* $\mathrm{d}q/\mathrm{d}\lambda$ (W/m) is described by Planck's law

$$\frac{\mathrm{d}q}{\mathrm{d}\lambda}(\lambda, T) = \frac{2\pi h c_0^2}{\lambda^5 \left[\exp\left(\frac{h c_0}{\lambda k T}\right) - 1\right]},$$

where $c_0 = 2.998 \times 10^8$ m/s is the speed of light in a vacuum, $h = 6.626 \times 10^{-34}$ J s is Planck's constant and $k = 1.381 \times 10^{-23}$ J/K is Boltzmann's constant.

Info box
Objects at the higher temperature radiate shorter wavelengths.

The spectral density of heat flow rate for three temperatures, most notably the temperature of the Sun $T \approx 5800$ K and room temperature $T \approx 300$ K, is shown in Fig. 2.18. Note that at higher temperatures, the total emission is larger, while the spectrum is moved to shorter wavelengths.

Bodies at temperatures similar to the temperature of the Sun emit radiation with a maximum in the visual spectrum, $4\,\mu\mathrm{m} > \lambda > 7.5\,\mu\mathrm{m}$. Incandescent light bulbs mimic the Sun in that a filament wire within the bulb is heated by electric current to temperature $T \approx 2700$ K (Fig. 2.19). At even lower temperatures, the typical centre of the fire and steel forging temperature is $T \approx 1200$ K, and only the red part of the visual spectrum is emitted. Ordinary objects at room temperatures primarily emit infrared radiation and hardly any visible light, which is why we cannot see them without the presence of light sources.

Usually, the most important physical quantity of interest is the total density of heat flow rate. We can get its value for the black body by integrating Planck's law for all wavelengths

$$q(T) = \int_0^\infty \frac{\mathrm{d}q}{\mathrm{d}\lambda}(\lambda, T)\mathrm{d}\lambda$$

$$\Longrightarrow q(T) = \sigma T^4. \tag{2.31}$$

Info box
Objects at higher temperatures radiate more intensely.

The obtained expression is called the *Stefan-Boltzmann law*, and constant $\sigma = 5.670 \times 10^{-8}$ W/(m^2 K^4) is called the Stefan-Boltzmann constant.

Example 2.1: Solar constant.

Calculate the total heat flow rate of the Sun, assuming that the temperature of Sun's surface is $T_{\text{Sun}} = 5780\,\text{K}$ and Sun's radius is $r_{\text{Sun}} = 6.96 \times 10^5\,\text{km}$. Calculate the value of the *solar constant*—density of heat flow rate at the outer edge of the Earth's atmosphere—if the distance from the Sun is about $r = 1.50 \times 10^8\,\text{km}$. Assume that the Sun is a perfect black body.

The density of heat flow rate of the Sun is (2.31)

$$q_{\text{Sun}} = \sigma\, T_{\text{Sun}}^4,$$

whereas the total flow rate is (2.3)

$$\Phi_{\text{Sun}} = A_{\text{Sun}}\, q_{\text{Sun}} = 4\pi r_{\text{Sun}}^2\, \sigma\, T_{\text{Sun}}^4 = 3.85 \times 10^{26}\,\text{W}.$$

We assume that radiation is spreading uniformly through the space, so the heat flow rate through any element of an imaginary spherical surface centred at the Sun must be the same. To get the heat flow rate on distance r from the Sun, we must therefore divide the total heat flow of the Sun with the area of the spherical surface of the same radius (2.3):

$$q = \frac{\Phi_{\text{Sun}}}{A} = \frac{\Phi_{\text{Sun}}}{4\pi r^2} = 1360\,\frac{\text{W}}{\text{m}^2}.$$

Note that the heat flow rate is considerably smaller at the surface of the Earth because the atmosphere absorbs and reflects a part of the incident radiation.

The whole Earth is in principle irradiated by the same density of heat flow rate; however, some parts of the Earth are considerably colder than others. The warmest parts are those where the Sun is close to the zenith, that is, for the angle of incidence 0°. For parts of the Earth north and south of this position, the angle of incidence increases, so the same area of the Earth's surface gets a smaller heat

Figure 2.19: An incandescent light bulb emits visual, short wavelength radiation due to the high temperature of the filament, which is heated by an electric current.

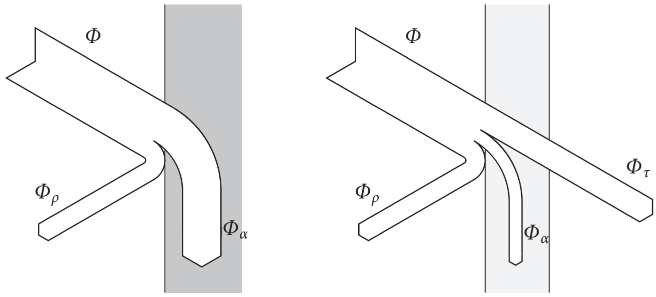


Figure 2.20: Radiation processes for nontransparent (left) and transparent (right) objects. Part of the incident radiation (Φ) is reflected (Φ_ρ), part is absorbed (Φ_α) and part is transmitted (Φ_τ).

flow rate (2.14). Further, the width of the atmosphere that has to be penetrated is larger there. This principle also explains the seasonal variation in temperatures.

Radiation also explains why mornings after clear sky nights are colder than mornings after cloudy nights. During the night, the Earth's surface radiation is much more intensive than the incoming radiation from stars. When radiation is not obstructed by the clouds, lost energy is much larger.

2.4.1 Grey bodies

So far we have been concerned with the idealised black body. In reality, most objects are *grey bodies* because part of the incident radiation is absorbed, part is transmitted and part is reflected (Fig. 2.20). If we denote the heat flow rates for incident radiation Φ, for reflected radiation Φ_ρ, for absorbed radiation Φ_α and for translated radiation Φ_τ, we can define *reflectance* ρ, *absorptance* α and *transmittance* τ as

$$\rho(\lambda) = \frac{\Phi_\rho(\lambda)}{\Phi(\lambda)}, \tag{2.32}$$

$$\alpha(\lambda) = \frac{\Phi_\alpha(\lambda)}{\Phi(\lambda)}, \tag{2.33}$$

$$\tau(\lambda) = \frac{\Phi_\tau(\lambda)}{\Phi(\lambda)}. \tag{2.34}$$

Here we took into account that all three physical quantities depend on the radiation wavelength. Note that their value range is $0 \leq \rho(\lambda), \alpha(\lambda), \tau(\lambda) \leq 1$.

Because energy is conserved in the process, the sum of the heat flow rates for the reflected, absorbed and translated radiation should equal the density of heat flow rate for incident radiation

$$\Phi_\rho(\lambda) + \Phi_\alpha(\lambda) + \Phi_\tau(\lambda) = \Phi(\lambda).$$

Together with the definition of reflectance, absorptance and transmittance, this leads to

$$\rho(\lambda) + \alpha(\lambda) + \tau(\lambda) = 1. \tag{2.35}$$

Figure 2.21: The same scene recorded by an optical camera (left) and an infrared camera (right). Because the glass transmits short wavelength (visual) radiation but does not transmit long wavelength (infrared) radiation, the infrared emitting objects are screened by the glass. Infrared thermography will be elaborated on in Section 2.4.4.

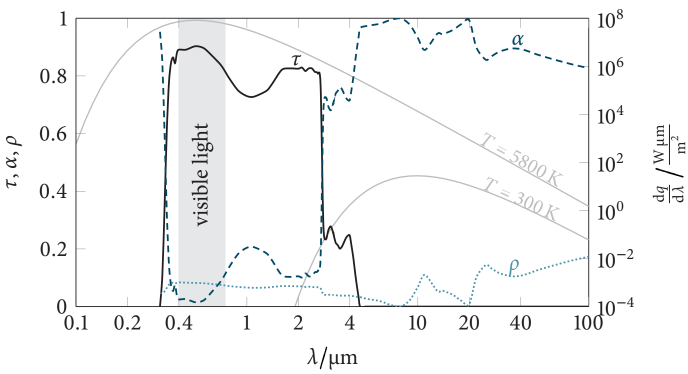


Figure 2.22: Spectral transmittance, absorptance and reflectance for common window glass [49]. Most short wavelength electromagnetic radiation (including visible light) emitted by the Sun is transmitted. Most long wavelength electromagnetic radiation is absorbed.

For an ideal black body, $\alpha(\lambda) = 1$, $\rho(\lambda) = \tau(\lambda) = 0$ for all wavelengths.

A common material with unique and prominent radiation properties is glass. Its transmittance for short wavelength (visual) radiation is rather large, $\tau \approx 0.85$, whereas transmittance for long wavelength (infrared) radiation is small, $\tau \approx 0$. We can demonstrate these features by photographing scenes behind glass using optical and infrared cameras, as shown in Fig. 2.21. For the infrared camera, infrared-emitting objects are screened by the glass.

The radiant properties of glass (Fig. 2.22) also explain the greenhouse effect. The Sun radiates mostly within the short wavelength spectrum, $\lambda < 3\,\mu\text{m}$, and this radiation is transmitted through the window glass from the exterior to the interior of the building (Fig. 2.23). The Sun's radiation is absorbed by objects in the building, increasing their temperature. Because the objects are at a much lower

Figure 2.23: Quintessential greenhouse with walls of glass [50]. Temperatures within the greenhouse are elevated due to the greenhouse effect of the glass.

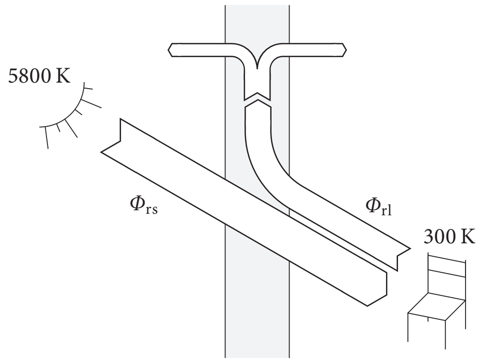


Figure 2.24: Greenhouse effect. Short wavelength radiation emitted by the Sun can enter essentially unobstructed, whereas long wavelength radiation emitted by room objects can escape the interior of the building only indirectly and partially.

temperature, they radiate mostly within the long wavelength spectrum, $\lambda > 3\,\mu m$, which is not transmitted but instead absorbed by the window glass. The absorbed radiation is then partially emitted back to the interior and partly to the exterior of the building. Therefore, by means of radiation, heat can enter essentially unobstructed, but it leaves the interior of the building only indirectly and partially (Fig. 2.24).

A similar effect can be observed in the Earth's atmosphere, where greenhouse gases (H_2O, CO_2, CH_4) act as a window glass.

Finally, an important property of grey bodies is that they emit less radiation than black bodies at the same temperature. To account for that phenomenon, we also define *emittance* ε as the ratio of heat flow rates, one emitted by grey body Φ to one emitted by black body Φ_0 at the same temperature T:

$$\varepsilon(\lambda) = \frac{\Phi(\lambda, T)}{\Phi_0(\lambda, T)}. \tag{2.36}$$

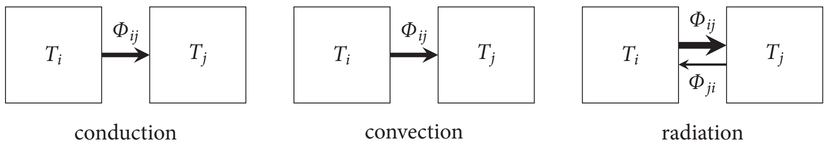


Figure 2.25: Differences between conduction, convection and radiation among two bodies, where $T_i > T_j$. In conduction and convection, heat is transferred only from higher to lower temperatures; in radiation heat, is transferred in both directions.

2.4.2 Net radiation exchange between surfaces

We have thus far only discussed radiation emitted by a single body. Now we will pay more attention to the problem of radiative exchanges between two or more bodies.

In conduction and convection, heat is transferred only from higher to lower temperatures. However, because all bodies radiate, radiation heat is always transferred in both directions—from the body with the higher temperature to the body with the lower temperature and vice versa (Fig. 2.25). Our concern is the *net radiation* Φ_{net} as the difference between radiation that goes from the i-th body to the j-th body and the radiation that goes from the j-th body to the i-th body:

Info box
For radiative heat exchange between two bodies, energy travels both ways, so we are interested in net value.

$$\Phi_{\text{net}} = \Phi_{ij} - \Phi_{ji}.$$

Net radiation is always directed from the body with the higher temperature to the body with the lower temperature. As we pointed out in Section 1.2, if both bodies possess the same temperature and are in thermal equilibrium, the heat flow rates in both directions are equal, and the net radiation heat flow rate is zero.

View factor

In order to address that problem properly, we will study the exchange between body surfaces instead of the exchange between bodies. The reason for this is quite intuitive. Energy is transferred by radiation only in an empty space, for example, in a vacuum and in gases, which means only bodies that share a well-defined enclosure can exchange heat by this mechanism. However, only those parts of surfaces that confine the enclosure, not the whole surface of the bodies, participate in the exchange, as shown in Fig. 2.26.

Net radiation depends on temperatures and radiative properties of both surfaces. However, it is also important to take into account that only part of the radiation that one surface emits is intercepted by another surface, which means that the net radiation also depends on the surface geometries and orientations. The physical quantity that takes into consideration surface geometries and orientations is called *view factor* F. It is defined as the ratio of the heat flow rate transferred from surface i to surface j Φ_{ij} to heat flow rate that is emitted from surface i Φ_i, that is,

$$\Phi_{ij} = F_{ij}\Phi_i. \tag{2.37}$$

Obviously, $1 \geq F_{ij} \geq 0$. Because radiation propagates in straight lines, no radiation that leaves a convex surface can strike back (see Example 2.2). Hence, for convex surfaces, $F_{ii} = 0$. On the other hand, for concave surfaces, $F_{ii} > 0$.

All radiation emitted by surface i must be received by other surfaces that confine the same enclose, hence

$$\Phi_i = \sum_j \Phi_{ij} = \sum_j F_{ij}\Phi_i.$$

Cancelling out Φ_i on both sides, we obtain the *summation rule* stating

$$\sum_j F_{ij} = 1. \tag{2.38}$$

The net radiation exchange between two black surfaces at the same temperature should be zero (Section 1.2):

$$\Phi_{\text{net}} = \Phi_{ij} - \Phi_{ji} = F_{ij}\Phi_i - F_{ji}\Phi_j = F_{ij}A_i q_i - F_{ji}A_j q_j = 0.$$

It follows from the Stefan-Boltzmann law (2.31) that in case of the same temperature, $T_i = T_j$, the densities of heat flow rates should also be the same, $q_i = q_j$, leading to the *reciprocity rule* of

$$F_{ij}A_i = F_{ji}A_j. \tag{2.39}$$

The net radiation exchange between two black surfaces with different temperatures is therefore

$$\Phi_{\text{net}} = F_{ij}A_i\sigma\left(T_i^4 - T_j^4\right). \tag{2.40}$$

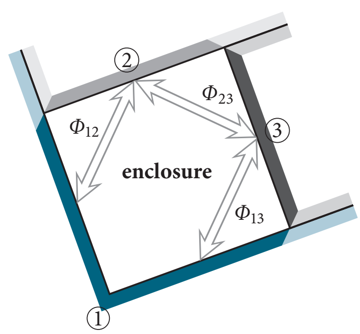
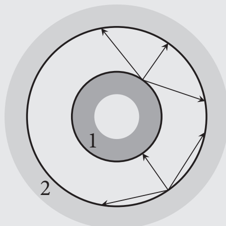


Figure 2.26: Bodies 1, 2 and 3 share a common enclosure and can therefore exchange heat by radiation. Note that only the darker shaded parts of the surfaces that confine the enclosure, not the whole surfaces of the bodies, participate in the exchange.

Example 2.2: Radiation between two spherical surfaces.

Calculate the view factors for a two-surface enclosure limited by two concentric spherical surfaces of areas A_1 and A_2.



Because radiation always follows straight lines, all radiation leaving the internal surface 1 must strike the external surface 2, so

$$F_{11} = 0, \; F_{12} = 1,$$
$$F_{11} + F_{12} = 1,$$

where the latter equation is simply confirmation of (2.38).
On the other hand, part of the radiation leaving the external surface 2 comes back to surface 2, hence

$$F_{22} > 0, \; F_{21} < 1.$$

Using (2.39), we get

$$F_{12}A_1 = F_{21}A_2 \implies F_{21} = \frac{A_1}{A_2}.$$

Finally, from (2.38), we get

$$F_{21} + F_{22} = 1 \implies F_{22} = 1 - \frac{A_1}{A_2}.$$

We will derive the exact expression for calculating view factors in Section 8.3.4.

Kirchhoff's law

By considering a simple radiation transfer between black and grey surfaces, we can derive the relation between emittance and absorptance. We consider an enclosure surrounded by two convex surfaces: one black surface and one nontransparent grey ($\alpha + \rho = 1$) surface in thermal equilibrium, that is, at the same temperature T. Because surfaces are convex, all emitted radiation from the black surface strikes the grey surface and vice versa (Fig. 2.27), that is, $F_{12} = F_{21} = 1$ and $F_{11} = F_{22} = 0$. The black surface emits heat flow rate Φ_0, of which $\alpha\Phi_0$ is absorbed by the grey surface, and $(1 - \alpha)\Phi_0$ is reflected back and fully absorbed by the black surface. The grey surface emits heat flow rate Φ, which is fully absorbed by the black surface. Because both bodies are in thermal equilibrium, absorbed and emitted energy should be equal

$$\begin{aligned} \Phi &= \alpha\Phi_0, && \text{grey surface,} \\ \Phi_0 &= \Phi + (1-\alpha)\Phi_0, && \text{black surface,} \\ \Longrightarrow \varepsilon &= \frac{\Phi}{\Phi_0} = \alpha. \end{aligned}$$

We see that emittance and absorptance are always equal:

$$\varepsilon(\lambda) = \alpha(\lambda). \tag{2.41}$$

That statement is called *Kirchhoff's law*.

Exchange between two grey surfaces

Although we have already calculated the net radiation exchange between two arbitrary black surfaces (2.40), the exchange between two grey surfaces would be much more useful. The general solution is extremely complex and not very instructive, so we will limit the discussion to a special case that meets two conditions:

1. The enclosure is surrounded by only two surfaces, $F_{11} + F_{12} = F_{21} + F_{22} = 1$.
2. Both surfaces are nontransparent, $\alpha_1 + \rho_1 = \alpha_2 + \rho_2 = 1$.

The schematic of the net radiation exchange between two nontransparent grey surfaces is presented in Fig. 2.28, where we assumed that the net heat transfer goes from body 1 to body 2. Note that because we have only two bodies, net heat transfer Φ_{net} is the same in the enclosure and within both surfaces. We denote q_{b1} and q_{b2} as the *black body* density of heat flow rate, q_{e1} and q_{e2} the *net emitted* density of heat flow rate and q_{i1} and q_{i2} the *net incident* density of heat flow rate for the first and second body, respectively. The net radiative exchange between the two bodies written in the enclosure is

$$\Phi_{net} = \Phi_{12} - \Phi_{21} = A_1F_{12}q_{e1} - A_2F_{21}q_{e2} = A_1F_{12}(q_{e1} - q_{e2}). \tag{2.42}$$

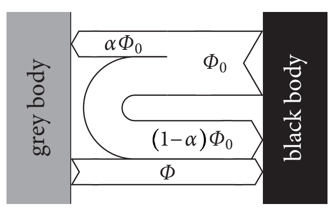


Figure 2.27: Transfer between two convex surfaces, one black and one grey, surrounding the same enclosure. The black surface emits heat flow rate Φ_0, of which $\alpha\Phi_0$ is absorbed by the grey surface, and $(1-\alpha)\Phi_0$ is reflected back and reabsorbed by the black surface. The grey surface emits heat flow rate Φ, which is completely absorbed by the black surface.

We want to express q_{e1} and q_{e2} in terms of body characteristics, ε_1, ε_2, q_{b1} and q_{b2}. As shown in Fig. 2.28, the emission consists of black body emitted radiation, multiplied by emittance, and the reflected part of the incident radiation:

$$q_{e1} = \varepsilon_1 q_{b1} + \rho_1 q_{i1} = \varepsilon_1 q_{b1} + (1-\varepsilon_1) q_{i1},$$
$$q_{e2} = \varepsilon_2 q_{b2} + \rho_2 q_{i2} = \varepsilon_2 q_{b2} + (1-\varepsilon_2) q_{i2}.$$

In order to eliminate q_{i1} and q_{i2}, we will write the net radiative exchange in both surfaces as

$$\Phi_{net} = A_1(\varepsilon_1 q_{b1} - \alpha_1 q_{i1}) = A_1\varepsilon_1\left(q_{b1} - \frac{q_{e1}-\varepsilon_1 q_{b1}}{1-\varepsilon_1}\right) = \frac{\varepsilon_1}{1-\varepsilon_1}A_1(q_{b1}-q_{e1}),$$
$$\Phi_{net} = A_2(\alpha_2 q_{i2} - \varepsilon_2 q_{b2}) = A_2\varepsilon_2\left(\frac{q_{e2}-\varepsilon_2 q_{b2}}{1-\varepsilon_2} - q_{b2}\right) = \frac{\varepsilon_2}{1-\varepsilon_2}A_2(q_{e2}-q_{b2}).$$

We can therefore rewrite the preceding expressions as

$$\Phi_{net} = \frac{q_{b1}-q_{e1}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}} = \frac{q_{e1}-q_{e2}}{\frac{1}{A_1F_{12}}} = \frac{q_{e2}-q_{b2}}{\frac{1-\varepsilon_2}{A_2\varepsilon_2}}.$$

Note the similarity of this equation to (2.12). The solution is

$$\Phi_{net} = \frac{q_{b1}-q_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}},$$

$$\boxed{\Phi_{net} = \frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}}.} \qquad (2.43)$$

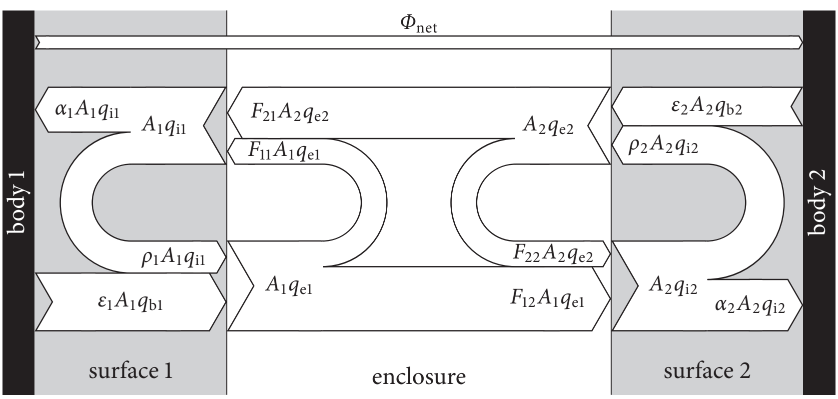


Figure 2.28: Net radiation exchange between two nontransparent grey surfaces.

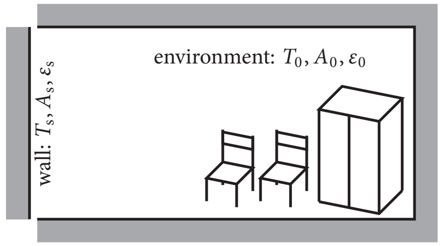


Figure 2.29: Radiative transfer between the wall surface and the environment. All radiation emitted by the wall surface must be intercepted by the room environment surfaces, hence, $F_{s0} = 1$.

2.4.3 Radiative transfer between the wall surface and the environment

Expression (2.43) can be substantially simplified for most typical civil engineering problems. The most important problem is the net radiation between the external wall of the building and its environment. For the wall external surface, the environment is the building vicinity, whereas for the wall internal surface, the environment is the rest of the room (Fig. 2.29).

We have already discussed in Section 2.3.1 that for *steady heat transfer*, we can assume that all objects in the room, including the air, except the radiator and external building elements, have the same temperature. Because the temperature of all objects in the environment is the same, the net exchange of radiation for each pair of objects is zero, and the only nonzero net exchange of radiation happens between the wall surface and the environment. Thus, we can consider the environment as a single surface of area A_0, average temperature T_0 and average emittance ε_0. On the other hand, the wall is a convex surface of area A_s, temperature T_s and emittance ε_s. All the radiation that leaves the surface of the wall is intercepted by the environment surface, so the view factor is $F_{s0} = 1$. Because $A_0 \gg A_s$, from (2.43), we get

Info box

In civil engineering, we are interested in radiation exchange between the wall surface and its adjacent environment.

$$\Phi_{\text{net}} = A_s \sigma \varepsilon_s (T_s^4 - T_0^4).$$

On the Kelvin scale, temperatures of the wall and environment surfaces do not differ significantly. We can linearise the difference in temperature to the fourth power by defining average temperature $\overline{T}$ and temperature difference ΔT, where $\Delta T \ll \overline{T}$:

$$\left.\begin{aligned} \overline{T} &= \frac{T_0 + T_s}{2} \\ \Delta T &= T_0 - T_s \end{aligned}\right\} \implies \left\{\begin{aligned} T_s &= \overline{T} + \tfrac{\Delta T}{2} \\ T_0 &= \overline{T} - \tfrac{\Delta T}{2}. \end{aligned}\right.$$

Using these expressions, we can write

$$\begin{aligned} T_s^4 &= \overline{T}^4 + 4\overline{T}^3 \tfrac{\Delta T}{2} + 6\overline{T}^2 \left(\tfrac{\Delta T}{2}\right)^2 + 4\overline{T}\left(\tfrac{\Delta T}{2}\right)^3 + \left(\tfrac{\Delta T}{2}\right)^4 \approx \overline{T}^4 + 4\overline{T}^3 \tfrac{\Delta T}{2} \\ T_0^4 &= \overline{T}^4 - 4\overline{T}^3 \tfrac{\Delta T}{2} + 6\overline{T}^2 \left(\tfrac{\Delta T}{2}\right)^2 - 4\overline{T}\left(\tfrac{\Delta T}{2}\right)^3 + \left(\tfrac{\Delta T}{2}\right)^4 \approx \overline{T}^4 - 4\overline{T}^3 \tfrac{\Delta T}{2} \\ \implies & T_s^4 - T_0^4 = 8\overline{T}^3 \tfrac{\Delta T}{2} = 4\overline{T}^3 (T_s - T_0). \end{aligned}$$

Taking this into account, we get

$$\Phi_{\text{net}} = A_s h_r (T_s - T_0),$$

where

$$h_r = 4\sigma\varepsilon_s \overline{T}^3 \tag{2.44}$$

is *radiative surface coefficient* h_r (W/(m^2 K)). Because the temperature differences on the Kelvin and Celsius temperature scales are the same, we finally write

$$\Phi = A\, h_r(\theta_s - \theta_0). \tag{2.45}$$

Info box

The radiation heat exchange is proportional to the temperature difference between the wall surface and the adjacent environment.

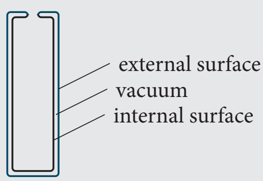
Note the similarity of this law to Newton's law of cooling (2.29). This statement also can be written in terms of the density of heat flow rate (2.3) as

$$q = h_r(\theta_s - \theta_0). \tag{2.46}$$

The obtained expression for the radiative surface coefficient is used by standard ISO 6946 [20]. Typical values of the radiative surface coefficient in building physics are 3 W/(m^2 K) to 5 W/(m^2 K).

Example 2.3: Vacuum flask.

A vacuum flask in the shape of a cylinder consists of two polished steel vessels, the internal of height h_1 = 30.0 cm and radius r_1 = 4.0 cm and the external of height h_2 = 32.0 cm and radius r_2 = 5.0 cm. The gap between the vessels is evacuated of air, and the emittance of the vessel walls is $\varepsilon_1 = \varepsilon_2 = 0.07$. Ice of mass m = 300 g at temperature θ_i = 0 °C is inserted into vacuum flask. Calculate the ice melting time if the external temperature is θ_e = 25 °C by using an exact and linearised expression. Neglect losses through the flask neck. The specific heat of fusion for ice is q_f = 336 kJ/kg.



The vacuum flask principle of operation is based on the vacuum in the gap between the two vessels. Without material particles, conduction and convection are disabled. The heat flow rate is greatly reduced because heat is transferred only by radiation. A smaller heat flow rate means that contents of the vacuum flask are cooled or warmed (depending on the contents) much slower than in other vessels.
The areas of two vessels amount to

$$A_1 = 2\pi r_1 h + 2\,\pi r_1^2 = 0.116\,\text{m}^2,$$
$$A_2 = 2\pi r_2 h + 2\,\pi r_2^2 = 0.085\,\text{m}^2.$$

The enclosure between the two vessels is surrounded only by two surfaces, so we can use expression (2.43). The internal surface 1 is at temperature $\theta_1 = \theta_i = 273\,\mathrm{K}$, the external surface 2 is at temperature $\theta_2 = \theta_e = 298\,\mathrm{K}$ and because internal surface is convex, $F_{12} = 1$ (see Example 2.2). Taking all this into account, we get

$$\Phi = \frac{\sigma\left(T_i^4 - T_e^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = -0.470\,\mathrm{W}.$$

The linearised form of this expression is

$$\Phi = \frac{4\sigma\overline{T}^3\left(T_i - T_e\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = -0.469\,\mathrm{W}.$$

The value of the heat flow rate is negative because the internal surface receives more heat than it emits. The heat required to melt the ice equals (1.27, 2.2)

$$Q = m\,q_f = |\Phi|\,t \implies t = \frac{m\,q_f}{|\Phi|} = \begin{cases} 59.6\,\mathrm{h}, & \text{(exact)} \\ 59.7\,\mathrm{h}. & \text{(linearised)} \end{cases}$$

As pointed out before, the difference between the results for exact and linearised expressions is negligible.

2.4.4 Infrared thermography

The fact that the quantity of emitted radiation depends on the temperature of the surface can be used for noncontact temperature measurement. In civil engineering (as well as in many other fields), we are primarily concerned with temperature ranges close to room temperatures and with spatial distribution of the temperatures. This interest led to the development of a special *infrared thermography* discipline.

Objects close to room temperatures emit most of the radiation in the infrared range. Thus, thermographic instruments detect radiation in the infrared range (commonly from 8 µm to 14 µm) and produce images of that radiation.

The study of the infrared thermal camera operation involves surfaces of three distinct entities, the object of interest at temperature T_{obj}, the instrument at temperature T_{ins} and the ambient at temperature T_{amb}. The situation is much more complicated than the case of radiative exchange between two surfaces, studied in Section 2.4.2, so we will take a simplified approach.

The object emitted density of the radiative heat flow rate is composed of thermal radiation due to the object and ambient radiation reflected from the object

$$q_{in} = \varepsilon\,\sigma\,T_{obj}^4 + (1-\varepsilon)\,\sigma\,T_{amb}^4.$$

Here we have assumed that the object is nontransparent, $\rho = 1 - \varepsilon$, and that ambient is the perfect black body. The total thermal exchange between instrument and object is

$$\Phi_{\mathrm{net}} = \Phi_{\mathrm{in}} - \Phi_{\mathrm{out}} = F_{\text{obj-ins}}\, A_{\mathrm{obj}} \left(q_{\mathrm{in}} - q_{\mathrm{out}}\right),$$

where $F_{\text{obj-ins}}$ is the view factor between the object and instrument, and A_{obj} is the area of the object. Assuming that the instrument is a perfect black body, the equation is transformed to

$$\Phi_{\mathrm{net}} = F_{\text{obj-ins}}\, A_{\mathrm{obj}}\, \sigma \left[\varepsilon\, T_{\mathrm{obj}}^4 + (1-\varepsilon)\, T_{\mathrm{amb}}^4 - T_{\mathrm{ins}}^4\right].$$

We can calculate the view factor by taking into account the reciprocity rule (2.39) and the view factor expression (8.31)

$$A_{\mathrm{obj}}\, F_{\text{obj-ins}} = A_{\mathrm{ins}}\, F_{\text{ins-obj}} = \frac{\Omega_{\text{ins-obj}}\, \cos\theta_{\mathrm{ins}}}{\pi}\, A_{\mathrm{ins}},$$

where A_{ins} is the sensor (or aperture) area of the thermographic camera, and θ_{ins} is the angle of incidence at the instrument. This leads to

$$\Phi_{\mathrm{net}} = \frac{\Omega_{\text{ins-obj}}\, \cos\theta_{\mathrm{ins}}}{\pi}\, A_{\mathrm{ins}}\, \sigma \left[\varepsilon\, T_{\mathrm{obj}}^4 + (1-\varepsilon)\, T_{\mathrm{amb}}^4 - T_{\mathrm{ins}}^4\right].$$

Finally, the instrument signal U is proportional to the total thermal exchange

$$\begin{aligned} U &= C\, \Phi_{\mathrm{net}} \\ &= C\, \frac{\Omega_{\text{ins-obj}}\, \cos\theta_{\mathrm{ins}}}{\pi}\, A_{\mathrm{ins}}\, \sigma \left[\varepsilon\, T_{\mathrm{obj}}^4 + (1-\varepsilon)\, T_{\mathrm{amb}}^4 - T_{\mathrm{ins}}^4\right]. \end{aligned} \tag{2.47}$$

Info box

Thermographical determination of the object's temperature requires knowledge of the object's emittance and the ambient and instrument temperatures.

Therefore, to determine its temperature, we do not have to know either the distance or the area of the measured surface, but only its solid angle and its direction, both easily obtainable from the perspective of the instrument.

Thermographic cameras measure the total thermal exchange in the specified infrared range without distinguishing between different wavelengths. The obtained numerical result can be represented as a monochromatic image. However, more often results are displayed in false colour, where changes in colour rather than changes in intensity are used to represent the result. Usually, but not necessarily, the parts of the image with the highest temperatures appear white, intermediate temperatures appear in reds and yellows, and lowest temperatures appear black (Fig. 2.30). The false colours in the image are therefore completely unrelated to what we usually perceive as a colour in the visual spectrum and merely represent the temperatures.

The thermographic image is very helpful in building physics in order to identify the temperatures of the internal and external building surfaces. As shown in Fig. 2.30, the building on a winter day with a thicker thermal insulation has lower surface temperatures. From equations (2.29) and (2.45), we see that lower surface temperatures and a smaller temperature differences imply smaller heat flow rate. Parts of the façade with higher temperatures reveal the weak spots in the thermal protection of the building. We will discuss this in more detail in Section 3.2.2.

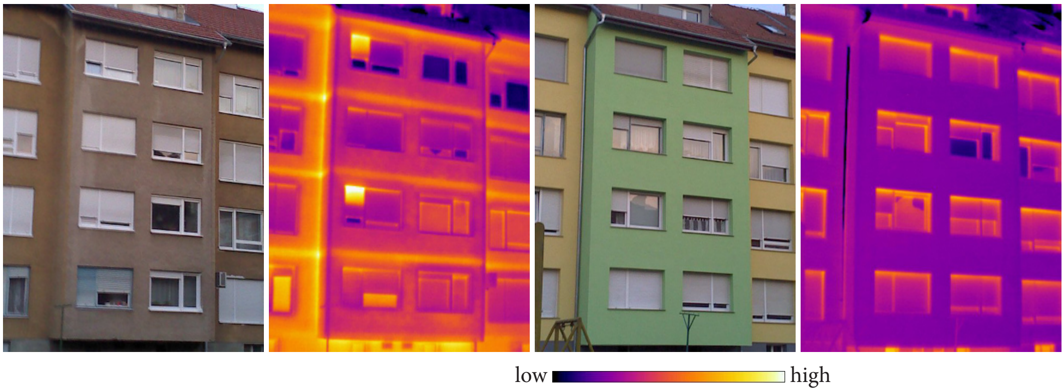


Figure 2.30: Thermographic camera image of the building on a winter day. The left side images present part of the building with a thin thermal insulation, and the right side images present part of the building with additional thermal insulation. Thicker and better implemented thermal insulation reduces the heat flow rate and lowers surface temperatures.

In most practical situations, the most crucial quantity in (2.47), apart from the temperature of the object, is the emittance of the surface. The exact value of this quantity is usually unknown and also differs for the surfaces captured in the same image. Fortunately, the emittance of most building components is close to one. One notable exception are all metals, which possess high reflectivity in both the visual and infrared range. On the right side, the thermographic image of Fig. 2.30, the metal roof drain pipe's apparent temperature is lower than the ambient temperature. Because of low emittance the emitted heat flow rate of the pipe is significantly smaller than the one surrounding the items despite being at similar temperatures.

Problems

2.1 A freezer of height 150 cm, width 60 cm and depth 60 cm has walls of 50 mm thick expanded polystyrene of heat conductivity 0.040 W/(m K). The freezer maintains an internal temperature of −18 °C, and the external temperature is 30 °C. Calculate the electrical power of the freezer if the efficiency of the heat pump is 300 %. (55 W)

2.2 A cuboid vessel, which edges measure 30 cm, 35 cm and 45 cm, has walls of 20 mm thick expanded polystyrene of heat conductivity 0.040 W/(m K). The vessel contains 2.0 kg of ice at 0 °C, and the external temperature is 30 °C. Calculate the melting time of the ice. We want to prolong melting time to 7.0 h. Calculate the thickness of the additional layer of extruded polystyrene of heat conductivity 0.035 W/(m K). Take the specific heat of the fusion of water to be 336 kJ/kg. (3.9 h, 14 mm)

2.3 A wall has two layers. The outer layer of heat conductivity 0.038 W/(m K) is 8.0 cm thick, and inner layer of heat conductivity 0.16 W/(m K) is 15.0 cm thick. What is the interface temperature (temperature at the boundary of the two layers) if the temperature on the external surface of the wall is −3.0 °C and the temperature on the internal surface of the wall is 17.0 °C? Calculate the distance between the external surface of the wall and the wall interior at temperature 0.0 °C. (10.8 °C, 1.7 cm)

2.4 What temperature would gain the flat black surface on the Earth's surface, neglecting radiation and convection with the environment, if (a) the black surface is perpendicular to the sunbeams, (b) the angle between the black surface and sunbeams is 60°? For the Sun at zenith, the solar density of heat flow rate at the surface of the Earth is approximately 1000 W/m^2. What temperature would gain the flat grey surface, whose emittance is independent of wavelength? (91 °C, 78 °C; the same)

2.5 What temperature would gain the flat black surface on the Earth's surface, if the black surface is perpendicular to the sunbeams? The environment temperature is 25 °C, for the Sun at zenith the density of heat flow rate at the surface of the Earth is approximately 1000 W/m^2, and the average wind velocity is 2.0 m/s. What temperature would gain the flat grey surface of emittance 0.1 independent of wavelength for the same conditions? (76 °C, 33 °C)

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